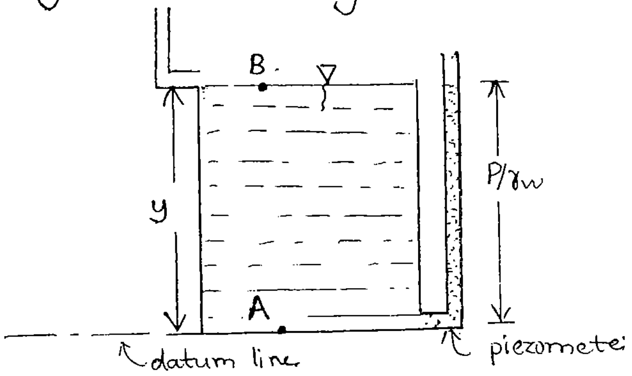
Sept,
ONDAY

6. PERMEABILITY

- Flow occurs only when there is difference in the total heads b/w the two points.

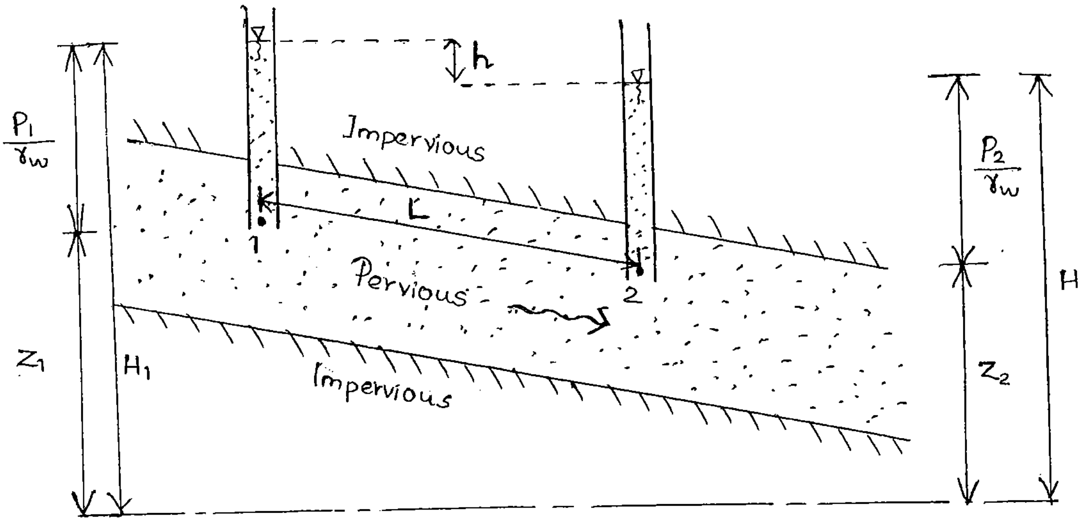
- Pressure head difference alone or elevation difference alone may not cause the flow.

- In the case of soils the velocity head is neglected (negligible),



At point A	At point B.
$\frac{P}{\gamma_w} = y$.	$\frac{P}{\gamma_w} = 0$.
$z = 0$.	$z = y$.
Total head, $H = y$	Total head, $H = y$.

∵ total head is same $(=y)$ at both A & B, no flow occurs b/w A B.



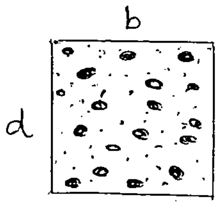
∴ Total head difference &or total head loss, $h = H_1 - H_2$

