

17. Increasing and Decreasing Functions

Exercise 17.1

1. Question

Prove that the function $f(x) = \log_e x$ is increasing on $(0, \infty)$.

Answer

let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$$\Rightarrow \log_e x_1 < \log_e x_2$$

$$\Rightarrow f(x_1) < f(x_2)$$

So, $f(x)$ is increasing in $(0, \infty)$

2. Question

Prove that the function $f(x) = \log_a x$ is increasing on $(0, \infty)$ if $a > 1$ and decreasing on $(0, \infty)$, if $0 < a < 1$.

Answer

case I

When $a > 1$

let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$$\Rightarrow \log_e x_1 < \log_e x_2$$

$$\Rightarrow f(x_1) < f(x_2)$$

So, $f(x)$ is increasing in $(0, \infty)$

case II

When $0 < a < 1$

$$f(x) = \log_a x = \frac{\log x}{\log a}$$

when $a < 1 \Rightarrow \log a < 0$

let $x_1 < x_2$

$$\Rightarrow \log x_1 < \log x_2$$

$$\Rightarrow \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a} [\because \log a < 0]$$

$$\Rightarrow f(x_1) > f(x_2)$$

So, $f(x)$ is decreasing in $(0, \infty)$

3. Question

Prove that $f(x) = ax + b$, where a, b are constants and $a > 0$ is an increasing function on \mathbb{R} .

Answer

we have,

$$f(x) = ax + b, a > 0$$

let $x_1, x_2 \in \mathbb{R}$ and $x_1 > x_2$

$\Rightarrow ax_1 > ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b > ax_2 + b$ for some b

$\Rightarrow f(x_1) > f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

So, $f(x)$ is increasing function of \mathbb{R}

4. Question

Prove that $f(x) = ax + b$, where a, b are constants and $a < 0$ is a decreasing function on \mathbb{R} .

Answer

we have,

$f(x) = ax + b, a < 0$

let $x_1, x_2 \in \mathbb{R}$ and $x_1 > x_2$

$\Rightarrow ax_1 < ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b < ax_2 + b$ for some b

$\Rightarrow f(x_1) < f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, $f(x)$ is decreasing function of \mathbb{R}

5. Question

Show that $f(x) = \frac{1}{x}$ is a decreasing function on $(0, \infty)$.

Answer

we have

$$f(x) = \frac{1}{x}$$

let $x_1, x_2 \in (0, \infty)$ We have, $x_1 > x_2$

$$\Rightarrow \frac{1}{x_1} < \frac{1}{x_2}$$

$\Rightarrow f(x_1) < f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, $f(x)$ is decreasing function

6. Question

Show that $f(x) = \frac{1}{1+x^2}$ decreases in the interval $[0, \infty)$ and increases in the interval $(-\infty, 0]$.

Answer

We have,

$$f(x) = \frac{1}{1+x^2}$$

Case 1

When $x \in [0, \infty)$

Let $x_1, x_2 \in (0, \infty]$ and $x_1 > x_2$

$$\Rightarrow x_1^2 > x_2^2$$

$$\Rightarrow 1 + x_1^2 > 1 + x_2^2$$

$$\Rightarrow \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$$

$$\Rightarrow f(x_1) < f(x_2)$$

$\therefore f(x)$ is decreasing on $[0, \infty)$.

Case 2

When $x \in (-\infty, 0]$

Let $x_1 > x_2$

$$\Rightarrow x_1^2 < x_2^2$$

$$\Rightarrow 1 + x_1^2 < 1 + x_2^2$$

$$\Rightarrow \frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$$

$$\Rightarrow f(x_1) > f(x_2)$$

$\therefore f(x)$ is increasing on $(-\infty, 0]$.

Thus, $f(x)$ is neither increasing nor decreasing on \mathbb{R} .

7. Question

Show that $f(x) = \frac{1}{1+x^2}$ is neither increasing nor decreasing on \mathbb{R} .

Answer

We have,

$$f(x) = \frac{1}{1+x^2}$$

Case 1

When $x \in [0, \infty)$

Let $x_1 > x_2$

$$\Rightarrow x_1^2 > x_2^2$$

$$\Rightarrow 1 + x_1^2 > 1 + x_2^2$$

$$\Rightarrow \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$$

$$\Rightarrow f(x_1) < f(x_2)$$

$\therefore f(x)$ is decreasing on $[0, \infty)$.

Case 2

When $x \in (-\infty, 0]$

Let $x_1 > x_2$

$$\Rightarrow x_1^2 < x_2^2$$

$$\Rightarrow 1 + x_1^2 < 1 + x_2^2$$

$$\Rightarrow \frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$$

$$\Rightarrow f(x_1) > f(x_2)$$

$\therefore f(x)$ is increasing on $(-\infty, 0]$.

Thus, $f(x)$ is neither increasing nor decreasing on \mathbb{R} .

8. Question

Without using the derivative, show that the function $f(x) = |x|$ is

A. strictly increasing in $(0, \infty)$

B. strictly decreasing in $(-\infty, 0)$.

Answer

We have,

$$f(x) = |x| = \begin{cases} x, & x > 0 \end{cases}$$

(a) Let $x_1, x_2 \in (0, \infty)$ and $x_1 > x_2$

$$\Rightarrow f(x_1) > f(x_2)$$

So, $f(x)$ is increasing in $(0, \infty)$

(b) Let $x_1, x_2 \in (-\infty, 0)$ and $x_1 > x_2$

$$\Rightarrow -x_1 < -x_2$$

$$\Rightarrow f(x_1) > f(x_2)$$

$\therefore f(x)$ is strictly decreasing on $(-\infty, 0)$.

9. Question

Without using the derivative show that the function $f(x) = 7x - 3$ is strictly increasing function on \mathbb{R} .

Answer

Given,

$$f(x) = 7x - 3$$

Lets $x_1, x_2 \in \mathbb{R}$ and $x_1 > x_2$

$$\Rightarrow 7x_1 > 7x_2$$

$$\Rightarrow 7x_1 - 3 > 7x_2 - 3$$

$$\Rightarrow f(x_1) > f(x_2)$$

$\therefore f(x)$ is strictly increasing on \mathbb{R} .

Exercise 17.2

1 A. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 10 - 6x - 2x^2$$

Answer

Given:- Function $f(x) = 10 - 6x - 2x^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$$f(x) = 10 - 6x - 2x^2$$

$$\Rightarrow f'(x) = \frac{d}{dx}(10 - 6x - 2x^2)$$

$$\Rightarrow f'(x) = -6 - 4x$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow -6 - 4x > 0$$

$$\Rightarrow -4x > 6$$

$$\Rightarrow x < -\frac{6}{4}$$

$$\Rightarrow x < -\frac{3}{2}$$

$$\Rightarrow x \in (-\infty, -\frac{3}{2})$$

Thus $f(x)$ is increasing on the interval $(-\infty, -\frac{3}{2})$

Again, For $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow -6 - 4x < 0$$

$$\Rightarrow -4x < 6$$

$$\Rightarrow x > -\frac{6}{4}$$

$$\Rightarrow x > -\frac{3}{2}$$

$$\Rightarrow x \in (-\frac{3}{2}, \infty)$$

Thus $f(x)$ is decreasing on interval $x \in (-\frac{3}{2}, \infty)$

1 B. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = x^2 + 2x - 5$$

Answer

Given:- Function $f(x) = x^2 + 2x - 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$$f(x) = x^2 + 2x - 5$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^2 + 2x - 5)$$

$$\Rightarrow f'(x) = 2x + 2$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow 2x + 2 > 0$$

$$\Rightarrow 2x < -2$$

$$\Rightarrow x < -\frac{2}{2}$$

$$\Rightarrow x < -1$$

$$\Rightarrow x \in (-\infty, -1)$$

Thus $f(x)$ is increasing on interval $(-\infty, -1)$

Again, For $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow 2x + 2 < 0$$

$$\Rightarrow 2x > -2$$

$$\Rightarrow x > -\frac{2}{2}$$

$$\Rightarrow x > -1$$

$$\Rightarrow x \in (-1, \infty)$$

Thus $f(x)$ is decreasing on interval $x \in (-1, \infty)$

1 C. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 6 - 9x - x^2$$

Answer

Given:- Function $f(x) = 6 - 9x - x^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 6 - 9x - x^2$$

$$\Rightarrow f'(x) = \frac{d}{dx}(6 - 9x - x^2)$$

$$\Rightarrow f'(x) = -9 - 2x$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow -9 - 2x > 0$$

$$\Rightarrow -2x > 9$$

$$\Rightarrow x < -\frac{9}{2}$$

$$\Rightarrow x < -\frac{9}{2}$$

$$\Rightarrow x \in (-\infty, -\frac{9}{2})$$

Thus $f(x)$ is increasing on interval $(-\infty, -\frac{9}{2})$

Again, For $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow -9 - 2x < 0$$

$$\Rightarrow -2x < 9$$

$$\Rightarrow x > -\frac{9}{2}$$

$$\Rightarrow x > -\frac{9}{2}$$

$$\Rightarrow x \in (-\frac{9}{2}, \infty)$$

Thus $f(x)$ is decreasing on interval $x \in (-\frac{9}{2}, \infty)$

1 D. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 2x^3 - 12x^2 + 18x + 15$$

Answer

Given:- Function $f(x) = 2x^3 - 12x^2 + 18x + 15$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 2x^3 - 12x^2 + 18x + 15$$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 12x^2 + 18x + 15)$$

$$\Rightarrow f'(x) = 6x^2 - 24x + 18$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 - 24x + 18 = 0$$

$$\Rightarrow 6(x^2 - 4x + 3) = 0$$

$$\Rightarrow 6(x^2 - 3x - x + 3) = 0$$

$$\Rightarrow 6(x - 3)(x - 1) = 0$$

$$\Rightarrow (x - 3)(x - 1) = 0$$

$$\Rightarrow x = 3, 1$$

clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$

and $f'(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases on $(-\infty, 1) \cup (3, \infty)$

and $f(x)$ is decreasing on interval $x \in (1, 3)$

1 E. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 5 + 36x + 3x^2 - 2x^3$$

Answer

Given:- Function $f(x) = 5 + 36x + 3x^2 - 2x^3$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 5 + 36x + 3x^2 - 2x^3$$

$$\Rightarrow f'(x) = \frac{d}{dx}(5 + 36x + 3x^2 - 2x^3)$$

$$\Rightarrow f'(x) = 36 + 6x - 6x^2$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 36 + 6x - 6x^2 = 0$$

$$\Rightarrow 6(-x^2 + x + 6) = 0$$

$$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$$

$$\Rightarrow -x^2 + 3x - 2x + 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow x = 3, -2$$

clearly, $f'(x) > 0$ if $-2 < x < 3$

and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, $f(x)$ increases on $x \in (-2, 3)$

and $f(x)$ is decreasing on interval $(-\infty, -2) \cup (3, \infty)$

1 F. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 8 + 36x + 3x^2 - 2x^3$$

Answer

Given:- Function $f(x) = 8 + 36x + 3x^2 - 2x^3$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 8 + 36x + 3x^2 - 2x^3$$

