CHAPTER 3

Geometry

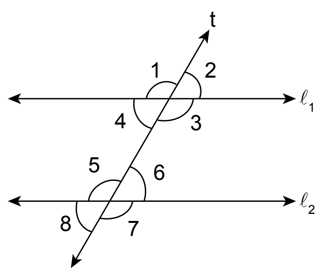
SYNOPSIS

- **Parallel lines:** Two co-planar lines that do not have a common point are called parallel lines.
  In the figure given above, ℓ_1 and ℓ_2 are parallel lines. We write $\ell_1 \parallel \ell_2$ and read as ℓ_1 is parallel to ℓ_2.

Properties of Parallel Lines

(1) The perpendicular distance between two parallel lines is equal everywhere.
(2) Two lines lying in the same plane and perpendicular to the same line are parallel to each other.
(3) If two lines are parallel to the same line, then they are parallel to each other.
(4) One and only one parallel line can be drawn to a given line through a given point which is not on the given line.

- **Transversal:** A straight line intersecting a pair of straight lines in two distinct points is a transversal for the two given lines.



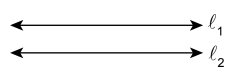
Let ℓ_1 and ℓ_2 be a pair of lines and t be a transversal.

As shown in the figure, totally eight angles are formed.

If ℓ_1 and ℓ_2 are parallel, then the

(i) corresponding angles are equal, i.e., $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$.
(ii) alternate interior angles are equal, i.e., $\angle 4 = \angle 6$ and $\angle 3 = \angle 5$.
(iii) alternate exterior angles are equal, i.e., $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$.
(iv) exterior angles on the same side of the transversal are supplementary, i.e., $\angle 1 + \angle 8 = 180°$ and $\angle 2 + \angle 7 = 180°$.
(v) interior angles on the same side of the transversal are supplementary, i.e., $\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$.

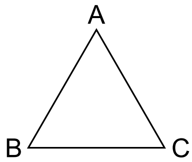
- **Intercepts:** If a transversal *t* intersects two lines ℓ_1 and ℓ_2 in distinct points P and Q, then the lines ℓ_1 and ℓ_2 are said to make an intercept PQ on t.



In the figure given above, $\overline{PQ}$ is an intercept on *t*.

A pair of parallel lines makes equal intercepts on all transversals which are perpendicular to them.

- **Triangles:** A triangle is a three sided simple closed plane figure.



Types of triangles: A. Based on sides:

(i) Scalene triangle: A triangle in which no two sides are equal.
(ii) Isosceles triangle: A triangle in which a minimum of two sides are equal.
(iii) Equilateral triangle: A triangle in which all the three sides are equal.

B. Based on angles:

(i) Acute-angled triangle: A triangle in which each angle is less than 90°.
(ii) Right-angled triangle: A triangle in which one of the angles is equal to 90°.
(iii) Obtuse-angled triangle: A triangle in which one of the angles is greater than 90°.

A triangle in which two sides are equal and one angle is 90°, is an isosceles right triangle. The hypotenuse is $\sqrt{2}$ times each equal side.

Important Properties of Triangles

1. The sum of the angles of a triangle is 180°.
2. The measure of an exterior angle is equal to the sum of the measures of its interior opposite angles.
3. If two sides of a triangle are equal, then the angles opposite to them are also equal.
4. If two angles of a triangle are equal, then the sides opposite to them are also equal.
5. Each angle in an equilateral triangle is equal to 60°.
6. In a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
7. The sum of any two sides of a triangle is always greater than the third side.
   In a ΔABC,
   (i) $AB + BC > AC$,
   (ii) $BC + AC > AB$ and
   (iii) $AB + AC > BC$.
8. The difference of any two sides of a triangle is less than the third side.
   In a ΔABC,
   (i) $(BC - AB) < AC$,
   (ii) $(AC - BC) < AB$ and
   (iii) $(AC - AB) < BC$.
9. In a triangle ABC, if $\angle B > \angle C$, then the side opposite to $\angle B$ is longer than the side opposite to $\angle C$, i.e., $AC > AB$.
10. In triangle ABC given above, if $AC > BC$, then the angle opposite to side AC is greater than the angle opposite to side BC, i.e., $\angle B > \angle A$.

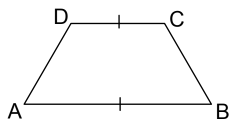
- **Congruence of triangles:** Two geometrical figures are congruent if they have the same shape and the same size.

  The three angles of a triangle determine its shape and its three sides determine its size. If the three angles and the three sides of a triangle are respectively equal to the corresponding angles and sides of another triangle, then the two triangles are congruent. However, it is not necessary that each of the six elements of one triangle is equal to the corresponding elements of the other triangle in order to conclude that the two triangles are congruent.

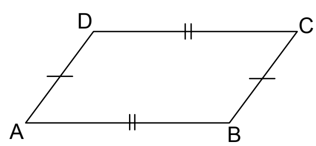
  Based on the study and experiments, the following results can be used to establish the congruence of two triangles.

Different Types of Quadrilaterals

1. **Trapezium:** In a quadrilateral, if two opposite sides are parallel to each other, then it is called a trapezium.
   In the given figure $\overline{AB} \parallel \overline{CD}$, hence ABCD is a trapezium.

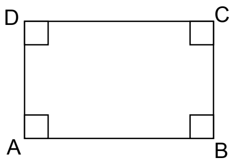


2. **Parallelogram:** In a quadrilateral, if both the pairs of opposite sides are parallel, then it is called a parallelogram.
   In the given figure,
   (i) $AB = CD$ and $BC = AD$.
   (ii) $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$.
   Hence, ABCD is a parallelogram.
   Note: In a parallelogram, diagonals need not be equal, but they bisect each other.



3. **Rectangle:** In a parallelogram, if each angle is a right angle (90°), then it is called a rectangle.

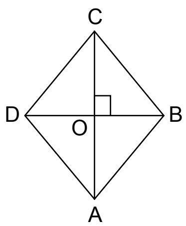
In the given figure, ∠A = ∠B = ∠C = ∠D = 90°, AB = CD and BC = AD.



Hence, ABCD is a rectangle.

Note: In a rectangle, the diagonals are equal, i.e., AC = BD.

4. **Rhombus:** In a parallelogram, if all the sides are equal, then it is called a rhombus. In the given figure, AB = BC = CD = AD, hence ABCD is a rhombus.



Note:

(1) In a rhombus, the diagonals need not be equal.

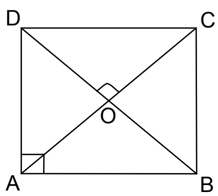
(2) In a rhombus, the diagonals bisect each other at right angles, i.e., AO = OC, BO = OD and $\overline{AC} \perp \overline{DB}$.

5. **Square:** In a rhombus, if each angle is a right angle, then it is called a square.

(OR)

In a rectangle, if all the sides are equal, then it is called a square. In the given figure, AB = BC = CD = DA and ∠A = ∠B = ∠C = ∠D = 90°.

Hence, ABCD is a square.

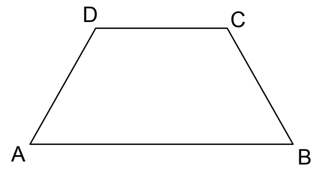


Note:

(1) In a square, the diagonals bisect each other at right angles.

(2) In a square, the diagonals are equal.

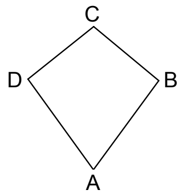
6. **Isosceles trapezium:** In a trapezium, if the non-parallel opposite sides are equal, then it is called an isosceles trapezium.



In the figure, $\overline{AB} \parallel \overline{CD}$ and BC = AD.

Hence, ABCD is an isosceles trapezium.

7. **Kite:** In a quadrilateral, if two pairs of adjacent sides are equal, then it is called a kite.

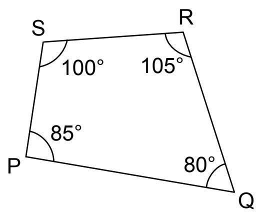


In the figure ABCD,

AB = AD and BC = CD

Hence, ABCD is a kite.

8. **Cyclic Quadrilateral:** A quadrilateral which can be inscribed in a circle is called a cyclic quadrilateral. The opposite angles of a cyclic quadrilateral are supplementary.



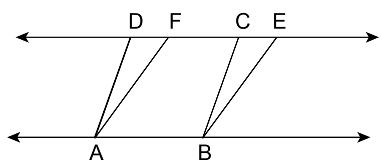
(We shall study more about such quadrilaterals in the next chapter circles)

Geometrical Results on Areas

1. Parallelograms on the same base and between the same parallels are equal in area.

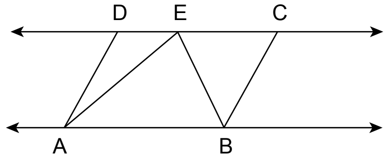
In the figure given above, parallelogram ABCD and parallelogram ABEF are on the same base $\overline{AB}$ and between the same parallels $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

∴ Area of parallelogram ABCD = Area of parallelogram ABEF.



