Coordinate Geometry

Exercise 7.1

Q. 1 A. Find the distance between the following pair of points

(2, 3) and (4, 1)

Answer : $(x_1, y_1) = (2,3)$ and $(x_2, y_2) = (4,1)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - 2)^2 + (1 - 3)^2}$$

$$= \sqrt{2^2 + (-2)^2}$$

$$= \sqrt{8}$$

$$d = 2\sqrt{2} \text{ units}$$

Q. 1 B. Find the distance between the following pair of points

(-5, 7) and (-1, 3)

Answer :

$(x_1, y_1) = (-5,7)$ and $(x_2, y_2) = (-1,3)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1 - -5)^2 + (3 - 7)^2}$$

$= \sqrt{(4)^2 + (4)^2}$

$= \sqrt{32}$

d = $4\sqrt{2}$

Q. 1 C. Find the distance between the following pair of points

(-2, -3) and (3, 2)

Answer : (x_1,y_1) = (-2,-3) and (x_2,y_2) = (3,2)

d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(3 - -2)^2 + (2 - -3)^2}$

= $\sqrt{(5)^2 + (5)^2}$

d = $5\sqrt{2}$

Q. 1 D. Find the distance between the following pair of points

(a, b) and (-a, -b)

Answer : (x_1,y_1) = (a,b) and (x_2,y_2) = (-a,-b)

d = $\sqrt{(-a - a)^2 + (-b - b)^2}$

= $\sqrt{(-2a)^2 + (-2b)^2}$

= $\sqrt{(2a)^2 + (2b)^2}$

d = $2\sqrt{a^2 + b^2}$

Q. 2. Find the distance between the points (0, 0) and (36, 15).

Answer : let (x_1,y_1) = (0,0) and (x_2,y_2) = (36,15)

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(36 - 0)^2 + (15 - 0)^2}$

= $\sqrt{(36)^2 + (15)^2}$

= $\sqrt{1521}$

d = 39

Q. 3. Verify whether the points (1, 5), (2, 3) and (-2, -1) are collinear or not.

Answer : let A = (1,5) B = (2,3) and c = (-2,-1)

⇒ AB = $\sqrt{(2 - 1)^2 + (3 - 5)^2}$

= $\sqrt{(1)^2 + (-2)^2}$

AB = $\sqrt{5}$

⇒ BC = $\sqrt{(-2 - 2)^2 + (-2 - 3)^2} = \sqrt{(4)^2 + (5)^2}$

BC = $\sqrt{41}$

⇒ AC = $\sqrt{(-2 - 1)^2 + (-1 - 5)^2}$

= $\sqrt{(-3)^2 + (-6)^2}$

AC = $\sqrt{45}$

⇒ AB + BC is not equal to AC.

∴ Points are not collinear

Q. 4. Check whether (5, -2), (6, 4) and (7, 2) are the vertices of an isosceles triangle.

Answer :

let A = (5,-2) B = (6,4) and c = (7,2)

$\Rightarrow AB = \sqrt{(6-5)^2 + (4--2)^2}$

$= \sqrt{(1)^2 + (6)^2}$

$AB = \sqrt{37}$

$\Rightarrow BC = \sqrt{(7-6)^2 + (2-4)^2}$

$= \sqrt{(1)^2 + (-2)^2}$

$BC = \sqrt{5}$

$\Rightarrow AC = \sqrt{(7-5)^2 + (2--2)^2}$

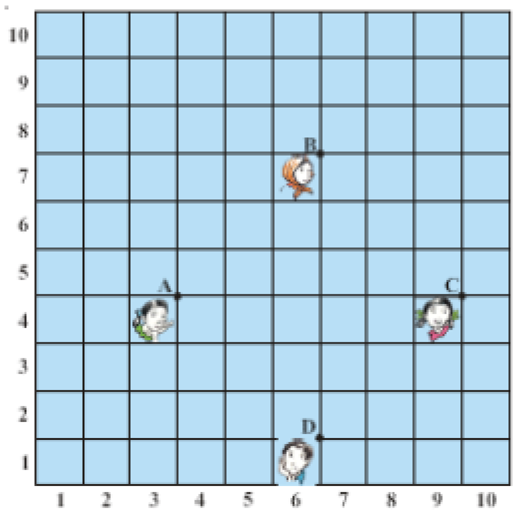
$= \sqrt{(2)^2 + (4)^2}$

$AC = \sqrt{20}$

As all the sides are unequal the triangle is not isosceles.

