Tangent Properties of Circles 18

STUDY NOTES

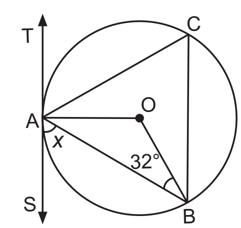
- A line which intersects a circle in two distinct points is called a **secant**.
- A line meeting a circle only in one point is called a **tangent** to the circle at that point.
- The point at which the tangent line meets the circle is called the **point of contact**.
- The length of the line segment of the tangent between a given point and the given point of contact with the circle is called the length of the tangent from the point to the circle.
- There is one and only one tangent passing through a point lying on a circle.
- There are exactly two tangents through a point lying outside a circle.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- If two circles touch each other, the point of contact lies on the straight line through their centres.
- If two tangents are drawn to a circle from an exterior point, then
  (i) The lengths of tangents are equal.
  (ii) The tangents subtend equal angles at the centre.
  (iii) The tangents are equally inclined to the line joining the point and the centre of the circle.
  (iv) The centre lies on the bisector of the angle between the two tangents.
- If two chords of a circle intersect internally or externally, then the product of the lengths of their segments are equal.
- If a chord and a tangent intersect internally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.
- The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.

QUESTION BANK

A. Multiple Choice Questions [1 Mark]

Choose the correct option:

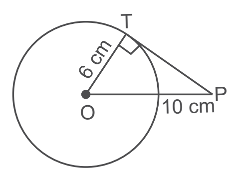
1. In the given figure, TAS is a tangent to the circle, with centre O, at point A. If $\angle OBA = 32^\circ$, then the value of x will be:



(a) 19° (b) 38°
(c) 58° (d) 76°

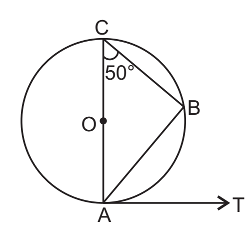
2. A point P is 10 cm from the centre O of a circle. The length of the tangent drawn from P to the circle is 8 cm. The radius of the circle is equal to :
(a) 4 cm (b) 5 cm
(c) 6 cm (d) 8 cm

3. In the given figure, PT is a tangent to the circle with centre O. If OT = 6 cm and OP = 10 cm, then the length of tangent PT is:



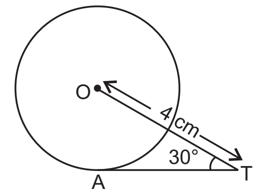
(a) 8 cm (b) 12 cm
(c) 10 cm (d) 16 cm

4. In the given figure, AC is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at point A, then $\angle BAT$ is equal to:



(a) 65° (b) 60°
(c) 50° (d) 40°

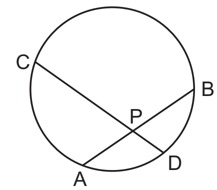
5. In the figure, AT is a tangent to the circle with centre O such that OT = 4 cm and $\angle OTA = 30°$. The length of AT is :



(a) 4 cm (b) 2 cm

(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

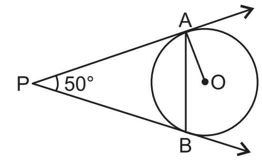
6. In the figure, chords AB and CD intersect at P. If PA = 6 cm, PB = 5 cm and CD = 13 cm, then the length of PC is :



(a) 10 cm (b) 8 cm

(c) 6 cm (d) 5 cm

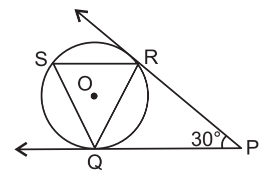
7. In the figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then the measure of $\angle OAB$ =



(a) 37° (b) 36°

(c) 50° (d) 25°

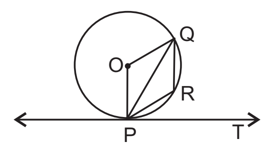
8. Tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Then $\angle RQS$ =



(a) 40° (b) 30°

(c) 25° (d) 20°

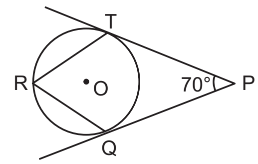
9. In the figure, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60°$, then $\angle PRQ$ =



