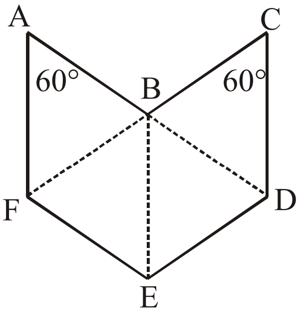
1 MARK

1. Point P is outside circle C on the plane. At most how many points on C are 3 cm from P ?

(1) 1 (2) 2 (3) 3 (4) 4

2. In the adjoining plane figure, sides AF and CD are parallel, as are sides AB and FE and sides BC and ED. Each side has length 1. Also, $\angle FAB = \angle BCD = 60°$. The area of the figure is



(1) $\frac{\sqrt{3}}{2}$

(2) 1

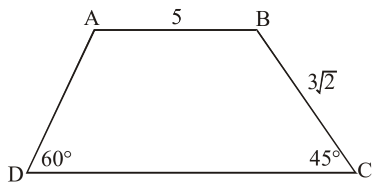
(3) $\frac{3}{2}$

(4) $\sqrt{3}$

3. Triangle ABC has a right angle at C. If sin A = 2/3 , then tan B is

(1) $\frac{3}{5}$ (2) $\frac{\sqrt{5}}{3}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{5}}{2}$

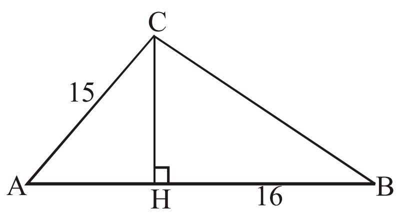
4. Figure ABCD is a trapezoid with AB || DC, AB = 5, BC = $3\sqrt{2}$, $\angle BCD = 45°$ and $\angle CDA = 60°$. The length of DC is



(1) $7+\frac{2}{3}\sqrt{3}$ (2) 8

(3) $9\frac{1}{2}$ (4) $8+\sqrt{3}$

5. A right triangle ABC with hypotenuse AB has side AC = 15. Altitude CH divides AB into segments AH and HB, with HB = 16. The area of ΔABC is



(1) 120 (2) 144 (3) 150 (4) 216

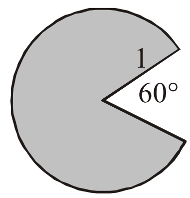
6. In an arcade game, the "monster" is the shaded sector of a circle of radius 1 cm, as shown in the figure. The missing piece (the mouth) has central angle 60°. What is the perimeter of the monster in cm ?

(1) $\pi + 2$

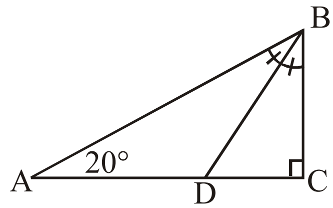
(2) 2π

(3) $\frac{5}{3}\pi$

(4) $\frac{5}{3}\pi+2$

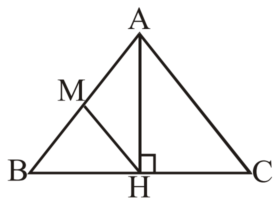


7. In the figure, ΔABC has a right angle at C and $\angle A = 20°$. If BD is the bisector of $\angle ABC$, then $\angle BDC =$



(1) 40° (2) 45° (3) 50° (4) 55°

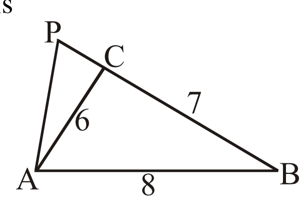
8. In ΔABC, AB = 13, BC = 14 and CA = 15. Also, M is the midpoint of side AB and H is the foot of the altitude from A to BC. The length of HM is -



(1) 6 (2) 6.5

(3) 7 (4) 7.5

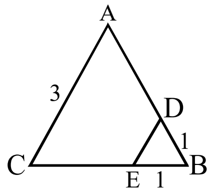
9. In ΔABC, AB = 8, BC = 7, CA = 6 and side BC is extended, as shown in the figure, to a point P so that ΔPAB is similar to ΔPCA. The length of PC is



(1) 7 (2) 8

(3) 9 (4) 10

10. As shown in figure, a triangular corner with side lengths DB = EB = 1 is cut from equilateral triangle ABC of side length 3. The perimeter of the remaining quadrilateral ADEC is



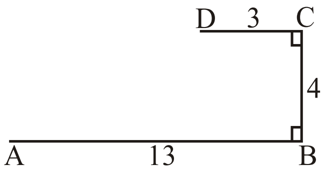
(1) 6

(2) $6\frac{1}{2}$

(3) 7

(4) 8

11. In the figure the sum of the distance AD and BD is



(1) between 10 and 11

(2) 12

(3) between 15 and 16

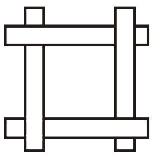
(4) between 16 and 17

12. Triangle ABC and XYZ are similar, with A corresponding to X and B to Y. If AB = 3, BC = 4 and XY = 5, then YZ is

(1) $3\frac{3}{4}$ (2) 6

(3) $6\frac{1}{4}$ (4) $6\frac{2}{3}$

13. Four rectangular paper strips of length 10 and width 1 are put flat on a table and overlap perpendicularly as shown. How much area of the table is covered ?



(1) 36 (2) 40

(3) 44 (4) 96

14. A quadrilateral is an equiangular parallelogram if and only if it is a

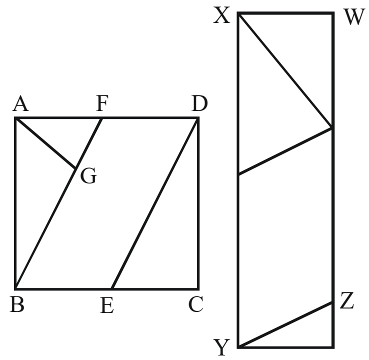
(1) rectangle

(2) regular polygon

(3) rhombus

(4) square

15. In one of the adjoining figures, a square of side 2 is dissected into four pieces so that E and F are the midpoints of opposite sides and AG is perpendicu-lar to BF. These four pieces can then be reassembled into a rectangle as shown in the second figure. The ratio of height to base, XY/YZ , in this rectangle is



(1) 4 (2) $1+2\sqrt{3}$

(3) $2\sqrt{5}$ (4) 5

16. A circle of radius r goes through two neighboring vertices of a square and is tangent to the side of the square opposite these vertices. In terms of r, the area of the square is -