Q. 5. In a class room, 4 friends are seated at the points A, B, C and D as shown in Figure. Jarina and Phani walk into the class and after observing for a few minutes Jarina asks Phani “Don’t you notice that ABCD is a square?” Phani disagrees.

Using distance formula, find which of them is correct. Why?



Answer :

let A = (3,4) B = (6,7) and C = (9,4) D = (6,1)

$\Rightarrow AB = \sqrt{(6-3)^2 + (7-4)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$AB = 3\sqrt{2}$

$\Rightarrow BC = \sqrt{(9-6)^2 + (4-7)^2}$

$= \sqrt{(3)^2 + (-3)^2}$

$BC = 3\sqrt{2}$

$\Rightarrow CD = \sqrt{(6-9)^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

CD = $3\sqrt{2}$

⇒ AD = $\sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{(3)^2 + (-3)^2}$

AD = $3\sqrt{2}$

As all the sides are equal the ABCD is a square. Jarina is correct.

Q. 6. Show that the following points form an equilateral triangle A(a, 0), B(-a, 0),

$C\left(0, \sqrt{3}\right)$

Answer :

let A = (a,0) B = (-a,0) and $C = (0, \sqrt{3}a)$

⇒ AB = $\sqrt{(-a-a)^2 + 0^2}$

$= \sqrt{(2a)^2}$

AB = 2a

⇒ BC = $\sqrt{(0--a)^2 + \left(\sqrt{3}a\right)^2}$

$= \sqrt{(a)^2 + 3a^2}$

$= \sqrt{4a^2}$

BC = 2a

$\Rightarrow AC = \sqrt{(0-a)^2 + \left((\sqrt{3})\right)^2}$

$= \sqrt{3a^2 + a^2}$

$= \sqrt{4a^2}$

AC = 2a

As all the sides are equal the triangle is Equilateral.

Q. 7. Prove that the point (-7, -3), (5, 10), (15, 8) and (3,-5) taken in order are the corners of a parallelogram. And find its area.

Answer : let A = (-7,-3) B = (5,10) and C = (15,8) D = (3,-5)

Let these points be a parallelogram.

So midpoints of AC and DB should be same.

⇒ To find midpoint of AC and DB

$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

$\Rightarrow$ For AC = $(\frac{-7+15}{2}, \frac{-3+8}{2})$

AC = $(\frac{8}{2}, \frac{5}{2})$

AC = $(4, \frac{5}{2})$

⇒ For DB

$$DB = (\frac{5+3}{2}, \frac{10-5}{2})$$

$$DB = (\frac{8}{2}, \frac{5}{2})$$

$$DB = (4, \frac{5}{2})$$

As midpoints of AC and DB are same the points form a parallelogram.

Let us divide the parallelogram into 2 triangle ΔABD and ΔBCD

Area of both triangles

$$\Rightarrow \text{Area } \Delta ABD = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|-7(10 - -5) + 5(-5 - -3) + 3(-3 - 10)|$$

$$= \frac{1}{2}|-7(15) + 5(-2) + 3(-13)|$$

$$= \frac{1}{2}|-7(15) + 5(-2) + 3(-13)|$$

$$= \frac{1}{2}|154|$$

= 77

$$\Rightarrow \text{Area } \Delta BCD = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|5(8 - -5) + 15(-5 - 10) + 3(10 - 8)|$$

$= \frac{1}{2}|5(13) + 15(-15) + 3(2)|$

$= \frac{1}{2}|154|$

= 77

Total area of parallelogram = Sum of Area of triangles

= ΔBCD + ΔABD

= 154 units

Q. 8. Show that the points (-4, -7), (-1, 2), (8, 5) and (5, -4) taken in order are the vertices of a rhombus.

(Hint: Area of rhombus $= \frac{1}{2} \times$ product of its diagonals)

Answer : Let the points be

A(-4, -7), B(-1, 2), C(8, 5) and D(5, -4)

