Comparing Quantities

Concept of Ratios

There are situations, when we need to compare two quantities.

For example, Swaminathan and Rohan both are in the same class. Their respective marks in mathematics are 96 and 48.

Marks scored by Swaminathan and Rohan can be compared by two methods.

1. Subtraction method

In this method, we subtract one quantity from other to find that one is how much more than the other.

Now,

Marks scored by Swaminathan – Marks scored by Rohan = 96 – 48 = 48

So, it can be said that Swaminathan scored 48 marks more than Rohan in mathematics.

2. Division method

In this method, we divide one quantity by other to find that one is how many times the other.

Now,

$$\frac{\text{Marks scored by Swaminathan}}{\text{Marks scored by Rohan}} = \frac{96}{48} = \frac{2}{1}$$

So, it can be said that the marks scored by Swaminathan are twice the marks scored by Rohan.

When two quantities are compared using division method, the quotient obtained is called "ratio".

First term of a ratio is called "**antecedent**" and the second term is called "**consequent**".

For example, in the ratio $x : y$, x is antecedent and y is consequent.

Remember

- **Comparison is made between the quantities carrying the same units.**
- **Comparison cannot be made between the quantities which are not similar.**
- **Ratio does not have any unit.**

If *x* and *y* are two quantities in a particular ratio, one should not be confused between *x* : *y* and *y* : *x*.

The ratio *x* : *y* means $\frac{x}{y}$ and *y* : *x* means $\frac{y}{x}$.

Conversion of a Fractional Ratio into a Whole Number Ratio

Example: Convert $\frac{1}{5} : \frac{1}{3}$ into ratio in simple form
There are two methods of converting a fractional ratio into a whole number ratio. They are:

Method I: Dividing the first quantity by the second

Solution: We are given the ratio as $\frac{1}{5} : \frac{1}{3}$.
We simply divide the first quantity by the second.

$$\frac{\frac{1}{5}}{\frac{1}{3}} = \frac{1}{5} \times \frac{3}{1} = \frac{3}{5} = 3 : 5$$

Method II:

(i) Find the LCM of the denominators.
So, LCM of 5 and 3 will be 15

(ii) Multiply the terms of the given ratio with the LCM and simplify.

$$\frac{1}{5} \times 15 : \frac{1}{3} \times 15 = 3 : 5$$

Let us now look at an example to understand this concept better.

Example:

Identify the cases out of the following in which a comparison can be

made using ratios.

1. **The ratio between the price of a book and the price of a shirt**
2. **The ratio between the age of a person and the amount of money he has**
3. **The ratio of the length of a park to its breadth**

   Solution:

1. The price of a book and the price of a shirt are of the same type. Therefore, in this case, comparison can be made using ratios.

2. The age and money are of different types. Therefore, in this case, we cannot compare the quantities.

3. The quantities length and breadth are of the same type. Therefore, in this case, comparison can be made using ratios.
   4. Concept of Equivalent Ratios
   5. Let *a*:*b* and *c*:*d* be two ratios. If *a*:*b* = *c*:*d,* i.e., if $\frac{a}{b} = \frac{c}{d}$, then *a*:*b* and *c*:*d* are called **equivalent ratios.**
   6. For example, consider the case where the number of boys in a class is 30 and the number of girls in the class is 35. Now, the ratio of the number of boys to the number of girls in the class is $\frac{30}{35} = \frac{30 \div 5}{35 \div 5} = \frac{6}{7}$. Now, we say that the required ratio is 6:7. Are both the ratios same?
   Yes. The two ratios 30:35 and 6:7 are same and are known as equivalent ratios. Thus, we can use either of the two ratios. However, we always express a ratio in its lowest terms. That is why we expressed the ratio as 6:7, and not as 30:35.

A ratio is always expressed in its lowest terms.

A ratio written in its lowest terms is said to be in its simplest form.

   7. Generally, if $\frac{a}{b}$ is in its lowest terms, then the ratio *a*:*b* is said to be in its simplest form*.*
   Now, how did we convert the ratio 30:35 to its equivalent ratio 6:7?
   We did this by dividing both the numerator and the denominator of the ratio by the same number. In fact, we can also multiply the numerator and the denominator of a ratio by the same number to get its equivalent ratio. However, the same is not true for addition and subtraction operations.

If we multiply or divide the numerator and the denominator of a ratio by the same non-zero number, then we will get the equivalent ratios of that ratio.

8. Mathematically, if we have a ratio *a*:*b* and a non zero-number *k,* then
$a:b = \frac{a}{b} = \frac{ak}{bk} = ak:bk$ and
9. $a:b = \frac{a}{b} = \frac{a \div k}{b \div k} = (a \div k):(b \div k)$
10. Let us find some equivalent ratios of 20:25.
$20:25 = \frac{20}{25} = \frac{20 \times 2}{25 \times 2} = \frac{40}{50} = 40:50$
$20:25 = \frac{20}{25} = \frac{20 \times 3}{25 \times 3} = \frac{60}{75} = 60:75$
11. $20:25 = \frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5} = 4:5$
12. Thus, 40:50, 60:75, and 4:5 are equivalent ratios of 20:25. In this way, we can find infinite number of equivalent ratios of any ratio.
13. Suppose we divide a number in the ratio *a*:*b* and in the ratio *c*:*d*. Here, we will get two sets in each case. An important point to be noted here is that, if *a*:*b* and *c*:*d* are equivalent ratios, then both the sets obtained in the two cases will be the same. Let us try to understand this with the help of an example.
14. We know that 1:2 and 2:4 are equivalent ratios. Suppose we need to divide the number 24 in each of the two ratios.
First consider the ratio 1:2. In this case, we can divide the number 24 into two parts as:
15. $\frac{1}{1+2} \times 24 = \frac{1}{3} \times 24 = 8$ and $\frac{2}{1+2} \times 24 = \frac{2}{3} \times 24 = 16$
16. Thus, the number 24 is divided in the ratio 1:2 as 8:16.
Let us now divide the number 24 in the ratio 2:4. Here, we can divide the number 24 as:
$\frac{2}{2+4} \times 24 = \frac{2}{6} \times 24 = 8$ and $\frac{4}{2+4} \times 24 = \frac{4}{6} \times 24 = 16$
17. As we can see, we again divided the number 24 into the same sets, i.e., 8 and 16. This is because the two ratios that we took were equivalent ratios.
18. **Note:** We can say that two ratios are equivalent, if the product of the numerator of the first ratio and the denominator of the other ratio is equal to the product of the denominator of first ratio and the numerator of the other ratio.
For example, to check the equivalence of the ratios, 14:49 and 6:21, we have to check whether $\frac{14}{49}$ and $\frac{6}{21}$are equivalent or not.
19. Then, 14 × 21 = 294 = 6 × 49
20. Therefore, $\frac{14}{49}$ and $\frac{6}{21}$are equivalent fractions.
Hence, 14:49 and 6:21 are equivalent ratios.

21. **Finding missing values and numbers in given ratio equations**
    If two or more equivalent ratios are given, we can find the missing numbers in the ratios.
    In order to do this, we make either the numerators or the denominators of the two fractions equal, and then obtain the missing value. Let us go through the following video to understand this concept better.
22. While comparing the fractions, if the two fractions are equal, then the given ratios are an example of **equivalent ratios**.
    Let us solve some examples based on the above discussed concepts.
23. **Example 1:**
    Find four equivalent ratios of 88:102.
24. **Solution:**
    $$88:102=\frac{88}{102}=\frac{88\times 2}{102\times 2}=\frac{176}{204}=176:204$$
    $$88:102=\frac{88}{102}=\frac{88\times 3}{102\times 3}=\frac{264}{306}=264:306$$
    $$88:102=\frac{88}{102}=\frac{88\times 4}{102\times 4}=\frac{352}{408}=352:408$$
    $$88:102=\frac{88}{102}=\frac{88\div 2}{102\div 2}=\frac{44}{51}=44:51$$
25. Thus, four equivalent ratios of 88:102 are 176:204, 264:306, 352:408, and 44:51.
26. **Example 2:**
    Find the missing numerator and denominator in the ratio equation
    $\frac{52}{39}=\frac{?}{3}=\frac{260}{?}$.
27. **Solution:**
    In $\frac{52}{39}=\frac{?}{3}$, division of 39 by 13 gives 3.
28. $\therefore \frac{52}{39}=\frac{52\div 13}{39\div 13}=\frac{4}{3}=\frac{?}{3}$
29. Thus, the missing numerator is 4.
30. Similarly, multiplication of 52 by 5 gives 260.
31. $\therefore \frac{52}{39}=\frac{52\times 5}{39\times 5}=\frac{260}{195}=\frac{260}{?}$
32. Thus, the missing denominator is 195.
33. **Example 3:**
    Arrange the following ratios in ascending order of magnitude.
    1:3, 4:5, 3:12, 3:8
34. **Solution:**
    The given ratios can be written as fractions as follows:
35. $\frac{1}{3},\frac{4}{5},\frac{3}{12}$, and $\frac{3}{8}$
36. LCM of 3, 5, 12, and 8 = 120

