# 1 Quadratic Equations

**Topics Covered**

1. Quadratic Equations
2. Solution of a Quadratic Equation by Factorisation
3. Solution of a Quadratic Equation by Quadratic Formula
4. Nature of Roots

## 1. Quadratic Equations

The second degree polynomial equations is commonly known as quadratic equation, i.e., if $p(x)$ is a quadratic polynomial, then $p(x) = 0$ is called a quadratic equation. The general form of quadratic equation in the variable $x$ is $ax^2 + bx + c = 0$, where $a$, $b$, $c$ are real numbers and $a \neq 0$.

For example, $2x^2 + x - 150 = 0$, $3x^2 - 2x + 5 = 0$, $4x - 3x^2 + 2 = 0$ are quadratic equations.

**Example 1.** Check whether the following are quadratic equations:

(*a*) $x(x + 2) - 3 = (x + 4)\, x$  (*b*) $(x + 2)^3 = x^3 - 4x^2 + 2$

**Solution.** (*a*) Since $x(x + 2) - 3 = x(x + 4)$

$\Rightarrow$ $x^2 + 2x - 3 = x^2 + 4x$

$\Rightarrow$ $2x + 3 = 0$

This is linear equation not a quadratic equation.

(*b*) $(x + 2)^3 = x^3 - 4x^2 + 2$

$\Rightarrow$ $x^3 + 6x^2 + 12x + 8 = x^3 - 4x^2 + 2$

$\Rightarrow$ $10x^2 + 12x + 6 = 0$

$\Rightarrow$ $5x^2 + 6x + 3 = 0$

This is a quadratic equation.

**Example 2.** Represent the following situation in the form a quadratic equation:

(*a*) Abdul and Sneha together have 30 oranges. Both of them ate 3 oranges each and the product of the number of oranges they have now is 120. We would like to find out how many oranges they had initially.

(*b*) The area of a rectangular plot is 428 $m^2$. The length of the plot (in metres) is two more than twice its breadth. We need to find the length and breadth of the plot.

**Solution.** (*a*) Let the number of number of left oranges with Abdul and Sneha be $x$ and $y$ respectively.

Then, $x + y = 30 \;\Rightarrow\; y = 30 - x$

The number of oranges left with both Abdul and Sneha are $x - 3$ and $y - 3$ respectively.

The product of number of left oranges = 120

$\Rightarrow$ $(x - 3)\,(y - 3) = 120$

$\Rightarrow$ $(x - 3)\,(30 - x - 3) = 120$  $(\because\; y = 30 - x)$

$\Rightarrow$ $(x - 3)\,(27 - x) = 120$

$\Rightarrow$ $27x - x^2 - 81 + 3x = 120 \Rightarrow x^2 - 30x + 201 = 0$

(*b*) Let the breadth of the plot be $x$. Then the length of the plot = $2x + 2$

Since, area of the plot = 428 $m^2$  (Given)

$\therefore$ $x(2x + 2) = 428$

$\Rightarrow$ $2x^2 + 2x - 428 = 0$

$\Rightarrow$ $x^2 + x - 214 = 0$

### Roots of a Quadratic Equation

A real number $\alpha$ is called a *root* of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$.

In other words, $x = \alpha$ is a root or solution of the quadratic equation $ax^2 + bx + c = 0$ as it satisfies it.

**Example 3.** Determine whether 3 is a root of the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$$

**Solution.** L.H.S. $= \sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{(3)^2 - 12 + 3} + \sqrt{9 - 9}$

$= 0 + 0 = 0$

R.H.S. $= \sqrt{4x^2 - 14x + 6} = \sqrt{36 - 42 + 6} = 0$

Since L.H.S. = R.H.S.

$\therefore$ 3 is the root of the given quadratic equation.

**Example 4.** If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then find the value of $k$.

**Solution.** It is given that $\frac{1}{2}$ is a root of quadratic equation.

$\therefore$ It must satisfy the quadratic equation:

$$x^2 + kx - \frac{5}{4} = 0 \Rightarrow \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\Rightarrow \quad \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0 \Rightarrow \frac{1 + 2k - 5}{4} = 0$$

$$\Rightarrow \quad 2k - 4 = 0 \Rightarrow k = 2$$

**Example 5.** If $a$ and $b$ are the roots of the equation $x^2 + ax - b = 0$, then find $a$ and $b$.

**Solution.** Here, A = 1, B = $a$, C = $-b$

$$\therefore \quad \text{Sum of the roots} = a + b = -\frac{\text{B}}{\text{A}} = -a \quad ...(i)$$

$$\text{Product of the roots} = ab = \frac{\text{C}}{\text{A}} = -b \quad ...(ii)$$

From ($i$) and ($ii$), $a + b = -a$ and $ab = -b$

$\Rightarrow$ $2a = -b$ and $a = -1$

$\Rightarrow$ $b = 2$ and $a = -1$

## Exercise 1.1

### I. Very Short Answer Type Questions [1 Mark]

1. **Multiple Choice Questions (MCQs)**

**Choose the correct answer from the given options:**

(1) Which of the following is a quadratic equation?

($a$) $x^2 + 2x + 1 = (4 - x)^2 + 3$ ($b$) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

($c$) $(k + 1)x^2 + \frac{3}{2}x = 7$ (where $k = -1$) ($d$) $x^3 - x^2 = (x - 1)^3$

(2) Which of the following equations has 2 as a root?

($a$) $x^2 - 4x + 5 = 0$ ($b$) $x^2 + 3x - 12 = 0$ ($c$) $2x^2 - 7x + 6 = 0$ ($d$) $3x^2 - 6x - 2 = 0$

(3) The roots of the quadratic equation $x^2 - 0.04 = 0$ are [CBSE Standard 2020]

($a$) $\pm 0.2$ ($b$) $\pm 0.02$ ($c$) 0.4 ($d$) 2

(4) The degree of quadratic equation is

($a$) 0 ($b$) 1 ($c$) 2 ($d$) 5

2. **Assertion-Reason Type Questions**

**In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

($a$) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

($b$) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

($c$) Assertion (A) is true but reason (R) is false.

($d$) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** The equation $x^2 + 3x + 1 = (x - 2)^2$ is a quadratic equation.

**Reason (R):** Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$, is a quadratic equation.

(2) **Assertion (A):** $(2x - 1)^2 - 4x^2 + 5 = 0$ is not a quadratic equation.

**Reason (R):** $x = 0, 3$ are the roots of the equation $2x^2 - 6x = 0$.

3. **Answer the following.**

(1) If $x = -\frac{1}{2}$ is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of $k$. [Delhi 2015]

(2) Find the value of $k$ for which $x = \sqrt{3}$ is a solution of the equation $kx^2 + \sqrt{3}x - 4 = 0$.

## II. Short Answer Type Questions - I [2 Marks]

4. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equations $ax^2 + 7x + b = 0$, find the values of $a$ and $b$. [Delhi 2016]

5. Show that $x = -2$ is a solution of the equation $3x^2 + 13x + 14 = 0$.

## III. Short Answer Type Questions - II [3 Marks]

*Represent the following situations in the form of a quadratic equation (Q. 6 & Q.7):*

6. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they have now is 124. We would like to find out how many marbles they had to start with.

7. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

8. If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, then find the value of $p$ and the other root of the equation.

## Case Study Based Questions

I. Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of $x$ km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km.

1. What will be the distance covered by Ajay's car in two hours?

(*a*) $2(x + 5)$ km (*b*) $(x - 5)$ km (*c*) $2(x + 10)$ km (*d*) $(2x + 5)$ km

2. Which of the following quadratic equations describes the speed of Raj's car?

(*a*) $x^2 - 5x - 500 = 0$ (*b*) $x^2 + 4x - 400 = 0$ (*c*) $x^2 + 5x - 500 = 0$ (*d*) $x^2 - 4x + 400 = 0$

3. The roots of the quadratic equation which describe the speed of Raj's car are

(*a*) 15, – 20 (*b*) 20, – 15 (*c*) 20, – 25 (*d*) 25, – 25

4. Which of the following quadratic equations has 2 as a root?

(*a*) $x^2 - 4x + 5 = 0$ (*b*) $x^2 + 3x - 12 = 0$ (*c*) $2x^2 - 7x + 6 = 0$ (*d*) $3x^2 - 6x - 2 = 0$

5. The positive root of $\sqrt{3x^2 + 6} = 9$ is

(*a*) 5 (*b*) –5 (*c*) 3 (*d*) –3

## Answers and Hints

**1.** (1) (*d*) $x^3 - x^2 = (x-1)^3$ (1)

(2) (*c*) $2x^2 - 7x + 6 = 0$ (1)

(3) (*a*) $\pm 0.2$ (1)

(4) (*c*) 2 (1)

**2.** (1) (*d*) Assertion (A) is false but reason (R) is true. (1)

(2) (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A). (1)

**3.** (1) $\because$ $x = \frac{-1}{2}$ is the solution of $3x^2 + 2kx - 3 = 0$

So, $3\left(\frac{-1}{2}\right)^2 + 2k\left(\frac{-1}{2}\right) - 3 = 0$

$\Rightarrow \quad \frac{3}{4} - k - 3 = 0$

$\Rightarrow \quad k = \frac{3}{4} - 3$

$\Rightarrow \quad k = -2\frac{1}{4}$ (1)

(2) We have $kx^2 + \sqrt{3}x - 4 = 0$

$\Rightarrow \quad k\left(\sqrt{3}\right)^2 + \sqrt{3}\left(\sqrt{3}\right) - 4 = 0$

$\Rightarrow \quad 3k + 3 - 4 = 0$

$\Rightarrow \quad 3k - 1 = 0$

$\Rightarrow \quad 3k = 1$

$\Rightarrow \quad k = \frac{1}{3}$

Hence, the required value of $k$ is $\frac{1}{3}$. (1)

**4.** Given quadratic equation is

$ax^2 + 7x + b = 0$ ...(*i*)

$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$

[$\because$ $x = \frac{2}{3}$ is the root of eq. (*i*)]

$\Rightarrow \quad \frac{4a + 42 + 9b}{9} = 0$

$\Rightarrow \quad 4a + 9b + 42 = 0$ ...(*ii*)

Also, $a(-3)^2 + 7(-3) + b = 0$

[$\because$ $x = -3$ is the root of eq. (*i*)]

$\Rightarrow \quad 9a + b - 21 = 0$

$\Rightarrow \quad b = 21 - 9a$ ...(*iii*)(1)

Putting the value of $b$ from (*iii*) in (*ii*), we get

$4a + 9[21 - 9a] + 42 = 0$

$\Rightarrow \quad 4a + 189 - 81a + 42 = 0$

$\Rightarrow \quad a = 3$

Putting $a = 3$ in (*iii*), we have

$b = -6$

So, $a = 3, \; b = -6.$ (1)

**6.** $x^2 - 45x + 324 = 0$ (3)

**7.** $x^2 - 55x + 750 = 0$ (3)

**8.** $3x^2 + px + 4 = 0$ (½)

$3\left(\frac{2}{3}\right)^2 + p\left(\frac{2}{3}\right) + 4 = 0$

$\frac{4}{3} + \frac{2p}{3} + 4 = 0$ (½)

$p = -8$ (½)

$3x^2 - 8x + 4 = 0$

$3x^2 - 6x - 2x + 4 = 0$ (½)

$x = \frac{2}{3}$ or $x = 2$ (½)

Hence, $x = 2$ (½)

### Case Study Based Questions

**I. 1.** (*a*) $2(x + 5)$ km **2.** (*c*) $x^2 + 5x - 500 = 0$

**3.** (*c*) 20, – 25 **4.** (*c*) $2x^2 - 7x + 6 = 0$

**5.** (*a*) 5

## 2. Solution of a Quadratic Equation by Factorisation

To find the solution of a quadratic equation by factorisation method, we first express the given equation as product of two linear factors by splitting the middle term. By equating each factor to zero, we get possible solutions/roots of the given quadratic equation.