Length of diagonals

AC = $\sqrt{(8 - -4)^2 + (5 - -7)^2}$

AC = $\sqrt{12^2 + 12^2}$

AC = $12\sqrt{2}$

BD = $\sqrt{(5 - -1)^2 + \left((-4 - 2)\right)^2}$

BD = $\sqrt{6^2 + 6^2}$

BD = $6\sqrt{2}$

⇒ Area of Rhombus = 1/2 × Product of diagonals

$= \frac{1}{2}(AC \times BD) = \frac{1}{2} \times 6\sqrt{2} \times 12\sqrt{2}$

= 72 units

⇒ Area of triangles

Area ΔABD $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|(-4)(2 - -4) + (-1)(-4 - -7) + 5(-7 - 2)|$

$= \frac{1}{2}|-24 - 3 - 45|$

$= \frac{1}{2} \times 72$

= 36

⇒ Are

a ΔBCD $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|(-1)(5 - -4) + (8)(-4 - 2) + 5(2 - 5)|$

$= \frac{1}{2}|-9 - 48 - 15|$

$= \frac{1}{2} \times 72$

= 36

Sum of area of triangles = 36 + 36 = 72 units

Thus proved

Q. 9 A. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(-1, -2), (1, 0), (-1, 2), (-3, 0)

Answer : Let A(-1, -2), B(1, 0), C(-1, 2) and D(-3, 0)

AB = $\sqrt{(1 - -1)^2 + (0 - -2)^2}$

AB = $\sqrt{2^2 + (-2)^2}$

AB = $\sqrt{8}$

AB = $2\sqrt{2}$

BC = $\sqrt{(-1 - 1)^2 + (0 - -2)^2}$

BC = $\sqrt{(-2)^2 + (-2)^2}$

BC = $\sqrt{8}$

BC = $2\sqrt{2}$

CD = $\sqrt{(-3 - -1)^2 + (0 - 2)^2}$

CD = $\sqrt{(-2)^2 + (-2)^2}$

CD = $2\sqrt{2}$

AD = $\sqrt{(-3 - -1)^2 + (0 - -2)^2}$

AD = $\sqrt{(-2)^2 + (-2)^2}$

AD = $2\sqrt{2}$

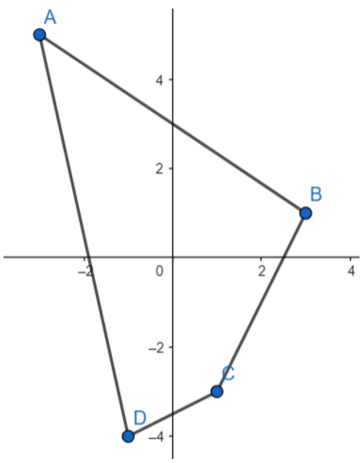
As all sides are equal the quadrilateral is a square

Q. 9 B. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(-3, 5), (3, 1), (1, -3), (-1,-4)

Answer : Let A(-3, 5), B(3, 1), C(1, -3) and D(-1,-4)

Let us see the points on coordinate axes.



Let us first calculate the length of the sides,We know that distance between two points A(x_1, y_1) and B(x_2, y_2) is given by,

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore,

$$AB = \sqrt{(3 - (-3))^2 + (1 - 5)^2}$$

$$AB = \sqrt{6^2 + (-4)^2}$$

$$AB = \sqrt{52}$$

Now calculating BC,

$$BC = \sqrt{(1 - 3)^2 + (-3 - 1)^2}$$

$$BC = \sqrt{(-2)^2 + (-4)^2}$$

$$BC = \sqrt{20}$$

Calculating CD,

$$CD = \sqrt{(-1-1)^2 + (-4 + 3)^2}$$

$$CD = \sqrt{(-2)^2 + (-1)^2}$$

$$CD = \sqrt{5}$$

Calculating DA,

$$DA = \sqrt{(-1 + 3)^2 + (-4 - 5)^2}$$

$$DA = \sqrt{(2)^2 + (-9)^2}$$

$$DA = \sqrt{85}$$

From the lengths we can see that, none of the sides are equal.

Hence, the quadrilateral formed is of no specific type.