(1) $\frac{8}{5}r^2$ (2) $2r^2$ (3) r^2 (4) $3r^2$

17. Two congruent 30° - 60° - 90° triangle are placed so that they overlap partly and their hypotenuses coincide. If the hypotenuse of each triangle is 12, the area common to both triangle is -

(1) $6\sqrt{3}$ (2) $8\sqrt{3}$

(3) $9\sqrt{3}$ (4) $12\sqrt{3}$

18. In ΔABC with right angle at C, altitude CH and median CM trisect the right angle. If the area of ΔCHM is K, then the area of ΔABC is -

(1) 6K (2) $4\sqrt{3}K$

(3) $3\sqrt{3}K$ (4) 4K

19. A sector with acute central angle θ is cut from a circle of radius 6. The radius of the circle circumscribed about the sector is -

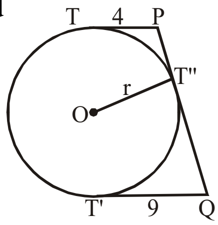
(1) $3\cos\theta$ (2) $3\sec\theta$

(3) $3\cos\left(\frac{\theta}{2}\right)$ (4) $3\sec\left(\frac{\theta}{2}\right)$

20. Given a quadrilateral ABCD inscribed in a circle with side AB extended beyond B to point E. If $\angle BAD = 92°$ and $\angle ADC = 68°$, find $\angle EBC$ -

(1) 66° (2) 68° (3) 70° (4) 88°

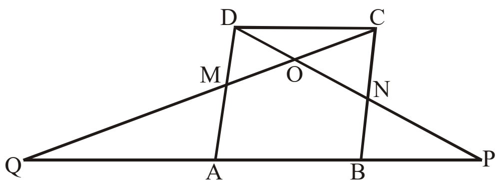
21. In the adjoining figure TP and TQ are parallel tangents to a circle of radius r, with T and T' the points of tangency. PT''Q is a third tangent with T'' as point of tangency. If TP = 4 and TQ = 9, then r is -



(1) 25/6 (2) 6

(3) 25/4 (4) a number other than 25/6, 6, 25/4

22. In parallelogram ABCD of the accompanying diagram, line DP is drawn disecting BC at N and meeting AB (extended) at P. From vertex C, line CQ is drawn bisecting side AD at M and meeting AB (extended) at Q. Lines DP and CQ meet at O. If the area of parallelogram ABCD is k. Then the area of triangle QPO is equal to-



(1) k (2) 6k/5 (3) 9k/8 (4) 5k/4

23. If the side of one square is the diagonal of a second square, what is the ratio of the area of the first square to the area of the second ?

(1) 2 (2) $\sqrt{2}$ (3) 1/2 (4) $2\sqrt{2}$

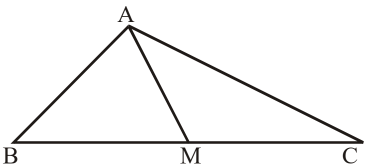
24. In the adjoining figure triangle ABC is such that AB = 4 and AC = 8. If M is the midpoint of BC and AM = 3, what is the length of BC ?

(1) $2\sqrt{26}$

(2) $2\sqrt{31}$

(3) 9

(4) $4+2\sqrt{13}$



25. The sum of the distances from one vertex of a square with side of length two to the midpoints of each of the sides of the square is -

(1) $2\sqrt{5}$ (2) $2+\sqrt{3}$

(3) $2+2\sqrt{3}$ (4) $2+2\sqrt{5}$

26. In triangle ABC, D is the midpoint of AB;E is the midpoint of DB and F is the midpoint of BC. If the area of ΔABC is 96, the area of ΔAEF is

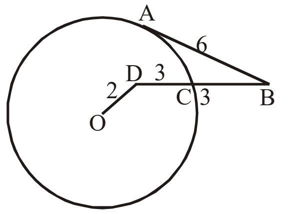
(1) 16 (2) 24 (3) 32 (4) 36

27. The measures of the interior angles of a convex polygon are in arithmetic progression. If the smallest angle is 100° and the largest angle is 140°. Then the number of sides the polygon has is-

(1) 6 (2) 8 (3) 10 (4) 11

28. In the adjoining figure.

AB is tangent at A to the circle O; point D is interior to the circle; and DB intersects the circle at C. If BC=DC = 3, OD = 2 and AB = 6, then the radius of the circle is -



(1) $3+\sqrt{3}$ (2) $15/\pi$

(3) 9/2 (4) $\sqrt{22}$

29. Which one of the following statements is false? All equilateral triangles are

(1) equiangular

(2) isosceles

(3) regular polygons

(4) congruent to each other

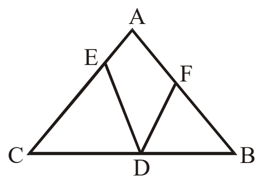
30. In triangle ABC, AB = AC and $\angle A = 80°$. If points D, E and F lies on sides BC,AC and AB respectively and CE = CD and DF = BD. then $\angle EDF$ equals

(1) 30°

(2) 40°

(3) 50°

(4) 65°



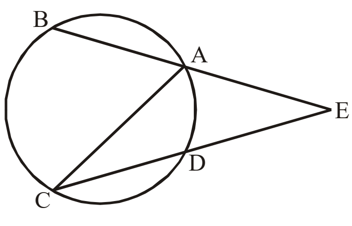
31. In the adjoining figure $\angle L = 40°$ and arc AB, arc BC and arc CD all have equal length. Find the measure of $\angle ACD$.