Let the given quadratic equation be $ax^2 + bx + c = 0$. Let the quadratic polynomial $ax^2 + bx + c$ be expressed as the product of two linear factors say $(px + q)$ and $(rx + s)$ where, $p, q, r, s$ are real numbers such that $p \neq 0, r \neq 0$.

Then $\quad ax^2 + bx + c = 0 \quad \Rightarrow \quad (px + q)(rx + s) = 0$

$\Rightarrow \quad$ Either $(px + q) = 0$ or $(rx + s) = 0$

$\Rightarrow \quad x = \frac{-q}{p}$ or $x = \frac{-s}{r}$

**Example 1.** Solve the following quadratic equations by the factorisation method.

(*a*) $7x^2 = 8 - 10x$ (*b*) $x(x + 9) = 52$

(*c*) $3(x^2 - 4) = 5x$ (*d*) $x(x + 1) + (x + 2)(x + 3) = 42$

(*e*) $3x^2 - 2\sqrt{6}x + 2 = 0$ [NCERT] **[Imp.]**

**Solution.** (*a*) $7x^2 = 8 - 10x \Rightarrow 7x^2 + 10x - 8 = 0$

$\Rightarrow \quad 7x^2 + 14x - 4x - 8 = 0 \Rightarrow 7x(x + 2) - 4(x + 2) = 0$

$\Rightarrow \quad (7x - 4)(x + 2) = 0$

$\therefore \quad x = \frac{4}{7}, x = -2$

(*b*) $x(x + 9) = 52 \Rightarrow x^2 + 9x - 52 = 0$

$\Rightarrow \quad x^2 + 13x - 4x - 52 = 0 \Rightarrow x(x + 13) - 4(x + 13) = 0$

$\Rightarrow \quad (x + 13)(x - 4) = 0$

$\therefore \quad x = -13, x = 4$

(*c*) $3(x^2 - 4) = 5x \Rightarrow 3x^2 - 5x - 12 = 0$

$\Rightarrow \quad 3x^2 - 9x + 4x + 12 = 0 \Rightarrow 3x(x - 3) + 4(x - 3) = 0$

$\Rightarrow \quad (3x + 4)(x - 3) = 0$

$\therefore \quad x = \frac{-4}{3}, x = 3$

(*d*) $x(x + 1) + (x + 2)(x + 3) = 42 \Rightarrow x^2 + x + x^2 + 3x + 2x + 6 - 42 = 0$

$\Rightarrow \quad 2x^2 + 6x - 36 = 0 \Rightarrow x^2 + 3x - 18 = 0$

$\Rightarrow \quad x^2 + 6x - 3x - 18 = 0 \Rightarrow x(x + 6) - 3(x + 6) = 0 \Rightarrow (x + 6)(x - 3) = 0$

$\therefore \quad x = -6, x = 3$

(*e*) $3x^2 - 2\sqrt{6}x + 2 = 0 \Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow \quad \sqrt{3}x\left(\sqrt{3}x - \sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}x - \sqrt{2}\right) = 0 \Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$

$\therefore \quad x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

**Example 2.** Solve the equation $\frac{4}{x} - 3 = \frac{5}{2x + 3}; x \neq 0, -\frac{3}{2}$, for $x$. [Delhi 2014]

**Solution.** $\frac{4}{x} - 3 = \frac{5}{2x + 3} \Rightarrow \frac{4 - 3x}{x} = \frac{5}{2x + 3}$

$(4 - 3x)(2x + 3) = 5x \Rightarrow 8x - 6x^2 + 12 - 9x = 5x$

$6x^2 + 6x - 12 = 0 \Rightarrow x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0 \Rightarrow x(x + 2) - 1(x + 2) = 0$

$\Rightarrow \quad (x - 1)(x + 2) = 0 \Rightarrow x - 1 = 0 \text{ or } x + 2 = 0$

$\Rightarrow \quad x = 1 \text{ or } x = -2$

**Example 3.** Solve for $x$: $\frac{16}{x} - 1 = \frac{15}{x + 1}; x \neq 0, -1$ [AI 2014]

**Solution.** $\frac{16}{x} - 1 = \frac{15}{x + 1} \Rightarrow \frac{16 - x}{x} = \frac{15}{x + 1}$

$\Rightarrow \quad (16 - x)(x + 1) = 15x \Rightarrow 16x - x^2 + 16 - x = 15x$

$\Rightarrow \quad x^2 + 15x - 15x - 16 = 0 \Rightarrow x^2 = 16$

$\Rightarrow \quad x = \pm 4$

**Example 4.** Solve for $x$ : $\frac{x - 4}{x - 5} + \frac{x - 6}{x - 7} = \frac{10}{3}; x \neq 5, 7$ [AI 2014]

**Solution.** $\frac{x - 4}{x - 5} + \frac{x - 6}{x - 7} = \frac{10}{3}$

$\Rightarrow \quad \frac{(x - 4)(x - 7) + (x - 6)(x - 5)}{(x - 5)(x - 7)} = \frac{10}{3} \Rightarrow \frac{x^2 - 7x - 4x + 28 + x^2 - 5x - 6x + 30}{x^2 - 7x - 5x + 35} = \frac{10}{3}$

$\Rightarrow \quad \frac{2x^2 - 22x + 58}{x^2 - 12x + 35} = \frac{10}{3} \Rightarrow \frac{x^2 - 11x + 29}{x^2 - 12x + 35} = \frac{5}{3}$

$\Rightarrow \quad 3x^2 - 33x + 87 = 5x^2 - 60x + 175$

$\Rightarrow \quad 2x^2 - 27x + 88 = 0 \Rightarrow 2x^2 - 16x - 11x + 88 = 0$

$\Rightarrow \quad 2x(x-8) - 11(x-8) = 0 \quad \Rightarrow \quad (2x-11)(x-8) = 0$

$\Rightarrow \quad 2x - 11 = 0$ or $x - 8 = 0$

$\Rightarrow \quad x = \dfrac{11}{2}$ or $x = 8$

**Example 5.** The sum of the squares of two consecutive odd numbers is 394. Find the numbers. [Foreign 2014]

**Solution.** Let the two consecutive odd numbers be $x$ and $x + 2$.

$\therefore \quad x^2 + (x+2)^2 = 394 \quad \Rightarrow \quad x^2 + x^2 + 4 + 4x = 394$

$\Rightarrow \quad 2x^2 + 4x + 4 = 394 \quad \Rightarrow \quad 2x^2 + 4x - 390 = 0$

$\Rightarrow \quad x^2 + 2x - 195 = 0 \quad \Rightarrow \quad x^2 + 15x - 13x - 195 = 0$

$\Rightarrow \quad x(x+15) - 13(x+15) = 0 \quad \Rightarrow \quad (x-13)(x+15) = 0$

Either $\quad x - 13 = 0$ or $x + 15 = 0 \quad \Rightarrow \quad x = 13$ or $x = -15$ (neglected)

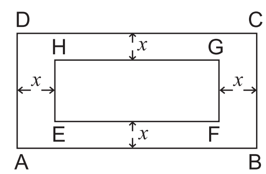
When first number $x = 13$, then second number $x + 2 = 13 + 2 = 15$.

**Example 6.** In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 $m^2$. Find the length and breadth of the pond. [NCERT Exemplar]

**Solution.** Let ABCD be a rectangular lawn and EFGH be rectangular pond. Let $x$ m be the width of grass area, same around the pond.

Now, length of lawn = 50 m, width of lawn = 40 m

$\therefore$ Length of pond = $(50 - 2x)$ m, width of pond = $(40 - 2x)$ m




Since area of grass surrounds the pond = 1184 $m^2$

$\Rightarrow$ Area of lawn – Area of pond = 1184 $m^2$

$\Rightarrow \quad 50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

$\Rightarrow \quad 2000 - (2000 - 80x - 100x + 4x^2) = 1184$

$\Rightarrow \quad 4x^2 - 180x + 1184 = 0$ or $x^2 - 45x + 296 = 0 \quad \Rightarrow \quad x^2 - 37x - 8x + 296 = 0$

$\Rightarrow \quad (x - 37)(x - 8) = 0 \quad \Rightarrow \quad x = 37, 8$

Since $x = 37$ is not possible, as otherwise the length of pond will be negative.

Hence, $x = 8$ is the required solution.

$\therefore \quad$ Length of pond $= 50 - 2 \times 8 = 34$ m

and $\quad$ breadth of pond $= 40 - 2 \times 8 = 24$ m

**Example 7.** The difference of two natural numbers is 5 and the difference of their reciprocals is $\dfrac{1}{10}$. Find the numbers. [Delhi 2014]

**Solution.** Let the two natural numbers be $x$ and $y$ such that $x > y$.

According to the question,

Difference of numbers, $\quad x - y = 5 \quad \Rightarrow \quad x = 5 + y \quad$ ...(*i*)

Difference of their reciprocals,

$$\frac{1}{y} - \frac{1}{x} = \frac{1}{10} \quad \text{...(ii)}$$

Putting the value of (*i*) in (*ii*)

$$\frac{1}{y} - \frac{1}{5+y} = \frac{1}{10} \quad \Rightarrow \quad \frac{5+y-y}{y(5+y)} = \frac{1}{10}$$

$\Rightarrow \quad 50 = 5y + y^2 \quad \Rightarrow \quad y^2 + 5y - 50 = 0$

$\Rightarrow \quad y^2 + 10y - 5y - 50 = 0 \quad \Rightarrow \quad y(y+10) - 5(y+10) = 0$

$\Rightarrow \quad (y-5)(y+10) = 0$

$\therefore \quad y = 5$ or $y = -10$

$\because \quad y$ is a natural number.

$\therefore \quad y = 5$

Putting the value of $y$ in (*i*), we get

$$x = 5 + 5 = 10$$

Thus, the required numbers are 10 and 5.

**Example 8.** The sum of two numbers is 15 and the sum of their reciprocals is $\dfrac{3}{10}$. Find the numbers.

**Solution.** Let the numbers be $x$ and $15 - x$.

According to given condition,

$$\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10} \Rightarrow \frac{15-x+x}{x(15-x)} = \frac{3}{10}$$

$\Rightarrow$ $150 = 3x(15-x) \Rightarrow 50 = 15x - x^2$

$\Rightarrow$ $x^2 - 15x + 50 = 0 \Rightarrow x^2 - 5x - 10x + 50 = 0$

$\Rightarrow$ $x(x-5) - 10(x-5) = 0 \Rightarrow (x-5)(x-10) = 0$

$\Rightarrow$ $x = 5$ or $10$.

When $x = 5$, then $15 - x = 15 - 5 = 10$

When $x = 10$, then $15 - x = 15 - 10 = 5$

Hence, the two numbers are 5 and 10.

