

- 1. A circle is a collection of all such points in a plane which are equidistant from a fixed point. This fixed point is called the **centre** of the circle while the distance of any point on the circle from the centre is called the **radius** of the circle.
- 2. A line segment joining any two points on a circle is called the chord of the circle.
- **3.** The chord passing through the centre of the circle is called the **diameter**. The diameter divides a circle into two equal parts called **semicircles**.



- 4. A line which intersects a circle in two distinct points is called a secant of the circle.
- 5. (i) A continuous piece of a circle is called an **arc** of the circle.
 - An arc whose length is less than the semicircular arc is called **minor arc** and if greater than semicircular arc is called **major arc**.



- (ii) A chord AB of a circle divides the circular region into two parts called segments of the circle.
 - The bigger part containing the centre of the circle is called the **major segment** and the smaller part which does not contain the centre is called the **minor segment** of the circle.

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- 6. A line touching a circle at a point is called the **tangent** to the circle.
- 7. Circles having the same centre are called **concentric circles**.
- 8. Two circles are said to congruent if and only if they have the same radii.
- **9.** The degree measure of an arc AB is denoted by m (AB). Two arcs of a circle (or of congruent circles) are said to be congruent if and only if they have the same degree measure.

10. Important Properties of Circles

- (i) If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.
 Conversely, if two chords of a circle (or of congruent circles) are equal, then their corresponding arcs are equal.
- (ii) The angle in a semi-circle is a right angle.

 $\angle ACB = 90^{\circ}$

(iii) The perpendicular from the centre to a chord bisects the chord.

If $OP \perp AB$, then AP = PB.

Conversely, the line joining the centre of the circle and the mid-point of the chord is perpendicular to the chord.

If P is the mid-point of AB, then $OP \perp AB$.

- (iv) There is one and only one circle passing through three non-collinear points.
- (v) Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centres).

If PQ = RS, then OA = OB.

Conversely, chords of a circle (or of congruent circles) which are equidistant from the centre or corresponding centres are equal.

If OA = OB, then PQ = RS.

- (vi) Equal chords subtend equal angles at the centre.
 If AB = CD, then ∠AOB = ∠ COD.
 If angles subtended by two chords of a circle at the centre are equal, then the two chords are equal.
 If ∠AOB = ∠ COD, then AB = CD
- (vii) The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.





(viii) Angles in the same segment of a circle are equal.

$$\angle PAQ = \angle PBQ = \angle PCQ$$

(ix) If all the vertices of a quadrilateral lie on a circle, it is called a **cyclic quadrilateral**. *ABCD* is a cyclic quadrilateral.













- (x) The opposite angles of a cyclic quadrilateral are supplementary. $\angle A + \angle C = 180^{\circ}$ and $\angle B + \angle D = 180^{\circ}$
- (xi) If the sum of any pair of opposite angles of a quadrilateral is 180°, then it is cyclic.
- (xii) If one side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle.

$$\angle BDE = \angle CAB$$

(xiii) The quadrilateral formed by the angle bisectors of a cyclic quadrilateral is also cyclic.

- (xiv) If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.
- (xv) An isosceles trapezium is cyclic.
- (xvi) Tangent to a circle at a point is perpendicular to the radius through the point of contact. $OR \perp l$
- (xvii) From a point lying outside a circle, two and only two tangents can be drawn to it.
- (xviii) The lengths of the two tangents drawn from an external point to a circle are equal

$$PA = PB$$

(xix) The angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.

$$\angle AOB + \angle APB = 180^{\circ}$$

(xx) The tangents are equally inclined to the line joining the external point (from where the tangents are drawn) and the centre of the circle.