Note: A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

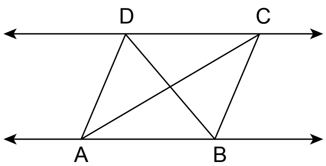
2. The area of a triangle is half the area of the parallelogram, if they lie on the same base and between the same parallels.



In the figure given above, parallelogram ABCD and Δ ABE are on the same base $\overline{AB}$ and between the same parallels $\overline{AB}$ and $\overline{CD}$.

∴ Area of ΔABE = 1/2 Area of parallelogram ABCD.

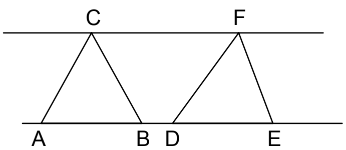
3. Triangles on the same base and between the same parallels are equal in area.



In the figure given above, ΔABC and ΔABD are on the same base $\overline{AB}$ and between the same parallels $\overline{AB}$ and $\overline{CD}$.

∴ Area of ΔABC = Area of ΔABD.

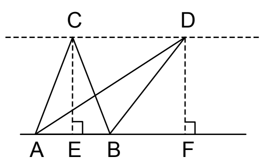
Note: Triangles with equal bases and between the same parallels are equal in area.



In the above given figure, in ΔABC and ΔDEF, AB = DE and $\overline{AE} \parallel \overline{CF}$.

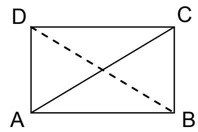
∴ Area of ΔABC = Area of ΔDEF.

4. Triangles with equal bases and with equal areas lie between the same parallels.
In the above figure, if Area of Δ ABC = Area of ΔABD, then $\overline{AB} \parallel \overline{CD}$.



Note: In this case, altitudes CE and DF are equal.

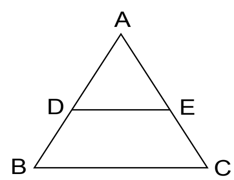
5. A diagonal of a parallelogram divides the parallelogram into two triangles of equal area.



In the figure given, diagonal $\overline{AC}$ divides parallelogram ABCD into two triangles; ΔABC and ΔACD.
Here, area of Δ ABC = area of Δ ACD.
Similarly, diagonal BD divides the parallelogram into two triangles, ΔABD and ΔBDC.
Hence, the area of ΔABD = area of ΔBCD.

- Mid-point Theorem: In a triangle, the line segment jointing the mid-points of any two sides is parallel to the third side and also half of it.
- Basic proportionality theorem: In a triangle, if a line is drawn parallel to one side of the triangle, then it divides the other two sides in the same ratio.
- Converse of basic proportionality theorem: If a line divides two sides of a triangle in the same ratio then that line is parallel to the third side.

In the figure given, AD/DB = AE/EC ⇒ $\overline{DE} \parallel \overline{BC}$.



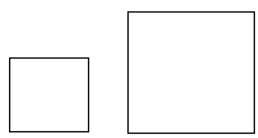
Note: The intercepts made by three or more parallel lines on any two transversals are proportional.

Similarity: Two figures are said to be congruent, if they have the same shape and same size. But the figures of the same shape need not have the same size. The figures of the same shape but not necessarily of the same size are called similar figures.

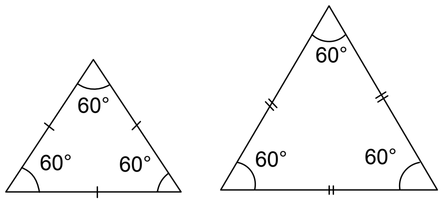
Examples:

1. Any two line segments are similar.

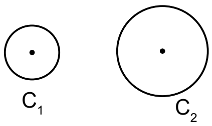
2. Any two squares are similar.



3. Any two equilateral triangles are similar.



4. Any two circles are similar.



Two polygons are said to be similar to each other if

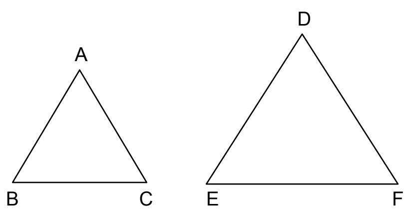
(i) their corresponding angles are equal and
(ii) the lengths of their corresponding sides are proportional.

Note: "~" is the symbol used for "is similar to".

If ΔABC is similar to ΔPQR, we denote it as ΔABC ~ ΔPQR. The relation 'is similar to' satisfies the following properties.

(1) It is reflexive as every figure is similar to itself.
(2) It is symmetric as, if A is similar to B, then B is also similar to A.
(3) It is transitive as, if A is similar to B and B is similar to C, then A is similar to C.
∴ The relation 'is similar to' is an equivalence relation.

❍ **Criteria for similarity of triangles:** In two triangles, if either the corresponding angles are equal or the ratio of corresponding sides are proportional, then the two triangles are similar to each other.
In ΔABC and ΔDEF,



(1) If ∠A = ∠D, ∠B = ∠E and ∠C = ∠F, then ΔABC ~ ΔDEF. This property is called **A.A.A. criterion**.

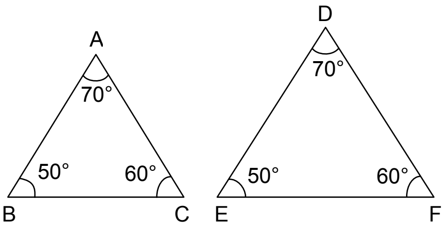
(2) If AB/DE = BC/EF = AC/DF, then ΔABC ~ ΔDEF. This property is called **S.S.S. criterion**.

(3) If AB/DE = AC/DF and ∠A = ∠D, then ΔABC ~ ΔDEF. This property is called **S.A.S. criterion**.

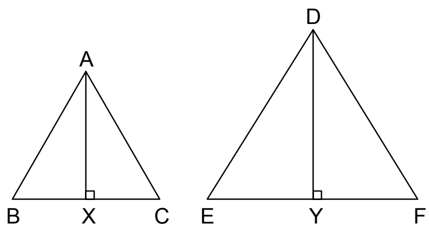
Results on Areas of Similar Triangles

(a) The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides of the triangles.
ΔABC ~ ΔDEF

$$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{CA^2}{FD^2}.$$



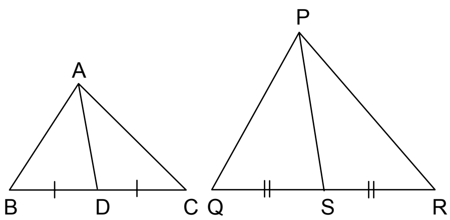
(b) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.



In the following figures, ΔABC ~ ΔDEF and AX, DY are the altitudes.

$$\text{Then, } \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AX^2}{DY^2}$$

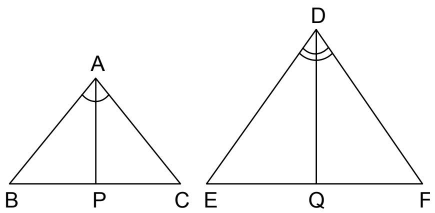
(c) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding medians.



In the above given figures, ΔABC ~ ΔPQR and AD and PS are medians.

$$\text{Then, } \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AD^2}{PS^2}$$

(d) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding angle bisector segments.

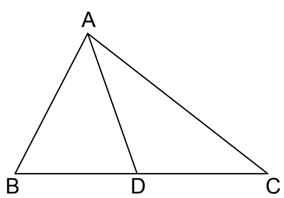


In the figure, ΔABC ~ ΔDEF and

AP, DQ are bisectors of ∠A and ∠D respectively, then

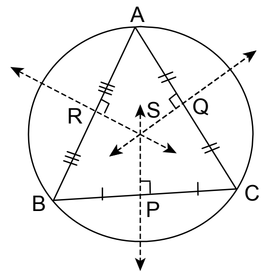
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AP^2}{DQ^2}$$

- **Pythagorean theorem:** In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
- **Converse of Pythagorean Theorem:** In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.
- **Polygons:** A closed plane figure bounded by three or more line segments is called a polygon.
- **Convex polygon and Concave polygon:** A polygon in which each interior angle is less than 180° is called a convex polygon.
  Otherwise it is called concave polygon.
- **Vertical angle bisector theorem:** The bisector of the vertical angle of a triangle divides the base in the ratio of the other two sides.
- **Converse of vertical angle bisector theorem:** If a line that passes through a vertex of a triangle, divides the base in the ratio of the other two sides, then it bisects the angle. In the adjacent figure, AD divides BC in the ratio $\frac{BD}{DC}$ and if $\frac{BD}{DC} = \frac{AB}{AC}$, then AD is the bisector of ∠A.