$$\Rightarrow f'(x) = \frac{d}{dx}(8 + 36x + 3x^2 - 2x^3)$$

$$\Rightarrow f'(x) = 36 + 6x - 6x^2$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 36 + 6x - 6x^2 = 0$$

$$\Rightarrow 6(-x^2 + x + 6) = 0$$

$$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$$

$$\Rightarrow -x^2 + 3x - 2x + 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow x = 3, -2$$

clearly, $f'(x) > 0$ if $-2 < x < 3$

and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, $f(x)$ increases on $x \in (-2, 3)$

and $f(x)$ is decreasing on interval $(-\infty, -2) \cup (3, \infty)$

1 G. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 5x^3 - 15x^2 - 120x + 3$$

Answer

Given:- Function $f(x) = 5x^3 - 15x^2 - 120x + 3$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 5x^3 - 15x^2 - 120x + 3$$

$$\Rightarrow f'(x) = \frac{d}{dx}(5x^3 - 15x^2 - 120x + 3)$$

$$\Rightarrow f'(x) = 15x^2 - 30x - 120$$

For $f(x)$ lets find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 15x^2 - 30x - 120 = 0$$

$$\Rightarrow 15(x^2 - 2x - 8) = 0$$

$$\Rightarrow 15(x^2 - 4x + 2x - 8) = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4, -2$$

clearly, $f'(x) > 0$ if $x < -2$ and $x > 4$

and $f'(x) < 0$ if $-2 < x < 4$

Thus, $f(x)$ increases on $(-\infty, -2) \cup (4, \infty)$

and $f(x)$ is decreasing on interval $x \in (-2, 4)$

1 H. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = x^3 - 6x^2 - 36x + 2$$

Answer

Given:- Function $f(x) = x^3 - 6x^2 - 36x + 2$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^3 - 6x^2 - 36x + 2$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 6x^2 - 36x + 2)$$

$$\Rightarrow f'(x) = 3x^2 - 12x - 36$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 3x^2 - 12x - 36 = 0$$

$$\Rightarrow 3(x^2 - 4x - 12) = 0$$

$$\Rightarrow 3(x^2 - 6x + 2x - 12) = 0$$

$$\Rightarrow x^2 - 6x + 2x - 12 = 0$$

$$\Rightarrow (x - 6)(x + 2) = 0$$

$$\Rightarrow x = 6, -2$$

clearly, $f'(x) > 0$ if $x < -2$ and $x > 6$

and $f'(x) < 0$ if $-2 < x < 6$

Thus, $f(x)$ increases on $(-\infty, -2) \cup (6, \infty)$

and $f(x)$ is decreasing on interval $x \in (-2, 6)$

1 I. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

Answer

Given:- Function $f(x) = 2x^3 - 15x^2 + 36x + 1$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 1)$$

$$\Rightarrow f'(x) = 6x^2 - 30x + 36$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 - 30x + 36 = 0$$

$$\Rightarrow 6(x^2 - 5x + 6) = 0$$

$$\Rightarrow 3(x^2 - 3x - 2x + 6) = 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 = 0$$

$$\Rightarrow (x - 3)(x - 2) = 0$$

$$\Rightarrow x = 3, 2$$

clearly, $f'(x) > 0$ if $x < 2$ and $x > 3$

and $f'(x) < 0$ if $2 < x < 3$

Thus, $f(x)$ increases on $(-\infty, 2) \cup (3, \infty)$

and $f(x)$ is decreasing on interval $x \in (2,3)$

1 J. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 2x^3 + 9x^2 + 12x + 20$$

Answer

Given:- Function $f(x) = 2x^3 + 9x^2 + 12x + 20$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 2x^3 + 9x^2 + 12x + 20$$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 + 9x^2 + 12x + 20)$$

$$\Rightarrow f'(x) = 6x^2 + 18x + 12$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 + 18x + 12 = 0$$

$$\Rightarrow 6(x^2 + 3x + 2) = 0$$

$$\Rightarrow 6(x^2 + 2x + x + 2) = 0$$

$$\Rightarrow x^2 + 2x + x + 2 = 0$$

$$\Rightarrow (x + 2)(x + 1) = 0$$

$$\Rightarrow x = -1, -2$$

clearly, $f'(x) > 0$ if $-2 < x < -1$

and $f'(x) < 0$ if $x < -2$ and $x > -1$

Thus, $f(x)$ increases on $x \in (-2, -1)$

and $f(x)$ is decreasing on interval $(-\infty, -2) \cup (-1, \infty)$

1 K. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 2x^3 - 9x^2 + 12x - 5$$

Answer

Given:- Function $f(x) = 2x^3 - 9x^2 + 12x - 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 2x^3 - 9x^2 + 12x - 5$$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 9x^2 + 12x - 5)$$

$$\Rightarrow f'(x) = 6x^2 - 18x + 12$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 - 18x + 12 = 0$$

$$\Rightarrow 6(x^2 - 3x + 2) = 0$$

$$\Rightarrow 6(x^2 - 2x - x + 2) = 0$$

$$\Rightarrow x^2 - 2x - x + 2 = 0$$

$$\Rightarrow (x - 2)(x - 1) = 0$$

$$\Rightarrow x = 1, 2$$

clearly, $f'(x) > 0$ if $x < 1$ and $x > 2$

and $f'(x) < 0$ if $1 < x < 2$

Thus, $f(x)$ increases on $(-\infty, 1) \cup (2, \infty)$

and $f(x)$ is decreasing on interval $x \in (1, 2)$

1 L. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 6 + 12x + 3x^2 - 2x^3$$

Answer

Given:- Function $f(x) = -2x^3 + 3x^2 + 12x + 6$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = -2x^3 + 3x^2 + 12x + 6$$

$$\Rightarrow f'(x) = \frac{d}{dx}(-2x^3 + 3x^2 + 12x + 6)$$

$$\Rightarrow f'(x) = -6x^2 + 6x + 12$$

For $f(x)$ lets find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow -6x^2 + 6x + 12 = 0$$

$$\Rightarrow 6(-x^2 + x + 2) = 0$$

$$\Rightarrow 6(-x^2 + 2x - x + 2) = 0$$

$$\Rightarrow x^2 - 2x + x - 2 = 0$$

$$\Rightarrow (x - 2)(x + 1) = 0$$

$$\Rightarrow x = -1, 2$$

clearly, $f'(x) > 0$ if $-1 < x < 2$

and $f'(x) < 0$ if $x < -1$ and $x > 2$

Thus, $f(x)$ increases on $x \in (-1, 2)$

and $f(x)$ is decreasing on interval $(-\infty, -1) \cup (2, \infty)$

1 M. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 2x^3 - 24x + 107$$

Answer

Given:- Function $f(x) = 2x^3 - 24x + 107$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 2x^3 - 24x + 107$$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 24x + 107)$$

$$\Rightarrow f'(x) = 6x^2 - 24$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 - 24 = 0$$

$$\Rightarrow 6(x^2 - 4) = 0$$

$$\Rightarrow (x - 2)(x + 2) = 0$$

$$\Rightarrow x = -2, 2$$

clearly, $f'(x) > 0$ if $x < -2$ and $x > 2$

and $f'(x) < 0$ if $-2 < x < 2$

Thus, $f(x)$ increases on $(-\infty, -2) \cup (2, \infty)$

and $f(x)$ is decreasing on interval $x \in (-2, 2)$

1 N. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

Answer

Given:- Function $f(x) = -2x^3 - 9x^2 - 12x + 1$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

$$\Rightarrow f'(x) = \frac{d}{dx}(-2x^3 - 9x^2 - 12x + 1)$$

$$\Rightarrow f'(x) = -6x^2 - 18x - 12$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow -6x^2 - 18x - 12 = 0$$

$$\Rightarrow 6x^2 + 18x + 12 = 0$$

$$\Rightarrow 6(x^2 + 3x + 2) = 0$$

$$\Rightarrow 6(x^2 + 2x + x + 2) = 0$$

$$\Rightarrow x^2 + 2x + x + 2 = 0$$

$$\Rightarrow (x + 2)(x + 1) = 0$$

$$\Rightarrow x = -1, -2$$

clearly, $f'(x) > 0$ if $x < -2$ and $x > -1$

and $f'(x) < 0$ if $-2 < x < -1$

Thus, $f(x)$ increases on $(-\infty, -2) \cup (-1, \infty)$

and $f(x)$ is decreasing on interval $x \in (-2, -1)$

1 O. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = (x - 1)(x - 2)^2$$

Answer

Given:- Function $f(x) = (x - 1)(x - 2)^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = (x - 1)(x - 2)^2$$

$$\Rightarrow f'(x) = \frac{d}{dx}((x - 1)(x - 2)^2)$$

$$\Rightarrow f'(x) = (x - 2)^2 + 2(x - 2)(x - 1)$$

$$\Rightarrow f'(x) = (x - 2)(x - 2 + 2x - 2)$$

$$\Rightarrow f'(x) = (x - 2)(3x - 4)$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow (x - 2)(3x - 4) = 0$$

$$\Rightarrow x = 2, \frac{4}{3}$$

clearly, $f'(x) > 0$ if $x < \frac{4}{3}$ and $x > 2$

and $f'(x) < 0$ if $\frac{4}{3} < x < 2$

Thus, $f(x)$ increases on $(-\infty, \frac{4}{3}) \cup (2, \infty)$

and $f(x)$ is decreasing on interval $x \in (\frac{4}{3}, 2)$

1 P. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = x^3 - 12x^2 + 36x + 17$$

Answer

Given:- Function $f(x) = x^3 - 12x^2 + 36x + 17$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^3 - 12x^2 + 36x + 17$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 12x^2 + 36x + 17)$$

$$\Rightarrow f'(x) = 3x^2 - 24x + 36$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 3x^2 - 24x + 36 = 0$$

$$\Rightarrow 3(x^2 - 8x + 12) = 0$$

$$\Rightarrow 3(x^2 - 6x - 2x + 12) = 0$$

$$\Rightarrow x^2 - 6x - 2x + 12 = 0$$

$$\Rightarrow (x - 6)(x - 2) = 0$$

$$\Rightarrow x = 2, 6$$

clearly, $f'(x) > 0$ if $x < 2$ and $x > 6$

and $f'(x) < 0$ if $2 < x < 6$

Thus, $f(x)$ increases on $(-\infty, 2) \cup (6, \infty)$

and $f(x)$ is decreasing on interval $x \in (2, 6)$

1 Q. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 2x^3 - 24x + 7$$

Answer

Given:- Function $f(x) = 2x^3 - 24x + 7$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 2x^3 - 24x + 7$$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 24x + 7)$$

$$\Rightarrow f'(x) = 6x^2 - 24$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow 6x^2 - 24 > 0$$

$$\Rightarrow x^2 < \frac{24}{6}$$

$$\Rightarrow x^2 < 4$$

$$\Rightarrow x < -2, +2$$

$$\Rightarrow x \in (-\infty, -2) \text{ and } x \in (2, \infty)$$

Thus $f(x)$ is increasing on interval $(-\infty, -2) \cup (2, \infty)$

Again, For $f(x)$ to be increasing, we must have

$$f'(x) < 0$$

$$\Rightarrow 6x^2 - 24 < 0$$

$$\Rightarrow x^2 > \frac{24}{6}$$

$$\Rightarrow x^2 < 4$$

$$\Rightarrow x > -1$$

$$\Rightarrow x \in (-1, \infty)$$

Thus $f(x)$ is decreasing on interval $x \in (-1, \infty)$

1 R. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

Answer

Given:- Function $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

$$\Rightarrow f'(x) = \frac{d}{dx} \left(\frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11 \right)$$

$$\Rightarrow f'(x) = 4 \times \frac{3}{10}x^3 - 3 \times \frac{4}{5}x^2 - 6x + \frac{36}{5}$$