 $\angle APO = \angle BPO$ and $\angle AOB = \angle BOP$



Question Bank-24

- 1. *O* is the centre of a circle with radius 5 cm. *LM* is the diameter of the circle. *P* is a point on the plane of the circle such that LP = 6 cm and MP = 8 cm. Then *P* lies.
 - (a) on LM
 - (b) outside the circle
 - (c) inside the circle
 - (d) on the circle.
- **2.** If the length of a chord of a circle is equal to its radius, then the angle subtended by it at the minor arc of the circle will be,

(a) 60°	(b)	75°
(c) 120°	(d)	150

3. Given a circle with centre *O*. The smallest chord PQ is of length 4 cm largest chord *AB* is of length 10 cm and chord *EF* is of length 7 cm. Then, the radius of the circle is

(a) 3 cm	(b) 2 cm
(c) 5 cm	(d) 3.5 cm

4. The radius of a circle is 6 cm. The perpendicular distance from the centre of the circle to the chord which is 8 cm in length is

(a)	$\sqrt{5}$ cm	(b)	$2\sqrt{5}$ cm
(c)	$2\sqrt{7}$ cm	(d)	$\sqrt{7}$ cm



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o



- 5. *PQ* and *RS* are two parallel chords of a circle with centre *C* such that PQ = 8 cm and RS = 16 cm. If the chords are on the same side of the centre and the distance between them is 4 cm, then the radius of the circle is :
 - (a) $3\sqrt{2}$ cm (b) $3\sqrt{5}$ cm
 - (c) $4\sqrt{5}$ cm (d) $5\sqrt{5}$ cm
- 6. In the given figure, *O* is the centre of the circle. The measure of $\angle ADB$ is



7. Given that *AOB* is a straight line and *O* is the centre of the circle. Find the value of *y*.



8. In the given diagram, AB is the diameter of the given circle with centre O. C and D are points on the circumference of the circle. If $\angle ABD = 35^{\circ}$ and $\angle CDB = 15^{\circ}$, then $\angle CBD$ equals.



9. In the diagram, *A*, *B*, *C*, *D*, *E* are points on the circle. $AD \parallel BC, \angle ABE = 39^{\circ}$ and $\angle ADC = 62^{\circ}$. Then the values of *x* and *y* respectively are:

(a) 23°, 51°	(b) 79°, 62°
(c) 62°, 79°	(d) 51°, 23°



10. In the given figure, *O* is the centre of the circle, $\angle ACB = 54^{\circ}$ and *BCE* is a straight line. Find *x*.



11. In the given figure, *BOD* is the diameter of the circle with centre *O*. $\angle COD = 92^{\circ}$ and $\angle ABD = 65^{\circ}$. Then *y* equals



12. O is the centre of the circle x and y respectively equal.



13. In the given figure,

AB and AC are tangents to the circle with centre O. Given that $\angle BAC = 70^{\circ}$ and P is a point on the



- **14.** The length of a tangent drawn from a point 10 cm away from the centre of the circle of radius 5 cm is
 - (a) 5 cm (b) $5\sqrt{3}$ cm
 - (c) $2\sqrt{3}$ cm (d) $\sqrt{15}$ cm
- **15.** In the figure shown here, a circle touches the side *BC* of a triangle *ABC* at *P* and *AB* and *AC* produced at *Q* and *R* respectively. What is *AQ* equal to?



- (a) One-third of the perimeter of $\triangle ABC$.
- (b) Half of the perimeter of $\triangle ABC$.
- (c) Two-third of the perimeter of ΔABC .
- (d) Three-fourth of the perimeter of $\triangle ABC$.
- **16.** In the given figure, *AB* and *AC* are tangents to the circle at *B* and *C* respectively and *O* is the centre of the circle, then *x* equals



17. *ABC* is an isosceles triangle (AB = AC) circumscribed about a circle. Then, which of the following statements is correct?



18. In the figure, *CDE* is a straight line and *A*, *B*, *C* and *D* are points on the circle. $\angle BCD = 44^{\circ}$, find the value of *x*.