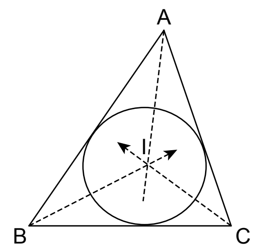


Concurrence – Geometric Centres of a Triangle

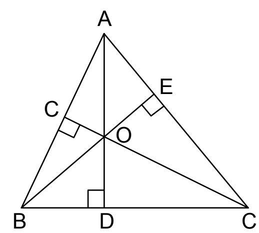
1. **Circumcentre:** The locus of the point equidistant from the end points of the line segment is the perpendicular bisector of the line segment. The three perpendicular bisectors of the three sides of a triangle are concurrent and the point of their concurrence is called the circumcentre of the triangle and is usually denoted by S. The circumcentre is equidistant from all the vertices of the triangle. The circumcentre of the triangle is the locus of the point in the plane of the triangle, equidistant from the vertices of the triangle.



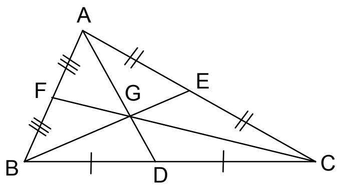
2. **Incentre:** The angle bisectors of the triangle are concurrent and the point of concurrence is called the incentre and is usually denoted by I. I is equidistant from the sides of the triangle. The incentre of the triangle is the locus of the point, in the plane of the triangle, equidistant from the sides of the triangle.



3. **Orthocentre:** The altitudes of the triangle are concurrent and the point of concurrence of the altitudes of a triangle is called orthocentre and is usually denoted by O.

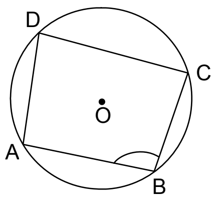


4. **Centroid:** The medians of a triangle are concurrent and the point of concurrence of the medians of a triangle is called the centroid and it is usually denoted by G. The centroid divides each of the medians in the ratio 2: 1, starting from vertex, i.e., in the figure given below, AG:GD = BG:GE = CG: GF = 2:1.



- The angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- An angle in a semicircle is a right angle.

  If an arc of a circle subtends a right angle at any point on the remaining part of the circle, it is a semi circle.
- **Cyclic quadrilateral:** If all the four vertices of a quadrilateral lie on one circle, then the quadrilateral is called a cyclic quadrilateral.

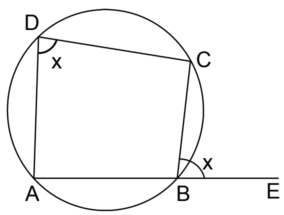


Note 1: Opposite angles in a cyclic quadrilateral are **supplementary**.

Note 2: In a quadrilateral, if the opposite angles are supplementary, then the quadrilateral is a cyclic quadrilateral.

In the above given figure, ABCD is a cyclic quadrilateral.

$$\angle A + \angle C = 180° \text{ and } \angle B + \angle D = 180°$$

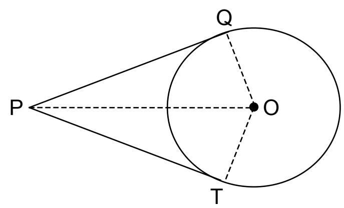


Note 3: Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

In the figure, ABCD is a cyclic quadrilateral. AB is produced to E to form an exterior angle, ∠CBE and it is equal to the interior angle at the opposite vertex, i.e., ∠ADC.

$$\therefore \angle CBE = \angle ADC$$

- Theorem 1: One and only one circle exists through three non-collinear points.
- Theorem 2: The perpendicular bisector of a chord of a circle passes through the centre of the circle.
- Theorem 3: Two equal chords of a circle are equidistant from the centre of the circle.
- Theorem 4: Equal chords subtend equal angles at the centre of the circle.
- Theorem 5: The opposite angles of a cyclic quadrilateral are supplementary.
- Theorem 6: The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- Theorem 7: The tangent at any point on a circle is perpendicular to the radius through the point of contact.
- Theorem 8: Two tangents drawn to a circle from an external print are equal in length.



- Theorem 9: If two chords of a circle intersect each other, then the products of the lengths of their segments are equal.
- Theorem 10: Alternate segment theorem: If a line touches the circle at a point and if a chord is drawn from the point of contact then the angles formed between the chord and the tangent are equal to the angles in the alternate segments.
- Appolonius theorem: In a triangle, the sum of the squares of two sides of a triangle is equal to twice the sum of the square of the median which bisects the third side and the square of half the third side.
- **Common tangent:** If the same line is tangent to two circles drawn on the same plane, then the line is called a common tangent to the circles. The distance between the point of contacts is called the length of the common tangent.

  In the figure, PQ is a common tangent to the circles, C_1 and C_2. The length of PQ is the length of the common tangent.

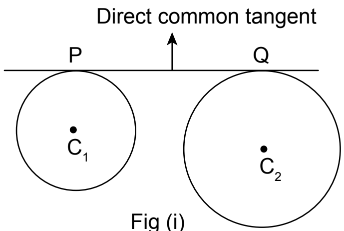
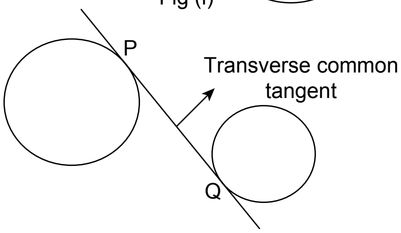


Fig (i)



In figure (i), we observe that both the circles lie on the same side of PQ. In this case, PQ is a ***direct*** common tangent and in figure (ii), we notice that the two circles lie on either side of PQ. Here PQ is a ***transverse*** common tangent.

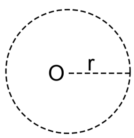
- **Locus**

  The collection (set) of all points and only those points which satisfy certain given geometrical conditions is called the **locus of a point** satisfying the given conditions.

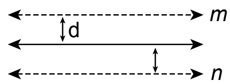
  Alternatively, a **locus** can be defined as the path or curve traced by a point in a plane when subjected to some geometrical conditions.

Consider the Following Examples

1. The locus of the point in a plane which is at a constant distance r from a fixed point O is a circle with centre O and radius r units.



2.



The locus of the point in a plane which is at a constant distance from a fixed straight line (ℓ) is a pair of lines, parallel to ℓ, lying on either side of ℓ. Let the fixed line be ℓ. The lines *m* and *n* form the set of all points which are at a constant distance from ℓ.

Before proving that a given path or curve is the desired locus, it is necessary to prove the following.

(i) Every point lying on the path satisfies the given geometrical conditions.

(ii) Every point that satisfies the given conditions lies on the path.

Some Important Points

(i) In an equilateral triangle, the centroid, the orthocentre, the circumcentre and the incentre coincide.

(ii) In an isosceles triangle, the centroid, the orthocentre, the circumcentre and the incentre all lie on the median to the base, i.e., they are collinear.

(iii) In a right-angled triangle the length of the median drawn to the hypotenuse is equal to half of the hypotenuse. The median is also equal to the circumradius. The mid-point of the hypotenuse is the circumcentre.

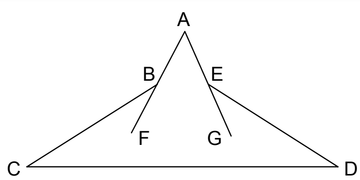
(iv) In an obtuse-angled triangle, the circumcentre and orthocentre lie outside the triangle and for an acute-angled triangle the circumcentre and the orthocentre lie inside the triangle.

(v) For all triangles, the centroid and the incentre lie inside the triangle.

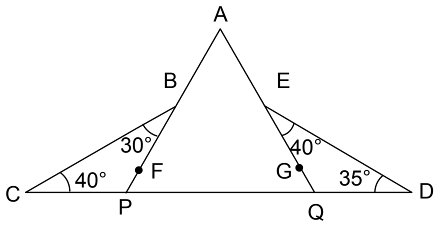
(vi) For all triangles, the excentre lies outside the triangle.

Solved Examples

1. In the above figure (not to scale), $\angle BCD = 40°$, $\angle EDC = 35°$, $\angle CBF = 30°$ and $\angle DEG = 40°$, find $\angle BAE$.

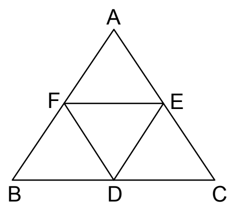


Solution: Given ∠BCD = 40°, ∠EDC = 35°
∠CBF = 30° and ∠DEG = 40°
Produce AF to meet CD at P and also produce AG to meet CD at Q.
In ΔBPC, ∠BPQ = ∠CBP + ∠BCD
(∵ exterior angle) ⇒ ∠BPQ = 70°
In ΔEQD, ∠EQP = ∠DEQ + ∠EQDQ
(∵ exterior angle)



⇒ ∠EQP = 40° + 35°
⇒ ∠EQP = 75°
In ΔAPQ, ∠PAQ + ∠APQ + ∠AQP = 180°
⇒ ∠PAQ = 180° – (70° + 75°) = 35°.
∴ ∠BAE = 35°

2. In the given triangle ABC, D, E and F are the mid-points of sides BC, CA and AB respectively. Prove that $\frac{AB-BC}{2} < AE < \frac{AB+BC}{2}$.