For $f(x)$ lets find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow \frac{12}{10}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5} = 0$$

$$\Rightarrow \frac{6}{5}(x-1)(x+2)(x-3) = 0$$

$$\Rightarrow x = 1, -2, 3$$

Now, lets check values of $f(x)$ between different ranges

Here points $x = 1, -2, 3$ divide the number line into disjoint intervals namely, $(-\infty, -2)$, $(-2, 1)$, $(1, 3)$ and $(3, \infty)$

Lets consider interval $(-\infty, -2)$

In this case, we have $x - 1 < 0$, $x + 2 < 0$ and $x - 3 < 0$

Therefore, $f'(x) < 0$ when $-\infty < x < -2$

Thus, $f(x)$ is strictly decreasing on interval $x \in (-\infty, -2)$

consider interval $(-2, 1)$

In this case, we have $x - 1 < 0$, $x + 2 > 0$ and $x - 3 < 0$

Therefore, $f'(x) > 0$ when $-2 < x < 1$

Thus, $f(x)$ is strictly increases on interval $x \in (-2, 1)$

Now, consider interval $(1, 3)$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 < 0$

Therefore, $f'(x) < 0$ when $1 < x < 3$

Thus, $f(x)$ is strictly decreases on interval $x \in (1, 3)$

finally, consider interval $(3, \infty)$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 > 0$

Therefore, $f'(x) > 0$ when $x > 3$

Thus, $f(x)$ is strictly increases on interval $x \in (3, \infty)$

1 S. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = x^4 - 4x$$

Answer

Given:- Function $f(x) = x^4 - 4x$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^4 - 4x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^4 - 4x)$$

$$\Rightarrow f'(x) = 4x^3 - 4$$

For $f(x)$ lets find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 4x^3 - 4 = 0$$

$$\Rightarrow 4(x^3 - 1) = 0$$

$$\Rightarrow x = 1$$

clearly, $f'(x) > 0$ if $x > 1$

and $f'(x) < 0$ if $x < 1$

Thus, $f(x)$ increases on $(1, \infty)$

and $f(x)$ is decreasing on interval $x \in (-\infty, 1)$

1 T. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

Answer

Given:- Function $f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

$$\Rightarrow f'(x) = \frac{d}{dx} \left(\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7 \right)$$

$$\Rightarrow f'(x) = x^3 + 2x^2 - 5x - 6$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow x^3 + 2x^2 - 5x - 6 = 0$$

$$\Rightarrow (x+1)(x-2)(x+3) = 0$$

$$\Rightarrow x = -1, 2, -3$$

clearly, $f'(x) > 0$ if $-3 < x < -1$ and $x > 2$

and $f'(x) < 0$ if $x < -3$ and $-3 < x < -1$

Thus, $f(x)$ increases on $(-3, -1) \cup (2, \infty)$

and $f(x)$ is decreasing on interval $(\infty, -3) \cup (-1, 2)$

1 U. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = x^4 - 4x^3 + 4x^2 + 15$$

Answer

Given:- Function $f(x) = x^4 - 4x^3 + 4x^2 + 15$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$$f(x) = x^4 - 4x^3 + 4x^2 + 15$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^4 - 4x^3 + 4x^2 + 15)$$

$$\Rightarrow f'(x) = 4x^3 - 12x^2 + 8x$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 4x^3 - 12x^2 + 8x = 0$$

$$\Rightarrow 4(x^3 - 3x^2 + 2x) = 0$$

$$\Rightarrow x(x^2 - 3x + 2) = 0$$

$$\Rightarrow x(x^2 - 2x - x + 2) = 0$$

$$\Rightarrow x(x - 2)(x - 1)$$

$$\Rightarrow x = 0, 1, 2$$

clearly, $f'(x) > 0$ if $0 < x < 1$ and $x > 2$

and $f'(x) < 0$ if $x < 0$ and $1 < x < 2$

Thus, $f(x)$ increases on $(0, 1) \cup (2, \infty)$

and $f(x)$ is decreasing on interval $(-\infty, 0) \cup (1, 2)$

1 V. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$$

Answer

Given:- Function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$$

$$\Rightarrow f'(x) = \frac{d}{dx}(5x^{\frac{3}{2}} - 3x^{\frac{5}{2}})$$

$$\Rightarrow f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$$

$$\Rightarrow f'(x) = \frac{15}{2}x^{\frac{1}{2}}(1 - x)$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow \frac{15}{2}x^{\frac{1}{2}}(1 - x) = 0$$

$$\Rightarrow x^{\frac{1}{2}}(1 - x) = 0$$

$$\Rightarrow x = 0, 1$$

Since $x > 0$, therefore only check the range on the positive side of the number line.

clearly, $f'(x) > 0$ if $0 < x < 1$

and $f'(x) < 0$ if $x > 1$

Thus, $f(x)$ increases on $(0, 1)$

and $f(x)$ is decreasing on interval $x \in (1, \infty)$

1 W. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = x^8 + 6x^2$$

Answer

Given:- Function $f(x) = x^8 + 6x^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^8 + 6x^2$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^8 + 6x^2)$$

$$\Rightarrow f'(x) = 8x^7 + 12x$$

$$\Rightarrow f'(x) = 4x(2x^6 + 3)$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 4x(2x^6 + 3) = 0$$

$$\Rightarrow x(2x^6 + 3) = 0$$

$$\Rightarrow x = 0, \sqrt[6]{-\frac{3}{2}}$$

Since $x = \sqrt[6]{-\frac{3}{2}}$ is a complex number, therefore only check range on 0 sides of number line.

clearly, $f'(x) > 0$ if $x > 0$

and $f'(x) < 0$ if $x < 0$

Thus, $f(x)$ increases on $(0, \infty)$

and $f(x)$ is decreasing on interval $x \in (-\infty, 0)$

1 X. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = x^3 - 6x^2 + 9x + 15$$

Answer

Given:- Function $f(x) = x^3 - 6x^2 + 9x + 15$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 6x^2 + 9x + 15)$$

$$\Rightarrow f'(x) = 3x^2 - 12x + 9$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 3x^2 - 12x + 9 = 0$$

$$\Rightarrow 3(x^2 - 4x + 3) = 0$$

$$\Rightarrow 3(x^2 - 3x - x + 3) = 0$$

$$\Rightarrow x^2 - 3x - x + 3 = 0$$

$$\Rightarrow (x - 3)(x - 1) = 0$$

$$\Rightarrow x = 1, 3$$

clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$

and $f'(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases on $(-\infty, 1) \cup (3, \infty)$

and $f(x)$ is decreasing on interval $x \in (1, 3)$

1 Y. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \{x(x - 2)\}^2$$

Answer

Given:- Function $f(x) = \{x(x - 2)\}^2$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \{x(x - 2)\}^2$$

$$\Rightarrow f(x) = \{x^2 - 2x\}^2$$

$$\Rightarrow f'(x) = \frac{d}{dx}([x^2 - 2x]^2)$$

$$\Rightarrow f'(x) = 2(x^2 - 2x)(2x - 2)$$

$$\Rightarrow f'(x) = 4x(x - 2)(x - 1)$$

For $f(x)$ lets find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 4x(x - 2)(x - 1) = 0$$

$$\Rightarrow x(x - 2)(x - 1) = 0$$

$$\Rightarrow x = 0, 1, 2$$

Now, lets check values of $f(x)$ between different ranges

Here points $x = 0, 1, 2$ divide the number line into disjoint intervals namely, $(-\infty, 0)$, $(0, 1)$, $(1, 2)$ and $(2, \infty)$

Lets consider interval $(-\infty, 0)$ and $(1, 2)$

In this case, we have $x(x - 2)(x - 1) < 0$

Therefore, $f'(x) < 0$ when $x < 0$ and $1 < x < 2$

Thus, $f(x)$ is strictly decreasing on interval $(-\infty, 0) \cup (1, 2)$

Now, consider interval $(0, 1)$ and $(2, \infty)$

In this case, we have $x(x-2)(x-1) > 0$

Therefore, $f'(x) > 0$ when $0 < x < 1$ and $x < 2$

Thus, $f(x)$ is strictly increases on interval $(0, 1) \cup (2, \infty)$

1 Z. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

Answer

Given:- Function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

$$\Rightarrow f'(x) = \frac{d}{dx}(3x^4 - 4x^3 - 12x^2 + 5)$$

$$\Rightarrow f'(x) = 12x^3 - 12x^2 - 24x$$

$$\Rightarrow f'(x) = 12x(x^2 - x - 2)$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow 12x(x^2 - x - 2) > 0$$

$$\Rightarrow x(x^2 - 2x + x - 2) > 0$$

$$\Rightarrow x(x - 2)(x + 1) > 0$$

$$\Rightarrow -1 < x < 0 \text{ and } x > 2$$

$$\Rightarrow x \in (-1, 0) \cup (2, \infty)$$

Thus $f(x)$ is increasing on interval $(-1, 0) \cup (2, \infty)$

Again, For $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow 12x(x^2 - x - 2) < 0$$

$$\Rightarrow x(x^2 - 2x + x - 2) < 0$$

$$\Rightarrow x(x - 2)(x + 1) < 0$$

$$\Rightarrow -\infty < x < -1 \text{ and } 0 < x < 2$$

$$\Rightarrow x \in (-\infty, -1) \cup (0, 2)$$

Thus $f(x)$ is decreasing on interval $(-\infty, -1) \cup (0, 2)$

1 A1. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$$

Answer

Given:- Function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$$

$$\Rightarrow f'(x) = \frac{d}{dx} \left(\frac{3}{2}x^4 - 4x^3 - 45x^2 + 51 \right)$$

$$\Rightarrow f'(x) = 6x^3 - 12x^2 - 90x$$

$$\Rightarrow f'(x) = 6x(x^2 - 2x - 15)$$

$$\Rightarrow f'(x) = 6x(x^2 - 5x + 3x - 15)$$

$$\Rightarrow f'(x) = 6x(x - 5)(x + 3)$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow 6x(x - 5)(x + 3) > 0$$

$$\Rightarrow x(x - 5)(x + 3) > 0$$

$$\Rightarrow -3 < x < 0 \text{ or } 5 < x < \infty$$

$$\Rightarrow x \in (-3, 0) \cup (5, \infty)$$

Thus $f(x)$ is increasing on interval $(-3, 0) \cup (5, \infty)$

Again, For $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow 6x(x - 5)(x + 3) > 0$$

$$\Rightarrow x(x - 5)(x + 3) > 0$$

$$\Rightarrow -\infty < x < -3 \text{ or } 0 < x < 5$$

$$\Rightarrow x \in (-\infty, -3) \cup (0, 5)$$

Thus $f(x)$ is decreasing on interval $(-\infty, -3) \cup (0, 5)$

1 B1. Question

Find the intervals in which the following functions are increasing or decreasing.