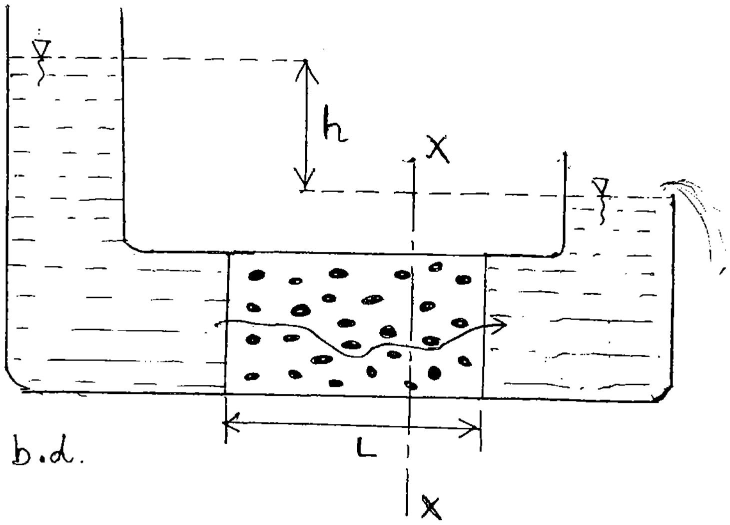
The total head loss b/w any two points in a soil mass is equal to the difference in the elevations of water in the two piezometers kept at those two points.

Hydraulic gradient, $i = \frac{h}{L}$; L → seepage length.

Hydraulic gradient is the head loss per unit seepage length.

c/s at XX :





Total c/s area of soil, $A = b.d.$

Area of voids or Actual area of flow } $= A_v = nA$. ; n → porosity of soil.

Discharge velocity or Apparent velocity } $V = \frac{Q}{A}$.

Actual velocity or Seepage velocity } $V_s = \frac{Q}{A_v}$

$$Q = AV = A_v V_s$$

$$\boxed{V_s = \frac{V}{n}}$$

$n < 1$ always, $\Rightarrow$ $V_s > V$.

→ Darcy's Law

Discharge velocity ∝ Hydraulic Gradient.

$$V \propto i$$

DARCY'S EQUATION $\boxed{V = ki}$

K → Coefficient of Permeability of soil.

If $i = 1$, $v = K \Rightarrow$ Coeffecient of ~~velocity~~ permeability is the discharge velocity occuring under unit hydraulic gradient.

Units of k : cm/s. or m/s or m/hour.

If $k > 10^{-1}$ cm/s $\Rightarrow$ 'Permeable Soil'

$k < 10^{-7}$ cm/s $\Rightarrow$ 'Impermeable Soil'

Permeable soil – gravel, coarse sand.
Impermeable soil – stiff clays.

Soil	Gravel	Sand	Silt	Clay.
K (cm/s)	10^{0} $(10^{-1} - 10^{1})$	10^{-2} $(10^{-1} - 10^{-3})$	10^{-4} $(10^{-3} - 10^{-5})$	10^{-6}. $(10^{-5} - 10^{-7})$

- Darcy's Law is valid for Laminar flow only

In soils, if reynold's number, $Re \leq 1$, it is laminar.

$$Re = \frac{\rho v d}{\mu}$$

$$Re \propto v \propto K$$

$\therefore$ as $K \downarrow$, $Re \downarrow$ and flow becomes laminar.

- Clay, silt & fine sand – flow is laminar.

$$V = Ki$$

$$\boxed{V_s = K_p i}$$

$K_p \rightarrow$ coeffecient of percolation.

$$\boxed{K_p = \frac{K}{n}}$$

$$\boxed{K = C . D_{10}^{2} \frac{e^3}{1+e} . \frac{\gamma_w}{\mu}}$$

$C \rightarrow$ shape constant ($c = 1$ for perfectly spherical particle)

$\mu \rightarrow$ dynamic viscosity.

→ Factors affecting K of soil :

- shape of particle
- size of particle
- void ratio
- properties of fluid
- degree of saturation.
- organic matter
- specific surface area.
- stratification etc.

* Effect of size , D_{10}

$$K \propto D_{10}^2$$

* Effect of Specific Surface Area

$$K \propto \frac{1}{SSA}$$

* Effect of shape.

K of rounded particles is more compared to angular particle since SSA of rounded particle is less.