(1) 10°

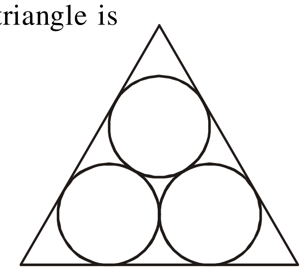
(2) 15°

(3) 20°

(4) $\left(\frac{45}{2}\right)^{o}$



32. Each of the three circles in the adjoining figure is externally tangent to the other two and each side of the triangle is tangent to two of the circles. If each circle has radius three, then the perimeter of the triangle is



(1) $36+9\sqrt{2}$

(2) $36+6\sqrt{3}$

(3) $36+9\sqrt{3}$

(4) $18+18\sqrt{3}$

33. Opposite sides of a regular hexagon are 12 inches apart. The length of each side in inches is

(1) 7.5 (2) $6\sqrt{2}$

(3) $5\sqrt{2}$ (4) $4\sqrt{3}$

34. If B is a point on circle C with center P, then the set of all points A in the plane of circle C such that the distance between A and B is less than or equal to the distance between A and any other point on circle C is-

(1) the line segment from P to B

(2) the ray beginning at P and passing through B

(3) a ray beginning at B

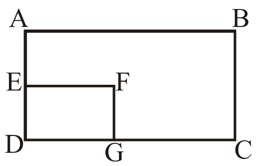
(4) a circle whose center is P

35. In ΔADE, $\angle ADE = 140°$. Points B and C lie on sides AD and AE, respectively, and points A,B,C,D,E are distinct. If lengths AB,BC,CD and DE are all equal, then the measure of $\angle EAD$ is-

(1) 5° (2) 6°

(3) 7.5° (4)10°

36. If rectangle ABCD has area 72 square meters and E and G are the midpoints of sides A D and CD. respectively, then the area of rectangle DEFG in square meters is :-



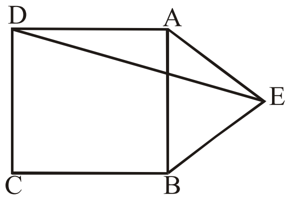
(1) 8

(2) 9

(3) 12

(4) 18

37. In the adjoining figure, ABCD is a square, ABE is an equilateral triangle and point E is outside square ABCD. What is the measure of $\angle AED$ in degrees ?



(1)10

(2)12.5

(3) 15

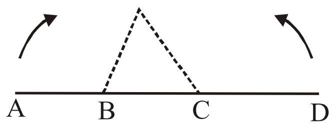
(4) 20

38. A circle with area A_1 is contained in the interior of a largest circle with area A_1+A_2. If the radius of the largest circle is 3 and if A_1, A_2, $A_1 + A_2$ is an arithmetic progression, then the radius of the smaller circle is :-

(1) $\frac{\sqrt{3}}{2}$ (2) 1 (3) $\frac{2}{\sqrt{3}}$ (4) $\sqrt{3}$

39. Points A, B, C and D are distinct and lie, in the given order, on a straight line. Line segment AB, AC and AD have lengths x, y and z respectively. If line segments AB and CD may be rotated about points B and C. respecticely, so that points A and D coincide, to form a triangle with positive area, then which of the following three inequalities must be satisfied ?

I. $x<\frac{z}{2}$ II. $y<x+\frac{z}{2}$ III. $y<\frac{z}{2}$



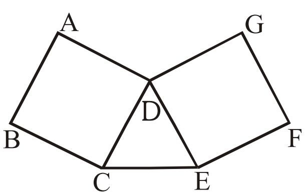
(1) I only

(2) II only

(3) I and II only

(4) II and III only

40. In the adjoining figure. CDE is an equilateral triangle and ABCD and DEFG are squares. The measure of $\angle GDA$ is :-



(1) 90°

(2) 105°

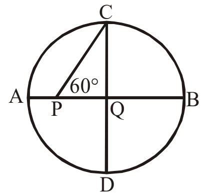
(3) 120°

(4) 135°

41. If AB and CD are perpendicular diameters of circle Q. and $\angle QPC = 60°$. then the length of PQ divided by, the length of AQ is :-

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{3}$

(3) $\frac{\sqrt{2}}{2}$ (4) $\frac{1}{2}$



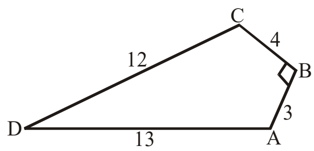
42. Sides AB,BC,CD and DA of convex quadrilateral ABCD have lengths 3, 4, 12 and 13 respectively and $\angle CBA$ is a right angle. The area of the quadrilateral is :-

(1) 32

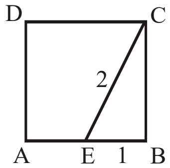
(2) 36

(3) 39

(4) 42



43. Point E is on side AB of square ABCD. If EB has length one and EC has length two. then the area of the square is :-



(1) $\sqrt{3}$ (2) $\sqrt{5}$ (3) 3 (4) $2\sqrt{3}$

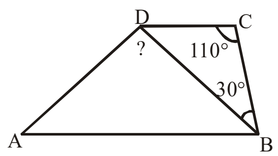
44. In trapezoid ABCD, sides AB and CD are parallel, and diagonal BD and side AD have equal length. If $\angle DCB = 110°$ and $\angle CBD = 30°$, then $\angle ADB$ =

(1) 80°

(2) 90°

(3) 100°

(4) 110°



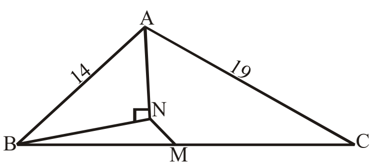
45. In ΔABC, M is the midpoint of side BC. AN bisects $\angle BAC. BN \perp AN$ and θ is the measure of $\angle BAC$. If sides AB and AC have lengths 14 and 19. respectively, then length MN equals :-

(1) 2

(2) 5/2

(3) $\frac{5}{2} - \sin\theta$

(4) $\frac{5}{2} - \frac{1}{2}\sin\theta$



46. In a triangle with sides of lengths a. b and c. (a + b + c)(a + b – c) = 3ab.
The measure of the angle opposite the side of length c is :-

(1) 15° (2) 30° (3) 45° (4) 60°

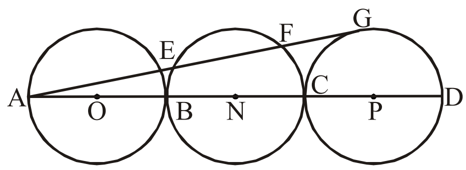
47. The perimeter of a semicircular region, measured in centimeters. is numerically equal to its area, measured in square centimeters- The radius of the semicircle, measured in centimeters, is :-

(1) π (2) $2/\pi$ (3) 1 (4) $\frac{4}{\pi}+2$

48. If sum of all but one of the interior angles of a convex polygon equals 2570°. The remaining angle is :-

(1) 90° (2) 105° (3) 120° (4) 130°

49. In the adjoining figure, points B and C lie on line segment AD, and AB, BC and CD are diameters of circles O, N and P. respectively. CIrcles O, N- and P all have radius 15. and the line AG is tangent to circle P at G. If AG intersects circle N at points E and F, then chord EF has length