(a) 130° (b) 120°

(c) 110° (d) 100°

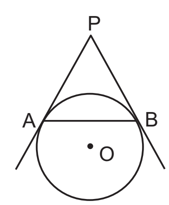
10. In the figure, O is the centre of the circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70°$, then $\angle TRQ$ is:



(a) 60° (b) 58.5°

(c) 55° (d) 50°

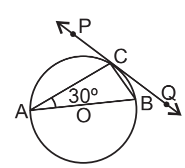
11. In the figure, PA and PB are tangents to a circle with centre O, such that AP = 5 cm and $\angle APB = 60°$, then the length of chord AB is:



(a) 9 cm (b) 5 cm

(c) 4.5 cm (d) 4 cm

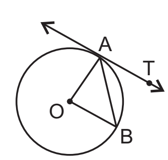
12. In the figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, then $\angle PCA$=



(a) 60° (b) 58°

(c) 57° (d) 56°

13. In the figure, O is the centre of the circle, AB is a chord and AT is the tangent at point A. If $\angle AOB = 100°$, then $\angle BAT$ =



(a) 30° (b) 40°

(c) 50° (d) 100°

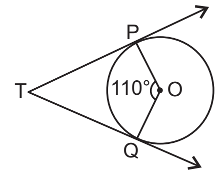
14. Point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. The radius of the circle is:

(a) 7 cm (b) 9 cm (c) 10 cm (d) 12 cm

15. In the figure, TP and TQ are two tangents to a circle with centre O such that, $\angle POQ = 110°$, the value of $\angle PTQ$ is:

(a) 60° (b) 65°

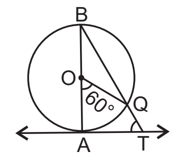
(c) 70° (d) 75°



16. In the figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 60°$, then $\angle ATQ =$

(a) 60° (b) 50.5°

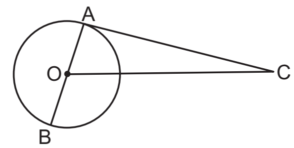
(c) 49° (d) 30°



17. In the figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, then $\angle ACO =$

(a) 35° (b) 38°

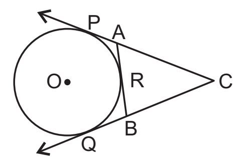
(c) 120° (d) 40°



18. In the figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then the length of BR is:

(a) 3.5 cm (b) 4 cm

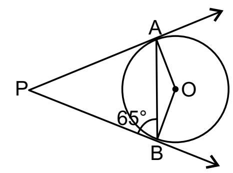
(c) 3 cm (d) 11 cm



19. In the figure, PA and PB are two tangents from an external point P to a circle with centre O. If $\angle PBA = 65°$, then $\angle OAB$:

(a) 15° (b) 25°

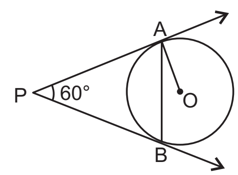
(c) 35° (d) 45°



20. In the figure, PA and PB are tangents to the circle with centre O. If $\angle APB = 60°$, then $\angle OAB$:

(a) 40° (b) 30°

(c) 25° (d) 20°

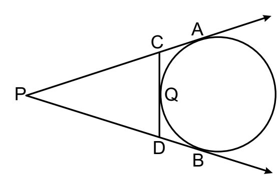


A. Answers

1. (c)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (b)	**10.** (c)
11. (b)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (c)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (b)

B. Short Answer Type Questions [3 Marks]

1. In the figure, PA and PB are tangents to the circle drawn from an external point P. CD is another tangent touching the circle at Q. If PA = PB = 12 cm and QD = 3 cm, find the length of PD.



Sol. PA and PB are tangents

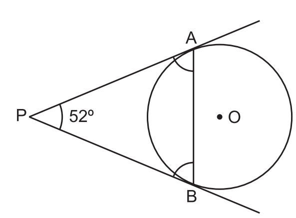
PA = PB = 12 cm

Also, DQ = DB

$\Rightarrow$ BD = 3 cm

PD = PB – BD = 12 cm – 3 cm = 9 cm.

2. In the figure, PA and PB is a pair of tangents drawn to a circle having its centre at O. If $\angle APB = 52°$, find $\angle PAB$ and $\angle PBA$.