Q. 9 C. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(4, 5), (7, 6), (4, 3), (1, 2)

Answer : Let A(4, 5), B(7, 6), C(4, 3) and D(1, 2)

If ABCD is parallelogram

Midpoint of diagonals AC and BD

$\Rightarrow$ For BD = $(\frac{7+1}{2}, \frac{6+2}{2})$

X(4,4)

$\Rightarrow$ For BD = $(\frac{7+1}{2}, \frac{6+2}{2})$

X(4,4)

As the midpoints are same the diagonals bisect each other

Thus, the points form a parallelogram

Q. 10. Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Answer : Let P(x,0) be the point

A(2,-5) and B(-2,9)

⇒ PA = $\sqrt{(x-2)^2 + (-5)^2}$

⇒ PB = $\sqrt{(-2-x)^2 + (9)^2}$

PA = PB

$PA^2 = PB^2$

$(x-2)^2 + (-5)^2 = (x + 2)^2 + 9^2$

$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

8x = -56

X = 7

Point is (-7,0)

Q. 11. If the distance between two points (x, 7) and (1, 15) is 10, find the value of x.

Answer : 7 or -5

Let A(x,7) and B(1,15)

be the point

⇒ AB = $\sqrt{(1-x)^2 + (15-7)^2}$

AB = 10

$AB^2 = 10^2$

⇒ $(1-x)^2 + 8^2 = 10^2$

⇒ $x^2-2x + 1 + 64 = 100$

⇒ $x^2-2x-35 = 0$

⇒ (x-7)(x + 5) = 0

⇒ X = 7 or x = -5

Q. 12. Find the value of y for which the distance between the points P(2, -3) and Q(10, y) is 10 units.

Answer : 3 or -9

Let P(2,-3) and Q(10,y)

be the point

⇒ PQ = $\sqrt{(10-2)^2 + (y + 3)^2}$

⇒ PQ = 10

$PQ^2 = 10^2$

⇒ $(y + 3)^2 + 8^2 = 10^2$

$y^2 + 6y + 9 + 64 = 100$

$y^2 + 6y-27 = 0$

⇒ (y + 9)(y-3) = 0

⇒ y = 9 or y = -3

Q. 13. Find the radius of the circle whose centre is (3, 2) and passes through (-5, 6).

Answer : Let P be center such that P(3,2) and Q be point on circumference Q(-5,6)

PQ = $\sqrt{(3--5)^2 + (6-2)^2}$

PQ = $\sqrt{(8)^2 + (4)^2}$

PQ = $\sqrt{80}$

PQ = $\sqrt{16 \times 5}$

PQ = $4\sqrt{5}$

Q. 14. Can you draw a triangle with vertices (1, 5), (5, 8) and (13, 14)? Give reason.

Answer : Let A(1, 5), B(5, 8) and C(13, 14)

$\Rightarrow$ AB = $\sqrt{(5-1)^2 + (8-5)^2}$

AB = $\sqrt{(4)^2 + (3)^2}$

AB = 5

$\Rightarrow$ BC = $\sqrt{(13-5)^2 + (14-8)^2}$

BC = $\sqrt{(8)^2 + (6)^2}$

BC = 10

$\Rightarrow$ AC = $\sqrt{(13-1)^2 + (14-5)^2}$

AC = $\sqrt{(12)^2 + (9)^2}$

Q. 15. Find a relation between x and y such that the point (x, y) is equidistant from the points (-2, 8) and (-3, -5)

Answer : Let P(x,y) be the point

A(-2,8) and B(-3,-5)

$\Rightarrow$ PA = $\sqrt{(-2-x)^2 + (8-y)^2}$

$\Rightarrow$ PB = $\sqrt{(x + 3)^2 + (y + 5)^2}$

PA = PB

$PA^2 = PB^2$

$(x + 2)^2 + (8-y)^2 = (x + 3)^2 + (y + 5)^2$

$x^2 + 4x + 4 + 64 - 16y + y^2 = x^2 + 6x + 9 + y^2 + 10y + 25$

-2x-26y = -34

$\Rightarrow$ X + 13y = 17

Exercise 7.2

Q. 1. Find the coordinates of the point which divides the line segment joining the points (-1,7) and (4, -3) in the ratio 2 : 3.