(a) 44°	(b) 68°
(c) 90°	(d) 56°

- **19.** From a point *P* which is at a distance of 13 cm from the centre *O* of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is :
 - (a) 60 cm^2 (b) 65 cm^2
 - (c) 30 cm^2 (d) 32.5 cm^2
- **20.** In the given figure, $\angle AFD = 25^{\circ}$. $\therefore \angle EFC$ equals



Answers									
1. (d)	2. (d)	3. (c)	4. (b)	5. (c)	6. (c)	7. (c)	8. (c)	9. (b)	10. (c)
11. (d)	12. (b)	13. (c)	14. (b)	15. (b)	16. (c)	17. (c)	18. (c)	19. (a)	20. (d)

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Hints and Solutions

1. (d) $LM = 2 \times OL = (2 \times 5) \text{ cm} = 10 \text{ cm}$ $LP^2 + PM^2 = 6^2 + 8^2 = 36 + 64 = 100$ and $LM^2 = 10^2 = 100$



- $\Rightarrow LP^2 + PM^2 = LM^2 \Rightarrow \Delta LPM \text{ is right angled}$ $triangle, rt. \angle d \text{ at } P \Rightarrow \angle LPM = 90^\circ$
- \Rightarrow *P* lies on the circumference of the circle
- (∴ Angle in a semi-circle is a rt. ∠)
 2. (d) Let AB be the chord, then the angle subtended by AB at the minor arc of the circle is ∠ACB. Given, OA = OB = AB



- $\Rightarrow \Delta OAB$ is equilateral
- $\Rightarrow \angle AOB = 60^{\circ}$
- \Rightarrow Reflex $\angle AOB = 360^{\circ} 60^{\circ} = 300^{\circ}$

$$\therefore \ \angle ACB = \frac{1}{2} \times \text{Reflex} \ \angle AOB = \frac{1}{2} \times 300^{\circ} = 150^{\circ}$$

(Angle at the centre = $2 \times$ angle at any pt. on remaining part of the circle)

- 3. (c) The diameter is the largest chord of a circle, so radius = $\frac{1}{2} \times AB = \frac{1}{2} \times 10$ cm = 5 cm.
- **4.** (b) Let AB = 8 cm be the given chord. Radius OA = 6 cm.



Since the perpendicular from the centre of the circle to the chord bisects the chord,

$$AP = PB = 4$$
 cm.
In rt. $\angle d \Delta OAP$, $OP^2 = OA^2 - AP^2$
 $= 36 - 16 = 20$

$$\Rightarrow OP = \sqrt{20} = 2\sqrt{5} \text{ cm}$$

5. (c) Given, chords PQ = 8 cm and RS = 16 cm and AB = 4 cm



Since, the perpendicular from the centre of the circle to the chord bisects the chord, RB = BS = 8 cm and PA = AO = 4 cm. Let OB = x cm, then OA = OB + AB = (x + 4) cm. In right triangle OBS, $OS^2 = OB^2 + BS^2 = OB^2 + 64$...(*i*) In right triangle OAQ, $OQ^2 = OA^2 + AQ^2 = OA^2 + 16$... (ii) : OS and OO are the radii of the circle $\therefore OB^2 + 64 = OA^2 + 16$ $\Rightarrow x^2 + 64 = (x + 4)^2 + 16$ $\Rightarrow x^2 + 64 = x^2 + 8x + 16 + 16$ $\Rightarrow 8x = 32 \Rightarrow x = 4$ \therefore From eqn (i) Radius of the circle $(OS)^2 = 16 + 64 = 80$ $\Rightarrow OS = 4\sqrt{5}$ cm **6.** (c) $\angle CEB = \angle DEA = 60^{\circ}$ (vert. opp. $\angle s$) \therefore In $\triangle CEB$, $\angle ECB = 180^\circ - (60^\circ + 25^\circ)$ = $180^{\circ} - 85^{\circ} = 95^{\circ}$ (Angle sum prop. of a Δ) $\therefore \ \angle ADB = \angle BCA \ (\ \angle BCE) = 95^{\circ}$ (Angles in the same segment) 7. (c) $\angle BOC = 2 \times \angle BAC = 2 \times 22^\circ = 44^\circ$ (Angle at the centre = $2 \times$ angle at any point on the remaining part of the circle) $\angle AOC = 180^{\circ} - \angle BOC = 180^{\circ} - 44 = 136^{\circ}$ (:: AOB is a straight line) $\Rightarrow \angle ADC = \frac{1}{2} \times \angle AOC = \frac{1}{2} \times 136^\circ = 68^\circ$ (Angle at any pt. on the remaining part of the circle = $\frac{1}{2}$ × angle at the centre) **8.** (c) $\angle ADB = 90^{\circ}$ (Angle in a semi-circle) In $\Delta ADB, \angle DAB = 180^{\circ} - (\angle ADB + \angle DBA)$ $= 180^{\circ} - (90^{\circ} + 35^{\circ}) = 180^{\circ} - 125^{\circ} = 55^{\circ}$ (Angle sum prop. of a Δ) In cyclic quad ABCD, $\angle A + \angle C = 180^{\circ} \Rightarrow \angle C = 180^{\circ} - 55^{\circ} = 125^{\circ}$ (Opp. $\angle s$ of a cyclic quad. are supp.)