(1) 20 (2) 15/2 (3) 24 (4) 25

50. The area of a square inscribed in a semicircle to the area inscribed in a quadrant of the same circle is as :-

(1) 2 : 1 (2) 3 : 2 (3) 5 : 3 (4) 8 : 5

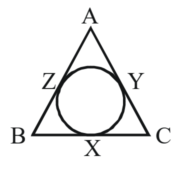
51. If in the figure AB = 4, BC 6, CA = 8 then AZ + BX + CY is :-

(1) 18

(2) 9

(3) 6

(4) 12



52. A triangle has sides of lengths 6, 8 and 10. find the distance between its incentre and circumcentre.

(1) $\sqrt{10}$ (2) $2\sqrt{5}$ (3) $\sqrt{2}$ (4) $\sqrt{5}$

53. A triangle (non degenerate) has integral sides and perimeter 8. If its area is A then A is :-

(1) less than 2

(2) Greater than 2 but less than 3

(3) Greater than 3 but less than 4

(4) None of these

2 MARKS

1. Segment AB is both a diameter of a circle of radius 2 and a side of an equilateral triangle ABC. The circle also intersects AC and BC at points D and E, respectively. The length of AE is

(1) $\frac{3}{2}$ (2) $\frac{5}{3}$

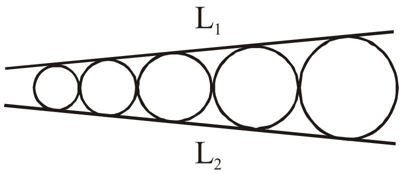
(3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

2. Point D is on side CB of triangle ABC. If $\angle CAD = \angle DAB = 60°$, AC = 3 and AB = 6, then the length of AD is

(1) 2 (2) 2.5

(3) 3 (4) 3.5

3. In the adjoining figure the five circles are tangent to one another consecutively and to the lines L_1 and L_2. If the radius of the largest circle is 18 and radius of smallest circle is 8 then the radius of the middle circle is



(1) 12 (2) 12.5

(3) 13 (4) 13.5

4. How many non-congruent right triangles are there such that the perimeter in cm and area in cm^2 are numerically equal ?

(1) none (2) 1

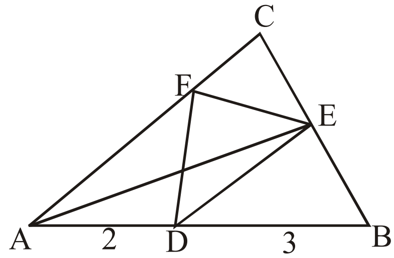
(3) 2 (4) infinitely many

5. A large sphere is on a horizontal field on a sunny day . At a certain time the shadow of the sphere reaches out a distance of 10 m from the point where the sphere touches the ground. At the same instant a meter stick (held vertically with one end on the ground) casts a shadow of length 2m. What is the radius of the sphere in meters ? (Assume the sun's rays are parallel and the meter stick is a line segment).

(1) 5/2 (2) $9-4\sqrt{5}$

(3) $8\sqrt{10}-23$ (4) $10\sqrt{5}-20$

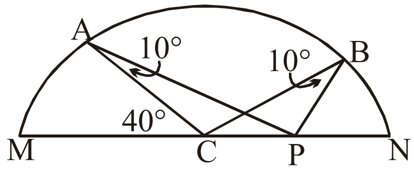
6. Triangle ABC in the figure has area 10. Points D, E and F all distinct from A, B and C are on sides AB, BC and CA respectively, and AD = 2, DB = 3. If triangle ABE and quadrilateral DBEF have equal areas, then that area is



(1) 4 (2) 5

(3) 6 (4) $\frac{5}{3}\sqrt{10}$

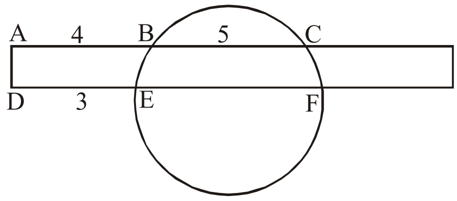
7. Distinct points A and B are on a semicircle with diameter MN and center C. The point P is on CN and $\angle CAP = \angle CBP = 10°$. If MA = 40°, then BN equals



(1) 10° (2) 15°

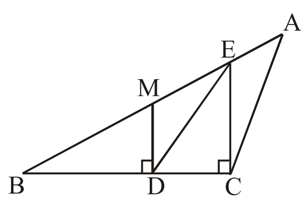
(3) 20° (4) 25°

8. A rectangle intersects a circle as shown : AB = 4, BC = 5 and DE = 3. Then EF equals



(1) 6 (2) 7 (3) 20/3 (4) 8

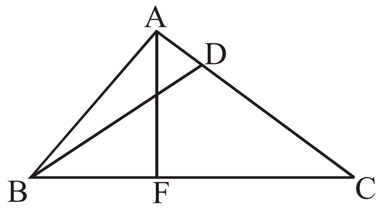
9. In the obtuse triangle ABC, AM = MB, MD $\perp$ BC, EC $\perp$ BC. If the area of ΔABC is 24, then the area of ΔBED is



(1) 9 (2) 12

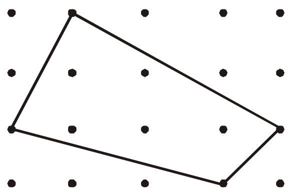
(3) 15 (4) 18

10. In ΔABC, D is on AC and F is on BC. Also, $AB \perp AC$, $AF \perp BC$, and $BD = DC = FC = 1$. Find AC



(1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) $\sqrt[3]{2}$ (4) $\sqrt[3]{3}$

11. Pegs are put in a board 1 unit apart both horizontally and vertically. A rubber band is stretched over 4 pegs as shown in the figure , forming a quadrilateral. Its area in square units is

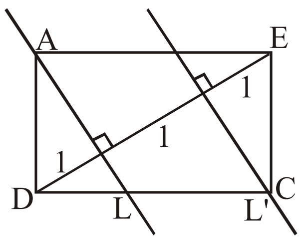


(1) 4 (2) 4.5 (3) 5 (4) 6

12. Exactly three of the interior angles of a convex polygon are obtuse. What is the maximum number of sides of such a polygon ?