Answer : Let P(x,y) be the point

$$P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

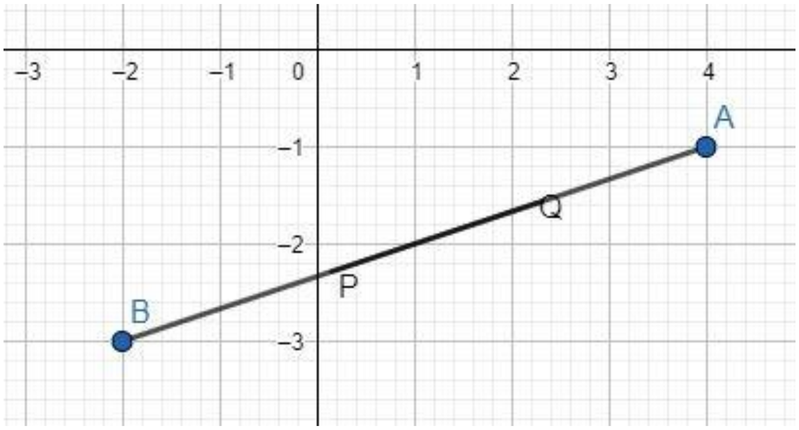
$$P(x,y) = \left(\frac{2\times 4 + 3\times(-1)}{2 + 3}, \frac{2\times(-3) + 3\times 7}{2 + 3}\right)$$

$$P(x,y) = \left(\frac{5}{5}, \frac{15}{5}\right)$$

P(x,y) = (1,3)

Q. 2. Find the coordinates of the points of trisection of the line segment joining (4,-1) and (-2, -3).

Answer :



points A(4, -1) and B(-2, -3) are shown in the graph above
Now points of trisection means the points which divides the line in three parts. From the figure it is clear that, there will be 2 points which will do that. Let us call them P and Q. Now clearly point P divides the BA in the ratio 1:2 and point Q divides the line in the ratio 2:1⇒ Let P and Q be points of trisection

∴ P divides BA internally in ratio 1:2

Such that AP = PQ = QB

Let us apply section formula to the points A and B such that P divides BA in the ratio 1:2

$$P(x,y) = (\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2})$$

$$P(x,y) = (\frac{1 \times (-2) + 2 \times (4)}{1 + 2}, \frac{1 \times (-3) + 2 \times (-1)}{1 + 2})$$

$$P(x,y) = (\frac{6}{3}, \frac{-5}{3})$$

$$P(x,y) = (2, \frac{-5}{3})$$

⇒ Q divides BA internally in ratio 2:1

$$Q(x,y) = (\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2})$$

$$Q(x,y) = (\frac{2 \times (-2) + 1 \times (4)}{2 + 1}, \frac{2 \times (-3) + 1 \times (-1)}{2 + 1})$$

$$Q(x,y) = (0, \frac{-7}{3})$$

$$Q(x,y) = (0, \frac{-7}{3})$$

Q. 3. Find the ratio in which the line segment joining the points (-3, 10) and (6, -8) is divided by (-1, 6).

Answer : Let P and Q be line

∴ A divides PQ internally in ratio a:b

$$A(x,y) = (\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2})$$

$$A(x,y) = (\frac{a \times (6) + b \times (-3)}{a + b}, \frac{a \times (-8) + b \times (10)}{a + b})$$

$$A(x,y) = (\frac{6a - 3b}{a + b}, \frac{10b - 8a}{a + b})$$

⇒ Given that A(-1,6)

6a-3b = -a-b , 10b-8a = 6a + 6b

7a = 2b, 14a = 4b

∴ a:b = 2:7

Q. 4. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer : Let A(1, 2), B(4, y), C(x, 6) and D(3, 5)

If ABCD is a parallelogram

AC and BD bisect each other

⇒ Midpoint of AC

$$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

$\Rightarrow$ For AB = $(\frac{1+x}{2}, \frac{2+6}{2})$

= $(\frac{1+x}{2}, 4)$

$\Rightarrow$ Midpoint of BD

For BD = $(\frac{4+3}{2}, \frac{5+y}{2})$

= $(\frac{7}{2}, \frac{5+y}{2})$

$\therefore \frac{1+x}{2} = \frac{7}{2}, 4 = \frac{5+y}{2}$

1 + x = 7 , 8 = 5 + y

x = 6 y = 3

Q. 5. Find the coordinates of point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

Answer : Let O be the center O(2,-3)

$\Rightarrow$ A(x,y) B(1,4)

$\therefore$ O divides AB internally in ratio 1:1

$$O(2,-3) = (\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2})$$

$$O(2,-3) = (\frac{1 \times (1) + 1 \times (x)}{1 + 1}, \frac{1 \times (4) + 1 \times (y)}{1 + 1})$$

$$O(2,-3) = (\frac{x + 1}{2}, \frac{y + 4}{2})$$

$$\frac{x + 1}{2} = 2, \frac{y + 4}{2} = -3$$

x + 1 = 4

x = 3

y + 4 = -6

y = -10

Q. 6. If A and B are (-2, -2) and (2, -4) respectively. Find the coordinates of P such that and P $AP = \frac{3}{7}$ AB lies on the segment AB.