**Example 9.** If Zeba was younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now? [NCERT Exemplar]

**Solution.** Let the present age of Zeba be $x$ years.

Age before 5 years = $(x - 5)$ years

According to given condition,

$(x-5)^2 = 5x + 11$

$\Rightarrow$ $x^2 + 25 - 10x = 5x + 11 \Rightarrow x^2 - 10x - 5x + 25 - 11 = 0$

$\Rightarrow$ $x^2 - 15x + 14 = 0 \Rightarrow x^2 - 14x - x + 14 = 0$

$\Rightarrow$ $x(x-14) - 1(x-14) = 0 \Rightarrow (x-1)(x-14) = 0$

$\Rightarrow$ $x - 1 = 0$ or $x - 14 = 0$

$\Rightarrow$ $x = 1$ or $x = 14$

But present age cannot be 1 year.

$\therefore$ Present age of Zeba is 14 years.

**Example 10.** Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream. [Delhi 2017]

**Solution.** Let the speed of stream be $x$ km/hr.

$\therefore$ Speed of boat in upstream = $(15 - x)$ km/hr.

Speed of boat in downstream = $(15 + x)$ km/hr.

$$\frac{30}{15-x} + \frac{30}{15+x} = 4\frac{1}{2} = \frac{9}{2} \Rightarrow \frac{30(15+x+15-x)}{(15-x)(15+x)} = \frac{9}{2}$$

$\Rightarrow$ $200 = 225 - x^2 \Rightarrow x = 5$ (Rejecting $-5$)

$\therefore$ Speed of stream = 5 km/hr

**Example 11.** A train travelling at a uniform speed for 360 km would have taken 48 minutes less to travel the same distance if its speed were 5 km/hour more. Find the original speed of the train. [CBSE Standard SP 2019-20]

**Solution.** Let original speed of the train be $x$ km/hr.

Time taken at original speed = $\frac{360}{x}$ hours

Time taken at increased speed = $\frac{360}{x+5}$ hours

ATQ, $$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60} \Rightarrow 360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5} \Rightarrow x^2 + 5x - 2250 = 0$$

$\Rightarrow$ $x = 45$ or $-50$ (as speed cannot be negative)

$\Rightarrow$ $x = 45$ km/h

**Example 12.** A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

**Solution.** Let the usual speed of the plane be $x$ km/hr. [CBSE 2018]

$\therefore$ $$\frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60} \Rightarrow x^2 + 100x - 300000 = 0$$

$\Rightarrow$ $x^2 + 600x - 500x - 300000 = 0 \Rightarrow (x+600)(x-500) = 0$

$x \neq -600$

$\therefore$ $x = 500$

Speed of plane = 500 km/hr

**Example 13.** Two pipes running together can fill a cistern in $3\frac{1}{13}$ hours. If one pipe takes 3 hours more than the other to fill it, find the time in which each pipe would fill the cistern.

[Delhi 2017] **[HOTS]**

**Solution.** Let time taken by faster pipe to fill the cistern be $x$ hrs.

Therefore, time taken by slower pipe to fill the cistern = $(x + 3)$ hrs

Since the faster pipe takes $x$ minutes to fill the cistern.

$\therefore$ Portion of the cistern filled by the faster pipe in one hour = $\frac{1}{x}$

Portion of the cistern filled by the slower pipe in one hour = $\frac{1}{x+3}$

Portion of the cistern filled by the two pipes together in one hour = $\frac{1}{\frac{40}{13}} = \frac{13}{40}$

According to question, $\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40} \Rightarrow \frac{x+3+x}{x(x+3)} = \frac{13}{40}$

$\Rightarrow \quad 40(2x+3) = 13x(x+3) \Rightarrow 80x + 120 = 13x^2 + 39x$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0 \Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow \quad 13x(x-5) + 24(x-5) = 0 \Rightarrow (x-5)(13x+24) = 0$

Either $\quad x - 5 = 0$ or $13x + 24 = 0$

$\Rightarrow \quad x = 5$ as $x = \frac{-24}{13}$ not possible.

$\therefore$ The time taken by the two pipes is 5 hours and 8 hours respectively.

**Example 14.** Solve for $x$: $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, \quad x \neq -1, -\frac{1}{5}, -4$. [AI 2017]

**Solution.** Here, $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4} \Rightarrow \frac{5x+1+3(x+1)}{(x+1)(5x+1)} = \frac{5}{x+4}$

$[(5x+1) + (x+1)3](x+4) = 5(x+1)(5x+1)$

$\Rightarrow \quad (8x+4)(x+4) = 5(5x^2 + 6x + 1)$

$\Rightarrow \quad 17x^2 - 6x - 11 = 0 \Rightarrow (17x+11)(x-1) = 0$

$\Rightarrow \quad x = \frac{-11}{17}, \ x = 1$

## Exercise 1.2

### I. Very Short Answer Type Questions [1 Mark]

**1. Multiple Choice Questions (MCQs)**

**Choose the correct answer from the given options:**

(1) The roots of the equation $\frac{4}{3}x^2 - 2x + \frac{3}{4} = 0$ are

(*a*) $\frac{2}{3}, \frac{3}{2}$ (*b*) $\frac{3}{4}, \frac{3}{4}$ (*c*) $\frac{1}{2}, -\frac{1}{2}$ (*d*) None of these

(2) The required solution of $4x^2 - 25x = 0$ are

(*a*) $x = 0, x = \frac{12}{7}$ (*b*) $x = 0, x = \frac{25}{4}$ (*c*) $x = 1, x = \frac{5}{9}$ (*d*) $x = 1, x = \frac{12}{7}$

**2. Assertion-Reason Type Questions**

**In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

($d$) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** When the quadratic equation $6x^2 - x - 2 = 0$ is factorised, we get its roots as $\frac{2}{3}$ and $-\frac{1}{2}$.

**Reason (R):** $6x^2 - x - 2 = 0 \Rightarrow 2x(3x-2) + (3x-2) = 0 \Rightarrow (3x-2)(2x+1) = 0 \Rightarrow x = \frac{2}{3}, -\frac{1}{2}$

(2) **Assertion (A):** If $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$, then $x^2 - \sqrt{3}x - x + \sqrt{3} = 0$

$\Rightarrow x(x-\sqrt{3}) - 1(x-\sqrt{3}) = 0 \Rightarrow (x-\sqrt{3})(x-1) = 0 \Rightarrow x = \sqrt{3}, 1$

**Reason (R):** If we can factorise $ax^2 + bx + c$, $a \neq 0$ into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

**3. Answer the following.**

*Find the roots of the following quadratic equations by factorisation* [(1) *to* (4)]:

(1) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$ **[Imp.]** (2) $(x-3)(2x+3) = 0$

(3) $3x^2 - 2ax - a^2 = 0$ (4) $3a^2x^2 + 8abx + 4b^2 = 0$

(5) Find the roots of the equation $x^2 + 7x + 10 = 0$. [CBSE Standard SP 2020-21]

## II. Short Answer Type Questions - I [2 Marks]

*Find the roots of the following quadratic equations by factorisation* (Q4 to Q10).

**4.** Solve for $x$: $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ [Delhi 2013]

**5.** Solve for $x$: $x^2 - (\sqrt{2}+1)x + \sqrt{2} = 0$ [Foreign 2013]

**6.** Solve for $x$: $\sqrt{2x+9} + x = 13$. [AI 2016]

**7.** Solve for $x$: $\sqrt{6x+7} - (2x-7) = 0$. [AI 2016]

**8.** Solve for $x$: $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$. [Foreign 2016]

**9.** Solve for $x$: $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$, $x \neq 3, -5$. [Foreign 2016]

**10.** Solve for $x$: $\sqrt{3}x^2 + 14x - 5\sqrt{3} = 0$

## III. Short Answer Type Questions - II [3 Marks]

**11.** Solve for $x$: $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$; $x \neq 1, -2, 2$. [Delhi 2016]

**12.** The difference of two natural numbers is 3 and the difference of their reciprocals is $\frac{3}{28}$. Find the numbers. [Delhi 2014]

**13.** The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{5}{14}$. Find the numbers. [Delhi 2014]

## IV. Long Answer Type Questions [5 Marks]

**14.** Solve the equation for $x$: $\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}$, $x \neq \frac{4}{3}$. [Foreign 2010]

**15.** Solve the equation for $x$: $\frac{1}{x+1} + \frac{2}{x+2} = \frac{5}{x+4}$, $x \neq -1, -2, -4$. [Foreign 2012]

**16.** Some students planned a picnic. The total budget for food was ₹2,000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by ₹20. How many students attended the picnic and how much did each student pay for the food? [Foreign 2010]

**17.** A two-digit number is such that the product of its digits is 14. When 45 is added to the number, the digits interchange their places. Find the number. [Foreign 2011]

**18.** Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. [Foreign 2012]

**19.** Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately. [AI 2016]

**20.** A pole has to be erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Find the distances from the two gates where the pole is to be erected. [Foreign 2016]

**21.** A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. [CBSE Standard 2020, CBSE 2018, AI 2013]

**22.** At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha. [NCERT Exemplar]

**23.** A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed? [CBSE 2018]

**24.** Solve the following equation:

$$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$$ [CBSE Standard SP 2019-20]

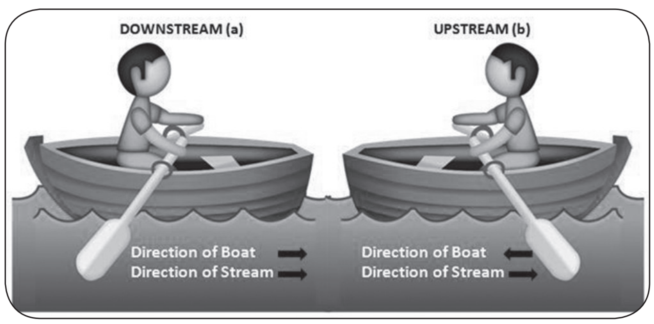
**25.** Find two consecutive positive integers sum of whose squares is 365. [CBSE Standard SP 2019-20]

**26.** A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the park. [CBSE 2016]

**27.** In a flight of 600 km, an aircraft was slowed down due to bad weather. The average speed of the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight. [CBSE Standard 2020]

## Case Study Based Questions

**I.** The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.




**1.** Let speed of the stream be $x$ km/hr, then speed of the motorboat in upstream will be

(*a*) 20 km/hr (*b*) $(20 + x)$ km/hr (*c*) $(20 – x)$ km/hr (*d*) 2 km/hr

**2.** What is the relation between speed, distance and time?

(*a*) $\text{speed} = \frac{\text{(distance)}}{\text{time}}$ (*b*) $\text{distance} = \frac{\text{(speed )}}{\text{time}}$

(*c*) time = speed × distance (*d*) speed = distance × time

**3.** Which is the correct quadratic equation for the speed of the stream?

(*a*) $x^2 + 30x - 200 = 0$ (*b*) $x^2 + 20x - 400 = 0$ (*c*) $x^2 + 30x - 400 = 0$ (*d*) $x^2 – 20x - 400 = 0$

**4.** What is the speed of stream?

(*a*) 20 km/hour (*b*) 10 km/hour (*c*) 15 km/hour (*d*) 25 km/hour

**5.** How much time boat took in downstream?

(*a*) 90 minutes (*b*) 15 minutes (*c*) 30 minutes (*d*) 45 minutes

## Answers and Hints

**1.** (1) (*b*) $\frac{3}{4}, \frac{3}{4}$ (1) (2) (*b*) $x = 0, x = \frac{25}{4}$ (1)

**2.** (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)

(2) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)