* Effect of void ratio

$$K \propto \frac{e^3}{1+e} \quad ; \quad \text{also} \quad K \propto e^2$$

also $\log K \propto e$ (latest one)

The more the void ratio, more will be permeability. But this cannot explain the reason for clay's low permeability and clay has low permeability though it has high void ratio,

$$K \propto \frac{Ce^3}{1+e}$$

Clay has lowest value of c. So the product of c & e will be low.

$$K \propto \gamma_w$$

$$K \propto \frac{1}{\mu} \propto \text{temperature} \qquad \mu \propto \frac{1}{\text{temp}}$$

$$\Rightarrow K \propto \text{temperature}.$$

$$\therefore K \propto \frac{\gamma_w}{\eta} \Rightarrow \boxed{\frac{K_2}{K_1} = \frac{\gamma_{w2}}{\gamma_{w1}} \cdot \frac{\mu_1}{\eta_2}}$$

During summer season day, permeability will be more.

* Effect of Degree of Saturation

k of partially saturated soil is relatively less compared to fully saturated soil. (air blocking)

* Effect of Organic matter.

Organic matter decreases k of soil. Due to low specific gravity of organic matter, it flows along with water and fills the voids.

$$k = \overbrace{C . D_{10}^{2} . \frac{e^{3}}{1+e}}^{\text{soil property}} . \underbrace{\frac{\gamma_w}{\mu}}_{\text{fluid properties.}}$$

$$k = k_0 . \frac{\gamma_w}{\mu}$$

$k_0 \rightarrow$ intrinsic permeability of soil (Inherent property).

Units of k_0 : cm^2 or m^2 or darcys

$$\boxed{1 \text{ darcy} = 9.87 \times 10^{-13} \ m^2}$$

→ Tests to Determine k of soil.

1. Constant Head Test.
2. Variable Head test } lab test.
3. Capillarity Permeability test
4. Consolidation test
5. Pumping out test } Field test.
6. Pumping in test.

Pumping out test : most accurate, used for large engg. projects.
Consolidation test : suitable for impermeable clays.
Capillarity Permeability test : for partially saturated soils.
Constant head test : for coarse grained soils
Variable head test : for fine grained soils
Pumping in test : to find k of individual layer of soil.