Answer : AP = 3/7AB

∴ P divides AB in ratio 3:4

$$P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$P(x,y) = \left(\frac{3 \times (2) + 4 \times (-2)}{3 + 4}, \frac{3 \times (-4) + 4 \times (-2)}{3 + 4}\right)$$

$$P(x,y) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$$

Q. 7. Find the coordinates of points which divide the line segment joining A(-4, 0) and B(0, 6) into four equal parts.

Answer :

$$\left(-3, \frac{3}{2}\right), (-2, 3), \left(-1, \frac{9}{2}\right)$$

⇒ Let x,y and z divide the line into 4 equal parts such that

AX = XY = YZ = ZB

⇒ X divides AB in ration 1:3

$$X(x,y) = (\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2})$$

$$X(x,y) = (\frac{1 \times (0) + 3 \times (-4)}{3 + 1}, \frac{1 \times (6) + 3 \times (0)}{3 + 1})$$

$$X(x,y) = (-3, \frac{3}{2})$$

⇒ Y divides AB in ratio 1:1

$$Y(x,y) = (\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2})$$

$$Y(x,y) = (\frac{1 \times (0) + 1 \times (-4)}{1 + 1}, \frac{1 \times (6) + 1 \times (0)}{1 + 1})$$

Y(-2,3)

⇒ Z divides AB in ratio 3:1

$$Z(x,y) = (\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2})$$

$$Z(x,y) = (\frac{3 \times (0) + 1 \times (-4)}{3 + 1}, \frac{3 \times (6) + 1 \times (0)}{3 + 1})$$

$$Z(x,y) = (-1, \frac{9}{2})$$

Q. 8. Find the coordinates of the points which divides the line segment joining A(-2, 2) and B(2, 8) into four equal parts.

Answer :

$$\left(1, \frac{13}{2}\right)$$

⇒ Let x,y and z divide the line into 4 equal parts such that

AX = XY = YZ = ZB

⇒ X divides AB in ration 1:3

$$X(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$X(x,y) = \left(\frac{1\times(2) + 3\times(-2)}{3 + 1}, \frac{1\times(8) + 3\times(2)}{3 + 1}\right)$$

$$X(x,y) = \left(-1, \frac{7}{2}\right)$$

⇒ Y divides AB in ratio 1:1

$$Y(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$Y(x,y) = \left(\frac{1\times(2) + 1\times(-2)}{1 + 1}, \frac{1\times(8) + 1\times(2)}{1 + 1}\right)$$

Y(0,5)

⇒ Z divides AB in ratio 3:1

$$Z(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$Z(x,y) = \left(\frac{3\times(2) + 1\times(-2)}{3 + 1}, \frac{3\times(8) + 1\times(2)}{3 + 1}\right)$$

$$Z(x,y) = \left(1, \frac{13}{2}\right)$$

Q. 9. Find the coordinates of the point which divides the line segment joining the points (a+b, a-b) and (a-b, a+b) in the ratio 3 : 2 internally.

Answer :

$$\left(\frac{5a - b}{5}, \frac{5a + b}{5}\right)$$

$$P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$P(x,y) = \left(\frac{3 \times (a-b) + 2 \times (a+b)}{3+2}, \frac{3 \times (a+b) + 2 \times (a-b)}{3+2}\right)$$

$$P(x,y) = \left(\frac{3a - 3b + 2a + 2b}{5}, \frac{3a + 3b + 2a - 2b}{5}\right)$$

$$P(x,y) = \left(\frac{5a - b}{5}, \frac{5a + b}{5}\right)$$

Q. 10 A. Find the coordinates of centroid of the triangle with vertices following:

(-1, 3), (6, -3) and (-3, 6)

Answer : The coordinates of centroid are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$\left(\frac{-1 + 6 - 3}{3}, \frac{3 - 3 + 6}{3}\right)$$

$$\left(\frac{2}{3}, 2\right)$$

Q. 10 B

Find the coordinates of centroid of the triangle with vertices following:

(6, 2), (0, 0) and (4, -7)

Answer :

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$\left(\frac{6 + 0 + 4}{3}, \frac{2 + 0 - 7}{3}\right)$$