**3.** (1) $-\sqrt{3}, -\frac{7}{\sqrt{3}}$ (1) (2) $3, \frac{-3}{2}$ (1)

(3) $a, \frac{-a}{3}$ (1) (4) $\frac{-2b}{a}, \frac{-2b}{3a}$ (1)

(5) $x^2 + 7x + 10 = 0$

$x^2 + 5x + 2x + 10 = 0$ (½)

$(x + 5)(x + 2) = 0$

$x = -5, x = -2$ (½)

**4.** Consider $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$ (1)

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow x = \frac{\sqrt{3}}{4}$ and $-\frac{2}{\sqrt{3}}$ (1)

**5.** Consider:

$x^2 - (\sqrt{2}+1)x + \sqrt{2} = 0$

$\Rightarrow x^2 - \sqrt{2}x - x + \sqrt{2} = 0$ (1)

$\Rightarrow (x - \sqrt{2})(x - 1) = 0$

$\Rightarrow x = \sqrt{2}$ or $x = 1$ (1)

**6.** $\sqrt{2x+9} + x = 13 \Rightarrow \sqrt{2x+9} = (13 - x)$

Squaring both sides, we get

$\Rightarrow 2x + 9 = 169 - 26x + x^2$ (1)

$\Rightarrow x^2 - 28x + 160 = 0$

$\Rightarrow (x - 20)(x - 8) = 0$

$\Rightarrow x = 20$ or $x = 8$

$\Rightarrow x = 8$

[as $x = 20$ does not satisfy the equation] (1)

**7.** $\sqrt{6x+7} - (2x - 7) = 0$

$\Rightarrow \sqrt{6x+7} = 2x - 7$

Squaring both sides, we get

$\Rightarrow 6x + 7 = 4x^2 - 28x + 49$ (1)

$\Rightarrow 4x^2 - 34x + 42 = 0$

$\Rightarrow 2x^2 - 17x + 21 = 0$

$\Rightarrow 2x^2 - 14x - 3x + 21 = 0$

$\Rightarrow (2x - 3)(x - 7) = 0$

$\Rightarrow x = 7$ or $x = \frac{3}{2} \Rightarrow x = 7$

[as $x = \frac{3}{2}$ does not satisfy the equation] (1)

**8.** $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 + \sqrt{2}x - 3\sqrt{2}x - 2\sqrt{3} = 0$ (1)

$\Rightarrow (\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$

$\therefore x = \frac{-\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{6}}{3}$ and $x = \sqrt{6}$ (1)

**9.** $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6} \Rightarrow \frac{x+5-x+3}{x^2+2x-15} = \frac{1}{6}$ (1)

$\Rightarrow x^2 + 2x - 15 = 48 \Rightarrow x^2 + 2x - 63 = 0$

$\Rightarrow (x + 9)(x - 7) = 0 \Rightarrow x = 7$ or $x = -9$ (1)

**10.** $\sqrt{3}x^2 + 14x - 5\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 + 15x - x - 5\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x(x + 5\sqrt{3}) - (x + 5\sqrt{3}) = 0$

$\Rightarrow (x + 5\sqrt{3})(\sqrt{3}x - 1) = 0$

Either $x + 5\sqrt{3} = 0$ (1)

or $\sqrt{3}x - 1 = 0$

$\Rightarrow x = -5\sqrt{3}$

or $x = \frac{1}{\sqrt{3}}$ (1)

**11.** $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$

$\Rightarrow \frac{x+1}{x-1} + \frac{x-2}{x+2} + \frac{2x+3}{x-2} = 4$ (1)

$\Rightarrow \frac{(x+1)(x+2)(x-2) + (x-2)^2(x-1) + (2x+3)(x-1)(x+2)}{(x-1)(x+2)(x-2)} = 4$

$\Rightarrow (x + 1)(x^2 - 4) + (x - 1)(x^2 + 4 - 4x) + (2x + 3)(x^2 + x - 2) = 4(x - 1)(x^2 - 4)$ (1)

$\Rightarrow 5x^2 + 19x - 30 = 0$

$\Rightarrow (x + 5)(5x - 6) = 0 \Rightarrow x = -5$

or $x = \frac{6}{5}$ (1)

**12.** 4, 7 (3)

**13.** 7, 2 (3)

**14.** Given that: $\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}, x \neq \frac{4}{3}$

Let us consider: $\frac{3x-4}{7} = y$

$\Rightarrow$ The given equation becomes $y + \frac{1}{y} = \frac{5}{2}$ (1)

$\Rightarrow 2y^2 - 5y + 2 = 0 \Rightarrow 2y^2 - 4y - y + 2 = 0$

$\Rightarrow (2y - 1)(y - 2) = 0 \Rightarrow y = \frac{1}{2}$ or 2 (1)

$\frac{3x-4}{7} = \frac{1}{2}$ or $\frac{3x-4}{7} = 2$ (1)

$\Rightarrow 6x - 8 = 7$ | $\Rightarrow 3x = 18$

$\Rightarrow x = \frac{15}{6}$ (1) | $\Rightarrow x = 6$ (1)

**15.** Given that:

$\frac{1}{x+1} + \frac{2}{x+2} = \frac{5}{x+4}; x \neq -1, -2, -4$

$\Rightarrow \frac{x+2+2x+2}{x^2+3x+2} = \frac{5}{x+4}$ (1)

$\Rightarrow (3x + 4)(x + 4) = 5(x^2 + 3x + 2)$ (1)

$\Rightarrow 3x^2 + 16x + 16 = 5x^2 + 15x + 10$ (1)

$\Rightarrow 2x^2 - x - 6 = 0$

$\Rightarrow (2x + 3)(x - 2) = 0$ (1)

$\Rightarrow x = 2$ or $x = \frac{-3}{2}$ (1)

**16.** **Case I.** Let number of students = $x$

and cost of food for each member = ₹ $y$

Then $x \times y = 2,000$ ...(*i*)(1)

**Case II.** New number of students = $x - 5$

New cost of food for each member = ₹$(y + 20)$

Then $(x - 5)(y + 20) = 2,000$

$\Rightarrow$ $xy + 20x - 5y - 100 = 2,000$ ...(*ii*)(1)

Solving (*i*) and (*ii*), we get

$\therefore$ $x = -20, 25$ (1)

$x = -20$ is rejected because number of students can't be negative.

So, $x = 25$

$\therefore$ $y = 80$ (1)

Number of students = 25

Cost of food for each student = ₹80. (1)

**17.** 27 (5)

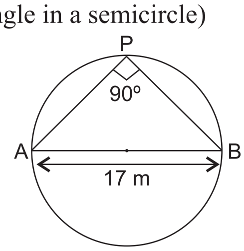
**18.** 9 hrs. (5)

**19.** 20 minutes, 25 minutes.

**Hints:** Solve same as Example 13. (5)

**20.** Let P be the position of the pole.

$\angle APB = 90°$ (angle in a semicircle)




By Pythagoras Theorem,

$AB^2 = AP^2 + PB^2$

$\Rightarrow$ $17^2 = AP^2 + PB^2$ ...(*i*)(1)

Now, $AP - PB = 7$ ...(*ii*)

$\Rightarrow$ $(AP - PB)^2 = 49$

$\Rightarrow$ $AP^2 + PB^2 - 2AP \cdot PB = 49$ ...(*iii*)(1)

From (*i*) and (*iii*), we have

$17^2 - 2AP \cdot PB = 49$

$\Rightarrow$ $AP \cdot PB = 120$ ...(*iv*)

From (*ii*) and (*iv*), we have

$120 = PB(7 + PB)$ (1)

Let $PB = x$

$120 = x(7 + x)$

$\Rightarrow$ $x^2 + 7x - 120 = 0$

$\Rightarrow$ $(x - 8)(x + 15) = 0$

$\Rightarrow$ $x = 8$

or $x = -15$ (Rejected)

$\therefore$ $PB = x = 8$ m,

$AP = 15$ m (2)

**21.** 6 km/h. (5)

**22.** Nisha's age = 5 years, Asha's age = 27 years (5)

**23.** Let the original average speed of train be $x$ km/hr.

Therefore $\frac{63}{x} + \frac{72}{x+6} = 3$ (1)

$\Rightarrow$ $x^2 - 39x - 126 = 0$ (1)

$\Rightarrow$ $(x - 42)(x + 3) = 0$ (1)

$x \neq -3$

$\therefore$ $x = 42$ (1)

Original speed of train is 42 km/hr. (1)

**24.** $\frac{1}{x} - \frac{1}{x-2} = 3$

$\frac{x - 2 - x}{x(x-2)} = \frac{3}{1}$ (1)

$3x^2 - 6x = -2$ (1)

$3x^2 - 6x + 2 = 0$ (1)

$x = \frac{6 \pm \sqrt{12}}{6}$ (1)

$= \frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}$ (1)

**25.** Let two consecutive positive integers be $x$ and $x + 1$

$\therefore$ $x^2 + (x + 1)^2 = 365$ (1)

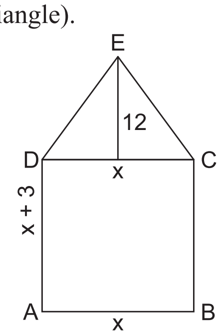
$\Rightarrow$ $x^2 + x - 182 = 0$ (1)

$(x + 14)(x - 13) = 0$ (1)

$\therefore$ $x = 13$ (1)

Hence, two consecutive positive integers are 13 and 14. (1)

**26.** Let ABCD is a rectangular park and CDE is a triangular park (isosceles triangle).




**Rectangle:**

Let breadth = $x$

Then its length = $x + 3$

So, area = $x(x + 3)$ (1)

**Triangle:**

Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Altitude}$

$= \frac{1}{2} \times CD \times \text{Altitude}$ (1)

$= \frac{1}{2} \times x \times 12$

$= 6x$

ATQ, $x(x + 3) = 4 + 6x$ (1)

$x^2 + 3x = 4 + 6x$

$x^2 - 3x - 4 = 0$

$x^2 - 4x + x - 4 = 0$

$x(x - 4) + 1(x - 4) = 0$

$(x - 4)(x + 1) = 0$

Either $x - 4 = 0$ or $x + 1 = 0$

$x = 4$ or $x = -1$ (1)

Since $x$ cannot be negative.

So, $x = 4$ is the solution

Thus, Breadth = $x$ = 4 m

and length = $x + 3 = 4 + 3 = 7$ m (1)

**27.** 1 hr (5)

### Case Study Based Questions

**I.** **1.** (*c*) $(20 - x)$km/hr **2.** (*a*) speed = $\frac{\text{(distance)}}{\text{time}}$

**3.** (*c*) $x^2 + 30x - 400 = 0$ **4.** (*b*) 10 km/hour

**5.** (*c*) 30 minutes

## 3. Solution of a Quadratic Equation by Quadratic Formula

The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, if $b^2 - 4ac \geq 0$.

This formula for finding the roots of a quadratic equation is often **referred to as the quadratic formula.**

The expression $b^2 - 4ac$ is called the **discriminant** of the quadratic equation and generally denoted by D. If $b^2 - 4ac < 0$, then the equation will have no real roots.