:. In $\Delta DCB, \angle CBD = 180^\circ - (15^\circ + 125^\circ) = 40^\circ$

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17. (c)
$$AB = AC$$

 $\Rightarrow AD + DB = AE + EC$
 $\Rightarrow DB = EC (\because AD = AE, \text{ tangents to a circle from the same external pt. are equal})$
 $\Rightarrow BF = FC (BD = BF \text{ and } CE = CF)$
18. (c) $\angle CDB = \frac{1}{2}(180^\circ - 44^\circ) = \frac{1}{2} \times 136^\circ = 68^\circ$
 $(\because BCD \text{ is an isos. } \Delta)$
 $\angle BAD = 180^\circ - 44^\circ = 136^\circ$
 $(\text{opp.} \angle s \text{ of a cyclic quad. are supp.})$
 $\therefore \angle ADB = \frac{1}{2}(180^\circ - 136^\circ) = \frac{1}{2} \times 44^\circ = 22^\circ$
 $(\because BAD \text{ is an isos. } \Delta)$
 $\therefore \angle ADC = \angle ADB + \angle BDC = 22^\circ + 68^\circ = 90^\circ$
 $\Rightarrow x = \angle ADE = 180^\circ - \angle ADC = 180^\circ - 90^\circ = 90^\circ$
 $(\because EDC \text{ is a st. line})$
19. (a) The radius \bot tangent at the pt. of contact, therefore, $OQ \bot PQ$ and $OR \bot PR$



Self Assessment Sheet-24

1. *O* is the centre of a circle. There is a point *P* in the region of the circle. If *PA* = *PB* = *PC* where *A*, *B* and *C* are points on the circumference of the circle, then *OP* must be equal to:

(a)
$$\frac{PA + PB + PC}{3}$$
 (b) $\frac{PA + PB + PC}{2}$
(c) $\frac{AB + BC}{2}$ (d) zero

2. In the given figure, *O* is the centre of the circle. Given that OD = OE = 3 cm and AD = 4 cm. Find the length of the longest chord.



3. *AOD* is a diameter of the circle with centre *O*. Given that $\angle BDA = 18^{\circ}$ and $\angle BDC = 38^{\circ} . \angle BCD$ equals



(a) 90°	(b) 108°
(c) 76°	(b) 52°

- 4. Tangents drawn at the end points of a diameter are(a) Perpendicular(b) Parallel
 - (c) Intersecting (b) None of these
- 5. In the given figure, AB is a chord of the circle with centre O and PQ is a tangent at point B of the circle. If $\angle AOB = 110^\circ$, then $\angle ABQ$ is



6. In the given figure, if *PA* and *PB* are tangents to the circle with centre *O* such that $\angle APB = 54^\circ$, then $\angle OAB$ equals