Solution: Difference of two sides is less than the third side.
∴ AB – BC < AC
Since E is the mid-point of AC, AE = AC/2
AB – BC < 2AE

$\frac{AB-BC}{2} < AE$ . . . . (1)

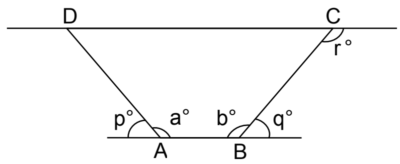
The sum of the two sides is greater than the third side.
AB + BC > AC. AC = 2AE. AB + BC > 2AE

$AE < \frac{AB+BC}{2}$ ….. (2)

From (1) ad (2), $\frac{AB-BC}{2} < AE < \frac{AB+BC}{2}$.

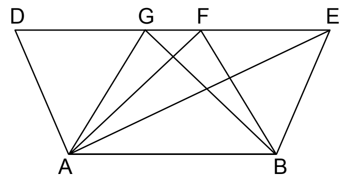
3. In the given quadrilateral ABCD, p° + q° = 100°, a° = 140° and $r° = \frac{1}{2}(a° + q°)$. Find the angles p°, q° and r°.

Solution: p° + a° = 180°



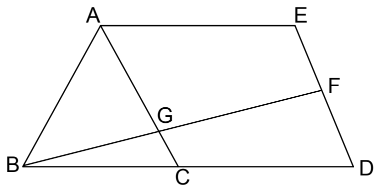
⇒ p + 140° = 180° ⇒ p = 40°
and given that p + q = 100°
⇒ 40° + q = 100° ⇒ q = 60°
But b° + q = 180° ⇒ b° + 60° = 180°
⇒ b° = 120°
given r = $^1/_2$(a + q) ⇒ r = $^1/_2$(140 + 60)
⇒ r = 100°

4. In the given figure, $\overline{AB} \parallel \overline{DE}$ and area of the parallelogram ABFD is 24 cm^2. Find the areas of ΔAFB, ΔAGB and ΔAEB.



Solution: Given $\overline{AB} \parallel \overline{DE}$ and the area of parallelogram ABFD is 24 cm^2.
ΔAFB, ΔAGB, ΔAEB and parallelogram ABFD are on the same base and between the same parallels.
∴ Area of ΔAFB = Area of ΔAGB
= Area of ΔAEB
= 1/2 x area of parallelogram ABFD
= 1/2 x 24 = 12 cm^2.

5. In the given figure (not to scale), ABC is an isosceles triangle in which AB = AC. AEDC is a parallelogram. If ∠CDF = 70° and ∠BFE = 100°, then find ∠FBA.



Solution: Given, ABC is an isosceles triangle in which AB = AC.

AEDC is a parallelogram.

$\angle CDF = 70°$ and $\angle BFE = 100°$.

Since AEDC is a parallelogram,

$\angle ACD + 70° = 180°$

$\Rightarrow \angle ACD = 110° \rightarrow (1)$

$\angle ACD + \angle GCB = 180°$ ($\because$ linear pair)

$\Rightarrow \angle GCB = 180° - 110° = 70° \rightarrow (2)$

$\angle GFD + \angle BFE = 180°$ (linear pair)

$\Rightarrow \angle GFD = 180° - 100° = 80° \rightarrow (3)$

In Δ BFD, $\angle FBD = 180° - (80° + 70°) = 30°$

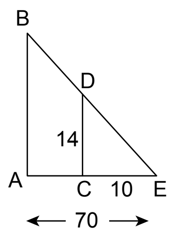
Since AB = AC, $\angle ABC = \angle ACB$

$\therefore \angle ABC = 70°$

$\Rightarrow \angle ABG = \angle ABC - \angle FBD = 70° - 30° = 40°$

6. A pole of height 14 m casts a shadow 10 m long on the ground. At the same time a tower casts the shadow 70 m long on the ground. Find the height of the tower.

Solution: Let CD be the pole and AB be the tower.



$\angle BAE = \angle DCE = 90°$

$\therefore$ DC || AB.

$\therefore \Delta ABE \sim \Delta CDE$ (AAA)

$\Rightarrow \frac{CE}{AE} = \frac{CD}{AB} \Rightarrow \frac{10}{70} = \frac{14}{AB}$

$\Rightarrow$ AB = 98 m

7. Find each interior and exterior angle of a regular polygon having 30 sides.

Solution: The number of sides (n) = 30

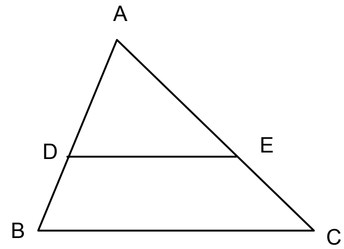
Each interior angle of a regular polygon with 'n' sides

$= \frac{(2n-4) \times 90°}{n} = \frac{(60-4) \times 90°}{30} = 56 \times 3$

$= 168°$

Each exterior angle $= \frac{360°}{n} = \frac{360°}{30} = 12°$

8. In the following figure, DE || BC, AD = 5.6 cm, AE = (x + 1) cm, DB = 2.8 cm and EC = (x – 1) cm. Find x.



Solution: DE || BC $\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{AE}{EC} \Rightarrow \frac{5.6}{2.8} = \frac{x+1}{x-1} \Rightarrow 2 = \frac{x+1}{x-1}$

$\Rightarrow 2x - 2 = x + 1 \Rightarrow x = 3$

9. One angle of a decagon is 90° and all the remaining nine angles are equal. What is the measure of the other angles?

Solution: One angle of the decagon = 90°

The number of sides (n) = 10

The sum of interior angles = $(2n - 4) \times 90°$

Let each of the other angles be equal to x°

$9x + 90° = (20 - 4) \times 90° = 16 \times 90° = 1440°$

$9x = 1440° - 90° = 1350°$

$x = \frac{1350°}{9} = 150°$. Each of the other angles = 150°.

10. In the above figure (not to scale), E and D are the mid-points of AB and BC respectively. Also $\angle B = 90°$, AD = $\sqrt{292}$ cm and CE = $\sqrt{208}$ cm. Find AC.

Solution: In Δ ABD, $AD^2 = AB^2 + BD^2$

$\Rightarrow AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2$ .... (1)

($\because$ D is the mid-point of BC)

In ΔBEC, $CE^2 = BC^2 + BE^2$

$$\Rightarrow CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2 . \; (2)$$

($\because$ E is mid-point of AB)

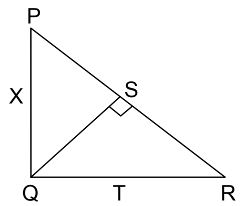
Adding (1) and (2), we get,

$$\Rightarrow 292 + 208 = \frac{5}{4}(AB^2 + BC^2)$$

$$\Rightarrow 400 = AB^2 + BC^2 \Rightarrow 400 = AC^2$$

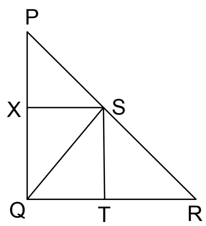
$$\Rightarrow AC = 20 \text{ cm}$$

11. In the following figure, angle PQR = 90°, and $\overline{SQ}$ is perpendicular to PR. Show that $QR^2 \times PS^2 = PQ^2 \times QR^2$.



👍 ***Solution:*** Given that PQR is a right angled triangle and QS is perpendicular to PR. ΔPQR and ΔPSQ are similar.

$$\frac{PQ}{PS} = \frac{PR}{PQ} \Rightarrow PQ^2 = PR \times PS$$



$$\Rightarrow PS = \frac{PQ^2}{PR} \quad \ldots\ldots.. (1)$$

In ΔPQR and ΔQSR, $\dfrac{QR}{RS} = \dfrac{PR}{QR}$

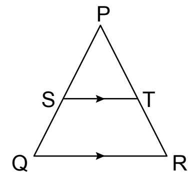
$$\Rightarrow QR^2 = PR \times RS \Rightarrow RS = \frac{QR^2}{PR} \quad \ldots (2)$$

and in ΔPQR, $QS^2 = PS \times SR$ .... (3)

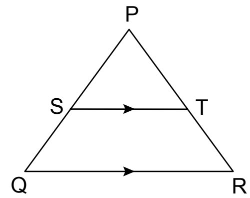
Substitute (1) and (2) in (3), $QS^2 = \dfrac{PQ^2}{PR} \times \dfrac{QR^2}{PR}$

$QS^2 \times PR^2 = PQ^2 \times QR^2$. Hence proved.