As this formula was given by an ancient Indian mathematician **Sridharacharya** around AD 1025, it is known as Sridharacharya's formula for determining the roots of the quadratic equations $ax^2 + bx + c = 0$.

**Example 1.** Find the solution of the quadratic equations by quadratic formula.

(*i*) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$ (*ii*) $-x^2 + 7x - 10 = 0$ **[Imp.]**

**Solution.** (*i*) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0 \Rightarrow x^2 - 2\sqrt{11}x + 2 = 0$

Here, $a = 1$, $b = -2\sqrt{11}$, $c = 2$

Using the formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get

$$x = \frac{+2\sqrt{11} \pm \sqrt{44 - 8}}{2} = \frac{2\sqrt{11} \pm 6}{2}$$

$$\Rightarrow \quad x = \sqrt{11} + 3, \sqrt{11} - 3$$

Hence, the roots of the given quadratic equation are $\sqrt{11} + 3$ and $\sqrt{11} - 3$

(*ii*) $-x^2 + 7x - 10 = 0$

Here, $a = -1$, $b = 7$, $c = -10$

Using the formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get

$$x = \frac{-7 \pm \sqrt{49 - 40}}{-2} = \frac{-7 \pm \sqrt{9}}{-2}$$

$$\Rightarrow \quad x = \frac{-7 + 3}{-2} = \frac{-4}{-2} = 2, \text{or } \frac{-7 - 3}{-2} = \frac{-10}{-2} = 5$$

Roots are 2 and 5.

**Example 2.** Using the quadratic formula, solve the equation

$a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$ [CBSE 2006]

**Solution.** Comparing given equation with $Ax^2 + Bx + C = 0$, we get

$A = a^2b^2$, $B = -(4b^4 - 3a^4)$ and $C = -12a^2b^2$.

$$\therefore B^2 - 4AC = (4b^4 - 3a^4)^2 - 4 \times a^2b^2 \times (-12a^2b^2)$$

$$= 16b^8 + 9a^8 - 24a^4b^4 + 48a^4b^4$$

$$= 16b^8 + 9a^8 + 24a^4b^4 = (4b^4 + 3a^4)^2$$

$$\Rightarrow \sqrt{B^2 - 4AC} = 4b^4 + 3a^4$$

Now,

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{+(4b^4 - 3a^4) \pm (4b^4 + 3a^4)}{2a^2b^2}$$

$$= \frac{4b^4 - 3a^4 + 4b^4 + 3a^4}{2a^2b^2} \text{ or } \frac{4b^4 - 3a^4 - 4b^4 - 3a^4}{2a^2b^2}$$

$$= \frac{8b^4}{2a^2b^2} \text{ or } \frac{-6a^4}{2a^2b^2} = \frac{4b^2}{a^2} \text{ or } \frac{-3a^2}{b^2}$$

**Example 3.** Solve the quadratic equation $2x^2 + ax - a^2 = 0$ for $x$ using quadratic formula. [Delhi 2011]

**Solution.** $2x^2 + ax - a^2 = 0$

Here, $a = 2$, $b = a$ and $c = -a^2$.

Using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we get}$$

$$x = \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a^2)}}{2 \times 2} = \frac{-a \pm \sqrt{9a^2}}{4} = \frac{-a \pm 3a}{4}$$

$$\Rightarrow \quad x = \frac{-a + 3a}{4} = \frac{a}{2}, \quad x = \frac{-a - 3a}{4} = -a$$

$\therefore$ Roots are

$$x = \frac{a}{2}, -a$$

**Example 4.** Solve the following quadratic equation:

$9x^2 - 9(a + b)x + [2a^2 + 5ab + 2b^2] = 0$ [Foreign 2016]

**Solution.** We have $9x^2 - 9(a + b)x + [2a^2 + 5ab + 2b^2] = 0$

Here, A = 9, B = –9 $(a + b)$ and $C = [2a^2 + 5ab + 2b^2]$

So, discriminant,

$$\begin{aligned} D &= B^2 - 4AC = \{-9(a+b)\}^2 - 4 \times 9(2a^2 + 5ab + 2b^2) \\ &= 9^2(a+b)^2 - 4 \times 9(2a^2 + 5ab + 2b^2) \\ &= 9\{9(a+b)^2 - 4(2a^2 + 5ab + 2b^2)\} \\ &= 9\{9a^2 + 9b^2 + 18ab - 8a^2 - 20ab - 8b^2\} \\ &= 9(a^2 + b^2 - 2ab) = 9(a-b)^2 \end{aligned}$$

Using the quadratic formula, $x = \dfrac{-B \pm \sqrt{D}}{2A}$, we get

$$x = \frac{9(a+b) \pm \sqrt{9(a-b)^2}}{2 \times 9}$$

$$\Rightarrow \quad x = \frac{9(a+b) \pm 3(a-b)}{2 \times 9} \quad \Rightarrow x = \frac{3(a+b) \pm (a-b)}{6}$$

$$\Rightarrow \quad x = \frac{(3a+3b) + (a-b)}{6} \text{ or } x = \frac{(3a+3b) - (a-b)}{6}$$

$$\Rightarrow \quad x = \frac{(4a+2b)}{6} \text{ or } x = \frac{(2a+4b)}{6}$$

$$\Rightarrow \quad x = \frac{2a+b}{3} \text{ or } x = \frac{a+2b}{3} \text{ are required solutions.}$$

## Exercise 1.3

### I. Very Short Answer Type Questions **[1 Mark]**

**1. Multiple Choice Questions (MCQs)**

**Choose the correct answer from the given options:**

(1) The discriminant of the equation $9x^2 + 6x + 1 = 0$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

(2) If D is the discriminant of the equation $x^2 + 2x - 4$, then 2D is:

(*a*) 20 (*b*) 40 (*c*) 60 (*d*) 80

(3) The discriminant of the quadratic equation $4x^2 - 6x + 3 = 0$ is:

(*a*) 12 (*b*) 84 (*c*) $2\sqrt{3}$ (*d*) –12

(4) The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$ if $b^2 - 4ac$

(*a*) $< 0$ (*b*) $\leq 0$ (*c*) $> 0$ (*d*) $\geq 0$

(5) The quadratic formula was given by an ancient Indian mathematician.

(*a*) Sridharacharya (*b*) Aryabhata (*c*) Brahmagupta (*d*) None of these

**2. Assertion-Reason Type Question**

**In the following question, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(*c*) Assertion (A) is true but reason (R) is false.
(*d*) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** The values of $x$ are $-\dfrac{a}{2}$, $a$ for a quadratic equation $2x^2 + ax - a^2 = 0$.

**Reason (R):** For quadratic equation $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**3. Answer the following:**

(1) Write the discriminant of the quadratic equation $(x + 5)^2 = 2(5x - 3)$.

(2) Find the discriminant of the quadratic equation: $4x^2 - \dfrac{2}{3}x - \dfrac{1}{16} = 0$.

### II. Short Answer Type Questions - I **[2 Marks]**

4. Find the roots of the equation $ax^2 + a = a^2x + x$. [CBSE 2012]
5. Solve the following quadratic equation for $x$: $4x^2 - 4a^2x + (a^4 - b^4) = 0$. [Delhi 2015]
6. Solve the following quadratic equation for $x$: $9x^2 - 6b^2x - (a^4 - b^4) = 0$. [Delhi 2015]
7. Solve the following quadratic equation for $x$: $4x^2 + 4bx - (a^2 - b^2) = 0$. [AI 2015]
8. Solve the following quadratic equation for $x$: $x^2 - 2ax - (4b^2 - a^2) = 0$. [AI 2015]

### III. Short Answer Type Questions - II **[3 Marks]**

*Solve the following using quadratic formula* (Q. 9 to 11):

9. $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$ **[Imp.]**
10. $3x^2 + 2\sqrt{5}x - 5 = 0$ [Foreign 2011] **[Imp.]**
11. $\dfrac{1}{x+4} - \dfrac{1}{x-7} = \dfrac{11}{30}$, $x \neq -4, 7$ [CBSE Standard 2020] **[Imp.]**
12. Find the roots of quadratic equation: $x^2 - 3\sqrt{5}x + 10 = 0$ [All India 2017]
13. Find the roots of quadratic equation: $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$
14. Solve for $x$: $4x^2 - 4ax + (a^2 - b^2) = 0$. [Delhi 2012]
15. Two water taps together can fill a tank in 9 hours 36 minutes. The tap of large diameter takes 8 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. [Foreign 2016]

## IV. Long Answer Type Questions **[5 Marks]**

16. A rectangular field is 20 m long and 14 m wide. There is a path of equal width all around it, having an area of 111 sq m. Find the width of the path.
[CBSE 2013, 2012] **[Imp.]**

17. At '$t$' minutes past 2 pm, the time needed by the minute hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find $t$. [NCERT Exemplar]

18. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number. [NCERT Exemplar]

19. Solve for $x$: $\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{6}; x \neq 4, 6$ [AI 2014]

20. Solve for $x$: $\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}; x \neq 3, 5$ [AI 2014]

21. Solve for $x$: $3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x \neq \frac{1}{3}, -\frac{3}{2}$ [Foreign 2014]

22. The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two numbers. [Delhi 2010]

23. Sum of the areas of two squares is 544 $m^2$. If the difference of their perimeters is 32 m, find the sides of the two squares.

## Case Study Based Questions

I. **Water Distribution System:** Delhi Jal Board (DJB) is the main body of the Delhi Government which supplies drinking water in the National Capital Territory of Delhi. Distribution system is well knit and properly planned. Maintenance of underground pipe and hose system is also performed at regular interval of time. Many rivers and canals are inter-connected in order to ensure un-interrupted water supply. It has been meeting the needs of potable water for more than 16 million people. It ensures availability of 50 gallons per capita per day of pure and filtered water with the help of efficient network of water treatment plants and pumping stations.

In our locality, DJB constructed two big reservoir labelled as Reservoir–A and Reservoir–B.

**Reservoir–A:** In order to fill it, department uses two pipes of different diameter.

**Reservoir–B:** Department uses two taps to store water in this reservoir.

**Refer to Reservoir-A**

1. Two pipes running together can fill the reservoir in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the reservoir, the time in which each pipe alone would fill the reservoir is
(*a*) 10 min, 12 min (*b*) 25 min, 20 min (*c*) 15 min, 18 min (*d*) 22 min, 28 min

2. Two pipes running together can fill a reservoir in 6 minutes. If one pipe takes 5 minutes more than the other to fill the reservoir, the time in which each pipe would fill the reservoir separately is
(*a*) 8 min, 6 min (*b*) 10 min, 15 min (*c*) 12 min, 16 min (*d*) 16 min, 18 min

**Refer to Reservoir-B**

3. Two water taps together can fill a reservoir in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the reservoir separately. The time in which each tap can separately fill the reservoir will be
(*a*) 15 hrs, 25 hrs (*b*) 20 hrs, 22 hrs (*c*) 14 hrs, 18 hrs (*d*) 18 hrs, 16 hrs

4. Two taps running together can fill the reservoir in $3\frac{1}{13}$ minutes. If one tap takes 3 minutes more than the other to fill it, how many minutes each tap would take to fill the reservoir?
(*a*) 12 min, 15 min (*b*) 6 min, 9 min (*c*) 18 min, 14 min (*d*) 5 min, 8 min

5. If two tapes function simultaneously, reservoir will be filled in 12 hours. One tap fills the reservoir 10 hours faster than the other. The time that the second tap takes to fill the reservoir is given by
(*a*) 25 hrs (*b*) 28 hrs (*c*) 30 hrs (*d*) 32 hrs

II. **A Hill Station:** In the last summer, I enjoyed a tour to a hill station at Shimla. I was accompanied by my five friends and enjoyed the natural beauties of mountains, rivers, streams, forests etc. The beginning of the tour was the most adventurous itself! How amazingly my group win the bet! Actually, the story is that my two friends along with me prefered train to go to Shimla, but other three were forcing for a car or a bus. At last the consensus was reached and we were divided ourselves in two groups of 3 each and started for Shimla at the same time. It was decided that the group who reach the destination first, would be declared as the winner, and runner up the group have to bear the expanses of the tour. I named my group, 'Group A' while the second group was named as 'Group B'. Luckily we reached Shimla 1 hour before the Group-B and enjoyed the trip for absolutely FREE!! How thrilling it was the tour!