$\frac{1}{3}=\frac{1\times 40}{3\times 40}=\frac{40}{120}$

$\frac{4}{5}=\frac{4\times 24}{5\times 24}=\frac{96}{120}$

$\frac{3}{12}=\frac{3\times 10}{12\times 10}=\frac{30}{120}$

$\frac{3}{8}=\frac{3\times 15}{8\times 15}=\frac{45}{120}$

37. $\frac{30}{120}<\frac{40}{120}<\frac{45}{120}<\frac{96}{120}$
38. Or, $\frac{3}{12}<\frac{1}{3}<\frac{3}{8}<\frac{4}{5}$
39. Thus, the given ratios in the ascending order are:
40. 3:12 < 1:3 < 3:8 < 4:5
41. **Example 4:**
    If $\frac{p}{q}=\frac{r}{s}$ then show that $\frac{7p-5r}{7q-5s}=\frac{9p+4r}{9q+4s}$.
42. **Solution:**
43. Let $\frac{p}{q}=\frac{r}{s}=k$
44. ∴ $p = qk$ and $r = sk$
45. We have to show that
46. $\frac{7p-5r}{7q-5s}=\frac{9p+4r}{9q+4s}$
47. L.H.S. = $\frac{7p-5r}{7q-5s}=\frac{7qk-5sk}{7q-5s}=\frac{k(7q-5s)}{7q-5s}=k$
48. R.H.S. = $\frac{9p+4r}{9q+4s}=\frac{9qk+4sk}{9q+4s}=\frac{k(9q+4s)}{9q+4s}=k$
49. So, L.H.S. = R.H.S.
50. ∴ $\frac{7p-5r}{7q-5s}=\frac{9p+4r}{9q+4s}$

Application of Ratios in Solving Problems

Ratios are used to compare quantities. They are widely applied in many day-to-day situations.

Consider the case where Seema and Sheetal wrote a test and scored 40 marks and 30 marks respectively.

Now, ratio of Seema's and Sheetal's scores $=\frac{\text{Seema's score}}{\text{Sheetal's score}}=\frac{40}{30}=\frac{4}{3}$

The ratio of two quantities is denoted by '**:**'.

Thus, the ratio of Seema's and Sheetal's scores $\left(\frac{4}{3}\right)$ can be written as 4:3.

Now, what information does this ratio give us?

Since the ratio of their scores is 4:3, it tells us that for every 4 marks that Seema scored, Sheetal scored 3 marks. Thus, even if we do not know their actual scores, but only the ratio of the scores, we can still tell who got more marks.

Let us take another case. Let us suppose Meenu has 60 marbles, out of which, 25 are red in colour, and the rest are black in colour. Now, we have to find the ratios of the numbers of red and black marbles out of the total number of marbles.

We know that the total number of marbles is 60.

Number of red marbles = 25

Thus, ratio of the number of red marbles to the total number of marbles

$$= \frac{\text{Number of red marbles}}{\text{Total number of marbles}} = \frac{25}{60} = \frac{5}{12} = 5:12$$

Now, we can find the number of black marbles by subtracting the number of red marbles from the total number of marbles.

Thus, number of black marbles = 60 – 25 = 35

Thus, ratio of the number of black marbles to the total number of marbles

$$= \frac{\text{Number of black marbles}}{\text{Total number of marbles}} = \frac{35}{60} = \frac{7}{12} = 7:12$$

We can, in fact, find the ratio of the number of black marbles to the total number of marbles in a different way. We know that the marbles are either red or black in colour.

Think of ratios as parts of a whole, just like fractions. Thus, the sum of the ratios of the numbers of red and black marbles to the total number of marbles would be 1.

Thus, ratio of the number of black marbles to the total number of marbles

= 1 – ratio of the number of red marbles to the total number of marbles

$$= 1 - \frac{5}{12} = \frac{12-5}{12} = \frac{7}{12} = 7:12$$

If the ratio between two quantities is *a* : *b*, then we cannot write the ratio as *b* : *a*.

∴ *a* : *b* ≠ *b* : *a*

Let us now solve some more problems to understand this concept better.

Example 1:

There are 35 boys and 30 girls in a class. Find the ratio of

1. **the number of boys to the total number of students in the class.**
2. **the total number of students to the number of girls in the class.**
3. **the number of boys to the number of girls in the class.**

Solution:

Number of boys in the class = 35

Number of girls in the class = 30

Thus, total number of students in the class = 35 + 30 = 65

(i) Ratio of number of boys to total number of students $= \frac{35}{65} = \frac{35 \div 5}{65 \div 5} = \frac{7}{13} = 7:13$

(ii) Ratio of total number of students to number of girls $= \frac{65}{30}$

$$= \frac{65 \div 5}{30 \div 5} = \frac{13}{6} = 13:6$$

(iii) Ratio of number of boys to number of girls $= \frac{35}{30}$

$$= \frac{35 \div 5}{30 \div 5} = \frac{7}{6} = 7:6$$

Example 2:

The length of a table is 1.5 m and its breadth is 75 cm. Find the ratio of the length of the table to its breadth.

Solution:

Length of the table = 1.5 m = (1.5 × 100) cm = 150 cm

Breadth of the table = 75 cm

Thus, ratio of the length of the table to its breadth $= \frac{\text{Length of table}}{\text{Breadth of table}}$

$= \frac{150 \text{ cm}}{75 \text{ cm}} = \frac{2}{1} = 2:1$

Example 3:

Naina has 15 chocolates. She wants to divide these chocolates between Prabha and Priyanka in the ratio 3:2. How many chocolates will each get?

Solution:

There are two parts. One is 3 and another is 2.

Therefore, there are a total of 3 + 2 = 5 parts.

This means that Naina has to divide 15 chocolates into 5 parts. Out of these 5 parts, Prabha will get 3 parts and Priyanka will get 2 parts.

Therefore, number of chocolates that Prabha got $= \frac{3}{5} \times 15 = 9$

Similarly, number of chocolates that Priyanka got $= \frac{2}{5} \times 15 = 6$

Example 4:

The ratio of Nayan's height to Tarun's height is 12:13. Who is taller?

Solution:

The ratio of their heights is 12:13. Now, 12 < 13.

Since the "13" part of the ratio corresponds to Tarun's height, Tarun is taller than Nayan.

Example 5:

Sanju and Rupam started a business. Sanju invested Rs.15000 and Rupam invested Rs.30000 in the business. If the profit of the business was Rs 60000, then divide the profit between them in the ratio of their investment.

Solution:

Amount invested by Sanju = Rs 15000

Amount invested by Rupam = Rs 30000

Thus, ratio of Sanju's investment to Rupam's investment $= \frac{\text{Rs } 15000}{\text{Rs } 30000}$

$= \frac{1}{2} = 1:2$

Therefore, there are a total of 1 + 2 = 3 parts, out of which, Sanju will get $\frac{1}{3}$ part and Rupam will get $\frac{2}{3}$ parts of the total profit.

Thus, Sanju's share of the profit $= \text{Rs } \left(\frac{1}{3} \times 60000\right) = \text{Rs } 20000$

Similarly, Rupam's share of the profit $= \text{Rs } \left(\frac{2}{3} \times 60000\right) = \text{Rs } 40000$

Example 6:

The ratio of milk and water in a 21 L solution is 5:2. Now, 3 L milk and 3 L water is added to the solution. What is the new ratio of milk and water in the solution?

Solution:

The two parts of the ratio are 5 and 2.

Therefore, sum of the parts = 5 + 2 = 7

This means that if there is a 7 L solution, then the amount of milk = 5 L and the amount of water = 2 L.