12. In a triangle PQR, ST is parallel to QR. Show that RT(PQ + PS) = SQ(PR + PT).



👍 ***Solution:*** PQR is a triangle. ST is parallel to QR.



By basic proportionality theorem, in ΔPQR.

$$\frac{PQ}{PS} = \frac{PR}{PT}$$

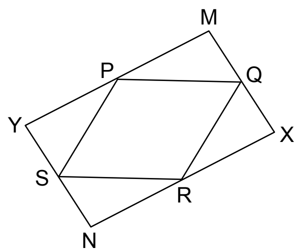
By componendo and dividendo,

$$\frac{PQ}{PS} = \frac{PR}{PT} \Rightarrow \frac{PQ + PS}{PQ - PS} = \frac{PR + PT}{PR - PT}$$

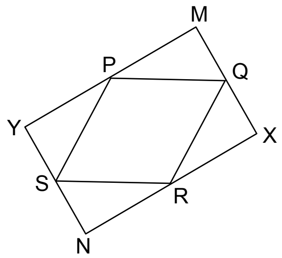
$$\frac{PQ + PS}{SQ} = \frac{PR + PT}{TR}$$

RT(PQ + PS) = SQ(PR + PT)

13. In the following figure, PQRS is a rhombus formed by joining the mid-points of a quadrilateral YMXN, show that $3PQ^2 = SN^2 + NR^2 + QX^2 + XR^2 + PY^2 + YS^2$.



👍 ***Solution:*** Given that, PQRS is a rhombus formed by joining mid-points of the sides of the quadrilateral YMXR.



$\therefore$ YMXR is a rectangle.

$\Rightarrow \angle Y = \angle M = \angle x = \angle R = 90°$

In triangle SNR, $SN^2 + NR^2 = SR^2$

In triangle QXR, $QX^2 + XR^2 = QR^2$

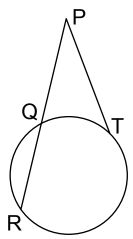
In triangle PYS, $PY^2 + YS^2 = PS^2$

Since PQRS is a rhombus, PQ = RQ = RS = SP

$SR^2 + QR^2 + PS^2 = 3PQ^2$

$\Rightarrow 3PQ^2 = SN^2 + NR^2 + QX^2 + XR^2 + PY^2 + YS^2$

14. In the following figure, PR is a secant and PT is a tangent to the circle. If PT = 6 cm and QR = 5 cm, then PQ = _______ cm.



Solution: PQR is a secant, PT is a tangent

$\therefore$ $PQ \times PR = PT^2$ .... (1)

Given, PT = 6, QR = 5

Let PQ = x

$\therefore$ PR = PQ + QR = x + 5

from (1)

$PQ \times PR = PT^2$

$x(x + 5) = 6^2$

$x^2 + 5x = 36$

$x^2 + 5x - 36 = 0$

$(x + 9)(x - 4) = 0$

$x + 9 = 0; x - 4 = 0$

$x = -9; x = 4$

Since length is always positive

$\therefore$ x = PQ = 4 cm

15. Find the locus of a point which is at a distance of 5 units from (–1, –2).

Solution: Let, P(x, y) be any point on locus and the given point be A(–1, –2)

Given that, AP = 5 units.

i.e., $\sqrt{(x-(-1))^2 + (y-(-2))^2} = 5$

$\Rightarrow \sqrt{(x+1)^2 + (y+2)^2} = 5$

$\Rightarrow x^2 + 2x + 1 + y^2 + 4y + 4 = 5$

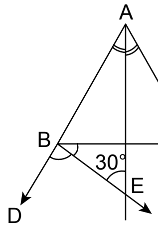
$\Rightarrow x^2 + y^2 + 2x + 4y = 0$

$\therefore$ The required locus is $x^2 + y^2 + 2x + 4y = 0$.

PRACTICE EXERCISE 3 (A)

Directions for questions 1 to 40: Select the correct alternative from the given choices.

1.



In the above figure, AB is produced to D. The bisectors of ∠DBC and ∠BAC meet at E. Find ∠ACB, if ∠BEA = 30°.

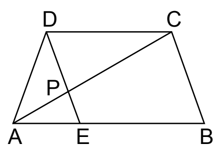
(1) 45° (2) 60°
(3) 75° (4) 90°

2. In a triangle ABC, AD, BE and CF are the altitudes. Find the value of

$$\frac{AD^2+BE^2+CF^2-(AF^2+BD^2+CE^2)}{CD.DB+BF.FA+AF.FB}$$

(1) 1 (2) 2
(3) 3 (4) 4

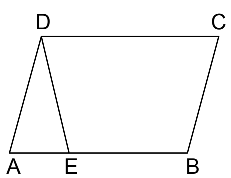
3.



In the figure above, ABCD is a trapezium and DEBC is a parallelogram such that AD = DE, ∠ DAP = 40° and ∠ABC = 75°. If AC and DE intersect at P, then find ∠PCD.

(1) 35° (2) 30°
(3) 40° (4) 45°

4.



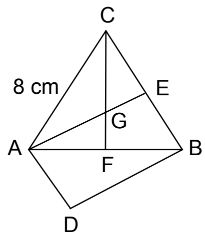
In the figure above, ABCD is a parallelogram and ∠DEB = ∠CBE. If ∠ADE = 40°, then find ∠ABC.

(1) 100° (2) 90°
(3) 80° (4) 110°

5. In a ΔABC, if ∠B = 90° and D is a point on AC such that $\overline{BD} \perp \overline{AC}$. Then AB^2 =

(1) BC × BD (2) BC × CD
(3) AC × BD (4) AC × AD

6.



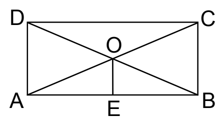
In the figure above, ABC is a triangle. AE and CF are the medians and ADBE is a rectangle. If AC = 8 cm and BE = 4 cm, then the length of BD is

(1) $3\sqrt{3}$ (2) $4\sqrt{3}$
(3) $5\sqrt{2}$ (4) $3\sqrt{2}$

7. ABCD is a rhombus in which ∠DAC = 30°. P is a point on $\overline{CD}$ and $\overline{BP}$ is perpendicular to CD. $\overline{AC}$ and $\overline{BP}$ intersect at T. If BD = 6 cm, then find the length of BT.

(1) $2\sqrt{2}$ (2) $2\sqrt{3}$
(3) $3\sqrt{2}$ (4) $3\sqrt{3}$

8.



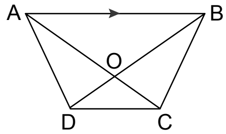
In the given rectangle ABCD, the sum of the lengths of two diagonals is equal to 52 cm and E is a point on AB such that $\overline{OE}$ is perpendicular to $\overline{AB}$. Find the area of the rectangle (in cm²), if OE = 5 cm.

(1) 120 (2) 240
(3) 260 (4) 90

9. In a ΔABC, E and F are points on the sides AB and AC respectively. If AE = 3 cm, AF = 4 cm, EB = 9 cm and FC = 12 cm, then find EF in terms of BC.

(1) $\frac{BC}{2}$ (2) $\frac{BC}{3}$
(3) $\frac{BC}{4}$ (4) $\frac{BC}{5}$

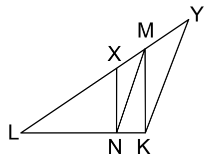
10.



In the following figure, show that area of ΔDCO: area of ΔCOB =

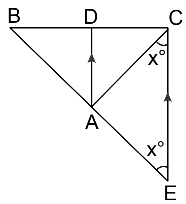
(1) OD: OC
(2) OC: OB
(3) OD: OB
(4) BC: CD

11. In the following figure, if $\overline{XN}$ is parallel to $\overline{MK}$ and $\overline{MN}$ is parallel to $\overline{KY}$. Then (LX) (MY) =



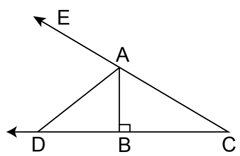
(1) (LM) (MX)
(2) (LM) (MN)
(3) (NK) (MK)
(4) (NK) (LN)

12. In the following figure, $\overline{CE}$ is parallel to $\overline{AD}$ and angle AEC = x° = angle ACE. If BEC is a triangle, Then AB × CD =



(1) BC × DE
(2) BC × CE
(3) AB × BC
(4) AC × BD

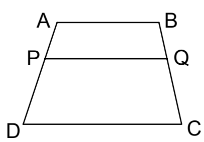
13.



In the figure above (not to scale), $\overline{AB} \perp \overline{CD}$ and AD is the bisector of $\angle BAE$. AB = 3 cm and AC = 5 cm. Find CD(in cm).

(1) 10
(2) 12
(3) 15
(4) 18

14.