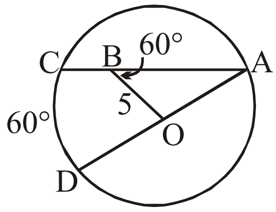
(1) 4 (2) 5 (3) 6 (4) 7

13. Diagonal DE of rectangle AECD is divided into three segments of length 1 by parallel lines L and L' that pass through A and C and are perpendicular to DE. The area of AECD, rounded to one decimal place, is



(1) 4.1 (2) 4.2 (3) 4.3 (4) 4.4

14. In a circle with center O, AD is a diameter, ABC is a chord, BO = 5 and $\angle ABO = \overset{\frown}{CD} = 60°$. Then the length of BC is



(1) 3 (2) $3+\sqrt{3}$

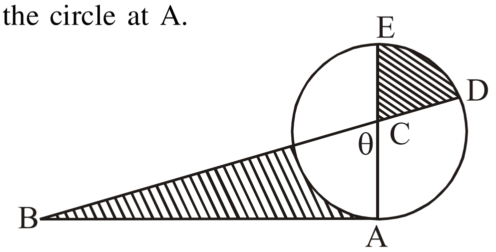
(3) $5-\frac{\sqrt{3}}{2}$ (4) 5

15. A park is in the shape of a regular hexagon 2k on a side. Starting at a corner, Alice walks along the perimeter of the park for distance of 5 k. How many kilometers is she from her starting point ?

(1) $\sqrt{3}$ (2) $\sqrt{14}$

(3) $\sqrt{15}$ (4) $\sqrt{16}$

16. In the configuration below, θ is measured in radians, C is the center of the circle , BCD and ACE are line segments, and AB is tangent to the circle at A.

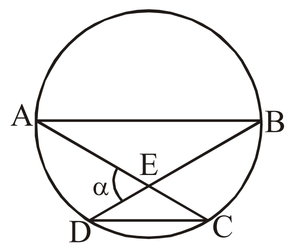


A necessary and sufficient condition for the equality of the two shaded areas, given $0 < \theta < \pi/2$, is

(1) $\tan \theta = \theta$ (2) $\tan \theta = 2\theta$

(3) $\tan \theta = 4\theta$ (4) $\tan 2\theta = \theta$

17. In the adjoining figure, AB is a diameter of the circle , CD is a chord parallel to AB, and AC intersects BD at E, with $\angle AED = \alpha$. The ratio of the area of ΔCDE to that of ΔABE is



(1) $\cos\alpha$ (2) $\sin\alpha$

(3) $\cos^2\alpha$ (4) $\sin^2\alpha$

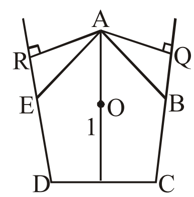
18. ABCDE is a regular pentagon. AP, AQ and AR are the perpendiculars dropped from A onto CD, CB extended and DE extended, respectively. Let O be the center of the pentagon . If OP = 1, then AP + AQ + AR equals

(1) 3

(2) $1+\sqrt{5}$

(3) 4

(4) $2+\sqrt{5}$



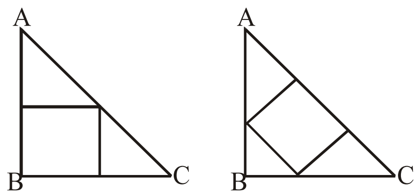
19. Two of the altitudes of the scalene triangle ABC have length 4 and 12. If the length of the third altitude is also an integer, what is the biggest it can be ?

(1) 4 (2) 5 (3) 6 (4) 7

20. A long piece of paper 5 cm wide is made into a roll for cash registers by wrapping it 600 times around a cardboard tube of diameter 2 cm, forming a roll 10 cm in diameter. Approximate the length of the paper in meters. (Pretend the paper forms 600 concentric circles with diameters evenly spaced from 2 cm to 10 cm.)

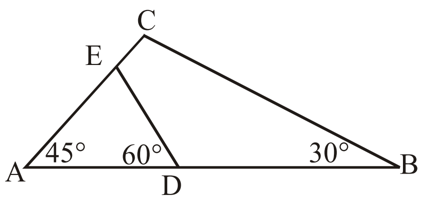
(1) 36π (2) 45π (3) 60π (4) 72π

21. There are two natural ways to inscribe a square in a given isosceles right triangle. If it is done as in figure 1 below, then one finds that the area of the square is 441 cm^2. What is the area (in cm^2) of the square inscribed in the same ΔABC as shown in figure 2 below ?



(1) 378 (2) 392 (3) 400 (4) 441

22. In the figure, ΔABC has $\angle A = 45°$ and $\angle B = 30°$. A line DE, with D on AB and $\angle ADE = 60°$, divides ΔABC into two pieces of equal area. (Note : the figure may not be accurate ; perhaps E is on CB instead of AC.) The ratio AD/AB is

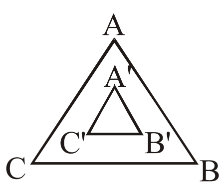


(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{2}{2+\sqrt{2}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{\sqrt[4]{12}}$

23. ABC and A'B'C' are equilateral triangles with parallel sides and the same center, as in the figure, The distance between side BC and side B'C' is 1/6 the altitude of ΔABC. The ratio of the area of $\Delta A'B'C'$ to the area of ΔABC is

(1) $\frac{1}{36}$ (2) $\frac{1}{6}$

(3) $\frac{1}{4}$ (4) $\frac{\sqrt{3}}{4}$



24. The six edges of tetrahedron ABCD measure 7,13,18,27,36 and 41 units. If the length of edge AB is 41, then the length of edge CD is

(1) 7 (2) 13

(3) 18 (4) 27

25. An isosceles trapezoid is circumscribed around a circle. The longer base of the trapezoid is 16, and one of the base angles is arcsin(.8). Find the area of the trapezoid.

(1) 72 (2) 75

(3) 80 (4) 90

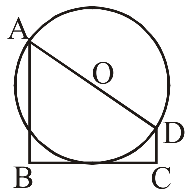
26. In the figure, $AB \perp BC, BC \perp CD$ and BC is tangent to the circle with centre O and diameter AD. In which one of the following cases is the area of ABCD an integer?