$$f(x) = \log(2 + x) - \frac{2x}{2 + x}$$

Answer

Given:- Function $f(x) = \log(2 + x) - \frac{2x}{2+x}$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \log(2 + x) - \frac{2x}{2 + x}$$

$$\Rightarrow f'(x) = \frac{d}{dx} \left(\log(2 + x) - \frac{2x}{2+x} \right)$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{(2+x)2 - 2x \times 1}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4+2x-2x}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{2+x-4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{x-2}{(2+x)^2}$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow \frac{x-2}{(2+x)^2} > 0$$

$$\Rightarrow (x - 2) > 0$$

$$\Rightarrow 2 < x < \infty$$

$$\Rightarrow x \in (2, \infty)$$

Thus $f(x)$ is increasing on interval $(2, \infty)$

Again, For $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow \frac{x-2}{(2+x)^2} < 0$$

$$\Rightarrow (x - 2) < 0$$

$$\Rightarrow -\infty < x < 2$$

$$\Rightarrow x \in (-\infty, 2)$$

Thus $f(x)$ is decreasing on interval $(-\infty, 2)$

2. Question

Determine the values of x for which the function $f(x) = x^2 - 6x + 9$ is increasing or decreasing. Also, find the coordinates of the point on the curve $y = x^2 - 6x + 9$ where the normal is parallel to the line $y = x + 5$.

Answer

Given:- Function $f(x) = x^2 - 6x + 9$ and a line parallel to $y = x + 5$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^2 - 6x + 9$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - 6x + 9)$$

$$\Rightarrow f'(x) = 2x - 6$$

$$\Rightarrow f'(x) = 2(x - 3)$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 2(x - 3) = 0$$

$$\Rightarrow (x - 3) = 0$$

$$\Rightarrow x = 3$$

clearly, $f'(x) > 0$ if $x > 3$

and $f'(x) < 0$ if $x < 3$

Thus, $f(x)$ increases on $(3, \infty)$

and $f(x)$ is decreasing on interval $x \in (-\infty, 3)$

Now, let's find coordinates of point

Equation of curve is

$$f(x) = x^2 - 6x + 9$$

slope of this curve is given by

$$\Rightarrow m_1 = \frac{dy}{dx}$$

$$\Rightarrow m_1 = \frac{d}{dx}(x^2 - 6x + 9)$$

$$\Rightarrow m_1 = 2x - 6$$

and Equation of line is

$$y = x + 5$$

slope of this curve is given by

$$\Rightarrow m_2 = \frac{dy}{dx}$$

$$\Rightarrow m_2 = \frac{d}{dx}(x + 5)$$

$$\Rightarrow m_2 = 1$$

Since slope of curve (i.e slope of its normal) is parallel to line

Therefore, they follow the relation

$$\Rightarrow \frac{-1}{m_1} = m_2$$

$$\Rightarrow \frac{-1}{2x-6} = 1$$

$$\Rightarrow 2x - 6 = -1$$

$$\Rightarrow x = \frac{5}{2}$$

Thus putting the value of x in equation of curve, we get

$$\Rightarrow y = x^2 - 6x + 9$$

$$\Rightarrow y = \left(\frac{5}{2}\right)^2 - 6\left(\frac{5}{2}\right) + 9$$

$$\Rightarrow y = \frac{25}{4} - 15 + 9$$

$$\Rightarrow y = \frac{25}{4} - 6$$

$$\Rightarrow y = \frac{1}{4}$$

Thus the required coordinates is $\left(\frac{5}{2}, \frac{1}{4}\right)$

3. Question

Find the intervals in which $f(x) = \sin x - \cos x$, where $0 < x < 2\pi$ is increasing or decreasing.

Answer

Given:- Function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \sin x - \cos x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\sin x - \cos x)$$

$$\Rightarrow f'(x) = \cos x + \sin x$$

For $f(x)$ let's find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow \cos x + \sin x = 0$$

$$\Rightarrow \tan(x) = -1$$

$$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Here these points divide the angle range from 0 to 2π since we have x as angle

clearly, $f'(x) > 0$ if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$

and $f'(x) < 0$ if $\frac{3\pi}{4} < x < \frac{7\pi}{4}$

Thus, $f(x)$ increases on $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

and $f(x)$ is decreasing on interval $(\frac{3\pi}{4}, \frac{7\pi}{4})$

4. Question

Show that $f(x) = e^{2x}$ is increasing on \mathbb{R} .

Answer

Given:- Function $f(x) = e^{2x}$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = e^{2x}$$

$$\Rightarrow f'(x) = \frac{d}{dx}(e^{2x})$$

$$\Rightarrow f'(x) = 2e^{2x}$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow 2e^{2x} > 0$$

$$\Rightarrow e^{2x} > 0$$

since, the value of e lies between 2 and 3

so, whatever be the power of e (i.e x in domain R) will be greater than zero.

Thus f(x) is increasing on interval R

5. Question

Show that $f(x) = e^{\frac{1}{x}}$, $x \neq 0$ is a decreasing function for all $x \neq 0$.

Answer

Given:- Function $f(x) = e^{\frac{1}{x}}$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b).

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then f(x) is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then f(x) is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to f(x)

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = e^{\frac{1}{x}}$$

$$\Rightarrow f'(x) = \frac{d}{dx} \left(e^{\frac{1}{x}} \right)$$

$$\Rightarrow f'(x) = e^{\frac{1}{x}} \cdot \left(\frac{-1}{x^2} \right)$$

$$\Rightarrow f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$$

As given $x \in R$, $x \neq 0$

$$\Rightarrow \frac{1}{x^2} > 0 \text{ and } e^{\frac{1}{x}} > 0$$

Their ratio is also greater than 0

$$\Rightarrow \frac{e^{\frac{1}{x}}}{x^2} > 0$$

$$\Rightarrow -\frac{e^{\frac{1}{x}}}{x^2} < 0 ; \text{ as by applying -ve sign change in comparison sign}$$

$$\Rightarrow f'(x) < 0$$

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing for all $x \neq 0$

6. Question

Show that $f(x) = \log_a x$, $0 < a < 1$ is a decreasing function for all $x > 0$.

Answer

Given:- Function $f(x) = \log_a x$, $0 < a < 1$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \log_a x, 0 < a < 1$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\log_a x)$$

$$\Rightarrow f'(x) = \frac{1}{x \log a}$$

As given $0 < a < 1$

$$\Rightarrow \log(a) < 0$$

and for $x > 0$

$$\Rightarrow \frac{1}{x} > 0$$

Therefore $f'(x)$ is

$$\Rightarrow \frac{1}{x \log a} < 0$$

$$\Rightarrow f'(x) < 0$$

Hence, condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing for all $x > 0$

7. Question

Show that $f(x) = \sin x$ is increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$ and neither increasing nor decreasing in $(0, \pi)$.

Answer

Given:- Function $f(x) = \sin x$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \sin x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\sin x)$$

$$\Rightarrow f'(x) = \cos x$$

Taking different region from 0 to 2π

$$\text{a) let } x \in (0, \frac{\pi}{2})$$

$$\Rightarrow \cos(x) > 0$$

$$\Rightarrow f'(x) > 0$$

Thus $f(x)$ is increasing in $(0, \frac{\pi}{2})$

$$\text{b) let } x \in (\frac{\pi}{2}, \pi)$$

$$\Rightarrow \cos(x) < 0$$

$$\Rightarrow f'(x) < 0$$

Thus $f(x)$ is decreasing in $(\frac{\pi}{2}, \pi)$

Therefore, from above condition we find that

$\Rightarrow f(x)$ is increasing in $(0, \frac{\pi}{2})$ and decreasing in $(\frac{\pi}{2}, \pi)$

Hence, condition for $f(x)$ neither increasing nor decreasing in $(0, \pi)$

8. Question

Show that $f(x) = \log \sin x$ is increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$.

Answer

Given:- Function $f(x) = \log \sin x$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \log \sin x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\log \sin x)$$

$$\Rightarrow f'(x) = \frac{1}{\sin x} \times \cos x$$

$$\Rightarrow f'(x) = \cot(x)$$

Taking different region from 0 to π

$$\text{a) let } x \in (0, \frac{\pi}{2})$$

$$\Rightarrow \cot(x) > 0$$

$$\Rightarrow f'(x) > 0$$

Thus $f(x)$ is increasing in $(0, \frac{\pi}{2})$

b) let $x \in (\frac{\pi}{2}, \pi)$

$$\Rightarrow \cot(x) < 0$$

$$\Rightarrow f'(x) < 0$$

Thus $f(x)$ is decreasing in $(\frac{\pi}{2}, \pi)$

Hence proved

9. Question

Show that $f(x) = x - \sin x$ is increasing for all $x \in \mathbb{R}$.

Answer

Given:- Function $f(x) = x - \sin x$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x - \sin x$$

$$\Rightarrow f'(x) = \frac{d}{dx} (x - \sin x)$$

$$\Rightarrow f'(x) = 1 - \cos x$$

Now, as given

$$x \in \mathbb{R}$$

$$\Rightarrow -1 < \cos x < 1$$

$$\Rightarrow -1 > \cos x > 0$$

$$\Rightarrow f'(x) > 0$$

hence, Condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x \in \mathbb{R}$

10. Question

Show that $f(x) = x^3 - 15x^2 + 75x - 50$ is an increasing function for all $x \in \mathbb{R}$.

Answer

Given:- Function $f(x) = x^3 - 15x^2 + 75x - 50$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^3 - 15x^2 + 75x - 50$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 15x^2 + 75x - 50)$$

$$\Rightarrow f'(x) = 3x^2 - 30x + 75$$

$$\Rightarrow f'(x) = 3(x^2 - 10x + 25)$$

$$\Rightarrow f'(x) = 3(x - 5)^2$$

Now, as given

$$x \in \mathbb{R}$$

$$\Rightarrow (x - 5)^2 > 0$$

$$\Rightarrow 3(x - 5)^2 > 0$$

$$\Rightarrow f'(x) > 0$$

hence, Condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x \in \mathbb{R}$

11. Question

Show that $f(x) = \cos^2 x$ is a decreasing function on $(0, \pi/2)$.

Answer

Given:- Function $f(x) = \cos^2 x$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \cos^2 x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\cos^2 x)$$

$$\Rightarrow f'(x) = 2\cos x(-\sin x)$$

$$\Rightarrow f'(x) = -2\sin(x)\cos(x)$$

$$\Rightarrow f'(x) = -\sin 2x ; \text{ as } \sin 2A = 2\sin A \cos A$$

Now, as given

$$x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow 2x \in (0, \pi)$$

$$\Rightarrow \sin(2x) > 0$$

$$\Rightarrow -\sin(2x) < 0$$

$$\Rightarrow f'(x) < 0$$

hence, Condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing on interval $\left(0, \frac{\pi}{2}\right)$

Hence proved

12. Question

Show that $f(x) = \sin x$ is an increasing function on $(-\pi/2, \pi/2)$.