Out of 1 L solution, the amount of milk is $\frac{5}{7}$ L and the amount of water is $\frac{2}{7}$ L.

Out of 21 L solution, the amount of milk is $\left(\frac{5}{7}\times 21\right)\text{L} = 15\text{ L}$ and the amount of water is $\left(\frac{2}{7}\times 21\right)\text{L} = 6\text{ L}$.

When 3 L of both milk and water is added to the solution, then the amount of milk = (15 + 3) L = 18 L and the amount of water = (6 + 3) L = 9 L.

Therefore, ratio of milk and water in new solution = $\frac{18\text{ L}}{9\text{ L}} = \frac{2}{1} = 2:1$

Example 7:

The population of two countries A and B are 800 lakhs and 1350 lakhs respectively. The respective areas of these countries are 4 lakh sq. km and 5 lakh sq. km. Which of these countries is less populated?

Solution:

The country will be less populated if its population per sq. km (population density) is less.

$$\text{Population density of country A} = \frac{\text{Population of country A}}{\text{Area of country A}}$$
$$= \frac{800\text{ lakh}}{4\text{ lakh sq. km}} = 200\text{ lakh/sq. km}$$

$$\text{Similarly, population density of country B} = \frac{\text{Population of country B}}{\text{Area of country B}}$$
$$= \frac{1350\text{ lakh}}{5\text{ lakh sq. km}} = 270\text{ lakh/sq. km}$$

From the above calculation, we can see that the population density of state A is less than that of state B. Therefore, state A is less populated.

Concept of Proportion

Ravi has 15 pens and Sumit has 10 pens. What is the ratio of the number of pens with Ravi to the number of pens with Sumit?

Ratio = $15:10=\frac{3}{2}$ = 3:2

This ratio is equivalent to a ratio 6:4 i.e., the ratio 3:2 is same as the ratio 6:4.

The numbers 3, 2, 6, and 4 are said to be in **proportion**.

> **"If two ratios are equal, then the numbers or values in the ratios are said to be in proportion".**

In general, if *a*, *b*, *c*, and *d* are any four numbers and $\frac{a}{b}=\frac{c}{d}$, then *a*, *b*, *c*, and *d* are said to be in proportion.

Proportion is denoted by the symbol '**=**' or '**::**' and is placed between two ratios.

For example, if 2, 4, 5, and 10 are in proportion, then we can denote this by writing

2:4 **=** 5:10

Or, 2:4 **::** 5:10

Now, consider one more example. Suppose there are 6 males and 2 females in a car. And there are 27 males and 9 females in a bus. Now, ratio of the number of males to the number of females in the car $=\frac{6}{2}=\frac{3}{1}$

Ratio of the number of males to the number of females in the bus

$$=\frac{27}{9}=\frac{3}{1}$$

Therefore, we can write, $\frac{6}{2}=\frac{27}{9}$

Thus, 6, 2, 27, and 9 are in proportion.

Here, the numbers 6, 2, 27, and 9 are called **respective terms.**

The four numbers or values involved in a statement of proportion when taken in order are called respective terms. If the numbers are not taken in order, then the numbers are not called respective terms.

The first and the fourth terms of the respective terms are called extreme terms or extremes. The second and the third terms are called middle terms or means.

For example, in the above example of proportion $\frac{6}{2}=\frac{27}{9}$, the numbers 6, 2, 27, and 9 are respective terms; however, 6, 27, 2, and 9 or 6, 2, 9, and 27 are not respective terms.

The numbers 6 and 9 are called extreme terms, while 2 and 27 are called middle terms.

Remember

- **If *p*, *q*, *r*, and *s* are four numbers and $p:q \neq r:s$, then *p*, *q*, *r*, and *s* are not in proportion.**

  For example, 2:4 ≠ 3:8, therefore, 2, 4, 3, and 8 are not in proportion.

- **If $x:y :: a:b$, then it is read as *x* is to *y* as *a* is to *b*.**

  Let us look at some more examples to understand this concept better.

Example 1:

Determine which of the following numbers are in proportion when taken in the given order.

1. **16, 32, 25, 50**
2. **– 22, 55, –35, 48**
3. **72, 24, 12, 36**
4. **– 84, – 60, – 56, – 40**

Solution:

1. 16, 32, 25, 50

$$16:32=\frac{16}{32}=\frac{1}{2}=1:2$$
$$25:50=\frac{25}{50}=\frac{1}{2}=1:2$$

Therefore, $16:32=25:50$

Hence, 16, 32, 25, and 50 are in proportion.

2. – 22, 55, –35, 48

$$-22:55=\frac{-22}{55}=-\frac{2}{5}=-2:5$$
$$-35:48=\frac{-35}{48}=-35:48$$

Therefore, $22:55\neq 35:48$

Hence, 22, 55, 35, and 48 are not in proportion.

3. 72, 24, 12, 36

$$72:24=\frac{72}{24}=\frac{3}{1}=3:1$$
$$12:36=\frac{12}{36}=\frac{1}{3}=1:3$$

Therefore, $72:24\neq 12:36$

Hence, 72, 24, 12, and 36 are not in proportion.

4. – 84, – 60, – 56, – 40

$$-84:-60=\frac{-84}{-60}=\frac{7}{5}=7:5$$
$$-56:-40=\frac{-56}{-40}=\frac{7}{5}=7:5$$

Therefore, $-84:-60=-56:-40$

Hence, – 84, – 60, – 56, and – 40 are in proportion.

Example 2:

Find the extreme terms and the middle terms of the following proportions.

1. $a:b = c:d$
2. 7:9 :: 28:36
3. $\frac{18}{30} = \frac{36}{60}$

Solution:

1. $a:b = c:d$

   Extreme terms = a, d

   Middle terms = b, c

2. 7:9 :: 28:36

   Extreme terms = 7, 36

   Middle terms = 9, 28

3. $\frac{18}{30} = \frac{36}{60}$

   Extreme terms = 18, 60

   Middle terms = 30, 36

Example 3:

The numbers 42, 54, *x*, and 108 are in proportion. Find the value of *x*.

Solution:

42, 54, x, and 108 are in proportion.

Therefore, $\frac{42}{54} = \frac{x}{108}$

Multiplication of 54 by 2 gives 108.

To find the value of x, we have to multiply 42 by 2.

$\therefore x = 42 \times 2 = 84$

Example 4:

Are the ratios 6 kg to 4800 g and Rs 75 to Rs 60 in proportion?

Solution:

6 kg to 4800 g $= \frac{6\,\text{kg}}{4800\,\text{g}} = \frac{6000\,\text{g}}{4800\,\text{g}} = \frac{6000}{4800} = \frac{5}{4} = 5:4$

Rs 75 to Rs 60 $= \frac{75}{60} = \frac{5}{4} = 5:4$

Therefore, the ratios 6 kg to 4800 g and Rs 75 to Rs 60 are in proportion.

Example 5:

Rita was asked that if a bus travelled 225 km in 5 hours, then what distance will it travel in 15 hours. She replied that the bus will travel 675 km in 15 hours. Was she correct?

Solution:

Rita will be correct, if the ratios 5 hours to 15 hours and 225 km to 675 km are in proportion.

5 hours to 15 hours $= \frac{5}{15} = \frac{1}{3} = 1:3$

225 km to 675 km $= \frac{225}{675} = \frac{1}{3} = 1:3$

Therefore, 5 hours to 15 hours and 225 km to 675 km are in proportion.

Thus, Rita's answer was **correct**.

Example 6:

If the cost of 6 umbrellas is Rs 144, then what is the cost of 8 umbrellas?

Solution:

Let the cost of 8 umbrellas be Rs *x.*

Now, the ratios of 8 umbrellas to 6 umbrellas and Rs *x* to Rs 144 must be in proportion,

i.e., 8 umbrellas to 6 umbrellas = Rs *x* to Rs 144

$$\Rightarrow \frac{8}{6} = \frac{x}{144}$$

Multiplication of 6 with 24 gives 144, therefore, we have to multiply 8 with 24 to find the value of *x.* On multiplying 8 with 24, we obtain

8 × 24 = 192

∴ *x* = 192

Thus, the cost of 8 umbrellas is Rs 192.