(a) 36°	(b) 18°
(c) 27°	(d) 36°

7. If two tangents inclined at an angle of 60° are drawn to a circle of radius 4 cm, then the length of each tangent is equal to:

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(a) $2\sqrt{3}$ cm (b) 8 cm

(c) 4 cm (d) $4\sqrt{3}$ cm

8. In the given figure , *RST* is the tangent to the circle with centre *O*, at *S*. *AOS* is a straight line *BO* || *RT* and $\angle ORS = 46^{\circ}$. Then $\angle BAC$ equals



9. In the diagram, *CB* and *CD* are tangents to the circle with centre *O*. *AOC* is a straight line and $\angle OCB = 34^\circ$. $\angle ABO$ equals.



10. *ED* is the tangent to the circle with centre *O*. $\angle BCD = 52^{\circ}$. Then, $\angle CAB$ equals



Answers						
1. (d)	2. (c)	3. (b)	4. (b)	5. (c)	6. (c)	7. (d) (Use trigonometrical ratios)
8. (c)	9. (b)	10. (c)				

Unit Test-4

1. *ABC* is a triangle whose altitudes *BE* and *CF* to sides *AC* and *AB* respectively are equal. Which of these conditions is not required to prove $\triangle ABE \cong \triangle ACF$? (a) $\angle B = \angle C$ (b) $\angle BAE = \angle FAC$

(c)
$$\angle AFC = \angle AEB$$
 (d) $BE = CF$

2. *ABCD* is a square and $\triangle DEC$ is an equilateral triangle. $\triangle ADE \cong \triangle BCE$ by



(a) RHS

- (c) AAS
- **3.** The centroid and the orthocentre are coincident for which one of the following triangles?

(d) SAS

- (a) Scalene triangle (b) Isosceles triangle
- (c) Equilateral triangle (d) Right angled triangle

- **4.** *ABC* is an isosceles triangle right angled at *B*. Similar triangles *ACD* and *ABE* are constructed on sides *AC* and *AB*. The ratio between the areas of $\triangle ABE$ and $\triangle ACD$ is
 - (a) $\sqrt{2}:1$ (b) 1:2 (c) 2:1 (d) $\sqrt{2}:1$
- 5. In $\triangle ABC$ and $\triangle DEF$, it is given that AB = 5 cm, BC = 4 cm, CA = 4.2 cm, DE = 10 cm, EF = 8 cm and FD = 8.4 cm. If AL is perpendicular to BC and DM is perpendicular to EF, then what is the ratio of AL to DM.

(a)
$$\frac{1}{2}$$
 (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) 1

- 6. In ΔPQR , PQ = 4 cm, QR = 3 cm, and RP = 3.5 cm. ΔDEF is similar to ΔPQR . If EF = 9 cm, then what is the perimeter of ΔDEF ?
 - (a) 10.5 cm
 - (b) 21 cm
 - (c) 31.5 cm
 - (d) Cannot be determined as data is insufficient

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- 7. In a $\triangle PQR$, perpendicular *PS* from *P* to *QR* meets *QR* at *S*. If *PS* : *QS* : *RS* = 2 : 4 : 1, then which of the following is correct?
 - (a) *PQR* is an equilateral triangle
 - (b) PQR is right angled at P
 - (c) *PQR* is an isosceles triangle
 - (d) PQ = 3PR
- **8.** *s* at *t* are transversals cutting a set of parallel lines such that a segment of length 3 in *s* corresponds to a segment of length 5 in *t*. What is the length of segment in *t* corresponding to a segment of length 12 in *s*?
 - (a) 20 (b) $\frac{36}{5}$ (c) 14 (d) $\frac{5}{4}$
- **9.** A point within an equilateral triangle, where perimeter is 18 m is 1 m from one side and 2 m from another side. Its distance from the third side is:
 - (a) $3\sqrt{3} + 3$ (b) $3\sqrt{3} 3$