Answer

Given:- Function $f(x) = \sin x$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \sin x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\sin x)$$

$$\Rightarrow f'(x) = \cos x$$

Now, as given

$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

That is 4th quadrant, where

$$\Rightarrow \cos x > 0$$

$$\Rightarrow f'(x) > 0$$

hence, Condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

13. Question

Show that $f(x) = \cos x$ is a decreasing function on $(0, \pi)$, increasing in $(-\pi, 0)$ and neither increasing nor

decreasing in $(-\pi, \pi)$.

Answer

Given:- Function $f(x) = \cos x$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \cos x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\cos x)$$

$$\Rightarrow f'(x) = -\sin x$$

Taking different region from 0 to 2π

a) let $x \in (0, \pi)$

$$\Rightarrow \sin(x) > 0$$

$$\Rightarrow -\sin x < 0$$

$$\Rightarrow f'(x) < 0$$

Thus $f(x)$ is decreasing in $(0, \pi)$

b) let $x \in (-\pi, 0)$

$$\Rightarrow \sin(x) < 0$$

$$\Rightarrow -\sin x > 0$$

$$\Rightarrow f'(x) > 0$$

Thus $f(x)$ is increasing in $(-\pi, 0)$

Therefore, from above condition we find that

$\Rightarrow f(x)$ is decreasing in $(0, \pi)$ and increasing in $(-\pi, 0)$

Hence, condition for $f(x)$ neither increasing nor decreasing in $(-\pi, \pi)$

14. Question

Show that $f(x) = \tan x$ is an increasing function on $(-\pi/2, \pi/2)$.

Answer

Given:- Function $f(x) = \tan x$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \tan x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\tan x)$$

$$\Rightarrow f'(x) = \sec^2 x$$

Now, as given

$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

That is 4th quadrant, where

$$\Rightarrow \sec^2 x > 0$$

$$\Rightarrow f'(x) > 0$$

hence, Condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

15. Question

Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is a decreasing function on the interval $(\pi/4, \pi/2)$.

Answer

Given:- Function $f(x) = \tan^{-1}(\sin x + \cos x)$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \tan^{-1}(\sin x + \cos x)$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\tan^{-1}(\sin x + \cos x))$$

$$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now, as given

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$\Rightarrow \cos x - \sin x < 0$; as here cosine values are smaller than sine values for same angle

$$\Rightarrow \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0$$

$$\Rightarrow f'(x) < 0$$

hence, Condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

16. Question

Show that the function $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$ is decreasing on $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

Answer

Given:- Function $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \sin\left(2x + \frac{\pi}{4}\right)$$

$$\Rightarrow f'(x) = \frac{d}{dx}\left\{\sin\left(2x + \frac{\pi}{4}\right)\right\}$$

$$\Rightarrow f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$$

$$\Rightarrow f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$$

Now, as given

$$x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

$$\Rightarrow \frac{3\pi}{8} < x < \frac{5\pi}{8}$$

$$\Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\Rightarrow \pi < 2x + \frac{\pi}{4} < \frac{3\pi}{2};$$

as here $2x + \frac{\pi}{4}$ lies in 3rd quadrant

$$\Rightarrow \cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow 2 \cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow f'(x) < 0$$

hence, Condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing on interval $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

17. Question

Show that the function $f(x) = \cot^{-1}(\sin x + \cos x)$ is decreasing on $(0, \pi/4)$ and increasing on $(\pi/4, \pi/2)$.

Answer

Given:- Function $f(x) = \cot^{-1}(\sin x + \cos x)$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = \cot^{-1}(\sin x + \cos x)$$

$$\Rightarrow f'(x) = \frac{d}{dx}\{\cot^{-1}(\sin x + \cos x)\}$$

$$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now, as given

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$\Rightarrow \cos x - \sin x < 0$; as here cosine values are smaller than sine values for same angle

$$\Rightarrow \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0$$

$$\Rightarrow f'(x) < 0$$

hence, Condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

18. Question

Show that $f(x) = (x - 1)e^x + 1$ is an increasing function for all $x > 0$.

Answer

Given:- Function $f(x) = (x - 1)e^x + 1$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = (x - 1)e^x + 1$$

$$\Rightarrow f'(x) = \frac{d}{dx}((x - 1)e^x + 1)$$

$$\Rightarrow f'(x) = e^x + (x - 1)e^x$$

$$\Rightarrow f'(x) = e^x(1 + x - 1)$$

$$\Rightarrow f'(x) = xe^x$$

as given

$$x > 0$$

$$\Rightarrow e^x > 0$$

$$\Rightarrow xe^x > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x > 0$

19. Question

Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on $(0, 1)$.

Answer

Given:- Function $f(x) = x^2 - x + 1$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = x^2 - x + 1$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - x + 1)$$

$$\Rightarrow f'(x) = 2x - 1$$

Taking different region from (0, 1)

a) let $x \in (0, \frac{1}{2})$

$$\Rightarrow 2x - 1 < 0$$

$$\Rightarrow f'(x) < 0$$

Thus $f(x)$ is decreasing in $(0, \frac{1}{2})$

b) let $x \in (\frac{1}{2}, 1)$

$$\Rightarrow 2x - 1 > 0$$

$$\Rightarrow f'(x) > 0$$

Thus $f(x)$ is increasing in $(\frac{1}{2}, 1)$

Therefore, from above condition we find that

$\Rightarrow f(x)$ is decreasing in $(0, \frac{1}{2})$ and increasing in $(\frac{1}{2}, 1)$

Hence, condition for $f(x)$ neither increasing nor decreasing in (0, 1)

20. Question

Show that $f(x) = x^9 + 4x^7 + 11$ is an increasing function for all $x \in \mathbb{R}$.

Answer

Given:- Function $f(x) = x^9 + 4x^7 + 11$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$$f(x) = x^9 + 4x^7 + 11$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^9 + 4x^7 + 11)$$

$$\Rightarrow f'(x) = 9x^8 + 28x^6$$

$$\Rightarrow f'(x) = x^6(9x^2 + 28)$$

as given

$$x \in \mathbb{R}$$

$$\Rightarrow x^6 > 0 \text{ and } 9x^2 + 28 > 0$$

$$\Rightarrow x^6(9x^2 + 28) > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x \in \mathbb{R}$

21. Question

Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is increasing on \mathbb{R} .

Answer

Given:- Function $f(x) = x^3 - 6x^2 + 12x - 18$

Theorem:- Let f be a differentiable real function defined on an open interval (a, b) .

(i) If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain, it is decreasing.

Here we have,

$$f(x) = x^3 - 6x^2 + 12x - 18$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 6x^2 + 12x - 18)$$

$$\Rightarrow f'(x) = 3x^2 - 12x + 12$$

$$\Rightarrow f'(x) = 3(x^2 - 4x + 4)$$

$$\Rightarrow f'(x) = 3(x - 2)^2$$

as given

$$x \in \mathbb{R}$$

$$\Rightarrow (x - 2)^2 > 0$$

$$\Rightarrow 3(x - 2)^2 > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x \in \mathbb{R}$

22. Question

State when a function $f(x)$ is said to be increasing on an interval $[a, b]$. Test whether the function $f(x) = x^2 - 6x + 3$ is increasing on the interval $[4, 6]$.

Answer

Given:- Function $f(x) = x^2 - 6x + 3$

Theorem:- Let f be a differentiable real function defined on an open interval (a,b) .

(i) If $f'(x) > 0$ for all $x \in (a,b)$, then $f(x)$ is increasing on (a, b)

(ii) If $f'(x) < 0$ for all $x \in (a,b)$, then $f(x)$ is decreasing on (a, b)

Algorithm:-

(i) Obtain the function and put it equal to $f(x)$

(ii) Find $f'(x)$

(iii) Put $f'(x) > 0$ and solve this inequation.

For the value of x obtained in (ii) $f(x)$ is increasing and for remaining points in its domain it is decreasing.

Here we have,

$$f(x) = f(x) = x^2 - 6x + 3$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - 6x + 3)$$

$$\Rightarrow f'(x) = 2x - 6$$

$$\Rightarrow f'(x) = 2(x - 3)$$

Here A function is said to be increasing on $[a,b]$ if $f'(x) > 0$

as given

$$x \in [4, 6]$$

$$\Rightarrow 4 \leq x \leq 6$$

$$\Rightarrow 1 \leq (x-3) \leq 3$$

$$\Rightarrow (x - 3) > 0$$

$$\Rightarrow 2(x - 3) > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x \in [4, 6]$

23. Question

Show that $f(x) = \sin x - \cos x$ is an increasing function on $(-\pi/4, \pi/4)$?

Answer

we have,

$$f(x) = \sin x - \cos x$$

$$f'(x) = \cos x + \sin x$$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right)$$

$$= \sqrt{2} \left(\frac{\sin \pi}{4} \cos x + \frac{\cos \pi}{4} \sin x \right)$$

$$= \sqrt{2} \sin \left(\frac{\pi}{4} + x \right)$$

Now,

$$x \in \left(-\frac{\pi}{4}, \frac{\pi}{4} \right)$$

$$\Rightarrow -\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$\Rightarrow 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$$

$$\Rightarrow \sin 0^\circ < \sin\left(\frac{\pi}{4} + x\right) < \sin \frac{\pi}{2}$$

$$\Rightarrow 0 < \sin\left(\frac{\pi}{4} + x\right) < 1$$

$$\Rightarrow \sqrt{2} \sin\left(\frac{\pi}{4} + x\right) > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $(-\pi/4, \pi/4)$

24. Question

Show that $f(x) = \tan^{-1} x - x$ is a decreasing function on \mathbb{R} ?

Answer

we have,

$$f(x) = \tan^{-1} x - x$$

$$f'(x) = \frac{1}{1+x^2} - 1$$

$$= -\frac{x^2}{1+x^2}$$

Now,

$$x \in \mathbb{R}$$

$$\Rightarrow x^2 > 0 \text{ and } 1 + x^2 > 0$$

$$\Rightarrow \frac{x^2}{1+x^2} > 0$$

$$\Rightarrow -\frac{x^2}{1+x^2} < 0$$

$$\Rightarrow f'(x) < 0$$

Hence, $f(x)$ is an decreasing function for \mathbb{R}

25. Question

Determine whether $f(x) = x/2 + \sin x$ is increasing or decreasing on $(-\pi/3, \pi/3)$?

Answer

we have,

$$f(x) = -\frac{x}{2} + \sin x$$

$$= f'(x) = -\frac{1}{2} + \cos x$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \cos\left(-\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \cos\left(\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \frac{1}{2} < \cos x < \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} + \cos x > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $(-\pi/3, \pi/3)$

26. Question

Find the interval in which $f(x) = \log(1+x) - \frac{x}{1+x}$ is increasing or decreasing ?