Example 7:

On a sunny day, Preetam observed that a 1 m long stick standing vertically on the ground makes a shadow of length 30 cm. What will be the height of a tree if the length of its shadow is 3 m 60 cm at that time?

Solution:

Let *x* be the height of the tree.

We know 1 m = 100 cm

∴ 3 m 60 cm = (3 × 100 + 60) cm = 360 cm

Ratio of height of stick and length of its shadow = 100 cm : 30 cm

Similarly, ratio of height of tree and length of its shadow = *x* : 360 cm

At the same time, the ratio of the height of the stick and the length of its shadow and the ratio of the height of the tree and the length of its shadow are equal. Thus, the two ratios are in proportion.

∴ 100 cm: 30 cm = *x* : 360 cm

$$\frac{100 \text{ cm}}{30 \text{ cm}} = \frac{x}{360 \text{ cm}}$$

$$x = \left(\frac{100 \times 360}{30}\right) \text{ cm}$$

$$x = 1200 \text{ cm}$$

$$x = 12 \text{ m}$$

Therefore, the height of the tree is 12 m.

Conversion of Fractions, Ratios, Decimals and Whole Numbers to Equivalent Percentages and Vice-versa

The following table shows the numbers of matches played and won in a season by two cricket clubs – Balmain Star and New Star.

	BALMAIN STAR	**NEW STAR**
Number of matches played	75	90
Number of matches won	48	54

Now, how can we tell which one is the better team? Though New Star had won more matches than Balmain Star, but New Star had also played more matches than Balmain Star. Therefore, we cannot compare the better performing team just by looking at the number of matches won by the two teams. We could have easily compared this data if the two teams had played the same number of matches. To solve such type of problems, we have to know the concept of **percentages.**

First of all, let us know what a percentage is and after that we will solve this problem.

Per cent is derived from Latin word 'per centum' meaning 'per hundred' and it is defined as

Percentages are numerators of fractions with denominator 100.

Now that we know how to convert whole numbers, fractions, decimals into percentages and vice-versa, let us use these concepts to find the better team between Balmain Star and New Star.

Before finding the percentage of match won by each team, we have to find the fraction of matches won by them out of the total number of matches played by the team.

Now, the fractions of matches won by Balmain Star and New Star are $\frac{48}{75}$ and $\frac{54}{90}$ respectively.

Therefore, percentage of matches won by Balmain Star $= \left(\frac{48}{75} \times 100\right)\% = 64\%$

Similarly, percentage of matches won by New Star $= \left(\frac{54}{90} \times 100\right)\% = 60\%$

Here, we can see that though New Star had won more matches than Balmain Star, the percentage of matches won by Balmain Star is more than that of New Star. Therefore, Balmain Star is a better team than New Star.

We know that a fraction represents a part of a whole and 1 represents the whole. Since 1 = 100%, 100% represents the whole of an object and the percentages below 100% represent parts of a whole.

Percentages can be added and subtracted, but the total percentage of a whole will be 100%.

For example, Sonu has a pizza and he ate 45% of it.

Now, percentage of remaining pizza = 100% − 45% = (100 − 45)% = 55%

Let us discuss some more examples based on the conversion of fractions, ratios, decimals, and whole numbers to equivalent percentage, and vice-versa.

Example 1:

Convert the percentages 325%, 9.5%, and 800% into fraction, whole number, and decimal.

Solution:

$325\% = \frac{325}{100} = \frac{13}{4}$

$\frac{13}{4}$ is a fraction. It cannot be a whole number.

Also, $325\% = \frac{325}{100} = 3.25$

$\therefore$ The fraction and decimal forms of 325% are $\frac{13}{4}$ and 3.25 respectively.

Now, $9.5\% = \frac{9.5}{100} = \frac{95}{1000} = \frac{19}{200}$

$\frac{19}{200}$ is a fraction. It cannot be a whole number.

Also, $9.5\% = \frac{9.5}{100} = 0.095$

$\therefore$ The fraction and decimal form of 9.5% are $\frac{19}{200}$ and 0.095 respectively.

Now, $800\% = \frac{800}{100} = 8$

Also, $800\% = \frac{800}{100} = 8.00$

$\therefore$ The whole number and the decimal form of 800% are 8 and 8.00 respectively.

Example 2:

Convert the numbers $\mathbf{0.0075, \frac{9}{25}, 2, \frac{31}{16}}$ into percentage form.

Solution:

$0.0075 = (0.0075 \times 100)\% = 0.75\%$

$\frac{9}{25} = \left(\frac{9}{25} \times 100\right)\% = 36\%$

$2 = (2 \times 100)\% = 200\%$

$\frac{31}{16} = \left(\frac{31}{16} \times 100\right)\% = 193.75\%$

Example 3:

In a hostel, 30% students like to watch BCB, 25% students like to watch NDVT, 27% students like to watch VT18 channel, and the remaining students like to watch NCN. If a student likes to watch only one channel, then what percentage of students like to watch NCN?

Solution:

Percentage of students who like to watch BCB = 30

Percentage of students who like to watch NDVT = 25

Percentage of students who like to watch VT18 = 27

Now, the percentage of students who like to watch these three channels = 30 + 25 + 27 = 82

We know that all the parts that form the whole when added together give the whole or 100%.

Therefore, percentage of students who like to watch NCN = 100 − 82 = 18

Thus, 18% students like to watch NCN.

Example 4:

A certain amount of money is divided among Pallavi, Manjri, and Payal in the ratio 3:9:8. What percentage of the amount distributed does each girl get?

Solution:

Ratio in which the amount is divided = 3:9:8

Sum of the parts of the ratio = 3 + 9 + 8 = 20

Therefore, Pallavi, Manjri, and Payal respectively got $\frac{3}{20}, \frac{9}{20}$ and $\frac{8}{20}$ of the total amount that was distributed among them.

Thus, Pallavi got $\left(\frac{3}{20}\times 100\right)\% = 15\%$ of the total amount that was distributed.

Similarly, Manjri got $\left(\frac{9}{20}\times 100\right)\% = 45\%$ of the total amount that was distributed.

Payal got $\left(\frac{8}{20}\times 100\right)\% = 40\%$ of the total amount that was distributed.

Example 5:

A child won 12 games out of a number of games. If the win percentage was 30, how many games were there in total?

Solution:

It is given that win percentage is 30. So,
30 games were won out of 100 games.

∴ 12 games were won out of $\frac{100}{30} \times 12 = 40$ games.
Hence, there were 40 games in total.

Example 6:

The salary of Rohan is increased by 20%. If his new salary is Rs 18,000, then what was his salary before increment?

Solution:

Let the salary before increment be Rs 100. Since his increment is 20%, so his salary after increment is Rs 100 + Rs 20 = Rs 120.
If the new salary is Rs 120, then the salary before increment is Rs 100.
So, if the new salary is Rs 18,000, then the salary before increment is
Rs $\frac{100}{120} \times 18000 = \text{Rs}\,15{,}000$
Thus, the salary of Rohan before the increment was Rs 15, 000.

Example 7:

Shyam's income is 40% more than that of Ram. What percent is Ram's income less than that of Shyam?

Solution:

Let Ram's income be Rs 100. Then Shyam's income will be Rs 140.
So, if Shyam's income is Rs 140, then Ram's income will be Rs 100.

If Shyam's income is Rs 1, then Ram's income will be Rs $\frac{100}{140}$.

If Shyam's income is Rs 100, then Ram's income will be Rs $\frac{100}{140} \times 100 = \text{Rs}\ \frac{5}{7} \times 100 = \text{Rs}\ 71\frac{3}{7}$

Hence, Ram's income is $\left(100 - 71\frac{3}{7}\right)\% = 28\frac{4}{7}\%$ less than that of Shyam's income.

Word Problems on Percentages

The following tables list the marks obtained by Bhaskar in his first unit test and the final term examinations.

1st Unit Test Mark Sheet

Subject	Marks Obtained	Total Marks
English	16	20
Mathematics	19.5	20
Science	16	20
French	18.5	20
Social Studies	16	20

Final Term Mark Sheet

Subject	Marks Obtained	Total Marks
English	75	100
Mathematics	95	100
Science	83	100
French	81	100
Social Studies	66	100

Now, how can we determine whether Bhaskar's overall performance was better in the 1st unit test or in the final term examination?