**Refer to Group-A**

1. An express train takes 1 hour less than a passenger train to travel 132 km between Delhi and Shimla (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/hr more than that of the passenger train, the average speeds of the two trains will be
   (*a*) 33 km/h, 44 km/hr (*b*) 40 km/h, 45 km/h
   (*c*) 30 km/h, 38 km/h (*d*) 42 km/h, 62 km/h
2. An express train makes a run of 240 km at a certain speed. Another train whose speed is 12 km/hr less takes an hour longer to make the same trip. The speed of the express train will be
   (*a*) 60 km/h (*b*) 50 km/h (*c*) 65 km/h (*d*) 48 km/h
3. A journey of 192 km from Delhi to Shimla takes 2 hours less by a super fast train than that by an ordinary passenger train. If the average speed of the slower train is 16 km/hr less than that of the faster train, average speed of super fast train is
   (*a*) 50 km/h (*b*) 48 km/h (*c*) 55 km/h (*d*) 60 km/h

**Refer to Group-B**

4. A deluxe bus takes 3 hours less than a ordinary bus for a journey of 600 km. If the speed of the ordinary bus is 10 km/hr less than that of the deluxe bus, the speeds of the two buses will be
   (*a*) 35 km/h, 42 km/h (*b*) 42 km/h, 52 km/h
   (*c*) 40 km/h, 50 km/h (*d*) 30 km/h, 58 km/h
5. A bus travels a distance of 300 km at a uniform speed. If the speed of the bus is increased by 5 km an hour, the journey would have taken two hours less. The original speed of the bus will be
   (*a*) 20 km/h (*b*) 15 km/h (*c*) 22 km/h (*d*) 25 km/h

## Answers and Hints

**1.** (1) (*a*) 0 (1) (2) (*b*) 40 (1)
(3) (*d*) –12 (1) (4) (*d*) ≥ (1)
(5) (*a*) Sridharacharya (1)

**2.** (1) (*d*) Assertion (A) is false but reason (R) is true. (1)

**3.** (1)
$$(x+5)^2 = 2(5x-3)$$
$$\Rightarrow x^2 + 25 + 10x = 10x - 6$$
$$\Rightarrow x^2 + 31 = 0$$
$$\Rightarrow x^2 + 0x + 31 = 0$$
$$\therefore \quad D = (0)^2 - 4 \times 1 \times 31 = 0 - 124 = -124 \quad (1)$$

(2) 3328 (1)

**4.** $a, \dfrac{1}{a}$ (2)

**5.** $4x^2 - 4a^2x + (a^4 - b^4) = 0$
$$\Rightarrow x = \frac{4a^2 \pm \sqrt{16a^4 - 4\times 4\times(a^4-b^4)}}{2\times 4} \qquad \because x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$\Rightarrow x = \frac{4a^2 \pm \sqrt{16b^4}}{2\times 4} \quad (1)$$
$$\Rightarrow x = \frac{4a^2 \pm 4b^2}{2\times 4} = \frac{a^2 \pm b^2}{2}$$
$$\Rightarrow x = \frac{a^2+b^2}{2} \text{ or } \frac{a^2-b^2}{2} \quad (1)$$

**6.** $9x^2 - 6b^2x - (a^4 - b^4) = 0$
$$\Rightarrow x = \frac{6b^2 \pm \sqrt{36b^4 + 4\times 9\times(a^4-b^4)}}{2\times 9} \qquad \because x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$\Rightarrow x = \frac{6b^2 \pm \sqrt{36b^4 + 36a^4 - 36b^4}}{2\times 9} \quad (1)$$
$$\Rightarrow x = \frac{6b^2 \pm \sqrt{36a^4}}{2\times 9} \Rightarrow x = \frac{6b^2 \pm 6a^2}{2\times 3\times 3}$$
$$\Rightarrow x = \frac{b^2 \pm a^2}{3} \Rightarrow x = \frac{b^2+a^2}{3} \text{ or } \frac{b^2-a^2}{3} \quad (1)$$

**7.** $4x^2 + 4bx - (a^2 - b^2) = 0$
$$x = \frac{-4b \pm \sqrt{16a^2}}{2\times 4} = \frac{-4b \pm 4a}{8} = \frac{-b \pm a}{2} \quad (1)$$
$$\Rightarrow x = \frac{-b+a}{2}, \frac{-b-a}{2} \quad (1)$$

**8.** $x^2 - 2ax - (4b^2 - a^2) = 0$
$$\therefore x = \frac{-(-2a) \pm \sqrt{16b^2}}{2\times 1} = \frac{2a \pm 4b}{2} = a \pm 2b \quad (1)$$

$\Rightarrow \quad x = a + 2b$ or $a - 2b$. (1)

**9.** $\dfrac{\sqrt{3}}{2}, \dfrac{1}{\sqrt{3}}$ (3) **10.** $\dfrac{\sqrt{5}}{3}, -\sqrt{5}$ (3)

**11.** 2, 1 (3)

**12.** $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{3\sqrt{5} \pm \sqrt{5}}{2 \times 1}$ (1)

$\Rightarrow \quad x = \dfrac{3\sqrt{5} + \sqrt{5}}{2}$ or $\dfrac{3\sqrt{5} - \sqrt{5}}{2}$ (1)

$\Rightarrow \quad x = \dfrac{4\sqrt{5}}{2}$ or $\dfrac{2\sqrt{5}}{2}$

$\Rightarrow \quad x = 2\sqrt{5}$ or $x = \sqrt{5}$ (1)

**13.** $5\sqrt{5}x^2 + \ 30x + 8\sqrt{5} = 0$

$\Rightarrow \quad 5\sqrt{5}x^2 + 20x + 10x + 8\sqrt{5} = 0$ (1)

$\Rightarrow \quad 5x(\sqrt{5}x + 4) + \ 2\sqrt{5}(\sqrt{5}x + 4) = 0$

$\Rightarrow \quad (\sqrt{5}x + 4)(5x + 2\sqrt{5}) = 0$ (1)

$x = \dfrac{-4\sqrt{5}}{5}$ or $\dfrac{-2\sqrt{5}}{5}$ (1)

**14.** Roots are $= \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \dfrac{-(-4a) \pm \sqrt{(-4a)^2 - 4 \times 4(a^2 - b^2)}}{2 \times 4}$ (1)

$= \dfrac{4a \pm \sqrt{16a^2 - 16a^2 + 16b^2}}{8}$ (1)

$= \dfrac{4a \pm 4b}{8} = \dfrac{a \pm b}{2}$ (1)

**15.** Let $x$ be the time taken by larger diameter tap.

$\therefore \ x + 8$ be the time taken by smaller diameter tap.

ATQ, $\dfrac{1}{x} + \dfrac{1}{x+8} = \dfrac{10}{96}$

$\left(\because \ 9 \text{ hrs } 36 \text{ min} = \dfrac{96}{10} \text{ hrs}\right)$ (1)

$\Rightarrow \quad 10x^2 - 112x - 768 = 0$

$\Rightarrow \quad 5x^2 - 56x - 384 = 0$

$\Rightarrow \quad x = \dfrac{56 \pm \sqrt{(56)^2 - 4 \times 5 \times (-384)}}{2 \times 5}$ (1)

$\Rightarrow \quad x = \dfrac{56 + 104}{10}$ or $\dfrac{56 - 104}{10}$

$\Rightarrow \quad x = 16$ or $x = -4.8$ (Rejected)

Hence, time taken by larger and smaller taps are 16 hrs and 24 hrs respectively. (1)

**16.** 1.5 m (5) **17.** 14 minutes (5)

**18.** 12 (5) **19.** $2 \pm \sqrt{10}$ (5)

**20.** $\dfrac{7}{2}, 6$ (5) **21.** 0, –7 (5)

**22.** 9 and 13 (5)

**23.** Let the sides of two squares in metres be $x$ and $y$ respectively (where $x > y$).

**Given:**

Sum of areas of two squares $= 544 \text{ m}^2$

$\Rightarrow \quad x^2 + y^2 = 544$ ...(*i*) (1)

Also, difference of their perimeters

$= 32$ m

$\Rightarrow \quad 4x - 4y = 32$

$\Rightarrow \quad x - y = 8$

$\Rightarrow \quad y = x - 8$ ...(*ii*) (1)

Substituting the value of $y$ for equation (*ii*) in equation (*i*), we get

$x^2 + (x - 8)^2 = 544$

$\Rightarrow \quad x^2 + x^2 - 16x + 64 - 544 = 0$

$\Rightarrow \quad 2x^2 - 16x - 480 = 0$

$\Rightarrow \quad x^2 - 8x - 240 = 0$ (1)

$\Rightarrow \quad x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 1 \times (-240)}}{2 \times 1}$

$= \dfrac{8 \pm \sqrt{64 + 960}}{2}$

$= \dfrac{8 \pm \sqrt{1024}}{2}$ (1)

$= \dfrac{8 \pm 32}{2} = 4 \pm 16$

$\Rightarrow \quad x = 4 + 16 = 20$

or $\quad x = 4 - 16 = -12$ (rejected)

From (*ii*), $y = 20 - 8 = 12$

Thus, the sides of two squares are 20 m and 12 m. (1)

## Case Study Based Questions

**I. 1.** (*b*) 25 min, 20 min **2.** (*b*) 10 min, 15 min

**3.** (*a*) 15 hrs, 25 hrs **4.** (*d*) 5 min, 8 min

**5.** (*c*) 30 hrs

**II. 1.** (*a*) 33 km/h, 44 km/hr

**2.** (*a*) 60 km/h **3.** (*b*) 48 km/h

**4.** (*c*) 40 km/h, 50 km/h

**5.** (*d*) 25 km/h

## 4. Nature of Roots

We know that the roots of the equation $ax^2 + bx + c = 0, a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where, } b^2 - 4ac = D$$

The quadratic equation $ax^2 + bx + c = 0$ has

($i$) two distinct real roots α and β, if $b^2 - 4ac > 0$ where $\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

($ii$) two equal real roots ($i.e.$, coincident roots), if $b^2 - 4ac = 0$. Roots are given by $\dfrac{-b}{2a}$.

($iii$) no real roots, if $b^2 - 4ac < 0$.