Answer

we have

$$f(x) = \log(1+x) - \frac{x}{1+x}$$

$$f'(x) = \frac{1}{1+x} - \left(\frac{(1+x) - x}{(1+x)^2} \right)$$

$$= \frac{1}{1+x} - \left(\frac{1}{(1+x)^2} \right)$$

$$= \frac{x}{(1+x)^2}$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \frac{x}{(1+x)^2} = 0$$

$$\Rightarrow x = 0, -1$$

Clearly, $f'(x) > 0$ if $x > 0$

And $f'(x) < 0$ if $-1 < x < 0$ or $x < -1$

Hence, $f(x)$ increases in $(0, \infty)$, decreases in $(-\infty, -1) \cup (-1, 0)$

27. Question

Find the intervals in which $f(x) = (x+2)e^{-x}$ is increasing or decreasing ?

Answer

we have,

$$f(x) = (x+2)e^{-x}$$

$$f'(x) = e^{-x} - e^{-x}(x+2)$$

$$= e^{-x}(1 - x - 2)$$

$$= -e^{-x}(x+1)$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow -e^{-x}(x+1) = 0$$

$$\Rightarrow x = -1$$

Clearly $f'(x) > 0$ if $x < -1$

$f'(x) < 0$ if $x > -1$

Hence $f(x)$ increases in $(-\infty, -1)$, decreases in $(-1, \infty)$

28. Question

Show that the function f given by $f(x) = 10^x$ is increasing for all x ?

Answer

we have,

$$f(x) = 10^x$$

$$\therefore f'(x) = 10^x \log 10$$

Now,

$$x \in \mathbb{R}$$

$$\Rightarrow 10^x > 0$$

$$\Rightarrow 10^x \log 10 > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all x

29. Question

Prove that the function f given by $f(x) = x - [x]$ is increasing in $(0, 1)$?

Answer

we have,

$$f(x) = x - [x]$$

$$\therefore f'(x) = 1 > 0$$

$\therefore f(x)$ is an increasing function on $(0, 1)$

30. Question

Prove that the following function is increasing on \mathbb{R} ?

i. $f(x) = 3x^5 + 40x^3 + 240x$

ii. $f(x) = 4x^3 - 18x^2 + 27x - 27$

Answer

(i) we have

$$f(x) = 3x^5 + 40x^3 + 240x$$

$$\therefore f'(x) = 15x^4 + 120x^2 + 240$$

$$= 15(x^4 + 8x^2 + 16)$$

$$= 15(x^2 + 4)^2$$

Now,

$$x \in \mathbb{R}$$

$$\Rightarrow (x^2 + 4)^2 > 0$$

$$\Rightarrow 15(x^2 + 4)^2 > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all x

(ii) we have

$$f(x) = 4x^3 - 18x^2 + 27x - 27$$

$$\therefore f'(x) = 12x^2 - 36x + 27$$

$$= 12x^2 - 18x - 18x + 27$$

$$= 3(2x - 3)^2$$

Now,

$$x \in \mathbb{R}$$

$$\Rightarrow (2x - 3)^2 > 0$$

$$\Rightarrow 3(2x - 3)^2 > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all x

31. Question

Prove that the function f given by $f(x) = \log \cos x$ is strictly increasing on $(-\pi/2, 0)$ and strictly decreasing on $(0, \pi/2)$?

Answer

we have,

$$f(x) = \log \cos x$$

$$\therefore f'(x) = \frac{1}{\cos x} (-\sin x) = -\tan x$$

$$\text{In Interval } (0, \frac{\pi}{2}), \tan x > 0 \Rightarrow -\tan x < 0$$

$$\therefore f'(x) < 0 \text{ on } (0, \frac{\pi}{2})$$

$$\therefore f \text{ is strictly decreasing on } (0, \frac{\pi}{2})$$

$$\text{In interval } (\frac{\pi}{2}, \pi), \tan x < 0 \Rightarrow -\tan x > 0$$

$$\therefore f'(x) > 0 \text{ on } (\frac{\pi}{2}, \pi)$$

32. Question

Prove that the function f given by $f(x) = x^3 - 3x^2 + 4x$ is strictly increasing on \mathbb{R} ?

Answer

$$\text{given } f(x) = x^3 - 3x^2 + 4x$$

$$\therefore f'(x) = 3x^2 - 6x + 4$$

$$= 3(x^2 - 2x + 1) + 1$$

$$= 3(x-1)^2 + 1 > 0 \text{ for all } x \in \mathbb{R}$$

Hence $f(x)$ is strictly increasing on \mathbb{R}

33. Question

33 Prove that the function $f(x) = \cos x$ is :

- i. strictly decreasing on $(0, \pi)$
- ii. strictly increasing in $(\pi, 2\pi)$
- iii. neither increasing nor decreasing in $(0, 2\pi)$

Answer

Given $f(x) = \cos x$

$$\therefore f'(x) = -\sin x$$

(i) Since for each $x \in (0, \pi)$, $\sin x > 0$

$$\Rightarrow \therefore f'(x) < 0$$

So f is strictly decreasing in $(0, \pi)$

(ii) Since for each $x \in (\pi, 2\pi)$, $\sin x < 0$

$$\Rightarrow \therefore f'(x) > 0$$

So f is strictly increasing in $(\pi, 2\pi)$

(iii) Clearly from (1) and (2) above, f is neither increasing nor decreasing in $(0, 2\pi)$

34. Question

Show that $f(x) = x^2 - x \sin x$ is an increasing function on $(0, \pi/2)$?

Answer

We have,

$$f(x) = x^2 - x \sin x$$

$$f'(x) = 2x - \sin x - x \cos x$$

Now,

$$x \in (0, \frac{\pi}{2})$$

$$\Rightarrow 0 \leq \sin x \leq 1, 0 \leq \cos x \leq 1,$$

$$\Rightarrow 2x - \sin x - x \cos x > 0$$

$$\Rightarrow f'(x) \geq 0$$

Hence, $f(x)$ is an increasing function on $(0, \frac{\pi}{2})$.

35. Question

Find the value(s) of a for which $f(x) = x^3 - ax$ is an increasing function on \mathbb{R} ?

Answer

We have,

$$f(x) = x^3 - ax$$

$$f'(x) = 3x^2 - a$$

Given that $f(x)$ is an increasing function

$$\therefore f'(x) \geq 0 \text{ for all } x \in \mathbb{R}$$

$$\Rightarrow 3x^2 - a \geq 0 \text{ for all } x \in \mathbb{R}$$

$$\Rightarrow a \leq 3x^2 \text{ for all } x \in \mathbb{R}$$

But the least value of $3x^2 = 0$ for $x = 0$

$$\therefore a \leq 0$$

36. Question

Find the values of b for which the function $f(x) = \sin x - bx + c$ is a decreasing function on \mathbb{R} ?

Answer

We have,

$$f(x) = \sin x - bx + c$$

$$f'(x) = \cos x - b$$

Given that $f(x)$ is a decreasing function on \mathbb{R}

$$\therefore f'(x) \leq 0 \text{ for all } x \in \mathbb{R}$$

$$\Rightarrow \cos x - b \leq 0 \text{ for all } x \in \mathbb{R}$$

$$\Rightarrow b \leq \cos x \text{ for all } x \in \mathbb{R}$$

But the least value of $\cos x$ is -1

$$\therefore b \geq -1$$

37. Question

Show that $f(x) = x + \cos x - a$ is an increasing function on \mathbb{R} for all values of a ?

Answer

We have,

$$f(x) = x + \cos x - a$$

$$f'(x) = 1 - \sin x = \frac{2 \cos^2 x}{2}$$

Now,

$$x \in \mathbb{R}$$

$$\Rightarrow \frac{\cos^2 x}{2} > 0$$

$$\Rightarrow \frac{2 \cos^2 x}{2} > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, $f(x)$ is an increasing function for $x \in \mathbb{R}$

38. Question

Let F defined on $[0, 1]$ be twice differentiable such that $|f''(x)| \leq 1$ for all $x \in [0, 1]$. If $f(0) = f(1)$, then show that $|f'(x)| < 1$ for all $x \in [0, 1]$?

Answer

As $f(0) = f(1)$ and f is differentiable, hence by Rolle's theorem:

$$f'(c) = 0 \text{ for some } c \in [0, 1]$$

let us now apply LMVT (as function is twice differentiable) for point c and $x \in [0,1]$,

hence,

$$\frac{|f'(x)-f'(c)|}{x-c} = f''(d)$$

$$\Rightarrow \frac{|f'(x)-0|}{x-c} = f''(d)$$

$$\Rightarrow \frac{|f'(x)|}{x-c} = f''(d)$$

As given that $|f''(d)| \leq 1$ for $x \in [0,1]$

$$\Rightarrow \frac{|f'(x)|}{x-c} \leq 1$$

$$\Rightarrow |f'(x)| \leq x - c$$

Now both x and c lie in $[0,1]$, hence $x - c \in [0,1]$

39. Question

Find the intervals in which f(x) is increasing or decreasing :

i. $f(x) = x|x|$, $x \in \mathbb{R}$

ii. $f(x) = \sin x + |\sin x|$, $0 < x \leq 2\pi$

iii. $f(x) = \sin x (1 + \cos x)$, $0 < x < \pi/2$

Answer

(i): Consider the given function,

$$f(x) = x|x|, x \in \mathbb{R}$$

$$\Rightarrow f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & x > 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -2x, & x < 0 \\ 2x, & x > 0 \end{cases}$$

$$\Rightarrow f'(x) > 0$$

Therefore, f(x) is an increasing function for all real values.

(ii): Consider the given function,

$$f(x) = \sin x + |\sin x|, 0 < x \leq 2\pi$$

$$\Rightarrow f(x) = \begin{cases} 2\sin x, & 0 < x \leq \pi \\ 0, & \pi < x \leq 2\pi \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2\cos x, & 0 < x \leq \pi \\ 0, & \pi < x \leq 2\pi \end{cases}$$

The function $2\cos x$ will be positive between $(0, \frac{\pi}{2})$

Hence the function f(x) is increasing in the interval $(0, \frac{\pi}{2})$

The function $2\cos x$ will be negative between $(\frac{\pi}{2}, \pi)$

Hence the function f(x) is decreasing in the interval $(\frac{\pi}{2}, \pi)$

The value of $f'(x) = 0$, when, $\pi < x \leq 2\pi$

Therefore, the function f(x) is neither increasing nor decreasing in the interval $(\pi, 2\pi)$

(iii): consider the function,

$$f(x) = \sin x(1 + \cos x), 0 < x < \frac{\pi}{2}$$

$$\Rightarrow f'(x) = \cos x + \sin x(-\sin x) + \cos x(\cos x)$$

$$\Rightarrow f'(x) = \cos x - \sin^2 x + \cos^2 x$$

$$\Rightarrow f'(x) = \cos x + (\cos^2 x - 1) + \cos^2 x$$

$$\Rightarrow f'(x) = \cos x + 2\cos^2 x - 1$$

$$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$$

for $f(x)$ to be increasing, we must have,

$$f'(x) > 0$$

$$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$$

$$\Rightarrow 0 < x < \frac{\pi}{3}$$

So, $f(x)$ to be decreasing, we must have,

$$f'(x) < 0$$

$$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$$

$$\Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$$

$$\Rightarrow x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

So, $f(x)$ is decreasing in $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

MCQ

1. Question

Mark the correct alternative in the following:

The interval of increase of the function $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$ is

- A. $(0, \infty)$
- B. $(-\infty, 0)$
- C. $(1, \infty)$
- D. $(-\infty, 1)$

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a, b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a, b)$

Given:-

$$f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$$

$$\frac{d}{dx}\left(\frac{f(x)}{dx}\right) = 1 - e^x = f'(x)$$

Now

$$f'(x) > 0$$

$$\Rightarrow 1 - e$$

$$x > 0$$

$$x < 0$$

$$x \in (-\infty, 0)$$

2. Question

Mark the correct alternative in the following:

The function $f(x) = \cos^{-1} x + x$ increases in the interval.