Total marks in the 1st unit test = $20 \times 5 = 100$

Total marks obtained by Bhaskar in the 1st unit test = $16 + 19.5 + 16 + 18.5 + 16$

$= 86$

Total marks in the final term examination = $100 \times 5 = 500$

Total marks obtained by Bhaskar in the final term examination = $75 + 95 + 83 + 81 + 66 = 400$

Here, the total marks in the unit test and the final term examination are not equal. Therefore, we cannot compare Bhaskar's performance just by looking at the marks obtained by him in the test and in the examination. We can solve this problem by calculating his percentage score in the unit test and in the final term examination.

'Percent' means 'per hundred' or 'out of hundred'. To express x as a percentage of y, we use the following formula:

To express x as a percentage of y, percentage = $\left(\frac{x}{y}\times 100\right)\%$

Thus, Bhaskar's percentage score in the 1st unit test $= \frac{\text{Marks obtained}}{\text{Total marks}}\times 100 = \frac{86}{100}\times 100 = 86$

Bhaskar's percentage score in the final term examination $= \frac{400}{500}\times 100 = 80$

Thus, Bhaskar scored 80% in the final term examination and 86% in the 1st unit test. Looking at this information, we can safely say that his performance was better in the 1st unit test as compared to that in the final term examination.

Let us consider another situation where Ramesh, Deepika, and Devyani are cousins. One day, their uncle gave them Rs 800 and told Deepika and Devyani to take 35% and 40% of this amount respectively and Ramesh to take the remaining amount. However, the three cousins were confused about how they would share this money. Let us try to help them out.

For this, first of all we have to know the method to find certain percentage of a quantity. For this, we use the following formula:

x% of a given quantity = $\left(\frac{x}{100} \times \text{ given quantity}\right)$

Deepika's share = 35%

Devyani's share = 40%

∴ Ramesh's share = [100 − (35 + 40)]% = (100 − 75)% = 25%

Thus, Deepika's share out of Rs 800 $= \text{Rs}\left(\frac{35}{100}\times 800\right) = \text{Rs } 280$

Similarly, Devyani's share out of Rs 800 $= \text{Rs}\left(\frac{40}{100}\times 800\right) = \text{Rs } 320$

Similarly, Ramesh’s share out of Rs 800 $= \text{Rs}\left(\frac{25}{100}\times 800\right) = \text{Rs } 200$

Therefore, the shares of Ramesh, Deepika, and Devyani are Rs 200, Rs 280, and Rs 320 respectively.

Now, let us consider another situation where Rohit scored 68% mark in an exam. If he scored 340 marks, then can we find the total marks for which the exam was held?

For this, we have to know the formula to calculate the total quantity, when its certain quantities are given in terms of percentage.

If *x* % of a given quantity is *y*, then quantity = $\frac{y}{x} \times 100$

Hence, the total marks for which the exam was held = $\frac{340}{68}\times 100 = 500$ marks

In this way, we can find the whole quantity or a part of the given quantity when the percentage is given. This concept is known as the “How Many” concept.

Let us discuss some more examples based on the discussed concepts.

Example 1:

Find the following values.

(i) 16% of 100

(ii) 25% of 325

(iii) 65% of 1.2 kg

Solution:

(i) 16% of 100 $= \frac{16}{100}\times 100 = 16$

(ii) 25% of 325 $= \frac{25}{100} \times 325 = 81.25$

(iii) 65% of 1.2 kg = 65% of 1200g = $\left(\frac{65}{100} \times 1200\right)$ g = 780 g = 0.78 kg

Example 2:

There were 25 chocolates in a bag, out of which Rohit ate 7 chocolates. What percentage of chocolates was eaten by Rohit?

Solution:

Number of chocolates eaten by Rohit = 7

Total number of chocolates in the bag = 25

∴ Fraction of chocolates eaten by Rohit = $\frac{7}{25}$

Therefore, percentage of chocolates eaten by Rohit = $\frac{7}{25} \times 100 = 28$

Thus, Rohit ate 28% chocolates in the bag.

Example 3:

A shopkeeper has a stock of 200 cricket bats of three brands – A, B, and C. Out of these, 65 bats are of brand A, 90 bats are of brand B, and the remaining bats are of brand C. Find the percentage of bats of each brand with the shopkeeper.

Solution:

Total number of cricket bats = 200

Number of bats of brand A = 65

Number of bats of brand B = 90

∴ Number of bats of brand C = 200 – (65 + 90) = 200 – 155 = 45

Thus, fraction of bats of brand A $= \frac{65}{200}$

Similarly, fraction of bats of brand B $= \frac{90}{200}$

Similarly, fraction of bats of brand C $= \frac{45}{200}$

∴ Percentage of bats of brand A $= \left(\frac{65}{200} \times 100\right)\% = 32.5\%$

∴ Percentage of bats of brand B $= \left(\frac{90}{200} \times 100\right)\% = 45\%$

∴ Percentage of bats of brand C $= \left(\frac{45}{200} \times 100\right)\% = 22.5\%$

Example 4:

There are 500 students in a school. Out of these, 325 students are day scholars while the remaining students are hostlers. Find the percentage of the students who are hostlers?

Solution:

Number of day scholars in the school = 325

Total number of students in the school = 500

Thus, number of hostlers in the school = 500 − 325 = 175

Thus, percentage of students who are hostlers $= \left(\frac{175}{500} \times 100\right)\% = 35\%$

This question can also be solved by another method.

Number of day scholars in the school = 325

Total number of students in the school = 500

Percentage of students who are day scholars $= \left(\frac{325}{500} \times 100\right)\% = 65\%$

Thus, percentage of students who are hostlers = 100% − 65% = 35%

Example 5:

Vicky spends Rs 4000 and saves Rs 1000 every month. What percentage of his monthly income constitutes his monthly savings?

Solution:

Vicky's monthly expenditure = Rs 4000

Vicky's monthly savings = Rs 1000

∴ Vicky's monthly income = Rs (4000 + 1000) = Rs 5000

∴ Percentage of money saved by Vicky every month $= \left(\frac{\text{Rs } 1000}{\text{Rs } 5000} \times 100\right)\% = 20\%$

Thus, 20% of Vicky's monthly salary constitutes his monthly savings.

Example 6:

In badminton, Sonia played 25 matches and lost 5 matches whereas Sunita played 32 matches and lost 6 matches. Who is a better player among them?

Solution:

Number of matches played by Sonia = 25

Number of matches lost by Sonia = 5

Therefore, number of matches won by Sonia = 25 − 5 = 20

∴ Percentage of matches won by Sonia $= \left(\frac{20}{25} \times 100\right)\% = 80\%$

Number of matches played by Sunita = 32

Number of matches lost by Sunita = 6

Therefore, number of matches won by Sunita = 32 − 6 = 26

∴ Percentage of matches won by Sunita $= \left(\frac{26}{32} \times 100\right)\% = 81.25\%$

Since the winning percentage of Sunita is more than that of Sonia, we can say that Sunita is the better player among the two.

Example 7:

What percentage of 1 minute is 21 seconds?

Solution:

1 minute = 60 seconds

Let *x*% of 60 seconds be 21 seconds.

∴ *x*% of 60 = 21

We know that, *x*% $= \frac{x}{100}$

$\Rightarrow \frac{x}{100}$ of $60 = 21$

$\Rightarrow \frac{x}{100} \times 60 = 21$

$\Rightarrow x \times \frac{3}{5} = 21$

Multiplying both sides by 5, we obtain

$x \times \frac{3}{5} \times 5 = 21 \times 5$

$\Rightarrow x \times 3 = 105$

Dividing both sides by 3, we obtain

$\frac{x \times 3}{3} = \frac{105}{3}$

$\Rightarrow x = 35$

Thus, 35% of 1 minute is 21 seconds.

Example 8:

In a class, 60% of all the students are boys. It was observed that 90 girls were present in the class on a specific day. If these girls were 75% of the entire girls, then how many boy students are there in the class?

Solution:

Let the total number of students in the class be *x*.

It is given that 60% of all the students are boys.

Therefore, number of boy students in the class = $\frac{60}{100}\times x=\frac{3x}{5}$

And, number of girl students in the class = $x-\frac{3x}{5}=\frac{2x}{5}$

It is also given that 75% of all the girls were present on the day.