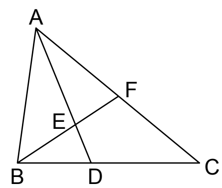
In the figure above (not to scale), ABCD is an isosceles trapezium. $\overline{AB} \parallel \overline{CD}$, AB = 9 cm and CD =12 cm. AP: PD = BQ: QC = 1: 2. Find the length of PQ (in cm).

(1) 8
(2) 9
(3) 6
(4) 10

15. In a ΔPQR, M lies on PR and between P and R such that QR = QM = PM. If $\angle MQR = 40°$, then find $\angle P$.

(1) 45°
(2) 40°
(3) 35°
(4) 50°

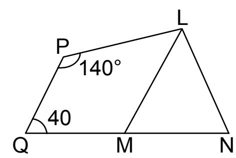
16.



In the figure given a triangle ABC with, AE = 3ED, 2BD = DC and $\sqrt{2}AB = AC$. If AB = 3BD. Find $\angle ABC$.

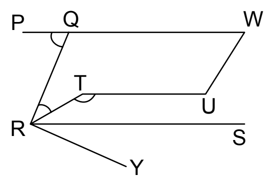
(1) 90°
(2) 70°
(3) 80°
(4) 72°

17. In the figure below, LM = LN and $\angle PLN = 130°$. Find $\angle MLN$.



(1) 70°
(2) 80°
(3) 85°
(4) 90°

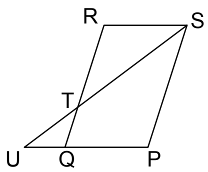
18.



In the figure above, TU||RS. Find $\angle WUT$, given $\angle PWU = 50°$, $\angle PQR = 70°$, $\angle QRT = 10°$ and $\angle UTR = 120°$.

(1) 110°
(2) 120°
(3) 130°
(4) 140°

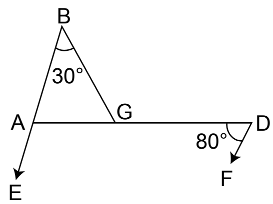
19. PQRS is a parallelogram. T is a point on QR satisfying $QT = \dfrac{QR}{3}$. If PQ and ST are produced to meet at U, as shown in the figure, then PU =



(1) 4UQ (2) 3UQ

(3) $\frac{5}{2}$UQ (4) $\frac{7}{2}$UQ

20. In the given figure, BE is parallel DF. Find ∠AGB.



(1) 70° (2) 75°

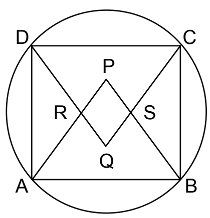
(3) 80° (4) 90°

21. Two parallel chords of equal length 18 cm are drawn inside a circle of radius 15 cm. Find the distance between the chords.

(1) 12 cm (2) 18 cm

(3) 24 cm (4) 30 cm

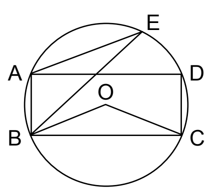
22. In the given figure, ABCD is a cyclic quadrilateral. AP, BP, CQ, DQ are the bisectors of angles A, B, C and D. The sum of the angles PRQ and PSQ is ______.



(1) 90° (2) 120°

(3) 180° (4) Data insufficient

23.

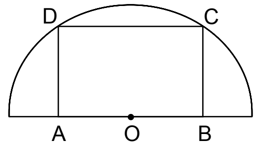


In the figure above, rectangle ABCD and triangle ABE are inscribed in the circle with centre O. If ∠AEB = 40°, then find ∠BOC.

(1) 100° (2) 60°

(3) 120° (4) 80°

24.

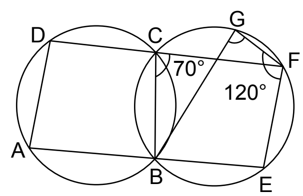


In the figure above, ABCD is a rectangle inscribed in a semi circle. If the length and the breadth of the rectangle are in the ratio 2: 1. What is the ratio of the perimeter of the rectangle to the diameter of the semicircle?

(1) $\sqrt{3}:\sqrt{2}$ (2) $3:\sqrt{2}$

(3) $2:\sqrt{3}$ (4) $\sqrt{3}:2$

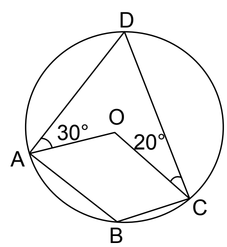
25. In the figure given below, if ∠BCF = 70° and ∠EFG = 120°, then find the value of ∠BAD + ∠ABG + ∠BGF.



(1) 260° (2) 360°

(3) 180° (4) 150°

26.

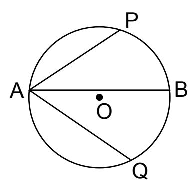


In the figure above, ABCD is a cyclic quadrilateral. O is the centre of the circle. If ∠OAD = 30° and ∠OCD = 20°, then find ∠ABC.

(1) 120° (2) 50°

(3) 130° (4) 80°

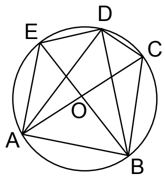
27.



In the figure above, AB is a diameter of the circle with the centre O. AP and AQ are equal chords. If ∠PAQ = 80°, then find ∠APO.

(1) 45°   (2) 40°
(3) 80°   (4) 100°

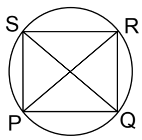
28.



In the figure above, A, B, C, D and E are concyclic points. O is the centre of the circle. ∠EAD = 25° and ∠CBD = 20°. Find ∠EDC.

(1) 145°   (2) 180°
(3) 135°   (4) 85°

29.



In the figure above, PQRS is a cyclic quadrilateral and PR is a diameter. If ∠RPQ = 50° and ∠SQP = 45°, then find ∠QRS.

(1) 85°   (2) 90°
(3) 60°   (4) 100°

30. Let A and B be two fixed points in a plane. Find the locus of a point P such that $PA^2 + PB^2 = AB^2$.

(1) Right triagle
(2) Circle with out A and B
(3) Semicircle with out A and B
(4) Square

31. Find the locus of the vertex of triangle with fixed base and having constant area.

(1) line parallel to the base
(2) circumcircle
(3) incircle
(4) the vertex

32. P is the point of intersection of the diagonals of a square READ. P is equidistant from

(1) the vertices R, E, A and D.
(2) $\overline{RE}$ and $\overline{EA}$.
(3) $\overline{EA}$ and $\overline{AD}$.
(4) All of these

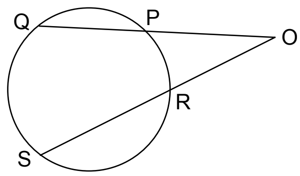
33. In a triangle ABC, D is a point on BC such that any point on AD is equidistant from the points B and C. Which of the following is necessarily true?

(1) AB = BC   (2) BC = AC
(3) AC = AB   (4) AB = BC = AC

34. In a ΔABC, AB = AC. P, Q and R are the mid-points of the sides AB, BC and CA respectively. A circle is passing through A, B, Q and R. Another circle is passing through A, P, Q and C. If AC = 6 cm, then find the distance between the centres of the circles.

(1) 4 cm   (2) 3 cm
(3) 5 cm   (4) 2 cm

35.



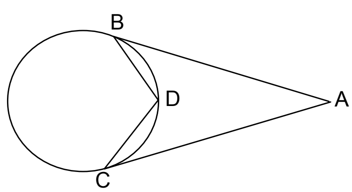
In the figure above (not to scale), QP and SR are two chords of the circle, produced to meet at the point O. OR = 3 cm, SR = 3x cm, OP = (x + 1) cm and PQ = (x + 2) cm. Find x.

(1) 3   (2) 6
(3) 9   (4) 12

36. In a triangle ABC, P and Q are the points on BC and AC respectively. If BP: PC = 2: 3 and $\overline{PQ} \parallel \overline{AB}$, then find AQ: AC.

(1) 2: 3
(2) 2: 5
(3) 3: 5
(4) None of these

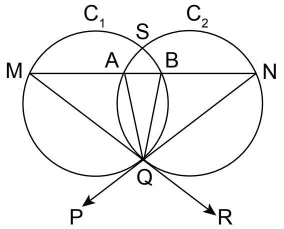
37.



In the figure above (not to scale), AB and AC are two tangents drawn to a circle at B and C respectively, ∠DCA = 35° and ∠DBA = 40°. Find the measure of ∠BAC.

(1) 15°   (2) 30°
(3) 45°   (4) 60°

38.



In the figure above (not to scale), two circles C_1 and C_2 intersect at S and Q. PQN and RQM are tangents drawn to C_1 and C_2 respectively at Q. MAB and ABN are the chords of the circles C_1 and C_2. If $\angle NQR = 85°$, then find $\angle AQB$.

(1) 10° (2) 15°
(3) 20° (4) 25°

39. Radii of two concentric circles are 40 cm and 41 cm. AB is a chord of the bigger circle and tangent to the smaller circle. Find the length of AB.