A. $(1, \infty)$

B. $(-1, \infty)$

C. $(-\infty, \infty)$

D. $(0, \infty)$

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = \cos^{-1} x + x$$

$$d\left(\frac{f(x)}{dx}\right) = \frac{x^2}{1+x^2} = f'(x)$$

Now

$$f'(x) > 0$$

$$\Rightarrow \frac{x^2}{1+x^2} > 0$$

$$x \in \mathbb{R}$$

$$\Rightarrow x \in (-\infty, \infty)$$

3. Question

Mark the correct alternative in the following:

The function $f(x) = x^x$ decreases on the interval.

A. $(0, e)$

B. $(0, 1)$

C. $(0, 1/e)$

D. $(1/e, e)$

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = x^x$$

$$d\left(\frac{f(x)}{dx}\right) = x^x(1 + \log x) = f'(x)$$

now for decreasing

$$f'(x) < 0$$

$$\Rightarrow x^x(1 + \log x) < 0$$

$$\Rightarrow (1 + \log x) < 0$$

$$\Rightarrow \log x < -1$$

$$\Rightarrow x < e^{-1}$$

$$x \in \left(0, \frac{1}{e}\right)$$

4. Question

Mark the correct alternative in the following:

The function $f(x) = 2\log(x - 2) - x^2 + 4x + 1$ increases on the interval.

A. (1, 2)

B. (2, 3)

C. ((1, 3)

D. (2, 4)

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = 2\log(x - 2) - x^2 + 4x + 1$$

$$d\left(\frac{f(x)}{dx}\right) = \frac{2}{x-2} - 2x + 4 = f'(x)$$

$$\Rightarrow f'(x) = -\frac{2(x-1)(x-3)}{x-2}$$

now for increasing

$$f'(x) > 0$$

$$\Rightarrow -\frac{2(x-1)(x-3)}{x-2} < 0$$

$$x - 3 < 0 \text{ and } x - 2 > 0$$

$$x < 3 \text{ and } x > 2$$

$$x \in (2, 3)$$

5. Question

Mark the correct alternative in the following:

If the function $f(x) = 2x^2 - kx + 5$ is increasing on [1, 2], then k lies in the interval.

A. $(-\infty, 4)$

B. $(4, \infty)$

C. $(-\infty, 8)$

D. $(8, \infty)$

Answer

Formula:- The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x) > 0$ for all $x \in (a,b)$

$$f(x) = 2x^2 - kx + 5$$

$$d\left(\frac{f(x)}{dx}\right) = 4x - k = f'(x)$$

$$f'(x) > 0$$

$$\Rightarrow 4x - k > 0$$

$$\Rightarrow k < 4x$$

For $x=1$

$$\Rightarrow k < 4$$

6. Question

Mark the correct alternative in the following:

Let $f(x) = x^3 + ax^2 + bx + 5 \sin^2 x$ be an increasing function on the set \mathbb{R} . Then, a and b satisfy.

A. $a^2 - 3b - 15 > 0$

B. $a^2 - 3b + 15 > 0$

C. $a^2 - 3b + 15 < 0$

D. $a > 0$ and $b > 0$

Answer

Formula:- (i) $ax^2+bx+c > 0$ for all $x \Rightarrow a > 0$ and $b^2-4ac < 0$

(ii) $ax^2+bx+c < 0$ for all $x \Rightarrow a < 0$ and $b^2-4ac < 0$

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = x^3 + ax^2 + bx + 5 \sin^2 x$$

$$d\left(\frac{f(x)}{dx}\right) = 3x^2 + 2ax + b + 5 \sin 2x = f'(x)$$

For increasing function $f'(x) > 0$

$$3x^2 + 2ax + b + 5 \sin 2x > 0$$

Then

$$3x^2 + 2ax + b - 5 < 0$$

And $b^2 - 4ac < 0$

$$\Rightarrow 4a^2 - 12(b-5) < 0$$

$$\Rightarrow a^2 - 3b + 15 < 0$$

$$\Rightarrow a^2 - 3b + 15 < 0$$

7. Question

Mark the correct alternative in the following:

The function $f(x) = \log_e \left(x^3 + \sqrt{x^6 + 1} \right)$ is of the following types:

- A. even and increasing
- B. odd and increasing
- C. even and decreasing
- D. odd and decreasing

Answer

Formula:- (i) if $f(-x)=f(x)$ then function is even

(ii) if $f(-x)=-f(x)$ then function is odd

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$$f(x) = \log_e(x^3 + \sqrt{x^6 + 1})$$

$$d\left(\frac{f(x)}{dx}\right) = \frac{1}{x^3(x^6 + 1)^{\frac{1}{2}}} \left(3x^2 + \frac{6x^5}{2(x^6 + 1)^{\frac{1}{2}}} \right)$$

$$f'(x) > 0$$

hence function is increasing function

$$f(-x) = -\log(\log_e(x^3 + \sqrt{x^6 + 1}))$$

$$\Rightarrow f(-x) = -f(x) \text{ is odd function}$$

8. Question

Mark the correct alternative in the following:

If the function $f(x) = 2\tan x + (2a + 1) \log_e |\sec x| + (a - 2)x$ is increasing on \mathbb{R} , then

- A. $a \in \left(\frac{1}{2}, \infty \right)$
- B. $a \in \left(-\frac{1}{2}, \frac{1}{2} \right)$
- C. $a = \frac{1}{2}$
- D. $a \in \mathbb{R}$

Answer

Formula:- (i) $ax^2+bx+c>0$ for all $x \Rightarrow a>0$ and $b^2-4ac<0$

(ii) $ax^2+bx+c<0$ for all $x \Rightarrow a<0$ and $b^2-4ac<0$

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$$f(x) = 2\tan x + (2a+1)\log_e |\sec x| + (a-2)x$$

$$\frac{d(f(x))}{dx} = 2 \sec^2 x + \frac{(2a+1)\sec x \cdot \tan x}{\sec x} + (a-2) = f'(x)$$

$$\Rightarrow f'(x) = 2\sec^2 x + (2a+1)\tan x + (a-2)$$

$$\Rightarrow f'(x) = 2(\tan^2 x + 1) + (2a+1)\tan x + (a-2)$$

$$\Rightarrow f'(x) = 2\tan^2 x + 2a\tan x + \tan x + a$$

For increasing function

$$f'(x) > 0$$

$$\Rightarrow 2\tan^2 x + 2a\tan x + \tan x + a > 0$$

From formula (i)

$$(2a+1)^2 - 8a < 0$$

$$\Rightarrow 4\left(a - \frac{1}{2}\right)^2 < 0$$

$$\Rightarrow a = \frac{1}{2}$$

9. Question

Mark the correct alternative in the following:

Let $f(x) = \tan^{-1}(g(x))$, where $g(x)$ is monotonically increasing for $0 < x < \frac{\pi}{2}$. Then, $f(x)$ is

A. increasing on $\left(0, \frac{\pi}{2}\right)$

B. decreasing on $\left(0, \frac{\pi}{2}\right)$

C. increasing on $\left(0, \frac{\pi}{4}\right)$ and decreasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

D. none of these

Answer

Formula:-

(i) The necessary and sufficient condition for differentiable function defined on (a, b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a, b)$

$$\text{Given:- } f(x) = \tan^{-1}(g(x))$$

$$\frac{d(f(x))}{dx} = \frac{g'(x)}{1 + (g(x))^2} = f'(x)$$

For increasing function

$$f'(x) > 0$$

$$x \in \left(0, \frac{\pi}{2}\right)$$

10. Question

Mark the correct alternative in the following:

Let $f(x) = x^3 - 6x^2 + 15x + 3$. Then,

- A. $f(x) > 0$ for all $x \in \mathbb{R}$
- B. $f(x) > f(x + 1)$ for all $x \in \mathbb{R}$
- C. $f(x)$ is invertible
- D. $f(x) < 0$ for all $x \in \mathbb{R}$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a, b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a, b)$

(ii) If $f(x)$ is strictly increasing function on interval $[a, b]$, then f^{-1} exist and it is also a strictly increasing function

Given:- $f(x) = x^3 - 6x^2 + 15x + 3$

$$\frac{d(f(x))}{dx} = 3x^2 - 12x + 15 = f'(x)$$

$$\Rightarrow f'(x) = 3(x - 2)^2 + \frac{1}{3}$$

$$\Rightarrow f'(x) = 3(x - 2)^2 + \frac{1}{3}$$

Therefore $f'(x)$ will be increasing

Also $f^{-1}(x)$ is possible

Therefore $f(x)$ is invertible function.