2nd Sept,
TUESDAY

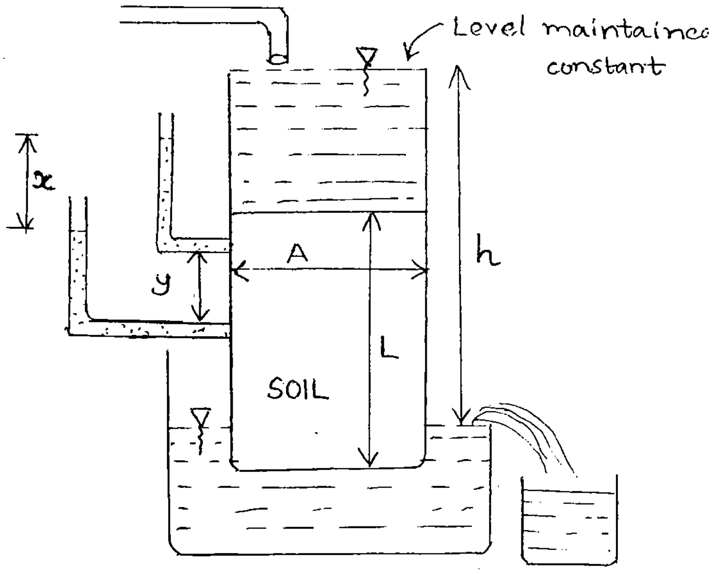
→ Constant Head Test

$$Q = k\,i\,A$$

$$Q = k\,\frac{h}{L}\,A$$

$$i = \frac{h}{L} = \frac{x}{y}$$

$$\boxed{k = \frac{QL}{Ah}}$$



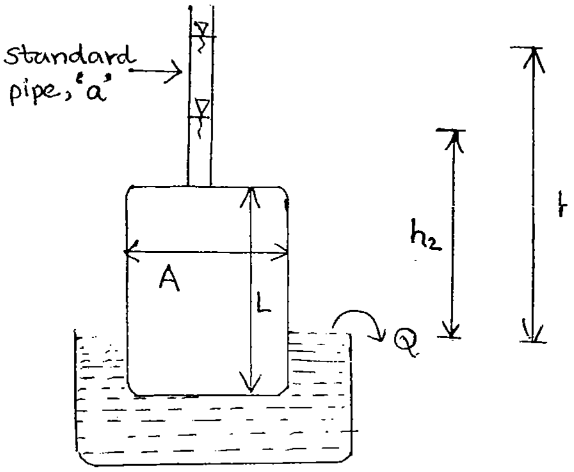
→ Variable Head Test

$$a\,dh = Q\cdot dt$$

$$= k\,\frac{h}{L}\,A\cdot dt$$

$$\int_{h_2}^{h_1}\frac{dh}{h} = \frac{kA}{La}\int_0^t dt$$

$$\therefore \quad \boxed{K = \frac{aL}{At}\log_e\left(\frac{h_1}{h_2}\right)}$$



→ Flow Parallel to Bedding Plates.

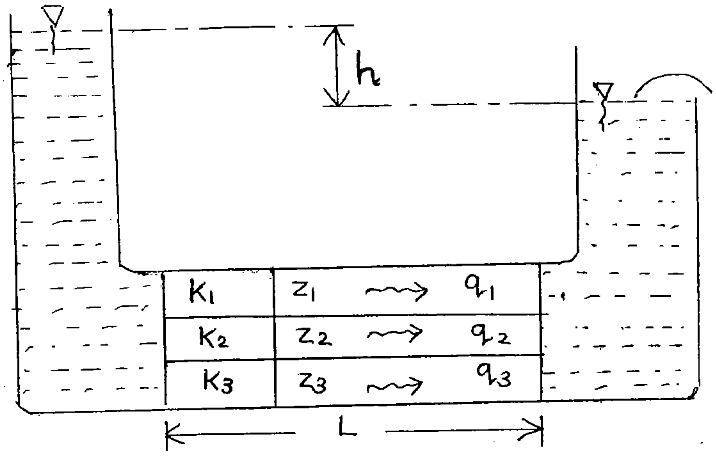
Total head loss = h

Let h_1, h_2, h_3 → head losses in layers 1, 2 & 3 rsptly.

$$h_1 = h_2 = h_3 = h.$$

$$\therefore \; i_1 = i_2 = i_3 = i = \frac{h}{L}$$

$$Q = q_1 + q_2 + q_3$$



$q_1 = k_1\,i_1\,A_1$; $q_2 = k_2\,i_2\,A_2$; $q_3 = k_3\,i_3\,A_3$

$= k_1\,\frac{h_1}{L}.(z_1 \times 1)$ $\quad = k_2\cdot\frac{h_2}{L}.(z_2 \times 1)$ $\quad = k_3\cdot\frac{h_3}{L}.(z_3 \times 1)$

Let k_H be average permeability for entire soil as a whole.

$$Q = k_H \cdot i \cdot A$$

$$= k_H \cdot \frac{h}{L} \cdot (z_1 + z_2 + z_3) \times 1$$

$$k_H \cdot i \cdot A = k_1 \cdot i_1 \cdot A_1 + k_2 \cdot i_2 \cdot A_2 + k_3\, i_3\, A_3.$$

$$k_H \cdot (z_1 + z_2 + z_3) \cdot 1 = k_1 \cdot z_1 \cdot 1 + k_2\, z_2 \cdot 1 + k_3\, z_3 \cdot 1.$$