## Formation of Quadratic Equation with given Roots

If α and β are the two roots of a quadratic equation, then the formula to construct the quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

That is $x^2$ – (sum of roots)x + product of roots = 0

**Note:** Let α and β be the two roots of quadratic equation $ax^2 + bx + c = 0$. Then the formula to get sum and product of the roots of a quadratic equation are:

$$\alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

**Example 1.** What will be the nature of roots of quadratic equation $2x^2 + 4x - 7 = 0$?

**Solution.** ∵ $2x^2 + 4x - 7 = 0$

Here, $a = 2$, $b = 4$, $c = -7$

$D = b^2 - 4ac = 16 - 4 \times 2 \times (-7) = 16 + 56 = 72 > 0$

Hence, roots of quadratic equation are real and unequal.

**Example 2.** If $ax^2 + bx + c = 0$ has equal roots, find the value of $c$.

**Solution.** For equal roots $D = 0$

$i.e.$, $b^2 - 4ac = 0 \Rightarrow b^2 = 4ac \Rightarrow c = \dfrac{b^2}{4a}$

**Example 3.** State whether the equation $(x + 1)(x - 2) + x = 0$ has two distinct real roots or not. Justify your answer.

**Solution.** We have $(x + 1)(x - 2) + x = 0 \Rightarrow x^2 - x - 2 + x = 0 \Rightarrow x^2 - 2 = 0$

∵ $D = b^2 - 4ac = 0 - 4(1)(-2) = 8 > 0$

∴ Given equation has two distinct real roots.

**Example 4.** Write the set of values of $k$ for which the quadratic equation $2x^2 + kx + 8$ has real roots. **[Imp.]**

**Solution.** For real roots, $D \geq 0$

$\Rightarrow b^2 - 4ac \geq 0 \Rightarrow k^2 - 4(2)(8) \geq 0$

$\Rightarrow k^2 - 64 \geq 0 \Rightarrow k^2 \geq 64 \Rightarrow k \leq -8$ and $k \geq 8$

**Example 5.** Find the value of $k$ for the quadratic equation $kx(x - 2) + 6 = 0$, so that it has two equal roots.

**Solution.** We have $kx(x - 2) + 6 = 0$ [NCERT] **[Imp.]**

$\Rightarrow kx^2 - 2kx + 6 = 0$

Here, $a = k$, $b = -2k$, $c = 6$

For equal roots, $D = 0$

$i.e.$, $b^2 - 4ac = 0 \Rightarrow (-2k)^2 - 4 \times k \times 6 = 0$

$\Rightarrow 4k^2 - 24k = 0 \Rightarrow 4k(k - 6) = 0$

Either $4k = 0$ or $k - 6 = 0 \Rightarrow k = 0$ or $k = 6$

But $k \neq 0$ (because if $k = 0$, then given equation will not be a quadratic equation).

So, $k = 6$.

**Example 6.** If –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the value of $k$. [Foreign 2014]

**Solution.** ∵ –5 is a root of the equation $2x^2 + px - 15 = 0$

∴ $2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow 50 - 5p - 15 = 0$ or $5p = 35$ or $p = 7$

Again $p(x^2 + x) + k = 0$ or $7x^2 + 7x + k = 0$ has equal roots

∴ $D = 0$

$i.e.$, $b^2 - 4ac = 0$ or $49 - 4 \times 7k = 0 \Rightarrow k = \dfrac{49}{28} = \dfrac{7}{4}$

**Example 7.** Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(*a*) $3x^2 - 4\sqrt{3}x + 4 = 0$ (*b*) $2x^2 - 6x + 3 = 0$ [NCERT] **[Imp.]**

**Solution.** (*a*) We have, $3x^2 - 4\sqrt{3}x + 4 = 0$

Here, $a = 3$, $b = -4\sqrt{3}$ and $c = 4$

Therefore, $D = b^2 - 4ac = (-4\sqrt{3})^2 - 4 \times 3 \times 4 = 48 - 48 = 0$

Hence, the given quadratic equation has real and equal roots.

Thus, $x = \dfrac{-b}{2a} = \dfrac{4\sqrt{3}}{6} = \dfrac{2\sqrt{3}}{3}$

Hence, equal roots of the given equation are $\dfrac{2\sqrt{3}}{3}$ and $\dfrac{2\sqrt{3}}{3}$

(*b*) We have, $2x^2 - 6x + 3 = 0$

Here $a = 2$, $b = -6$, $c = 3$

Therefore, $D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

Hence, given quadratic equation has real and distinct roots.

Thus, $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-6) + \sqrt{12}}{2 \times 2} = \dfrac{6 + 2\sqrt{3}}{4} = \dfrac{3 + \sqrt{3}}{2}$

and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-6) - \sqrt{12}}{2 \times 2} = \dfrac{6 - 2\sqrt{3}}{4} = \dfrac{3 - \sqrt{3}}{2}$

Hence, roots of given equation are $\dfrac{3 + \sqrt{3}}{2}$ and $\dfrac{3 - \sqrt{3}}{2}$.

**Example 8.** Find the value of $p$, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. [AI 2017]

**Solution.** Let the roots of the given equation be $\alpha$ and $6\alpha$.

Thus the quadratic equation is $(x - \alpha)(x - 6\alpha) = 0$

$\Rightarrow \quad x^2 - 7\alpha x + 6\alpha^2 = 0$ ...(*i*)

Given equation can be written as

$$x^2 - \frac{14}{p}x + \frac{8}{p} = 0 \quad \text{...}(ii)$$

Comparing the coefficients in (*i*) and (*ii*) $7\alpha = \dfrac{14}{p}$ and $6\alpha^2 = \dfrac{8}{p}$

Solving to get $p = 3$.

**Example 9.** Find the value(s) of $k$ for which the equation $x^2 + 5kx + 16 = 0$ has real and equal roots.

**Solution.** For roots to be real and equal, $b^2 - 4ac = 0$ [CBSE SP 2018-19]

$\Rightarrow \quad (5k)^2 - 4 \times 1 \times 16 = 0 \quad \Rightarrow \quad k = \pm\dfrac{8}{5}$

**Example 10.** If the roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.

**Solution.** Since the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ has equal roots, therefore discriminant

$D = (b - c)^2 - 4(a - b)(c - a) = 0$

$\Rightarrow \quad b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow \quad b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$

$\Rightarrow \quad 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$

$\Rightarrow \quad (2a)^2 + (-b)^2 + (-c)^2 + 2(2a)(-b) + 2(-b)(-c) + 2(-c)2a = 0$

$\Rightarrow \quad (2a - b - c)^2 = 0$

$\Rightarrow \quad 2a - b - c = 0$

$\Rightarrow \quad 2a = b + c$. Hence Proved.

**Example 11.** If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1 + m^2)$. [Delhi 2017]

**Solution.** The given equation is $(1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$

Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$

Since the given equation has equal roots, therefore

$D = 0 \quad \Rightarrow \quad B^2 - 4AC = 0$

$\Rightarrow \quad (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow \quad 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$\Rightarrow \quad m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$ [Dividing throughout by 4]

$\Rightarrow \quad -c^2 + a^2(1 + m^2) = 0 \quad \Rightarrow \quad c^2 = a^2(1 + m^2)$ Hence Proved.

**Example 12.** Form a quadratic equation whose roots are 2 and 3.

**Solution.** Sum of roots = 2 + 3 = 5

Product of roots = 2 × 3 = 6

So, quadratic equation can be formed as $x^2$ – (sum of roots)$x$ + product of roots = 0

$x^2 - 5x + 6 = 0$

## Exercise 1.4

### I. Very Short Answer Type Questions [1 Mark]

**1. Multiple Choice Questions (MCQs)**

**Choose the correct answer from the given options:**

(1) For what value of $k$, the equation $9x^2 - 24x + k = 0$ has equal roots?
(*a*) 12 (*b*) 16 (*c*) 18 (*d*) 20

(2) The values of $k$ for which the quadratic equation $(k + 1)x^2 + 2(k - 1)x + (k - 2) = 0$ has equal roots, is:
(*a*) $k = 2$ (*b*) $k = 3$ (*c*) $k = 0$ (*d*) None of these

(3) The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is
(*a*) 4 (*b*) ± 4 (*c*) – 4 (*d*) 0

(4) The value(s) of $k$, for which the roots of the equation $3x^2 + 2k + 27 = 0$ are real and equal are
(*a*) $k = 9$ (*b*) $k = \pm 9$ (*c*) $k = -9$ (*d*) $k = 0$

(5) The value of $k$, for which the equation $2x^2 - 10x + k = 0$ has real roots is
(*a*) $k \le \frac{25}{2}$ (*b*) $k \ge \frac{25}{2}$ (*c*) $k = \frac{25}{2}$ (*d*) $k > \frac{25}{2}$

(6) If one root of the equation $(k - 1)x^2 - 10x + 3 = 0$ is the reciprocal of the other, then the value of $k$ is
(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

(7) If the quadratic equation $x^2 - 2x + k = 0$ has equal roots, then value of $k$ is
(*a*) *1* (*b*) 2 (*c*) 3 (*d*) 0

(8) If quadratic equation $3x^2 - 4x + k = 0$ has equal roots, then the value of $k$
(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$ (*c*) 3 (*d*) $\frac{4}{3}$

**2. Assertion-Reason Type Questions**

**In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(*c*) Assertion (A) is true but reason (R) is false.
(*d*) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** The equation $8x^2 + 3kx + 2 = 0$ has equal roots, then the value of $k$ is $\pm \frac{8}{3}$.
**Reason (R):** The equation $ax^2 + bx + c = 0$ has equal roots if $D = b^2 - 4ac = 0$.

(2) **Assertion (A):** The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary.
**Reason (R):** If discriminant $D = b^2 - 4ac < 0$, then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

**3. Answer the following.**

(1) Find the value of $p$, so that the quadratic equation $px(x - 3) + 9 = 0$ has equal roots. [CBSE 2014]

(2) For what values of $k$, the roots of the equation $x^2 + 4x + k = 0$ are real? [Delhi 2019]

(3) Find the value of $k$ for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other. [Delhi 2019]

(4) Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, hence find the nature of its roots. [NCERT]

(5) If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of $k$. [CBSE 2018]

(6) For what values of $k$, the equation $9x^2 + 6kx + 4 = 0$ has equal roots? [CBSE Standard SP 2020-21]

(7) For what value(s) of '$a$' quadratic equation $3ax^2 - 6x + 1 = 0$ has no real roots? [CBSE Standard SP 2020-21]

### II. Short Answer Type Questions - I [2 Marks]

**4.** State whether the quadratic equation $4x^2 - 5x + \frac{25}{16} = 0$ has two distinct real roots or not. Justify your answer. [NCERT Exemplar]

5. Find the value of $k$ so that the quadratic equation $kx(3x-10)+25=0$, has two equal roots. [Delhi 2011]
6. For what value of $k$ does the quadratic equation $(k-5)x^2+2(k-5)x+2=0$ have equal roots? [Foreign 2011]
7. Find the value(s) of $k$ so that the quadratic equation $2x^2+kx+3=0$ has equal roots. [Delhi 2012]
8. Find the value(s) of $k$ so that the quadratic equation $x^2-4kx+k=0$ has equal roots. [Delhi 2012]
9. Find the value(s) of $k$ so that the quadratic equation $3x^2-2kx+12=0$ has equal roots. [Delhi 2012]
10. Find the values of $k$ for which the quadratic equation $9x^2-3kx+k=0$ has equal roots. [AI 2014]