(1) AB=3,CD=1

(2) AB=5,CD=2

(3) AB=7,CD=3

(4) AB=9,CD=4



27. If the sum of all the angles except one of a convex polygon, is 2190°, then number of sides of the polygon must be -

(1) 13 (2) 15

(3) 17 (4) 19

28. A cowboy is 4 miles south of a stream which flows due east. He is also 8 miles west and 7 miles north of the cabin. He wishes to water his horse at the stream and return home. The shortest distance (in miles) he can travel and accomplish this is -

(1) $4 \div \sqrt{185}$ (2) 16

(3) 17 (4) 18

29. A circular grass plot 12 feet in diameter is cut by a straight gravel path 3 feet wide, one edge of which passes through the center of the plot. The number of square feet in the remaining grass area is -

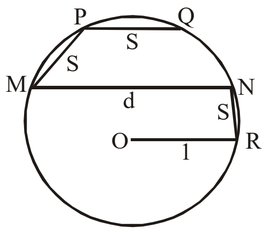
(1) $36\pi - 34$ (2) $30\pi - 15$

(3) $36\pi - 33$ (4) $30\pi - 9\sqrt{3}$

30. In the unit circle shown in the figure to the right. Chords PQ and MN are parallel to the unit radius OR of the circle with center at O. Chords MP,PQ and NR are each s units long and chord MN is d units long. Of the three equations.

I. d-s= 1. II. ds = 1. III $d^2 - s^2 = \sqrt{5}$

those which are necessarily true are -



(1) I only (2) II only

(3) III only (4) I, II and III

31. A circle of radius r is inscribed in a right isosceles triangle, and a circle of radius R is circumscribed about the triangle. Then R/r equals -

(1) $1+\sqrt{2}$ (2) $\frac{2+\sqrt{2}}{2}$

(3) $\frac{\sqrt{2}-1}{2}$ (4) $\frac{1+\sqrt{2}}{2}$

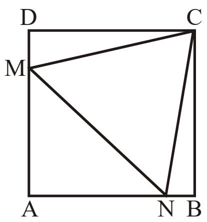
32. In the adjoining figure ABCD is a square and CMN is an equilateral triangle. If the area of ABCD is one square inch. then the area of CMN in square inches is -

(1) $2\sqrt{3}-3$

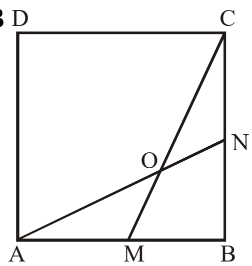
(2) $1-\sqrt{3}/3$

(3) $\sqrt{3}/4$

(4) $\sqrt{2}/3$



33. In the adjoining figure A B and BC are adjacent sides of square ABCD : M is the midpoint of A B; N is the midpoint of BC : and AN and CM intersect at O. Ratio of the area of AOCD to the area of ABCD is -



(1) 516 (2) 3/4 (3) 2/3 (4) $\sqrt{3}/2$

34. In triangle ABC. ∠C = θ and ∠B = 2θ. Where 0° < θ < 60°. The circle with center A and radius AB intersects AC at D and inersects BC. extended if necessary at B and at E(E may coincide with B). Then EC = AD

(1) for no values of θ

(2) only if θ = 45°

(3) only if 0° < θ < 45°

(4) for all θ such that 0° < θ < 60°

35. In acute triangle ABC the bisector of ∠A meets side BC at D. The circle with center B and radius BD intersects side AB at M; and the circle with center C and radius CD intersects side AC at N. Then it is always true that -

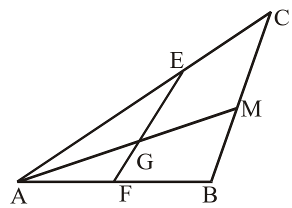
(1) ∠CND + ∠BMD = ∠DAC = 120°

(2) AMDN is a trapezoid

(3) BC is parallel to MN

(4) $AM - AN = \frac{3(DB-DC)}{2}$

36. In triangle ABC shown in the adjoining figure, M is the mid-point of side BC.AB = 12 and AC = 16. Points E and



F are taken on AC and AB, respectively, and lines EF and AM intersect at G. If AE = 2AF then EG/GF equals -

(1) 3/2 (2) 4/3

(3) 5/4 (4) 6/5

37. In triangles ABC and DEF, lengths AC,BC,DF and EF are all equal length. AB is twice the length of the altitude of ΔDEF from F to DE. Which of the following statements is (are) true ?

I. ∠ACB and ∠DFE must be complementary

II. ∠ACB and ∠DFE must be supplementary.

III. The area of ΔABC must equal the area of ΔDEF.

(1) II only (2) III only

(3) IV only (4) II and III only

38. Given an equilateral triangle with side of length s. consider the locus of all points P in the plane of the triangle such that the sum of the squares of the distance from P to the vertices of the triangle is a fixed number a. This locus -

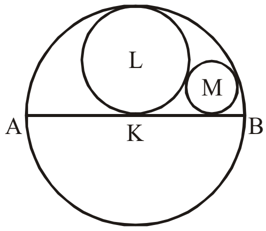
(1) is a circle if $a > s^2$

(2) contains only three points if $a = 2s^2$ and is a circle if $a > 2s^2$

(3) is a circle with positive radius only if $s^2 < a < 2s^2$

(4) contains only a finite number of points for any value of a

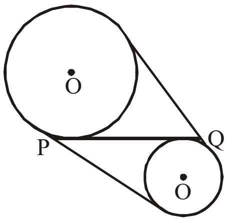
39. In the adjoining figure, circle K has diameter AB; circle L is tangent to circle K and to AB at the center of circle K and circle M is tangent to circle K; to circle L and to AB. The ratio of the area of circle K to the area of circle M is -



(1) 12 (2) 14

(3) 16 (4) 18

40. In the adjoining figure, every point of circle O' is exterior to circle O. Let P and Q be the points of intersection of an internal common tangent with the two external common tangents. Then the length of PQ is-



(1) The average of the lengths of the internal and external common tangents.

(2) Equal to the length of an external common tangent if and only if circles O and O' have equal radii

(3) Always equal to the length of an external common tangent.

(4) Greater than the length of an external common tangent.