(c)
$$3-\sqrt{3}$$
 (d) $3+\sqrt{3}$

- **10.** The perimeter of two similar triangles are 24 cm and 16 cm respectively. If one side of the first triangle is 10 cm, then the corresponding side of the second triangle is
 - (a) 9 cm (b) 20/3 cm
 - (c) 16/3 cm (d) 5 cm
- **11.** In a circle of radius 10 cm, a chord is drawn 6 cm from the centre. If a chord half the length of the original chord were drawn, its distance in centimeters from the centre would be

(a) $\sqrt{84}$	(b) 9
(c) 8	(d) 3π

12. The number of tangents that can be drawn to two non-intersecting circles is

(a) 4	(b) 3
(c) 2	(d) 1

13. ABCD is a parallelogram. A circle passes through A and D and cuts AB at E and DC at F. Given $\angle BEF = 80^\circ$, find $\angle ABC$.



14. In the given figure, *PR* is the diameter of the circle. PQ = 7 cm, QR = 6 cm and RS = 2 cm. The perimeter of the cyclic quadrilateral *PQRS* is



(a) 18 cm (b) $20\sqrt{2}$ cm

(c) 24 cm (d) $22\sqrt{3}$ cm

15. In the given figure, $\angle y$ equals



16. *TP* and *TQ* are tangents from *T* to the circle with centre *O*. Then is it possible to draw a circle through the points *P*, *O*, *Q* and *T*?



(a) No(c) Cannot say

(d) Data insufficient

17. *BC*, *AB* and *AC* are tangents to the circle at *D*, *E* and *F* respectively. $\angle EBD = x^{\circ}$, $\angle FCD = y^{\circ}$. Then $\angle EDF$ equals



18. Find $\angle y$.





(a) 32°	(b) 72°
(c) 64°	(d) 44°

19. *TP* and *TQ* are the tangents to a circle, with centre *O*. Find *x*.





(a) 22°

(c) 20°

20. *AB* and *AC* are tangents to the circle with centre *O*. Then *x* equals.



- **21.** Diagonals of a quadrilateral bisect each other. If $\angle A = 45^\circ$, then $\angle B$ equals
 - (a) 45° (b) 55°
 - (c) 135° (d) 115°
- **22.** If *APB* and *CQD* are two parallel lines, then the bisectors of the angles *APQ*, *BPQ*, *CQP* and *PQD* form:
 - (a) square (b) a rhombus
 - (c) a rectangle (d) kite
- **23.** The interior angle of a regular polygon with *n* sides is 6 times that of an exterior angle of a regular

polygon with
$$\frac{3}{2}n$$
 sides. Then *n* equals
(a) 12 (b) 20

- (c) 10 (d) 18
- **24.** In the given figure, *ABCD* is a square. *M* is the midpoint of *AB* and $PQ \perp CM$. Which of the following statements is not true?



25. In the diagram, ACDE is a trapezium with $AC \parallel ED$. Given that $\angle EAB = 52^\circ$, $\angle CDR = 126^\circ$ and $\angle PBC = 90^\circ$ and $EQ \parallel DR$. Then $\angle BCD$ equals



Answers

1.	(a)	2. (d)	3. (c)	4. (b)	5. (a)	6. (c)		
7.	(b) [Hint	t. Find sides	PQ and PR ,	given PS	$= 2x, \ QS = 4x$	x and RS = x].	8. (a)	9. (b)
10.	(b) [Hint	t. Ratio of p	erimeters of	two $\Delta s = 1$	Ratio of corres	sponding sides.]	11. (a)	12. (a)
13.	(c)	14. (c)	15. (d)	16. (b)				
17.	7. (d) [Hint. Show $\triangle EBD$ and $\triangle FDC$ are isosceles and find $\angle EDB$ and $\angle FDC$.]							
18.	(c)	19. (c)	20. (b)	21. (c)	22. (c)			
23.	(c) [Hint	t. Size of an	exterior ang	le of a reg	ular polygon v	with $\frac{3}{2}n$ sides $=\frac{360^\circ}{\frac{3n}{2}}$.]	24. (c)	25. (b)