$(\frac{10}{3}, -\frac{5}{3})$

Q. 10 C. Find the coordinates of centroid of the triangle with vertices following:

(1, -1), (0, 6) and (-3, 0)

Answer :

$(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3})$

$(\frac{1 + 0 - 3}{3}, \frac{-1 + 6 + 0}{3})$

$(\frac{-2}{3}, -\frac{5}{3})$

Q. 1 A. Find the area of the triangle whose vertices are

(2, 3) (-1, 0), (2, -4)

Answer :

$\text{Area } \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|2(0 - -4) - 1(-4 - 3) + 2(3 - 0)|$

$= \frac{1}{2}|2(4) - 1(-7) + 3(2)|$

$= \frac{21}{2}$

Q. 1 B. Find the area of the triangle whose vertices are

(-5, -1), (3, -5), (5, 2)

Answer :

$$\text{Area } \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|-5(-5 - 2) + 3(2 - -1) + 5(-1 - -5)|$$

$$= \frac{1}{2}|-5(-7) + 3(3) + 5(4)|$$

$$= \frac{1}{2}|64|$$

$$= 32$$

Q. 1 C. Find the area of the triangle whose vertices are

(0, 0) (3, 0) and (0, 2)

Answer :

$$\text{Area } \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|0(0 - 2) + 3(2 - 0) + 0(0 - 2)|$$

$$= \frac{1}{2}|0(-2) + 3(2) + 0(-2)|$$

$$= \frac{1}{2}|6|$$

$$= 3$$

Q. 2 A. Find the value of 'K' for which the points are collinear.

(7, -2) (5, 1) (3, K)

Answer : For points to be collinear area of ΔABC = 0

$$\text{Area } \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|7(1 - k) + 5(k - -2) + 3(-2 - 1)|$$

$$= \frac{1}{2}|7(1 - k) + 5(k + 2) + 3(-3)|$$

$$= \frac{1}{2}|7 - 7k + 5k + 10 - 9|$$

$$= \frac{1}{2}|8 - 2k|$$

∴ 8 - 2k = 0

K = 4

Q. 2 B. Find the value of 'K' for which the points are collinear.

(8, 1), (k, -4), (2, -5)

Answer : For points to be collinear area of ΔABC = 0

$$\text{Area } \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|8(-4 - -5) + k(-5 - 1) + 2(1 - -4)|$$

$$= \frac{1}{2}|8(1) - k(6) + 2(5)|$$

$$= \frac{1}{2}|18 - 6k|$$

$18\text{-}6k = 0$

$K = 3$

Q. 2 C. Find the value of 'K' for which the points are collinear.

(K, K) (2, 3) and (4, -1).

Answer : For points to be collinear area of ΔABC = 0

$$\text{Area } \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|k(3 - -1) + k(-1 - k) + 4(k - 3)|$$

$$= \frac{1}{2}|k(4) - k(k + 1) + 4k - 12|$$

$$= \frac{1}{2}|8k - k^2 - k + 4k - 12|$$

$$= \frac{1}{2}|k^2 - 11k - 12|$$

⇒ $K^2 - 11k - 12 = 0$

⇒ K(k-12) + (k-12) = 0

⇒ (K-12)(k + 1) = 0

⇒ K = 12 or k = -1

Q. 3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area of the area of the given triangle.

Answer : 1 sq. unit; 1 : 4

⇒ Let A(0,-1),B(2,1),C(0,3)

So midpoints of AB and BC and CD

⇒ Midpoint formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

⇒ For AB = $\left(\frac{0 + 2}{2}, \frac{1 - 1}{2}\right)$

AB = $\left(\frac{2}{2}, 0\right)$

AB = $(1,0)$

⇒ For BC

BC = $\left(\frac{2 + 0}{2}, \frac{1 + 3}{2}\right)$

BC = $\left(\frac{2}{2}, \frac{4}{2}\right)$

BC = $(1,2)$

⇒ For AC

AC = $\left(\frac{0 + 0}{2}, \frac{-1 + 3}{2}\right)$

AC = $\left(\frac{0}{2}, \frac{2}{2}\right)$

AC = $(0,1)$

(1,0)(1,2)(0,1)

Area ΔXYZ = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|1(2-1) + 1(1-0) + 0(0-2)|$

$= \frac{1}{2}|1(2) + 1(1) + 0|$

$= \frac{1}{2}|2|$

= 1 sq cm

⇒ Let A(0,-1),B(2,1),C(0,3)