Sol. $\angle APB = 52°$

PA = PB

$\therefore \angle PAB = \angle PBA$

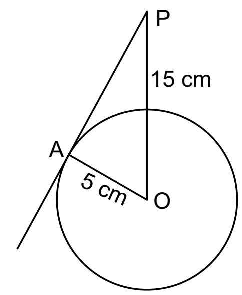
Also, $\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow \angle PAB + \angle PAB + 52° = 180° \Rightarrow 2\angle PAB = 180° - 52°$

$\Rightarrow \angle PAB = \dfrac{128°}{2}$ $\angle PAB = 64°$

$\angle PAB = \angle PBA = 64°$.

3. A point P is 15 cm from the centre of a circle. The radius of the circle is 5 cm. Find the length of the tangent drawn to the circle from the point P.



Sol. $\angle OAP = 90°$ [Radius is perpendicular to tengent]

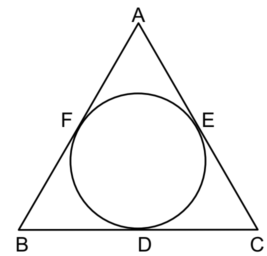
By Pythagoras Theorem

$OP^2 = OA^2 + AP^2$

$15^2 = 5^2 + AP^2 \Rightarrow 225 - 25 = AP^2 \Rightarrow \sqrt{200} = AP$

$\Rightarrow AP = 10\sqrt{2}$ cm.

4. In the figure, the circle touches the sides BC, CA and AB of ΔABC at D, E and F respectively. If AB = AC, prove that BD = CD.



Sol. **Proof** $AB = AC$ ...(i)

$AF = AE$ [Tangents are equal] ...(ii)

So, $AB - AF = AC - AE$

$\Rightarrow$ $BF = CE$ ...(iii)

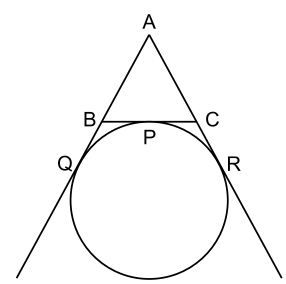
$\Rightarrow$ $BF = BD$ ...(iv)

$CE = CD$ ...(v)

From (iii), (iv) and (v), we have

$BD = CD$ **Proved**

5. A circle is touching the side BC of a ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that $AQ = \frac{1}{2}$ (Perimeter of ΔABC).



Sol.

$AQ = AR$

$BQ = BP$

$CP = CR$

Perimeter of $\Delta ABC = AB + BC + CA$

$= AB + (BP + PC) + (AR - CR)$

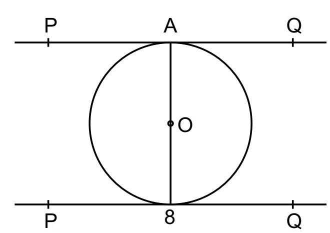
$= (AB + BP) + PC + (AQ - PC)$

$= AQ + AQ$

$= 2AQ$

$\Rightarrow AQ = \frac{1}{2}$ perimeter of ΔABC. **Proved.**

6. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



Sol. PQ is tangent at point A

$OA \perp PQ$

or $\angle OAP = 90°$ ...(i)

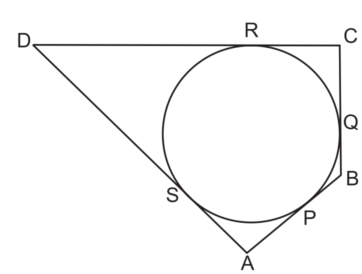
Similarly, $\angle OBS = 90°$ ...(ii)

$\angle OAP = \angle OBS$

$\angle BAP = \angle ABS$ [Alternate interior angles are equal]

So, $PQ \parallel RS$. **Proved.**

7. A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that AB + CD = AD + BC.



Sol. $AP = AS$ ...(i)

$BP = BQ$ ...(ii)

$CR = CQ$ ...(iii)

$DR = DS$ ...(iv)

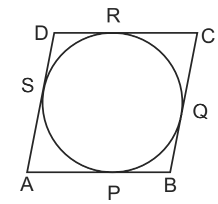
Adding eq (i), (ii), (iii) and (iv), we have

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC.$ **Proved**

8. In the figure, a circle touches all the four sides of a quadrilateral ABCD with AB = 6 cm, BC = 7 cm and CD = 4 cm. What is the length of AD?