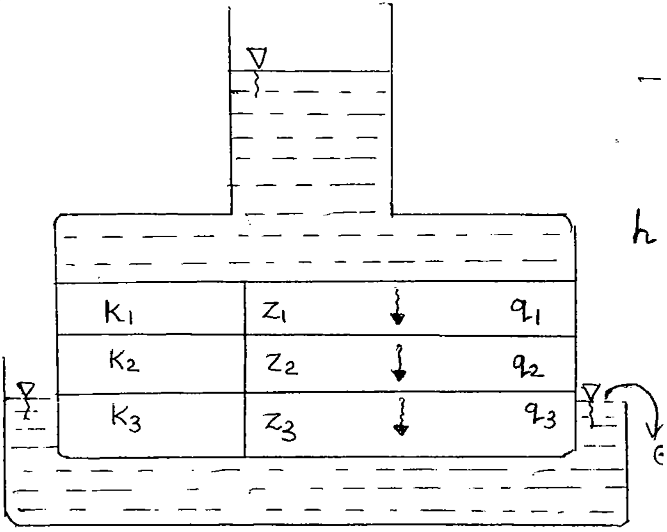
$$\therefore \boxed{k_H = \frac{k_1 z_1 + k_2 z_2 + k_3 z_3}{z_1 + z_2 + z_3}}$$

→ Flow Perpendicular to Bedding Plane.

$h \rightarrow$ total head loss.

$$h = h_1 + h_2 + h_3$$

$$q_1 = q_2 = q_3 = Q.$$



Let K_v be average permeability for entire soil.

$$q_1 = k_1 \cdot i_1 \cdot A_1$$

$$= k_1 \cdot \frac{h_1}{z_1} \cdot A.$$

$$q_2 = k_2\, i_2\, A_2 \quad ; \quad q_3 = k_3 \cdot \frac{h_3}{z_3} \cdot A.$$

$$= k_2 \cdot \frac{h_2}{z_2} \cdot A$$

$$Q = k_v \cdot i \cdot A$$

$$= k_v \cdot \frac{h}{z_1 + z_2 + z_3} \cdot A$$

$$\therefore \boxed{K_V = \frac{z_1 + z_2 + z_3}{\frac{z_1}{K_1} + \frac{z_2}{K_2} + \frac{z_3}{K_3}}}$$

$$\boxed{K_H > K_V}$$

P-30

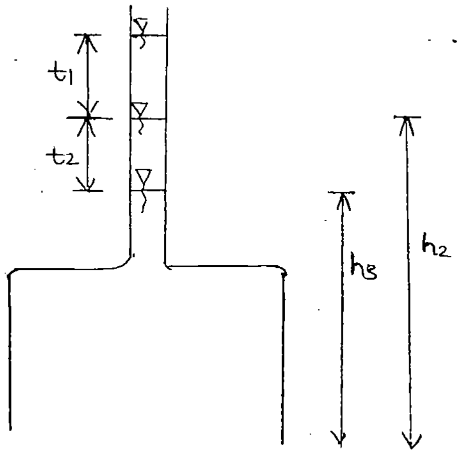
1.

$$\frac{k_2}{k_1} = \frac{\gamma_{w2}}{\gamma_{w1}} , \frac{\mu_1}{\mu_2}$$

$$= \frac{0.9\,\gamma_{w1}}{\gamma_{w1}} , \frac{\mu_1}{0.75\,\mu_1} = 1.2 .$$

$k_2 = 1.2\ k_1 \implies 20\%$ increase.

2. Given $t_1 = t_2$.



$$\frac{aL}{AK} \log_e \frac{h_1}{h_2} = \frac{aL}{A \cdot K} \log_e \frac{h_2}{h_3}.$$

$$\frac{h_1}{h_2} = \frac{h_2}{h_3}$$

$$h_2^2 = h_1 h_3$$

3.

$k_1 = 2$ $\quad z_1 = 2$

$k_2 = 3$ $\quad z_2 = 1$

$k_3 = 1$ $\quad z_3 = 2.$

$$K_v = \frac{z_1 + z_2 + z_3}{\frac{z_1}{K_1} + \frac{z_2}{K_2} + \frac{z_3}{K_3}} = \frac{5}{1 + \frac{1}{3} + 2} = \underline{\underline{\frac{3}{2}}}$$

4.

$$k_v = \frac{6 + 6 + 6}{\frac{6}{30 \times 10^{-5}} + \frac{6}{4 \times 10^{-4}} + \frac{6}{6 \times 10^{-4}}} = \overset{4.}{2}.667 \times 10^{-4}\ \text{cm/s}$$