Therefore, number of girls present in the class = $\frac{75}{100}\times\frac{2x}{5}=\frac{3x}{10}$

It is also given that 90 girls were present in the class.

$$\therefore \frac{3x}{10}=90$$
$$\Rightarrow 3x=900$$
$$\Rightarrow x=\frac{900}{3}=300$$

Therefore, number of boy students in the class $=\frac{3x}{5}=\frac{3\times 300}{5}=180$

Example 9:

Sujit saved 33% of his salary last month. If he saved Rs 4950 that month, then what is his monthly salary?

Solution:

Let Rs *n* be his monthly salary.

Now, 33% of *n* = 4950

$\Rightarrow 33\% \times n = 4950$

$$\Rightarrow \frac{33}{100} n = 4950$$

Multiplying both sides by 100, we obtain

$33n = 495000$

Now, dividing both sides by 33, we obtain

$$\frac{33n}{33} = \frac{495000}{33}$$

$\Rightarrow n = 15000$

Therefore, his monthly salary is Rs 15000.

Example 10:

In a carton of 250 bulbs, 4.8% bulbs are found to be defective. How many bulbs in that carton are in working condition?

Solution:

Total number of bulbs in the carton = 250

Percentage of defective bulbs = 4.8%

$$\therefore \text{Number of defective bulbs in the carton} = 4.8\% \text{ of } 250 = \frac{4.8}{100} \times 250 = \frac{1200}{100} = 12$$

Therefore, number of bulbs that are in working condition = 250 − 12 = 238

Percentage Increase and Decrease

In many cases, it is better if we represent increase or decrease in quantities in percentage terms rather than describing them numerically.

Let us take the case where Manoj got a raise of Rs 5000 in his monthly salary. Now, we also have to consider his original salary in order to ascertain whether the raise was high or low. For example, if his original salary was Rs 10000, then this means that he got a raise of 50% in his monthly salary, which is quite high. However, let us suppose that his original monthly salary was Rs 50000. Now this would mean that Manoj got a raise of only 10%, which is quite low.

Thus, if we talk only about the increase or decrease in a quantity without referring to its original value, then it will cause confusion in some cases. However, this problem does not arise if we use percentages.

In the same way, we can find the percentage decrease, using the following formula.

$$\text{Percentage decrease} = \frac{\text{Decrease in quantity}}{\text{Original quantity}} \times 100$$

Let us consider an example to understand percentage decrease.

In an athletic event, 60 students of a school participated last year. This year, the number of students of that school taking part is decreased by 5%. Find the number of students taking part in the athletic event this year.

Here, we have percentage decrease = 5%

Original number of students = 60

$$\therefore \text{Percentage decrease} = \frac{\text{Decrease in number of students}}{\text{Original number of students}} \times 100$$
$$\Rightarrow 5 = \frac{\text{Decrease in number of students}}{60} \times 100$$
$$\Rightarrow \text{Decrease in number of students} = \frac{5 \times 60}{100}$$

⇒ Decrease in number of students = 3

Thus, number of students taking part this year = 60 – 3 = 57

Thus, 57 students participated this year.

Let us solve some examples to understand the concept better.

Example 1:

The price of a toy was decreased by 20%. If this meant a decrease of Rs 125 in its price, then what were the original and the reduced prices of that toy?

Solution:

Let the original price of the toy to be *x*.

Percentage decrease in the price of the toy = 20

Decrease in the price of the toy = Rs 125

In this case, $\text{percentage decrease} = \dfrac{\text{Decrease in price of the toy}}{\text{Original price of the toy}} \times 100$

$$\therefore 20 = \frac{\text{Rs } 125}{x} \times 100$$
$$x = \text{Rs}\left(\frac{125}{20} \times 100\right) = \text{Rs } 625$$

Thus, original price of the toy = Rs 625

And, reduced price of the toy = Rs 625 − Rs 125 = Rs 500

Example 2:

The population of a city in 2002 was 2.5 crores. If it increased by 15% in a year, then what was the population in 2003?

Solution:

Original population of the city = 2.5 crores

Percentage increase in population = 15 %

Let *x be* the increase in population of the city.

In this case, $\text{percentage increase in population} = \dfrac{\text{Increase in population}}{\text{Original population}} \times 100$

$$\therefore 15 = \frac{x}{2.5 \text{ crores}} \times 100$$
$$\Rightarrow x = \frac{15 \times 2.5}{100} \text{ crores} = 0.375 \text{ crores}$$

Thus, the population increased by 0.375 crores over the year.

Thus, population of the town in 2003 = (2.5 + 0.375) crores = 2.875 crores

Example 3:

The enrolment at a school increased from 1400 to 1500 in one year. What is the percentage increase in the enrolment?

Solution:

Increase in the enrolment = 1500 - 1400 = 100

Percentage increase in the enrolment $= \frac{100}{1400} \times 100$

$= \frac{100}{14}$
$= 7.14\%$

Thus, the enrolment is increased by 7.14%.

Example 4:

The selling price of a DVD player was dropped by 20% in one year. If the selling price of the DVD player is Rs 8000 now, then find the selling price of the DVD player one year before.

Solution:

Let x be the selling price of the DVD player before one year. The selling price of the DVD player was dropped by 20%. Therefore, now the selling price of the DVD is (100 - 20)% of x = 80% of x

∴ 80% of x = Rs 8000

$\frac{80}{100} \times x = \text{Rs } 8000$
$x = \text{Rs } \frac{8000 \times 100}{80}$
$x = \text{Rs } 10000$

Thus, the selling price of the DVD player before one year was Rs 10000.

Concept of Profit and Loss

What is every shopkeeper's motto? "To buy an item cheap and to sell it at a higher price". The extra money that he makes in the process is known as profit. However, a shopkeeper is not always lucky and sometimes has to sell an item at a lesser price than the one he bought it at. This is a case of loss for the shopkeeper. Let us try and understand the concept of profit and loss with the help of some examples.

Generally when an item is purchased, some additional expenses such a labour charges, transportation charges, maintenance charges etc are made before the selling of the item. These expenses are known as overhead charges. These expenses have to be added in the cost price of the item.

∴ Real cost price = Price for purchasing the goods + Overhead charges

Let us discuss some examples based on the above concept.

Example 1:

Find out the profit or loss in the following transactions.

(i) Rahul bought a bicycle for Rs 1200 and sold it for Rs 1150.

(ii) Tanmay bought a pair of trousers for Rs 700 and sold them for Rs 725.

Solution:

(i) Here, C.P. = Rs 1200

S.P. = Rs 1150

∴ Loss incurred = C.P. − S.P. = Rs 1200 − Rs 1150 = Rs 50

Hence, a loss of Rs 50 was incurred in this transaction.

(ii) Here, C.P. = Rs 700

S.P. = Rs 725

∴ Profit made = S.P. − C.P. = Rs 725 − Rs 700 = Rs 25

Hence, a profit of Rs 25 was made in this transaction.

Example 2:

Javed bought 10 pens for Rs 120 and sold them for Rs 80. Find out the loss incurred on five pens.

Solution:

Cost price (C.P.) of 1 pen = $\text{Rs}\left(\frac{120}{10}\right)$ = Rs 12

Selling price (S.P.) of 1 pen = $\text{Rs}\left(\frac{80}{10}\right) = \text{Rs } 8$

∴ Loss incurred on 1 pen = Rs 12 − Rs 8 = Rs 4

Hence, loss incurred on 5 pens = 5 × Rs 4 = Rs 20

Example 3:

Kanika sold three bottles for Rs 135 and incurred a loss of Rs 15. What is the cost price of one bottle?

Solution:

SP of 3 bottles = Rs 135

∴ SP of 1 bottle = $\text{Rs } \frac{135}{3} = \text{Rs } 45$

Loss incurred on 3 bottles = Rs 15

∴ Loss incurred on 1 bottle = $\text{Rs } \frac{15}{3} = \text{Rs } 5$

CP = Loss + SP

= Rs 5 + Rs 45

= Rs 50

Thus, the cost price of one bottle is Rs 50.

Concept of Profit Percent and Loss Percent

In buying and selling articles, sometimes there is loss and sometimes profit. We can also write profit and loss as a percentage. Profit percent or loss percent is always calculated on the cost price of the article.

In the same way, we can find loss percent.

Remember the following formulae.