Sol. **Proof**

AP = AS ...(i)

BP = BQ ...(ii)

DR = DS ...(iii)

CR = CQ ...(iv)

Adding (i), (ii), (iii) and (iv), we have

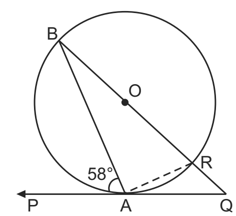
$$AP + BP + DR + CR = AS + BQ + DS + CQ$$

$$\Rightarrow \quad AB + CD = BC + DA$$

$$\Rightarrow \quad 6 + 4 = 7 + AD$$

$$\Rightarrow \quad AD = 3 \text{ cm.}$$

9. In the figure, O is the centre of the circle, PQ is a tangent to the circle at A. If $\angle PAB = 58°$, find $\angle ABQ$ and $\angle AQB$.



Sol. Join OA

$$\angle OAP = 90°$$

$$\Rightarrow \quad \angle OAB = \angle OAP - \angle PAB$$

$$\Rightarrow \quad \angle OAB = 90° - 58° = 32°$$

$$OA = OB \text{ [radii]}$$

$$\Rightarrow \quad \angle OAB = \angle OBA$$

$$\Rightarrow \quad \angle OBA = 32°$$

$$\angle AOQ = \angle OAB + \angle OBA \text{ [By exterior angle property]}$$

$$= 32° + 32° = 64°$$

In ΔOAQ,

$$\angle OQA + \angle AOQ + \angle OAQ = 180°$$

$$\Rightarrow \quad \angle OQA = 180° - \angle AOQ - \angle OAQ$$

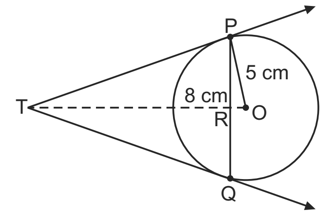
$$= 180° - 64° - 90°$$

$$\Rightarrow \quad \angle OQA = 26°.$$

So, $\angle ABQ = 32°$ and $\angle AQB = 26°$

C. Long Answer Type Questions [4 Marks]

1. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



Sol.

$$TP = TQ$$

$$OT \perp PQ$$

OT bisects PQ

So, PR = RQ = 4 cm

$$OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2} \Rightarrow OR = 3 \text{ cm}$$

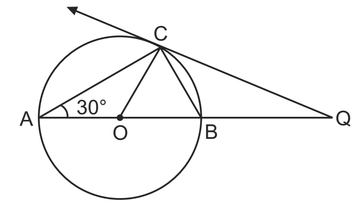
$$\angle TPR + \angle RPO = 90° = \angle TPR + \angle PTR$$

$$\angle RPO = \angle PTR$$

So, $\Delta TRP \sim \Delta PRO$

$$\Rightarrow \frac{TP}{PO} = \frac{RP}{RO} \Rightarrow \frac{TP}{5} = \frac{4}{3} \Rightarrow TP = \frac{20}{3} \text{ cm.}$$

2. In the figure, AB is diameter of a circle with centre O and QC is a tangent to the circle at C. If ∠CAB = 30º, find (a) ∠CQA (b) ∠CBA.



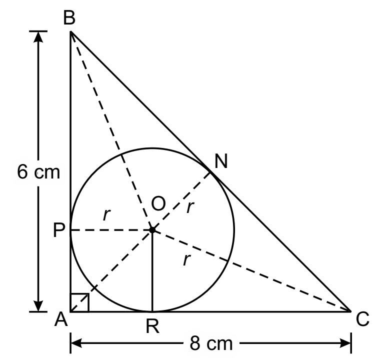
Sol. (a) AOB is the diameter

$$\angle ACB = 90^\circ$$
$$\angle OCQ = 90^\circ$$
$$\angle ACO = 30^\circ \quad [OA = OC]$$
$$\angle OCB = 60^\circ$$
$$\angle BCQ = 30^\circ$$
$$\angle CQA = 60^\circ - 30^\circ = 30^\circ$$

(b) Exterior $\angle CBQ = 30^\circ + 90^\circ = 120^\circ$

$\Rightarrow$ $\angle CBA = 180^\circ - \angle CBQ = 180^\circ - 120^\circ = 60^\circ$

3. ABC is a right triangle, right angled at B. A circle is inscribed in it. The lengths of the two sides containing the right angle are 6 cm and 8 cm. Find the radius of the incircle.