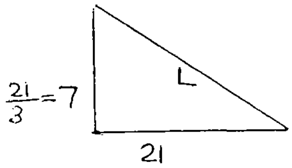
$$t = \frac{aL}{A \cdot K_v} \log_e \frac{h_1}{h_2} = \frac{2 \times 18}{22 \times \overset{4.}{2}.67 \times 10^{-4}} \log_e \frac{25}{10}$$

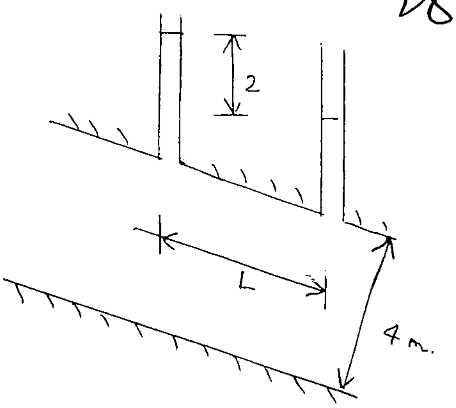
$$= 3748.46\ \text{s}$$

$$= \underline{\underline{62.474}}\ \text{min}$$

05. $L = \sqrt{21^2 + 7^2}$

$= 22.135$ m.





$Q = kiA$

$= k\frac{h}{L} \cdot dx1$

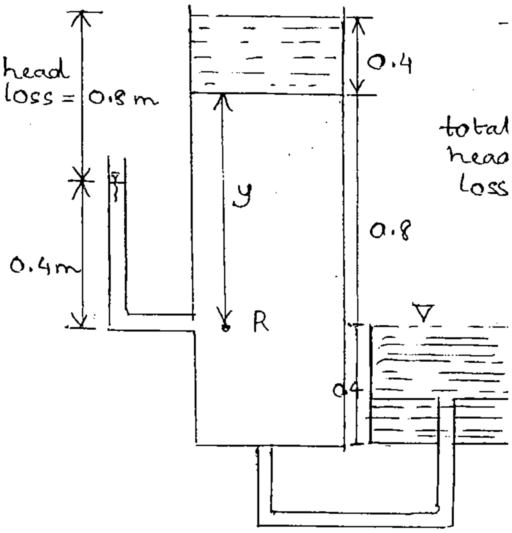
$\frac{5}{1000} = k \cdot \frac{2}{22.135} \times 4 \times 1$

$k = 0.0138$ m/hr $= \underline{3.85 \times 10^{-6}}$ m/s

06. Total head loss $= 0.8 + 0.4 = 1.2$ m.

Total seepage length $= 0.8 + 0.4 = 1.2$ m.

$i = \frac{h}{L} = \frac{1.2}{1.2} = 1$



For a seepage length of y,

head loss $= i \times y = 1 \times 0.8 = \underline{0.8 \text{ m}}$

∴ Pressure head at R $= 1.2 - 0.8 = \underline{0.4 \text{ m}}$

∵ datum line is not given, let as assume,

datum line at the d/s water surface

If d/s water surface is datum, then datum head at R = 0.

Total head at R = Pressure head + datum head

$= 0.4 + 0 = \underline{0.4 \text{ m}}$

* If datum is taken to be bottom of soil,

datum head at R = 0.4 m.

Total head at R = pressure head + datum head

$= 0.4 + 0.4 = \underline{0.8 \text{ m}}$

7 Discharge velocity, $V = ki = k$.

Seepage velocity, $V_s = \frac{V}{n} = \frac{k}{0.5} = \underline{\underline{2k}}$

→ Allen Hazen's Equation :

$$K \approx 100 \; D_{10}^2 \qquad D \rightarrow cm$$
$$K \rightarrow cm/s$$

Q The hydraulic conductivity of a soil at a void ratio of 0.8 is 0.047. cm/s. Estimate the hydraulic conductivity at a void ratio of 0.5.