⇒ Area ΔABC = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|0(1-3) + 2(3--1) + 0(-1-1)|$

$= \frac{1}{2}|0 + (8) + 0|$

$= \frac{1}{2}|8|$

= 4 sq cm

∴ Ratio of Area of triangles is 1:4

Q. 4. Find the area of the quadrilateral whose vertices, taken in order, are (-4, -2), (-3, -5), (3, -2) and (2, 3)

Answer : 28 sq. units

⇒ Let A = (-4, -2), B = (-3, -5), C = (3, -2) and D = (2, 3)

We draw Line BD and divide the quadrilateral into 2 triangles

ΔABD and ΔBDC

Area of both triangles

⇒ Area ΔBDC = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|-3(-2-3) + 3(3--5) + 2(-5--2)|$

$= \frac{1}{2}|-3(-5) + 3(8) + 2(-3)|$

$= \frac{1}{2}|(15) + (24) + (-6)|$

$= \frac{1}{2}|33|$

$= \frac{33}{2}$

= 16.5 units

A = (-4, -2), B = (-3, -5), D = (2, 3)

Area ΔABD = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|-4(-5-3)-3(3--2)\ +\ 2(-2--5)|$

$= \frac{1}{2}|4(8)-3(5)\ +\ 2(3)|$

$= \frac{1}{2}|32-15\ +\ 6|$

$= \frac{1}{2}|23|$

= 11.5 units

⇒ Total area of quadrilateral = Sum of Area of triangles

= ΔBCD + ΔABD

= 16.5 + 11.5 units

= 28 sq units.

Q. 5. Find the area of the triangle formed by the points (8, -5), (-2, -7) and (5, 1) by using Heron's formula.

Answer : Not possible

Let A = (8,-5) B = (-2,-7) and C = (5,1)

AB = $\sqrt{(-2-8)^2\ +\ (-7--5)^2}$

AB = $\sqrt{(-10)^2\ +\ (-2)^2}$

AB = $\sqrt{104}$

BC = $\sqrt{(5--2)^2\ +\ (1--7)^2}$

BC = $\sqrt{(7)^2\ +\ (8)^2}$

$BC = \sqrt{113}$

$AC = \sqrt{(5-8)^2 + (1--5)^2}$

$AC = \sqrt{(-3)^2 + (6)^2}$

$AC = \sqrt{45}$

Exercise 7.4

Q. 1 A. Find the slope of the line passing the two given points

(4, -8) and (5, -2)

Answer : Slope

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2 - -8}{5 - 4}$

$= \frac{6}{1}$

$= 6$

Q. 1 B. Find the slope of the line passing the two given points

(0, 0) and $\left(\sqrt{3}, 3\right)$

Answer :

$= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{3-0}{\sqrt{3}-0}$$

$$= \frac{3}{\sqrt{3}}$$

$$= \sqrt{3}$$

Q. 1 C. Find the slope of the line passing the two given points

(2a, 3b) and (a, -b)

Answer :

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-b-3b}{a-2a}$$

$$= \frac{-4b}{-a}$$

$$= \frac{4b}{a}$$

Q. 1 D. Find the slope of the line passing the two given points

(a, 0) and (0, b)

Answer :

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{b-0}{0-a}$$

$$= \frac{b}{-a}$$

$$= \frac{-b}{a}$$

Q. 1 E. Find the slope of the line passing the two given points

A(-1.4, -3.7), B(-2.4, 1.3)

Answer :

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1.3 - -3.7}{-2.4 - -1.4}$$

$$= \frac{5}{-1}$$

$$= -5$$

Q. 1 F. Find the slope of the line passing the two given points

A(3, -2), B(-6, -2)

Answer :

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - -2}{-6 - -3}$$

$$= \frac{0}{-3}$$

$$= 0$$

Q. 1 G. Find the slope of the line passing the two given points

$$A\left(-3\frac{1}{2}, 3\right), B\left(-7, 2\frac{1}{2}\right)$$

Answer :

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2.5 - 3}{-7 - -3.5}$$

$$= \frac{1}{7}$$

Q. 1 H. Find the slope of the line passing the two given points

A(0, 4), B(4, 0)

Answer :

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 4}{4 - 0}$$

$$= \frac{-4}{4}$$

$$= -1$$