11. Question

Mark the correct alternative in the following:

The function $f(x) = x^2 e^{-x}$ is monotonic increasing when

- A. $x \in \mathbb{R} - [0, 2]$
- B. $0 < x < 2$
- C. $2 < x < \infty$
- D. $x < 0$

Answer

$$f(x) = x^2 e^{-x}$$

$$\frac{d(f(x))}{dx} = x e^{-x} (2 - x) = f'(x)$$

for

$$f'(x) = 0$$

$$\Rightarrow x^2 e^{-x} = 0$$

$$\Rightarrow x(2 - x) = 0$$

$$x=2, x=0$$

$f(x)$ is increasing in $(0,2)$

12. Question

Mark the correct alternative in the following:

Function $f(x) = \cos x - 2\lambda x$ is monotonic decreasing when

A. $\lambda > \frac{1}{2}$

B. $\lambda < \frac{1}{2}$

C. $\lambda < 2$

D. $\lambda > 2$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly decreasing on (a,b) is that $f'(x) < 0$ for all $x \in (a,b)$

Given:-

$$f(x) = \cos x - 2\lambda x$$

$$\frac{d(f(x))}{dx} = -\sin x - 2\lambda = f'(x)$$

for decreasing function $f'(x) < 0$

$$-\sin x - 2\lambda < 0$$

$$\Rightarrow \sin x + 2\lambda > 0$$

$$\Rightarrow 2\lambda > -\sin x$$

$$\Rightarrow 2\lambda > 1$$

$$\Rightarrow \lambda > \frac{1}{2}$$

13. Question

Mark the correct alternative in the following:

In the interval $(1, 2)$, function $f(x) = 2|x - 1| + 3|x - 2|$ is

A. monotonically increasing

B. monotonically decreasing

C. not monotonic

D. constant

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly decreasing on (a,b) is that $f'(x) < 0$ for all $x \in (a,b)$

Given:-

$$f(x) = 2(x-1) + 3(2-x)$$

$$f(x) = -x + 4$$

$$\frac{d(f(x))}{dx} = -1 = f'(x)$$

Therefore $f'(x) < 0$

Hence decreasing function

14. Question

Mark the correct alternative in the following:

Function $f(x) = x^3 - 27x + 5$ is monotonically increasing when

A. $x < -3$

B. $|x| > 3$

C. $x \leq -3$

D. $|x| \geq 3$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = x^3 - 27x + 5$$

$$\frac{d(f(x))}{dx} = 3x^2 - 27 = f'(x)$$

for increasing function $f'(x) > 0$

$$3x^2 - 27 > 0$$

$$\Rightarrow (x+3)(x-3) > 0$$

$$\Rightarrow |x| > 3$$

15. Question

Mark the correct alternative in the following:

Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when

A. $x < 2$

B. $x > 2$

C. $x > 3$

D. $1 < x < 2$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly decreasing on (a, b) is that $f'(x) < 0$ for all $x \in (a,b)$

Given:-

$$f(x) = 2x^3 - 9x^2 + 12x + 29$$

$$\frac{d(f(x))}{dx} = f'(x) = 6(x-1)(x-2)$$

for decreasing function $f'(x) < 0$

$$f'(x) < 0$$

$$\Rightarrow 6(x-1)(x-2) < 0$$

$$\Rightarrow 1 < x < 2$$

16. Question

Mark the correct alternative in the following:

If the function $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then

A. $k < 3$

B. $k \leq 3$

C. $k > 3$

D. $k < 3$

Answer

Formula:- (i) $ax^2+bx+c > 0$ for all $x \Rightarrow a > 0$ and $b^2-4ac < 0$

(ii) $ax^2+bx+c < 0$ for all $x \Rightarrow a < 0$ and $b^2-4ac < 0$

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = kx^3 - 9x^2 + 9x + 3$$

$$\frac{d(f(x))}{dx} = f'(x) = 3kx^2 - 18x + 9$$

for increasing function $f'(x) > 0$

$$f'(x) > 0$$

$$\Rightarrow 3kx^2 - 18x + 9 > 0$$

$$\Rightarrow kx^2 - 6x + 3 > 0$$

using formula (i)

$$36 - 12k < 0$$

$$\Rightarrow k > 3$$

17. Question

Mark the correct alternative in the following:

$f(x) = 2x - \tan^{-1} x - \log \left\{ x + \sqrt{x^2 + 1} \right\}$ is monotonically increasing when

A. $x > 0$

B. $x < 0$

C. $x \in \mathbb{R}$

D. $x \in \mathbb{R} - \{0\}$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a,b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = 2x - \tan^{-1}x - \log\{x + \sqrt{x^2 + 1}\}$$

$$\frac{df(x)}{dx} = 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{x^2+1}} = f'(x)$$

For increasing function $f'(x) > 0$

$$\Rightarrow 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{x^2+1}} > 0$$

$x \in \mathbb{R}$

18. Question

Mark the correct alternative in the following:

Function $f(x) = |x| - |x - 1|$ is monotonically increasing when

- A. $x < 0$
- B. $x > 1$
- C. $x < 1$
- D. $0 < x < 1$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a, b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a, b)$

Given:-

For $x < 0$

$$f(x) = -1$$

for $0 < x < 1$

$$f(x) = 2x - 1$$

for $x > 1$

$$f(x) = 1$$

Hence $f(x)$ will be increasing in $0 < x < 1$

19. Question

Mark the correct alternative in the following:

Every invertible function is

- A. monotonic function
- B. constant function
- C. identity function
- D. not necessarily monotonic function

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a, b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a, b)$

If $f(x)$ is strictly increasing function on interval $[a, b]$, then f^{-1} exists and it is also a strictly increasing function

20. Question

Mark the correct alternative in the following:

In the interval (1, 2), function $f(x) = 2|x - 1| + 3|x - 2|$ is

- A. increasing
- B. decreasing
- C. constant
- D. none of these

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly decreasing on (a, b) is that $f'(x) < 0$ for all $x \in (a,b)$

Given:-

$$f(x) = 2(x-1) + 3(2-x)$$

$$\Rightarrow f(x) = -x + 4$$

$$\frac{d(f(x))}{dx} = f'(x) = -1$$

Therefore $f'(x) < 0$

Hence decreasing function

21. Question

Mark the correct alternative in the following:

If the function $f(x) = \cos|x| - 2ax + b$ increases along the entire number scale, then

- A. $a = b$
- B. $a = \frac{1}{2}b$
- C. $a \leq -\frac{1}{2}$
- D. $a > -\frac{3}{2}$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = \cos|x| - 2ax + b$$

$$\frac{d(f(x))}{dx} = -\sin x - 2a = f'(x)$$

For increasing $f'(x) > 0$

$$\Rightarrow -\sin x - 2a > 0$$

$$\Rightarrow 2a < -\sin x$$

$$\Rightarrow 2a \leq -1$$

$$\Rightarrow a \leq -\frac{1}{2}$$

22. Question

Mark the correct alternative in the following:

The function $f(x) = \frac{x}{1+|x|}$ is

- A. strictly increasing
- B. strictly decreasing
- C. neither increasing nor decreasing
- D. none of these

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a,b)$

$$f(x) = \frac{x}{1+|x|}$$

For $x > 0$

$$\frac{d(f(x))}{dx} = \frac{1}{1+x^2} = f'(x)$$

For $x < 0$

$$\frac{d(f(x))}{dx} = \frac{1}{1-x^2} = f'(x)$$

Both are increasing for $f'(x) > 0$

23. Question

Mark the correct alternative in the following:

The function $f(x) = \frac{\lambda \sin x + 2 \cos x}{\sin x + \cos x}$ is increasing, if

- A. $\lambda < 1$
- B. $\lambda > 1$
- C. $\lambda < 2$
- D. $\lambda > 2$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = \frac{\lambda \sin x + 2 \cos x}{\sin x + \cos x}$$

For increasing function $f'(x) > 0$

$$\frac{d(f(x))}{dx} = f'(x) = \frac{\lambda - 2}{(\sin x + \cos x)^2} > 0$$

$$\Rightarrow \lambda > 2$$

24. Question

Mark the correct alternative in the following:

Function $f(x) = a^x$ is increasing on \mathbb{R} , if

- A. $a > 0$
- B. $a < 0$
- C. $a > 1$
- D. $a > 0$

Answer

Let $x_1 < x_2$ and both are real number

$$a^{x_1} < a^{x_2}$$

$$\Rightarrow f(x_1) < f(x_2)$$

$$\Rightarrow x_1 < x_2 \in \mathbb{R}$$

only possible on $a > 1$

25. Question

Mark the correct alternative in the following:

Function $f(x) = \log_a x$ is increasing on \mathbb{R} , if

- A. $0 < a < 1$
- B. $a > 1$
- C. $a < 1$
- D. $a > 0$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a, b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a, b)$

$$f(x) = \log_a x$$

$$\frac{d(f(x))}{dx} = \frac{1}{x \log_e a} = f'(x)$$

For increasing $f'(x) > 0$

$$\Rightarrow \frac{1}{x \log_e a} > 0$$

For $\log a > 1$

26. Question

Mark the correct alternative in the following:

Let $\phi(x) = f(x) + f(2a - x)$ and $f''(x) > 0$ for all $x \in [0, a]$. The, $\phi(x)$

- A. increases on $[0, a]$
- B. decreases on $[0, a]$
- C. increases on $[-a, 0]$
- D. decreases on $[a, 2a]$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a,b)$

$$\phi(x) = f(x) + f(2a - x)$$

$$\Rightarrow \phi'(x) = f'(x) - f'(2a - x)$$

$$\Rightarrow \phi''(x) = f''(x) + f''(2a - x)$$

checking the condition

$\phi(x)$ is decreasing in $[0, a]$

27. Question

Mark the correct alternative in the following:

If the function $f(x) = x^2 - kx + 5$ is increasing on $[2, 4]$, then

A. $k \in (2, \infty)$

B. $k \in (-\infty, 2)$

C. $k \in (4, \infty)$

D. $k \in (-\infty, 4)$

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = x^2 - kx + 5$$

$$\frac{d(f(x))}{dx} = 2x - k = f'(x)$$

For increasing function $f'(x) > 0$

$$2x - k > 0$$

$$\Rightarrow k < 2x$$

Putting $x=2$

$$k < 4$$

$$\Rightarrow k \in (-\infty, 4)$$

28. Question

Mark the correct alternative in the following:

The function $f(x) = -\frac{x}{2} + \sin x$ defined on $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ is

A. increasing

B. decreasing

C. constant

D. none of these

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a,b)$

Given:-

$$f(x) = -\frac{x}{2} + \sin x$$

$$\frac{d(f(x))}{dx} = -\frac{1}{2} + \cos x = f'(x)$$

checking the value of x

$$\cos - \frac{1}{2} > 0$$

hence increasing

29. Question

Mark the correct alternative in the following:

If the function $f(x) = x^3 - 9kx^2 + 27x + 30$ is increasing on \mathbb{R} , then

A. $-1 \leq k < 1$

B. $k < -1$ or $k > 1$

C. $0 < k < 1$

D. $-1 < k < 0$

Answer

Formula:- (i) $ax^2+bx+c>0$ for all $x \Rightarrow a>0$ and $b^2-4ac<0$

(ii) $ax^2+bx+c<0$ for all $x \Rightarrow a<0$ and $b^2-4ac<0$

(iii) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x)>0$ for all $x \in (a,b)$

Given:-

$$f(x) = x^3 - 9kx^2 + 27x + 30$$

$$\frac{d(f(x))}{dx} = f'(x) = 3x^2 - 18kx + 27$$

for increasing function $f'(x)>0$

$$3x^2 - 18kx + 27 > 0$$

$$\Rightarrow x^2 - 6kx + 9 > 0$$

Using formula (i)

$$36k^2 - 36 > 0$$

$$\Rightarrow k^2 > 1$$

Therefore $-1 < k < 1$

30. Question

Mark the correct alternative in the following:

The function $f(x) = x^9 + 3x^7 + 64$ is increasing on

A. \mathbb{R}

B. $(-\infty, 0)$

C. $(0, \infty)$

D.R₀

Answer

Formula:- (i) The necessary and sufficient condition for differentiable function defined on (a,b) to be strictly increasing on (a, b) is that $f'(x) > 0$ for all $x \in (a, b)$

Given:-

$$f(x) = x^9 + 3x^7 + 64$$

$$\frac{d(f(x))}{dx} = 9x^8 + 21x^6 = f'(x)$$

For increasing $f'(x) > 0$

$$\Rightarrow 9x^8 + 21x^6 > 0$$

$$\Rightarrow x \in \mathbb{R}$$