(1) 9 (2) 18
(3) 13 (4) 26

40. In ΔABC, $\angle ABC = 90°$, AB = 3 cm and BC = 4 cm. If $\overline{BD} \perp \overline{AC}$ where D is a point on $\overline{AC}$, then find BD.

(1) 2.4 cm (2) 3.6 cm
(3) 4.8 cm (4) 1.2 cm

PRACTICE EXERCISE 3 (B)

Directions for questions 1 to 40: Select the correct alternative from the given choices.

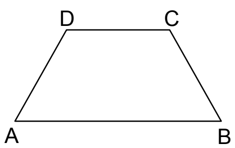
1. The perpendicular bisectors of AB and AC of the triangle ABC meet BC at D. If the bisectors meet AB at P and AC at Q, then find $\angle BDP + \angle CDQ$.

(1) 80° (2) 90°
(3) 60° (4) 100°

2. In ΔABC, D, E and F are the mid-points of BC, CA and AB respectively. $\frac{(AB + BE + CF)}{(AB + BC + CA)}$ is

(1) $> \frac{3}{4}$ (2) $< \frac{1}{2}$
(3) $< \frac{3}{5}$ (4) $< \frac{3}{4}$

3.



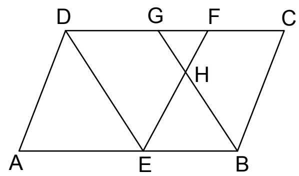
In the figure above (not to scale), ABCD is a trapezium in which $\overline{AB} \,||\, \overline{CD}$, AD = CD and AB = 2CD. If $\angle ADC = 100°$, then find $\angle ABC$.

(1) 40° (2) 50°
(3) 60° (4) 70°

4. ABCD is an isosceles trapezium in which $\overline{AB} \,||\, \overline{CD}$. If the bisectors of $\angle BAD$ and $\angle ADC$ intersect at P. Find $\angle APD$.

(1) 45° (2) 60°
(3) 90° (4) 120°

5.



In the figure above, ABCD is a parallelogram $\overline{DE}$, $\overline{EF}$ and $\overline{BG}$ are the bisectors of $\angle ADC$, $\angle DEB$ and $\angle ABC$ respectively. BG and EF intersect at H. If $\angle DAB = 70°$, then find $\angle BHE$.

(1) 50° (2) $\left(62\frac{1}{2}\right)^{\circ}$
(3) 75° (4) $\left(87\frac{1}{2}\right)^{\circ}$

6. Match the Column A with Column B.

Column A (Name of the polygon)		Column B (Number of diagonals)
(a) Octagon	( )	(p) 5
(b) Decagon	( )	(q) 9
(c) Pentagon	( )	(r) 10
(d) Hexagon	( )	(s) 15
	( )	(t) 20
		(u) 35

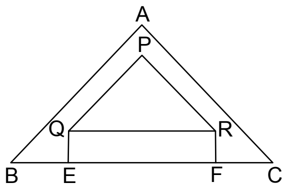
(1) (a) $\rightarrow$ t, (b) $\rightarrow$ u, (c) $\rightarrow$ p, (d) q

(2) (a) $\rightarrow$ t, (b) $\rightarrow$ u, (c) $\rightarrow$ P, (d) r

(3) (a) $\rightarrow$ t, (b) $\rightarrow$ u, (c) q, (d) r

(4) None of these

7.

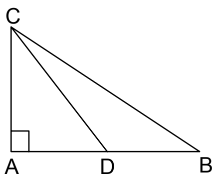


In the figure above, AC = 4 cm, PR = 2.5 cm and QR = 4 cm. If $\overline{AB} \,||\, \overline{PQ}$, $\overline{BC} \,||\, \overline{QR}$, $\overline{AC} \,||\, \overline{PR}$ and $\overline{QE} \,||\, RF$, then find the length of BE + FC.

(1) 2.4 cm (2) 3.6 cm

(3) 4.8 cm (4) 6 cm

8.



In the above figure ΔABC is a right triangle. $\angle A = 90°$ and CD is the median then $BC^2 - CD^2 =$ _________.

(1) $\frac{1}{2}AB^2$ (2) $\frac{3}{2}AB^2$

(3) $\frac{3}{4}AB^2$ (4) $\frac{1}{4}AB^2$

9. If the sides AB, BC, CD and DA of a trapezium ABCD measure 10 cm, 20 cm, 18 cm and 16 cm respectively, then find the length of the longer diagonal, given that AB is parallel to CD.

(1) $\sqrt{760}$ (2) $\sqrt{231}$

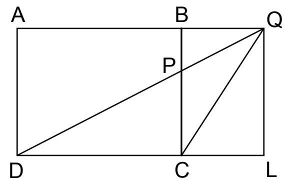
(3) $\sqrt{54}$ (4) $\sqrt{930}$

10. PQR is a right triangle. The length of its greatest side is $24\sqrt{3}$ cm. Find the length of the line segment joining the vertex of the right angle and the mid-point of the greatest side (in cm).

(1) $6\sqrt{3}$ (2) $9\sqrt{3}$

(3) $12\sqrt{3}$ (4) $18\sqrt{3}$

11. In the figure, ABCD is a parallelogram. P is the point on BC such that BP/PC = 1/4 and DP is produced to meet AB produced at Q. The area of triangle BPQ = times of (area of triangle CPD).



(1) 1/4 (2) 1/8

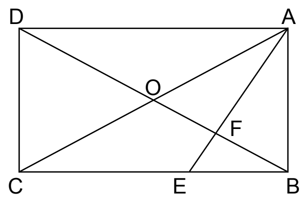
(3) 1/16 (4) 1/32

12. The distance between two buildings is 24 m. The heights of the buildings are 12 m and 22 m. Find the distance between the tops (in m).

(1) 20 (2) 26

(3) 30 (4) 32

13.



In the figure above (not to scale), ABCD is a rectangle with diagonals intersecting at O. If OA = AB and $\angle AEC = 120°$, then $\angle OAF =$ _________ .

(1) 30° (2) 45°

(3) 50° (4) Cannot be determined

14. Diagonal AC of a rectangle ABCD is produced to the point E such that AC: CE = 2: 1. AB = 8 cm and BC = 6 cm. Find the length of DE(in m)

(1) $\sqrt{17}$ (2) $3\sqrt{17}$

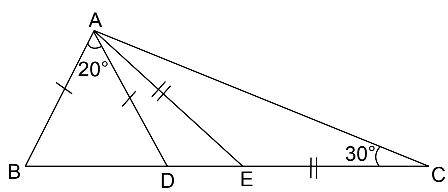
(3) $2\sqrt{17}$ (4) $4\sqrt{17}$

15. In ΔPQR, PQ = 6 cm, PR = 9 cm and M is a point on QR such that it divides QR in the ratio 1: 2. $PM \perp QR$. Find the length of QR.

(1) $\sqrt{15}$ (2) $2\sqrt{15}$

(3) $3\sqrt{15}$ (4) $4\sqrt{15}$

16. In the given figure ABC and ADE are triangles such that AB = AD, AE = EC. Find the measure of angle DAE.



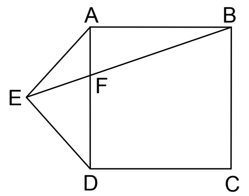
(1) 10° (2) 20°

(3) 25° (4) 15°

17. In a right angled triangle ABC, AB = $10\sqrt{3}$ cm and BC = 20 cm, $\angle A = 90°$. An equilateral triangle ABD is constructed with base AB and with vertex D, at a maximum possible distance from C. Find the length of CD.

(1) $10\sqrt{7}$ cm (2) $10\sqrt{11}$ cm
(3) $10\sqrt{12}$ cm (4) $10\sqrt{14}$ cm

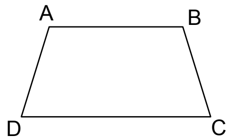
18.



In the above figure, ABCD is a square and triangle ADE is equilateral. Find $\angle EFD$.

(1) 65° (2) 70°
(3) 80° (4) 75°

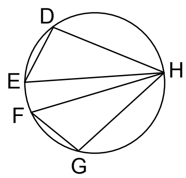
19.



In the figure above, AB is parallel to CD×AD = BC. If $\angle DAB = 100°$, find $\angle BCD$

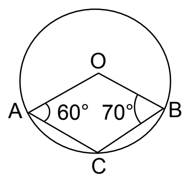
(1) 80° (2) 60°
(3) 70° (4) 50°

20. In the given figure, $\overline{DE}$ and $\overline{FG}$ are equal chords of the circle subtending $\angle DHE$ and $\angle FHG$ at the point H on the circle. If $\angle DHE = 23\frac{1}{2}°$, then find $\angle FHG$.