At $e_1 = 0.8, \quad K_1 = 0.047$

$e_2 = 0.5, \quad K_2 = ?$

$$K \propto \frac{e^3}{1+e.} \Rightarrow \frac{K_1}{K_2} = \frac{e_1^3/1+e_1}{e_2^2/1+e_2.}$$

$$\frac{0.047}{K_2}, = \frac{0.8^3/1.8}{0.5^3/1.5}$$

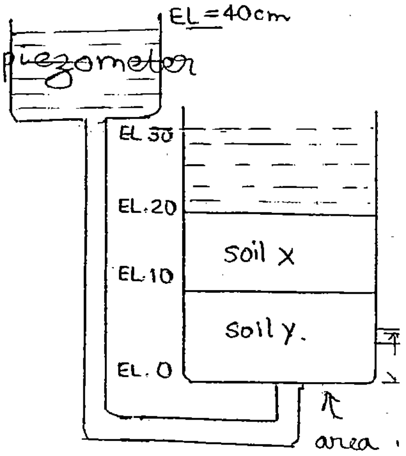
$K_2 = \underline{\underline{0.0137}}$ cm/s

If $K \propto e^2$, $K_2 = \underline{\underline{0.01835}}$ cm/s

Q. In fig. shown below, the soil X has a permeability of 4×10^{-3} cm, and the head loss in soil Y is 9 times the head loss in soil X a) What is the permeability of the soil Y?

b) What is seepage rate per hour?

c) To what elevation would water rise in a piezometer inserted in soil Y at elevation 5 cm ?



a) Total head loss, h = 40−30 = 10 cm.

$$h_x + h_y = h. = 10 \text{ cm} \longrightarrow ①$$

Given, $h_y = 9 \, h_x.$

$\therefore \; h_x = 1$ cm

$h_y = 9$ cm

$$Q_x = Q_y$$

$$k_x . \frac{h_x}{Z_x} . A_x = k_y . \frac{h_y}{Z_y} . A_y .$$

$$4 \times 10^{-3} \times \frac{1}{10} \times 10 = k_y . \frac{9}{10} . 10$$

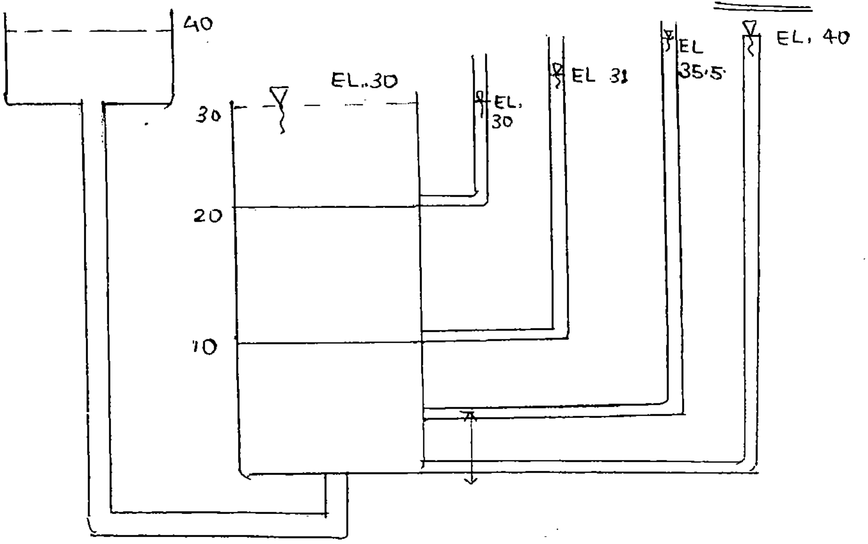
$$k_y = 4.4 \times 10^{-4} \text{ cm/s}$$

b). $Q_x = k_x . \dfrac{h_x}{Z_x} . A_x . = Q .$

$$= 4 \times 10^{-3} \times \frac{1}{10} \times 10 = 4 \times 10^{-3} \text{ cm}^3/\text{s}$$

$$= 4 \times 10^{-3} \times 60 \times 60 = \underline{\underline{14.4}} \text{ cm}^3/\text{hr}$$

c).



Pressure head = height of water column in piezometer.

H