41. Let E be the point of intersection of the diagonals of convex quadrilateral ABCD and let P, Q, R and 5 be the centers of the circles circumscribing triangles ABE, BCE, CDE and ADE respectively. Then

(1) PQRS is a parallelogram

(2) PQRS is a parallelogram if and only if ABCD is a rhombus

(3) PQRS is a parallelogram if and only if ABCD is rectangle

(4) PQRS is a parallelogram if and only if ABCD is a parallelogram

42. Let a, b, c and d be the lengths of sides MN, NP, PQ and QM respectively, of quadrilateral MNPQ. If A is the area of MNPQ, then -

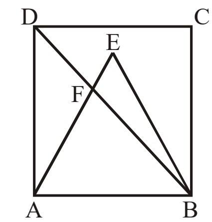
(1) $A = \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if only if MNPQ is convex

(2) $A = \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if an only if, MNPQ is a rectangle

(3) $A \le \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if only if MNPQ is a rectangle

(4) $A \le \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if and only if MNPQ is parallelogram

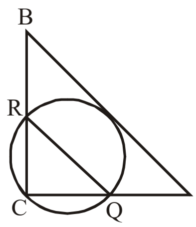
43. Vertex E of equilateral triangle ABE is in the interior of square ABCD and F is the point of intersection of diagonal BD and line segment AE. If length AB is $\sqrt{1+\sqrt{3}}$ then the area of ΔABF is-



(1) 1 (2) $\frac{\sqrt{2}}{2}$

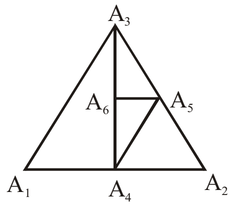
(3) $\frac{\sqrt{3}}{2}$ (4) $4-2\sqrt{3}$

44. In ΔABC, AB = 10. AC = 8 and BC = 6. Circle P is the circle with smallest radius which passes through C and is tangent to AB. Let Q and R be the points of intersection, distinct from C of circle P with sides AC and BC respectively. The length of segment QR is-



(2) 4.75 (2) 4.8 (3) 5 (4) $\sqrt{\ }$

45. If $\Delta A_1A_2A_3$ is equilateral and A_{n+3} is the midpoint of line segment A_nA_{n+1} for all positive integers n, then the measure of $\angle A_{44}A_{45}A_{43}$ equals -



(1) 30° (2) 45° (3) 60° (4) 120°

46. Sides AB, BC, CD and DA, respectively, of convex quadrilateral ABCD are extended past B,C,D and A to points B', C', D' and A', Also AB = BB' =6. BC = CC = 7, CD = DD' = 8 and DA = AA' = 9; and the area of ABCD is 10. The area of A'B'C'D' is -

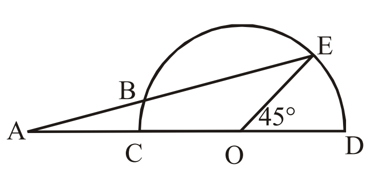
(1) 20 (2) 40 (3) 45 (4) 50

47. If $P_1 P_2 P_3 P_4 P_5 P_6$ is a regular hexagon whose apothem (distance from the centre to the modpoint of a side) is 2, and Q_i is the midpoint of side $P_i P_{i+1}$ for i = 1, 2, 3, 4. then the area of quadrilateral $Q_1 Q_2 Q_3 Q_4$ is :-

(1) 6 (2) $2\sqrt{6}$

(3) $\frac{8\sqrt{3}}{3}$ (4) $3\sqrt{3}$

48. In the adjoining figure, CD is the diameter of a semi-circle with centre O. Point A lies on the extension of DC past C; point E lies on the semi-circle, and B is the point of intersection (distinct from E) of line segment AE with the semi-circle. If length AB equals length OD, and the measure of $\angle EOD$ is 45°. then the measure of $\angle BAO$ is :-



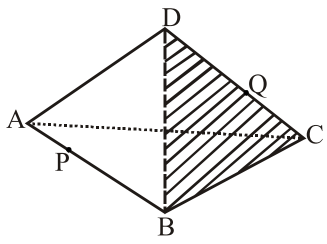
(1) 10° (2) 15°

(3) 20° (4) 25°

49. The length of the hypotenuse of a right triangle is h, and the radius of the inscribed circle is r. The ratio of the area of the circle to the area of the triangle is :-

(1) $\frac{\pi r}{h+2r}$ (2) $\frac{\pi r}{h+r}$ (3) $\frac{\pi r}{2h+r}$ (4) $\frac{\pi r^2}{h^2+r^2}$

50. The edges of a regular tetrahedron with vertices A, B, C and D each have length one. Find the least possible distance between a pair of points P and Q. where P is on edge AB and Q is on edge CD.



(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{\sqrt{2}}{2}$ (4) $\frac{\sqrt{3}}{2}$

51. Sides AB. BC and CD of (simple*) quadrilateral A BCD have length 4. 5 and 20, respectively. If vertex angles B and C are obtuse and sinC = –cosB = 1/5. then side AD has length

(1) 24 (2) 24.5

(3) 24.6 (4) 25

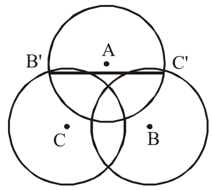
52. Circle with centers A, B and C each have radius r, where i < r < 2. The distance between each pair of centres is 2. If B' is the point of intersection of circle A and circle C which is outside circle B, and if C' is the point of intersection of circle A and circle B which is outside circle C. then length B'C' equals

(1) 3r–2

(2) r^2

(3) $r + \sqrt{3(r-1)}$

(4) $1 + \sqrt{3(r^2-1)}$



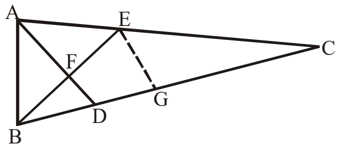
53. Let C_1, C_2 and C_3, be three parallel chords of circle on the same side of the centre. The distance between C_1, and C_2, is the same as the distance between C_2 and C_3. The lengths of the chords are 20, 16 and 8. The radius of the circle is :-

(1) 12 (2) $4\sqrt{7}$ (3) $\frac{5\sqrt{65}}{3}$ (4) $\frac{5\sqrt{22}}{2}$

54. In triangle ABC, $\angle CBA = 72°$ E is the midpoint of side AC. and D is a point on side BC such that 2BD = DC : AD and BE intersect at F. The ratio of the area of ΔBDF to the area of quadrilateral FDCE is :-

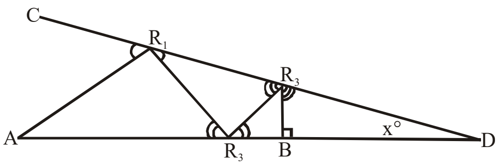
(1) $\frac{1}{5}$ (2) $\frac{1}{4}$

(3) $\frac{1}{3}$ (4) $\frac{5}{2}$



55. A ray of light originates from point A and travels in a plane, being reflected n times between lines AD and CD. before striking a point R (which may be on AD or CD) perpendicularly and retracing its path to A (At each point of reflection the light makes two equal angles as indicated in the adjoining figure. The figure shows the light path for n = 3.) If $\angle CDA = 8°$. what is the largest value n can have ?