Profit % $= \frac{\text{Profit}}{\text{C.P}} \times 100$

Loss % $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$\text{S.P.} = \frac{(100 + \text{Profit}\,\%)}{100} \times \text{C.P.}$$

$$\text{S.P.} = \frac{(100 - \text{Loss}\,\%)}{100} \times \text{C.P.}$$

$$\text{C.P.} = \frac{100}{(100 + \text{Profit}\,\%)} \times \text{S.P.}$$

$$\text{C.P.} = \frac{100}{(100 - \text{Loss}\,\%)} \times \text{S.P.}$$

Now, let us solve some examples.

Example 1:

Apurva sold his bike for Rs 30000 at a loss of 40%. At what price did Apurva buy the bike?

Solution:

Let the price at which Apurva bought the bike be *x*.

C.P. = *x*

S.P. = Rs 30000

Loss = C.P. − S.P. = Rs (*x* − 30000)

We know that, Loss % $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$40=\frac{(x-30000)}{x}\times 100$$

$40x = 100x - 3000000$

$100x - 40x = 3000000$

$60x = 3000000$

$$x=\frac{3000000}{60}$$

x = Rs 50000

Thus, Apurva bought the bike for Rs 50000.

Example 2:

Javed sold a refrigerator and a washing machine for Rs 15000 and Rs 10000 respectively. He made a gain of 25% on the refrigerator and a loss of 20% on the washing machine. Find his overall gain or loss.

Solution:

Let the cost price of refrigerator be x and that of washing machine be y.

S.P. of refrigerator = Rs 15000

Gain on refrigerator = S.P. − C.P. = Rs $(15000 - x)$

We know that, gain % $=\frac{\text{Gain}}{\text{C.P.}}\times 100$

$$25=\frac{(15000-x)}{x}\times 100$$

$25x = (1500000 - 100x)$

$125x = 1500000$

$$x=\frac{1500000}{125}$$

x = Rs 12000

C.P. of refrigerator = Rs 12000

Now, selling price of washing machine = Rs 10000

Loss on washing machine = Rs (y − 10000)

We know that, Loss% $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$20 = \frac{(y - 10000)}{y} \times 100$$

$20y = 100y - 1000000$

$100y - 20y = 1000000$

$80y = 1000000$

$$y = \frac{1000000}{80}$$

y = Rs 12500

C.P. of washing machine = Rs 12500

Now, total cost price of refrigerator and washing machine is, (Rs 12000 + Rs 12500)

= Rs 24500

Total S.P. = Rs 15000 + Rs 10000 = Rs 25000

Overall gain = Rs 25000 − Rs 24500 = Rs 500

Thus, there is an overall gain of Rs 500 on the selling of the refrigerator and the washing machine.

Example 3:

Rahul purchased a television for Rs 20000 and its transportation cost was Rs 100. For how much should the television be sold so that he makes a profit of 7%?

Solution:

C.P. of the television = Rs 20000

Overhead charges = Rs 100

∴ Total cost of TV = Rs 20000 + Rs 100 = Rs 20100

Profit % = 7%

We know that, Profit % $= \frac{\text{Profit}}{\text{C.P.}} \times 100$

$$7 = \frac{\text{Profit}}{20100} \times 100$$

7 × 201 = Profit

∴ Profit = Rs 1407

Now,

S.P. = C.P. + Profit = Rs 20100 + Rs 1407 = Rs 21507

Thus, the television should be sold for Rs 21507.

Example 4:

Arun bought an umbrella for Rs 125 and sold it for a profit of Rs 20. What was the selling price of the umbrella and the profit percent of the transaction?

Solution:

Cost price (C.P.) of the umbrella = Rs 125

Profit made in the transaction = Rs 20

∴ Selling price (S.P.) of the umbrella = C.P. + Profit = Rs 125 + Rs 20 = Rs 145

Hence, profit percent of the transaction $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{20}{125} \times 100 = 16$%

Example 5:

Julie bought a washing machine for Rs 20000. She then sold it at 10% profit. At what price did she sell the washing machine?

Solution:

Profit per cent = 10%

Cost price (C.P.) of the washing machine = Rs 20000

Profit per cent of the transaction $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$

$$\therefore 10 = \frac{\text{S.P.} - 20000}{20000} \times 100$$
$$\text{S.P.} - 20000 = \frac{10 \times 20000}{100}$$
$$\text{S.P.} - 20000 = 2000$$
$$\text{S.P.} = 20000 + 2000 = 22000$$

Hence, Julie sold the washing machine for Rs 22000.

Example 6:

If the cost price of 16 chocolates is equal to selling price of 12 chocolates, then find the profit percent.

Solution:

Let the C.P. of each chocolate be Re 1. Then the C.P. of 16 chocolates will be Rs 16.

By the given data, S.P. of 12 chocolates = C.P. of 16 chocolates = Rs 16

∴ S.P. of 1 chocolate = Rs $\frac{16}{12}$

Profit = S.P. – C.P. = $\text{Rs}\frac{16}{12} - \text{Re}1 = \text{Re}\frac{4}{12} = \text{Re}\frac{1}{3}$

Thus, there is a profit of Rs $\frac{1}{3}$ on each chocolate.

Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100\% = \frac{\text{Re}\frac{1}{3}}{\text{Re}1} \times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$

Thus, the profit percent is $33\frac{1}{3}\%$.

Example 7:

On selling a bicycle for Rs 5,600, a dealer loses 20%. For how much should he sell it to gain 15%?

Solution:
Selling price of the bicycle is Rs 5,600 and the loss is 20%. Therefore,

$$\begin{aligned}\text{C.P.} &= \frac{100}{(100-\text{loss\%})}\times \text{S.P.}\\ &= \frac{100}{(100-20)}\times \text{Rs } 5{,}600\\ &= \frac{100}{80}\times \text{Rs } 5{,}600\\ &= \text{Rs } 7{,}000\end{aligned}$$

Expected profit = 15%

$$\begin{aligned}\text{S.P.} &= \frac{(100+\text{profit\%})}{100}\times \text{C.P.}\\ &= \frac{(100+15)}{100}\times \text{Rs } 7{,}000\\ &= \frac{115}{100}\times \text{Rs } 7{,}000\\ &= \text{Rs } 8{,}050\end{aligned}$$

Thus, the selling price of the bicycle to gain 15% is Rs 8,050.

Example 8:

The cost price of a laptop is Rs 24,000. An additional Rs 1000 was spent on installing a software. If it is sold at 15% profit, then find the selling price of the laptop.
Solution:
Cost price of the laptop = Rs 24,000 + Rs 1,000 (overhead charges) = Rs 25,000
The laptop is sold at a profit of 15%. Therefore,

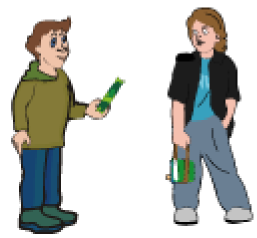
$$\begin{aligned}\text{S.P.} &= \frac{(100+\text{Profit\%})}{100}\times \text{C.P.}\\ &= \frac{(100+15)}{100}\times \text{Rs } 25{,}000\\ &= \text{Rs } \frac{115}{100}\times 25{,}000\\ &= \text{Rs } 28{,}750\end{aligned}$$

Thus, the selling price of the laptop is Rs 28, 750.

Concept of Simple Interest

Sometimes, we need a large amount of money for bigger purposes such as buying a house, buying a car, paying fee for higher education etc. If we do not readily have that much of money then we have to borrow it from money lenders, banks or co-operative credit societies for a particular period of time along with a condition of paying some extra money at a particular rate. This borrowed money is called **loan**.

For example, Jatin borrowed Rs 10000 from Shashank. He promised to give him back Rs 11000 after one year.



Here, Rs 10000 is loan taken by Jatin from Shashank.

Now, the question is that why did Jatin promise to pay more money than he borrowed?

Have you heard about interest? Let us suppose that someone borrows some money for a specific time period. Then, the borrower has to pay some extra money along with the original amount after the passage of that fixed time period. This extra money that is paid by the borrower is called **interest**.

In the given example, extra money paid = Rs (11000 − 10000) = Rs 1000

Thus, here, he paid Rs 1000 as the interest for one year.

The original amount of money borrowed is called **principal**. In this case, Rs 10000 is the principal.

The total amount that has to be paid back after the specific time period is called **amount**. In the above example, Rs 11000 is the amount.