Sol. AB, BC and CA are the tangents of circle at P, N and R recpectively.

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

By Pythagoras Theorem

$$BC^2 = AB^2 + AC^2 = 6^2 + 8^2$$
$$BC = \sqrt{100} = 10 \text{ cm}$$

Area of ΔABC = area of ΔOAB + area of ΔOBC + area of ΔAOC

$$\Rightarrow \quad 24 = \frac{1}{2} AB \times r + \frac{1}{2} BC \times r + \frac{1}{2} AC \times r$$
$$\Rightarrow \quad 24 = \frac{1}{2} r\,(AB + BC + AC)$$
$$\Rightarrow \quad 24 = \frac{1}{2} r\,(6 + 8 + 10)$$
$$\Rightarrow \quad 48 = r \times 24$$
$$\Rightarrow \quad r = 2 \text{ cm.}$$

4. Two tangents PA and PB are drawn to the circle with centre O, such that ∠APB = 120º. Prove that OP = 2AP.

Sol. O is the centre of the circle.

$\angle OAP = \angle OBP = 90^\circ$

OA = OB [Radii]

In ΔOAP and ΔOBP

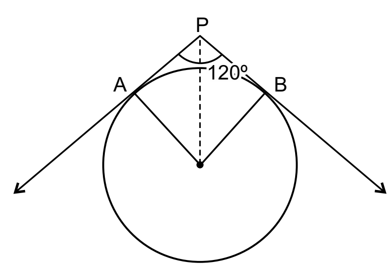
OA = OB [Radii]

$\angle OAP = \angle OBP = 90^\circ$ [Given]

OP = OP [Common]

$\Delta OAP \cong \Delta OBP$ [SAS]

$\angle OPA = \angle OPB$ [CPCT]



$\angle OPA = \frac{1}{2} \angle APB = \frac{1}{2} \times 120^\circ = 60^\circ$

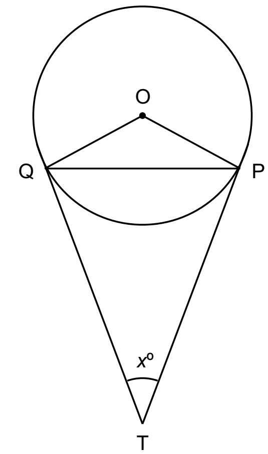
$\angle OPA = 60^\circ$

$\angle OPB = 60^\circ$

In ΔOAP,

$\cos 60^\circ = \frac{AP}{OP} \Rightarrow \frac{1}{2} = \frac{AP}{OP} \Rightarrow OP = 2AP$ **Proved**.

5. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Sol. Let $\angle PTQ = x$

By angle sum property of a triangle

$\angle TQP + \angle TPQ = 180° - x$ ...(i)

$TP = TQ$

$\angle TPQ = \angle TPQ = \frac{1}{2}(180° - x) = 90° - \frac{x}{2}$

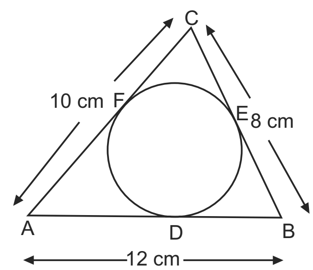
$\angle OPQ = \angle OPT - \angle TPQ$

$\Rightarrow \angle OPQ = 90° - \angle TPQ$

$\Rightarrow \angle OPQ = \frac{x}{2} \Rightarrow x = 2\angle OPQ$

$\Rightarrow \angle PTQ = 2\angle OPQ$ **Proved**

6. A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm as shown in the figure. Find AD, BE and CF.



Sol. $AC = 10$ cm

$AB = 12$ cm

$BC = 8$ cm

Let $CF = x$

$\Rightarrow CF = CE = x$

$\Rightarrow AF = 10 - x = AD$

$\Rightarrow BE = 8 - x = BD$

Now, $AD + BD = 12$

$\Rightarrow 10 - x + 8 - x = 12$

$\Rightarrow 18 - 2x = 12$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$ cm

So, $AD = 10 - x = 10 \text{ cm} - 3 \text{ cm} = 7$ cm

$BE = 8 - x = 8 \text{ cm} - 3 \text{ cm} = 5$ cm

$CF = x = 3$ cm