## III. Short Answer Type Questions-II [3 Marks]

11. Find the nature of the roots of the following quadratic equations. If the real roots exist, then also find them.
(*a*) $4x^2+12x+9=0$ (*b*) $3x^2+5x-7=0$ (*c*) $7y^2-4y+5=0$ **[Imp.]**
12. If 2 is a root of the quadratic equation $3x^2+px-8=0$ and the quadratic equation $4x^2-2px+k=0$ has equal roots, find the value of $k$. [CBSE (F) 2014]
13. Find the value of $p$ for which the quadratic equation $(p+1)x^2-6(p+1)x+3(p+9)=0$, $p\neq-1$ has equal roots. Hence, find the roots of the equation. [Delhi 2015]
14. Find that non-zero value of $k$, for which the quadratic equation $kx^2+1-2(k-1)x+x^2=0$ has equal roots. Hence, find the roots of the equation. [Delhi 2015]
15. The roots $\alpha$ and $\beta$ of the quadratic equation $x^2-5x+3(k-1)=0$ are such that $\alpha-\beta=1$. Find the value $k$. [CBSE Standard SP 2020-21]
16. Find the values of $k$ for which the quadratic equation $(3k+1)x^2+2(k+1)x+1=0$ has equal roots. Also find these roots. [Delhi 2014]

## IV. Long Answer Type Questions [5 Marks]

17. Find whether the equation $\frac{1}{2x-3}+\frac{1}{x-5}=1, x\neq\frac{3}{2}, 5$ has real roots. If real roots exist, find them. [NCERT Exemplar]
18. Check whether the equation $5x^2-6x-2=0$ has real roots and if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation. [CBSE SP 2018]

## Answers and Hints

**1.** (1) (*b*) 16 (1) (2) (*b*) $k=3$ (1)
(3) (*b*) $\pm 4$ (1) (4) (*b*) $k=\pm 9$ (1)
(5) (*a*) $k\le\frac{25}{2}$ (1) (6) (*d*) 4 (1)
(7) (*a*) 1 (1) (8) (*d*) $\frac{4}{3}$ (1)

**2.** (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(2) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)

**3.** (1) $px(x-3)+9=0$
$\Rightarrow px^2-3px+9=0$
When roots are equal,
$D=b^2-4ac=0$
$9p^2-36p=0$
$\Rightarrow 9p(p-4)=0$
$\Rightarrow p=0, p=4$
But $p\neq 0$
[$\because$ In quadratic equation, $a\neq 0$]
$\therefore p=4$ (1)

(2) For real roots, $D\ge 0$
$\Rightarrow b^2-4ac\ge 0$
$\Rightarrow (4)^2-4\times 1\times k\ge 0$
$\Rightarrow 16-4k\ge 0$
$\Rightarrow 16\ge 4k, k\le 4$ (1)

(3) Let the roots of the given equation be $\alpha$ and $\frac{1}{\alpha}$.
$\alpha.\frac{1}{\alpha}=\frac{c}{a}=\frac{k}{3}\Rightarrow k=3$ (1)

(4) $-8$, no real roots (1)

(5) $x=3$ is one root of the equation
$\therefore 9-6k-6=0$
$\Rightarrow k=\frac{1}{2}$ (1)

(6) $9x^2+6kx+4=0$
$(6k)^2-4\times 9\times 4=0$ (½)
$36k^2=144$
$\Rightarrow k^2=4$
$k=\pm 2$ (½)

(7) $3ax^2-6x+1=0$ (½)
$(-6)^2-4(3a)(1)<0$
$12a>36$
$\Rightarrow a>3$ (½)

**4.** No, $D=0$ (2)

**5.** $kx(3x-10)+25=0$
$\Rightarrow 3kx^2-10kx+25=0$

$$D = (-10k)^2 - 4 \times 3k \times 25$$
$$= 100k^2 - 300k$$

For equal roots, $D = 0$ (1)

$\Rightarrow \quad 100k^2 - 300k = 0$

$\Rightarrow \quad 100k(k - 3) = 0$

$\Rightarrow \quad k = 0$ or $k = 3$

But $\quad k \neq 0,$

So, $\quad k = 0$ (Rejected)

[$\because$ In quadratic equation, $a \neq 0$]

Hence, $\quad k = 3$ (1)

**6.** $k = 7$ (2)

**7.** $2x^2 + kx + 3 = 0$

For equal roots, $D = 0$ (1)

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad k^2 - 24 = 0$

$\Rightarrow \quad k = \pm 2\sqrt{6}$ (1)

**8.** $x^2 - 4kx + k = 0$

Since given equation has equal roots,

$\therefore \quad D = 0$ (1)

$16k^2 - 4k = 0$

$\Rightarrow \quad 4k(4k - 1) = 0$

$\Rightarrow \quad k = 0$ and $k = \frac{1}{4}$ (1)

**9.** $3x^2 - 2kx + 12 = 0$

Since given equation has equal roots, so

$D = 0$ (1)

$\Rightarrow \quad 4k^2 - 144 = 0$

$\Rightarrow \quad 4(k^2 - 36) = 0$

$\Rightarrow \quad k = \pm 6$

$\Rightarrow \quad k = 6$ and $k = -6$ (1)

**10.** For equal roots, $D = 0$

$\Rightarrow \quad 9k^2 - 36k = 0$

$\Rightarrow \quad 9k(k - 4) = 0$ (1)

$\Rightarrow \quad k = 0$ or $k = 4$ (1)

**11.** (*a*) Real and equal roots: $\frac{-3}{2}, \frac{-3}{2}$ (1)

(*b*) Real and distinct roots:

$\frac{-5 + \sqrt{109}}{6}, \frac{-5 - \sqrt{109}}{6}$ (1)

(*c*) No real roots (1)

**12.** $3(2)^2 + p(2) - 8 = 0$

$\Rightarrow \quad 12 + 2p - 8 = 0$

$\Rightarrow \quad p = -2$ ...(*i*)(1)

So, equation becomes

$4x^2 + 4x + k = 0$ [using (*i*)](1)

For equal roots, $D = 0$

$\Rightarrow \quad (4)^2 - 4 \times 4 \times k = 0$

$\Rightarrow \quad 16 = 16k$

$\Rightarrow \quad k = 1$ (1)

**13.** 3, 3. (3)

**14.** $kx^2 + 1 - 2(k - 1)x + x^2 = 0$

$\Rightarrow \quad (k + 1)x^2 - 2(k - 1)x + 1 = 0$

$\because$ Above equation has equal roots, (1)

So, discriminant, $D = 0$

$\Rightarrow \quad \{-2(k - 1)\}^2 - 4 \times (k + 1) \times 1 = 0$ (1)

$\Rightarrow \quad 4(k^2 - 2k + 1) - 4(k + 1) = 0$

$\Rightarrow \quad 4k^2 - 12k = 0$

$\Rightarrow \quad 4k(k - 3) = 0$

$\Rightarrow \quad k = 3$ (as $k \neq 0$) (1)

**15.** $k = 3$ (3)

**16.** For equal roots, $D = 0$

$\{2(k + 1)\}^2 - 4(3k + 1) \cdot 1 = 0$ (1)

$\Rightarrow \quad 4(k^2 + 2k + 1) - 12k - 4 = 0$

$\Rightarrow \quad 4k^2 + 8k + 4 - 12k - 4 = 0$ (1)

$\Rightarrow \quad 4k^2 - 4k = 0$

$\Rightarrow \quad 4k(k - 1) = 0$

$\Rightarrow \quad k = 0, 1$ (1)

**17.** Yes, $\frac{8 \pm 3\sqrt{2}}{2}$ (5)

**18.** Discriminant $= b^2 - 4ac$

$= 36 - 4 \times 5 \times (-2)$

$= 76 > 0$

So, the given equation has two distinct real roots

$5x^2 - 6x - 2 = 0$ (1)

Multiplying both sides by 5, we get

$(5x)^2 - 2 \times (5x) \times 3 = 10$

$\Rightarrow \quad (5x)^2 - 2 \times (5x) \times 3 + 3^2 = 10 + 3^2$ (1)

$\Rightarrow \quad (5x - 3)^2 = 19$

$\Rightarrow \quad 5x - 3 = \pm\sqrt{19}$

$\Rightarrow \quad x = \frac{3 \pm \sqrt{19}}{5}$ (1)

Verification: $5\left(\frac{3 + \sqrt{19}}{5}\right)^2 - 6\left(\frac{3 + \sqrt{19}}{5}\right) - 2$

$= \frac{9 + 6\sqrt{19} + 19}{5} - \frac{18 + 6\sqrt{19}}{5} - \frac{10}{5} = 0$ (1)

Similarly, $5\left(\frac{3 - \sqrt{19}}{5}\right)^2 - 6\left(\frac{3 - \sqrt{19}}{5}\right) - 2 = 0$

(1)

## EXPERTS' OPINION

**Questions based on following types are very important for Exams. So, students are advised to revise them thoroughly.**

1. To find the roots of a quadratic equation by factorisation.
2. To find the roots of a quadratic equation by quadratic formula.
3. To find the nature of roots of quadratic equation.
4. To find the value of unknown when nature of roots is given.

## IMPORTANT FORMULAE

For quadratic equation $ax^2 + bx + c = 0$,

- Roots are given by $x = \frac{-b \pm \sqrt{D}}{2a}$, where $D = b^2 - 4ac$
- If $D > 0$, there are two distinct real roots $\alpha$ and $\beta$ which are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a}, \beta = \frac{-b - \sqrt{D}}{2a}$$

- If $D = 0$, there are two equal roots and each equal root is given by $x = \frac{-b}{2a}$
- If $D < 0$, then there is no real roots.

## QUICK REVISION NOTES

- A quadratic equation in the variable $x$ is of the form $ax^2 + bx + c = 0$, where $a, b, c$ are real numbers and $a \neq 0$.
- A real number $\alpha$ is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$. The zeroes of the quadratic polynomial $ax^2 + bx + c = 0$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.
- If we can factorise $ax^2 + bx + c$, $a \neq 0$ into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- **Quadratic formula:** The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided $b^2 - 4ac \geq 0$.
- A quadratic equation $ax^2 + bx + c = 0$ has
  - (*i*) two distinct real roots, if $b^2 - 4ac > 0$
  - (*ii*) two equal roots (coincident roots) if $b^2 - 4ac = 0$.
  - (*iii*) no real roots, if $b^2 - 4ac < 0$.

## COMMON ERRORS

| Errors | Corrections |
|---|---|
| (*i*) While finding the discriminant, taking incorrectly the value of $a$ and $b$ with variables. | (*i*) Remember that coefficient of $x^2$ is $a$ and coefficient of $x$ is $b$. |
| (*ii*) Incorrectly considering two consecutive even integers as $x$ and $x + 1$. | (*ii*) Remember that difference between two consecutive even integers is 2. So, we should take $x$ and $x + 2$. |
| (*iii*) Incorrectly writing difference between two positive integers and difference between their reciprocals as $x - y =$ ___ and $\frac{1}{x} - \frac{1}{y} =$ ___. | (*iii*) Attention that when $x > y$, $\frac{1}{x} < \frac{1}{y}$ and will be negative. |
| (*iv*) In factorisation method, equating factors incorrectly. For example: $(x - 4)(x - 2) = 4$ $\Rightarrow x - 4 = 4, x - 2 = 4$. | (*iv*) It is not correct. We should equating factors to zero only. For example, if $(x - 4)(x - 2) = 0$, then $x - 4 = 0, x - 2 = 0$. |

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