Thus, we can conclude that

Amount = Principal + Interest

Interest = Amount − Principal

Principal = Amount − Interest

The interest is always calculated on the principal and is given in the form of percentage for a certain period of time. This percentage is known as the rate of interest.

Let us now calculate the rate of interest for the above example.

Principal = Money borrowed by Jatin for one year = Rs 10000

Amount = Money returned by Jatin after one year = Rs 11000

Interest = Extra money given by Jatin = Rs (11000 − 10000) = Rs 1000

Now, to express the rate of interest, we need to calculate the interest per Rs 100 as principal.

Interest on Rs 10000 in one year = Rs 1000

Interest on Re 1 in one year = $\frac{\text{Rs } 1000}{\text{Rs } 10000}$

Therefore, interest per Rs 100 in one year = $\frac{\text{Rs } 1000}{\text{Rs } 10000} \times \text{Rs } 100$ = Rs 10

Thus, the rate of interest is 10% per year, which can also be expressed as 10% p.a. (per annum).

Now, how will you calculate the interest applicable on Rs 7000, at the rate of 12% p.a., at the end of one year?

Rate of interest = 12% p.a.

This means, if Rs 100 is borrowed, then interest after one year = Rs 12

Thus, if Re 1 is borrowed, then interest after 1 year = $\frac{\text{Rs } 12}{\text{Rs } 100} = \frac{12}{100}$

And, if Rs 7000 is borrowed, then interest after 1 year = $\frac{12}{100} \times$ (Rs 7000) = Rs 840

Thus, the interest on Rs 7000 at the end of one year = Rs 840

Now, what is the amount?

Amount = Principal + Interest = Rs 7000 + Rs 840 = Rs 7840

Therefore, from the above example, it is clear that

Interest = Rate of interest × Principal

If we express interest as I, principal as P, and rate of interest as R%, then the above statement can be expressed as follows:

$$I = P\% \text{ of } R = \frac{P}{100} \times R = \frac{PR}{100}$$

Now, if the amount is expressed as A, then $A = P + I$

$$\therefore A = P + \frac{PR}{100} = P\left(1 + \frac{R}{100}\right)$$

Remember:

1. Interest = I $= \frac{PR}{100}$

2. Amount = A $= P\left(1 + \frac{R}{100}\right)$

Note that the above formulae are applicable only in cases where **the money has to be returned after 1 year, i.e., the time period is 1 year.**

These formulae are not applicable if the time period is, say, 1 month, 2 months, 3 months, etc, or it is 2 years, 3 years, etc.

Let us apply the above formulae in an example.

What amount is to be paid at the end of one year for Rs 10000 at the rate of 10% p.a.?

Principal = P = Rs 10000

Rate of interest = R = 10%

Thus, amount payable at the end of one year = $A = P\left(1+\frac{R}{100}\right)$

$$= 10000\left(1+\frac{10}{100}\right)$$
$$= 10000\left(\frac{100+10}{100}\right)$$
$$= 10000\times\frac{110}{100}$$
$$= 11000$$

Therefore, amount payable at the end of one year = Rs 11000

Now, how should we proceed if we have to find the interest when the time period is more than one year?

Let us go through some examples to understand the concepts of simple interest better.

Example 1:

Rahul borrowed some money from Parul. In return, Rahul had to pay Rs 350 as interest, along with the actual sum. If he paid a total of Rs 4850 to Parul, then find the amount that he borrowed.

Solution:

Interest = Rs 350

Amount = Rs 4850

∴ Principal = Amount − Interest = Rs 4850 − Rs 350 = Rs 4500

Thus, Rahul borrowed Rs 4500 from Parul.

Example 2:

What will be the interest on Rs 8700 at the end of one year at the rate of 20% per year? Also find the amount payable at the end of the year.

Solution:

Principal (P) = Rs 8700

Rate of interest (R) = 20%

Time period (T) = 1 year

$$\therefore I = \frac{P \times R \times T}{100} = \text{Rs } \frac{8700 \times 20 \times 1}{100} = \text{Rs } \frac{174000}{100} = \text{Rs } 1740$$

Therefore, interest (I) payable at the end of the year = Rs 1740

Now, amount (A) payable at the end of the year = $P + I$

= Rs 8700 + Rs 1740

= Rs 10440

Example 3:

To buy a car, Jitendra borrowed a sum of Rs 200000 for 12 years at the rate of 4% p.a. What is the total amount that he has to pay to repay the loan?

Solution:

Principal (P) = Rs 200000

Rate of interest (R) = 4%

Time period (T) = 12 years

Now, amount (A) payable at the end of the time period $= P\left(1 + \frac{R \times T}{100}\right)$

$$= 200000\left(1 + \frac{4 \times 12}{100}\right)$$
$$= 200000\left(\frac{100 + 48}{100}\right)$$
$$= 200000 \times \frac{148}{100}$$
$$= 296000$$

Therefore, Jitendra has to pay Rs 296000 in order to repay his loan.

Example 4:

Kiran invested a sum of Rs 15000 for 4 years and received a total amount of Rs 18000 at the end of this time period. Find the applicable rate of interest.

Solution:

Principal (P) = Rs 15000

Time period (T) = 4 years

Amount (A) = Rs 18000

∴ Interest = $A - P$ = Rs 18000 − Rs 15000 = Rs 3000

Let the rate of interest be R% p.a.

We know that $A = P + \frac{P \times R \times T}{100}$.

$$\Rightarrow 18000 = 15000 + \frac{15000 \times R \times 4}{100}$$
$$\Rightarrow 15000 + \frac{15000 \times R \times 4}{100} = 18000$$
$$\Rightarrow \frac{15000 \times R \times 4}{100} = 18000 - 15000$$
$$\Rightarrow \frac{15000 \times R \times 4}{100} = 3000$$
$$\Rightarrow 15000 \times R \times 4 = 3000 \times 100$$
$$\Rightarrow R = \frac{300000}{15000 \times 4} = 5$$

Therefore, the applicable rate of interest is 5% p.a.

Example 5:

Mr. Sharma borrowed some money from Mr. Gupta for $\frac{3}{2}$ years at the rate of 11% per annum. Also, he borrowed the same amount of money from Mr. Verma for same time period at the rate of 12% per annum. If Mr. Sharma had to pay Rs 6210 as total interest to repay the whole debt then what was the total money that he borrowed?

Solution:

Case I: Money borrowed from Mr. Gupta

Time period (T) = $\frac{3}{2}$ years

Rate (R) = 11% per annum

Let principal be P and interest be I_1.

$$I_1 = \frac{P \times R \times T}{100}$$
$$\Rightarrow I_1 = \frac{P \times 11 \times 3}{100 \times 2}$$
$$\Rightarrow I_1 = \frac{33P}{200}$$

Case II: Money borrowed from Mr. Verma

Time period (T) = $\frac{3}{2}$ years

Rate (R) = 12% per annum

Principal = P

Let interest be I_2.

$$I_2 = \frac{P \times R \times T}{100}$$
$$\Rightarrow I_2 = \frac{P \times 12 \times 3}{100 \times 2}$$
$$\Rightarrow I_2 = \frac{36P}{200}$$

Total interest = Rs 6210

$\therefore$ $I_1 + I_2$ = Rs 6210

$$\frac{33P}{200} + \frac{36P}{200} = \text{Rs } 6210$$
$$\Rightarrow \frac{69P}{200} = \text{Rs } 6210$$
$$\Rightarrow P = \text{Rs} \frac{6210 \times 200}{69}$$
$$\Rightarrow P = \text{Rs } 18000$$

Thus, Mr. Sharma borrowed Rs 18000 from Mr. Gupta and the same amount from Mr. Verma.

Therefore, the total money borrowed by Mr. Sharma = Rs 18000 + Rs 18000 = Rs 36000

Example 6:

Surya deposited Rs 25000 in the bank at the rate of 8% per annum. After how much time the money will get doubled?

Solution:

Principal (P) = Rs 25000

Amount (A) = 2P = Rs (2 × 25000) = Rs 50000

Interest = A – P = Rs 50000 – Rs 25000 = Rs 25000

Rate (R) = 8%

$$I = \frac{P \times R \times T}{100}$$
$$\Rightarrow 25000 = \frac{25000 \times 8 \times T}{100}$$
$$\Rightarrow T = \frac{100}{8}$$
$$\Rightarrow T = 12.5$$

Thus, after 12 years and 6 months, the money will get doubled.