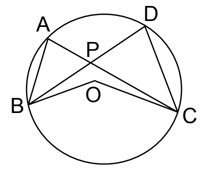
(1) $(11^3/_4)°$ (2) 30°
(3) $(23^1/_2)°$ (4) 47°

21. In the given figure, O is the centre of the circle and $\angle OAC = 60°$ and $\angle OBC = 70°$. Find $\angle AOB$.



(1) 130° (2) 80°
(3) 65° (4) 100°

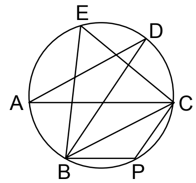
22.



In the given figure (not to scale), O is the centre of the circle. $\overline{AC}$ and $\overline{BD}$ interest at P. PB = PC, $\angle PBO = 25°$ and $\angle BOC = 130°$, then find $\angle ABP + \angle DCP$.

(1) 15° (2) 30°
(3) 45° (4) None of these

23.

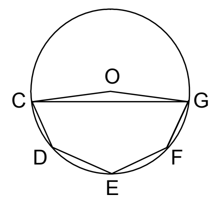


In the figure above (not to scale), AC is the diameter of the circle and $\angle ADB = 20°$, then find $\angle BPC$.

(1) 80° (2) 90°
(3) 100° (4) 110°

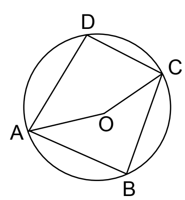
24. In the figure given below, O is the centre of the circle and CD = DE = EF = GF.

If $\angle COD = 40°$, then find reflex $\angle COG$.



(1) 200° (2) 220°
(3) 250° (4) 280°

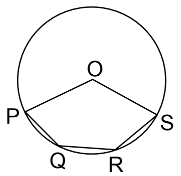
25.



In the figure above, ABCD is a cyclic quadrilateral and O is the centre of the circle. B, O and D are collinear points. If $\angle ABC = 70°$ and $\angle OAD = 50°$, then find $\angle OCB$.

(1) 15° (2) 20°
(3) 30° (4) 45°

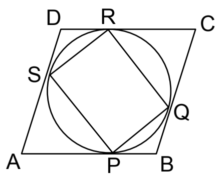
26.



In the figure above P, Q, R and S are concyclic points. O is the centre of the circle PQ = QR = RS. If ∠OPS = 15°, then find ∠OQR.

(1) 50° (2) 55°
(3) 60° (4) 65°

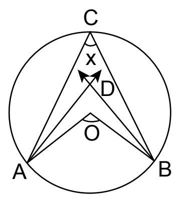
27.



In the figure above, a circle is inscribed in a parallelogram ABCD. PQRS is also a parallelogram and $\overline{AB} \perp \overline{PR}$. If ∠SRP = 60°, then find ∠BPQ.

(1) 60° (2) 45°
(3) 30° (4) 25°

28.



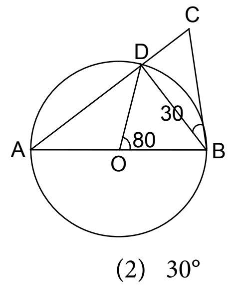
In the figure above, O is the centre of the circle. A, B and C are the points on the circle $\overrightarrow{AD}$ and $\overrightarrow{BD}$ are the angle bisectors of ∠OAC and ∠OBC respectively. If ∠ACB = x and ∠ADB = y, then x: y

(1) 3: 2 (2) 2: 3
(3) 1: 2 (4) 2: 1

29. ABCD is a parallelogram. A circle is passing through B, C, D and intersecting $\overline{AB}$ at E. If ∠DAE = 65°, then find ∠CDE.

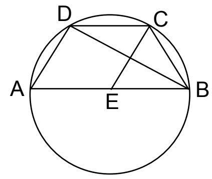
(1) 65° (2) 50°
(3) 55° (4) 40°

30. In the figure given below, if O is the centre of the circle, $\overline{AB}$ is the diameter and AD is produced to the point C, then find ∠ACB.



(1) 15° (2) 30°
(3) 45° (4) 60°

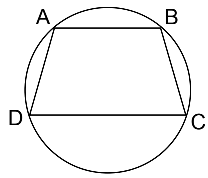
31.



In the figure above, ABCD is a trapezium in which $\overline{AB} \parallel \overline{DC}$. E is the centre of the circle. If AD = EC, then find ∠BDC.

(1) 15° (2) 20°
(3) 25° (4) 30°

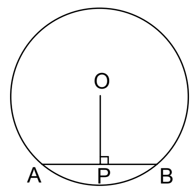
32. In the given figure, ABCD is a cyclic quadrilateral and AD = BC. If ∠A = 110°, then find ∠B.



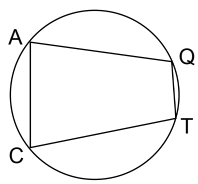
(1) 70° (2) 80°
(3) 100° (4) 110°

33. In the given figure, O is the centre of the circle. If OP = 3 cm and radius of the circle is 5 cm, then find the length of AB.

(1) 8 cm (2) 6 cm
(3) 4 cm (4) 2 cm



34.

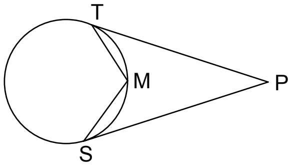


In the figure above, AQ = CT and ∠QAC = 70°. Find ∠ACT.

(1) 70° (2) 80°

(3) 90° (4) 110°

35.

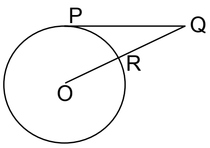


In the figure above (not to scale), PT and PS are tangents segments drawn to a circle at T and S respectively. TM and SM are chords of the circle. If ∠TMS = 100°, then find the angle between the tangents.

(1) 10° (2) 15°

(3) 20° (4) 25°

36.



In the figure above (not to scale), PQ is a tangent, drawn to the circle with centre O at P and QR = RO. If PQ = $3\sqrt{3}$ cm, and ORQ is a line segment, then find the radius of the circle. (in cm).

(1) $\sqrt{3}$ (2) $2\sqrt{3}$

(3) 3 (4) $3\sqrt{3}$

37. P, Q and R are on AB, BC and AC of the equilateral triangle ABC respectively. AP: PB = CQ: QB = 1: 2. G is the centroid of the triangle PQB and R is the mid-point of AC. Find BG: GR.

(1) 4: 3 (2) 3: 4

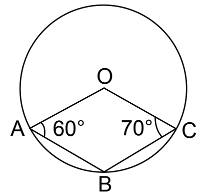
(3) 5: 4 (4) 4: 5

38. Three congruent circles of radius r units are inscribed in an equilateral triangle in such way that, each circle touches two sides of the triangle. Each circle also touches the other two circles. Find the length of side of the triangle (in units).

(1) $\left(\sqrt{3}+1\right)$ (2) $2\left(\sqrt{3}+1\right)$

(3) $3\left(\sqrt{3}+1\right)$ (4) $4\left(\sqrt{3}+1\right)$

39.

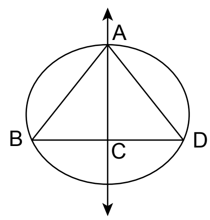


In the figure above (not to scale), O is the centre of the circle, ∠OAB = 60° and ∠OCB = 70°. Then find the ∠AOC.

(1) 150° (2) 120°

(3) 100° (4) 80°

40. The following figure shows that ΔABD is symmetrical figure about the line AC. If ∠ABD = 60°, then ∠BAD = ________ .



(1) 60° (2) 80°

(3) 100° (4) C an

ANSWER KEYS

PRACTICE EXERCISE 3 (A)

1. 2	**2.** 2	**3.** 1	**4.** 4	**5.** 4	**6.** 2	**7.** 2	**8.** 2	**9.** 3	**10.** 3
11. 1	**12.** 4	**13.** 1	**14.** 4	**15.** 3	**16.** 1	**17.** 2	**18.** 3	**19.** 2	**20.** 1
21. 3	**22.** 3	**23.** 1	**24.** 2	**25.** 1	**26.** 3	**27.** 2	**28.** 3	**29.** 1	**30.** 2
31. 1	**32.** 4	**33.** 3	**34.** 2	**35.** 1	**36.** 2	**37.** 2	**38.** 1	**39.** 2	**40.** 1

PRACTICE EXERCISE 3 (B)

1. 2	**2.** 1	**3.** 2	**4.** 3	**5.** 2	**6.** 1	**7.** 1	**8.** 3	**9.** 1	**10.** 3
11. 3	**12.** 2	**13.** 1	**14.** 2	**15.** 3	**16.** 2	**17.** 1	**18.** 4	**19.** 1	**20.** 3
21. 4	**22.** 2	**23.** 4	**24.** 1	**25.** 3	**26.** 4	**27.** 3	**28.** 2	**29.** 1	**30.** 4
31. 4	**32.** 4	**33.** 1	**34.** 1	**35.** 3	**36.** 3	**37.** 4	**38.** 2	**39.** 3	**40.** 1