(1) 6 (2) 10 (3) 38 (4) 98



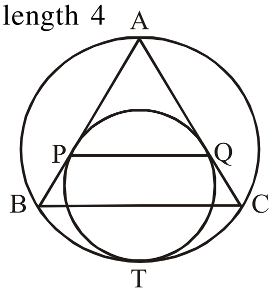
56. Equilateral ΔABC is inscribed in a circle. A second circle is tangent internally to the circumcircle at T and tangent to sides AB and AC at points P and Q. If side BC has length 12, then segment PQ has length 4

(1) 6

(2) $6\sqrt{3}$

(3) 8

(4) $8\sqrt{3}$



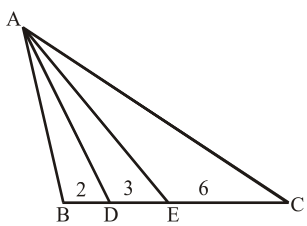
57. In triangle ABC in the adjoining figure. AD and AE trisect $\angle BAC$. The length of BD, DE and EC are 2, 3 and 6, respectively. The length of the shortest side of ΔABC is :-

(1) $2\sqrt{10}$

(2) 11

(3) $6\sqrt{6}$

(4) 6



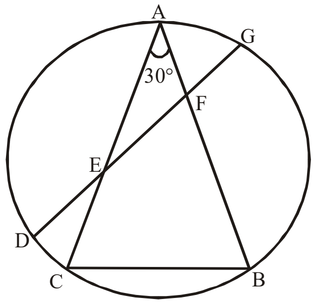
58. In the adjoining figure traingle ABC is inscribed in a circle. Point D lies on AC with DC = 30°. and point G lies on BA with BG > GA. Side AB and side XC each have length equal to 10 the length of chord DG. and $\angle CAB$ =30°. Chord DG intersects sides AC and AB at E and F. respectively. The ratio of the area of ΔAFE to the area of ΔABC is :-

(1) $\frac{2-\sqrt{3}}{3}$

(2) $\frac{2\sqrt{3}-3}{3}$

(3) $7\sqrt{3}-12$

(4) $3\sqrt{3}-5$



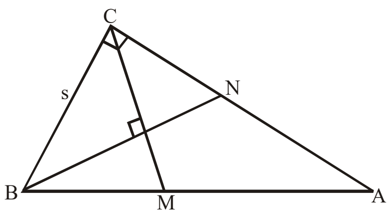
59. In the adjoining figure, the triangle ABC is a right triangle with $\angle BCA = 90°$. Median CM is perpendicular to median BN, and side BC = s. The length of BN is :-

(1) $s\sqrt{2}$

(2) $\frac{3}{2}s\sqrt{2}$

(3) $2s\sqrt{2}$

(4) $\frac{s\sqrt{6}}{2}$



60. The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle. The cosine of the smallest angle is :-

(1) $\frac{3}{4}$ (2) $\frac{7}{10}$ (3) $\frac{2}{3}$ (4) $\frac{9}{14}$

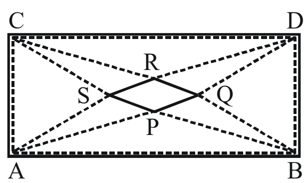
61. In the figure, quadrilateral PQRS is formed by the points of intersection of trisectors of the angles of the rectangle ABCD. The quadrilateral PQRS is a :-

(1) Square

(2) Rhombus

(3) Rectangle

(4) Parallelogram



62. Let ABC be a triangle and the points D, E, F are in the plane of the triangle such that :-

(i) B and E are separated by AC

(ii) D and C are separated by AB

(iii)A and F are separated by BC

(iv)$\Delta ADB \;|||\; \Delta CEA \;|||\; \Delta CFB$ (similar)

Then the quadrilateral AFED must be :-

(1) a parallelogram (2) cyclic

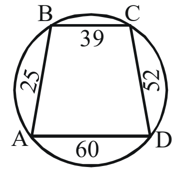
(3) a rectangle (4) a rhombus

63. Let ABC be a triangle with BC = 5, CA = 8, AB = 7. If G is the centroid of ΔABC then $GA^2 + GB^2 + GC^2$ is :-

(1) 46 (2) 138

(3) 69 (4) 40

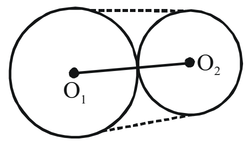
64. The quadrilateral ABCD is inscribed in a circle. The diameter of the circle is :-



(1) $38\frac{14}{15}$ (2) $31\frac{3}{5}$

(3) 65 (4) 55

65. O_1 and O_2 are the centres of two circles with radii 9 cm and 3 cm respectively. The length of the shortest rope that could be wound around the circles is of length (in cm.)

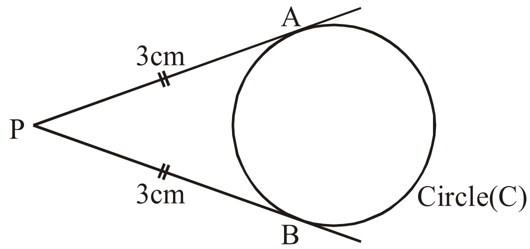


(1) $12\pi + 14\sqrt{3}$ (2) $14\pi+12\sqrt{3}$

(3) $6\pi+6\sqrt{3}$ (4) $7\pi+12\sqrt{3}$

1 MARK

1.



Since we know from any external point two tangents are drawn which are of equal lengths.
So Atmost 2 points on circle (C)

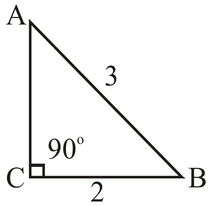
2. Area of equilateral $\Delta ABF = \frac{\sqrt{3}}{4} \times (1)^2$

$= \frac{\sqrt{3}}{4}$

So area of Figure = 4 × Area of ΔABF

$= 4 \times \frac{\sqrt{3}}{4} = \sqrt{3}$

3. $\sin A = \frac{2}{3}$

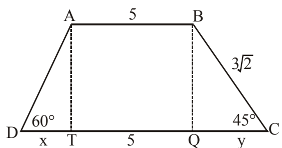


$AC = \sqrt{3^2 - 2^2} = \sqrt{5}$

then $\tan B = \frac{AC}{BC}$

$= \frac{\sqrt{5}}{2}$

4. Draw a perpendicular from A & B on line CD
Let DT = x
CQ = y
In ΔBQC



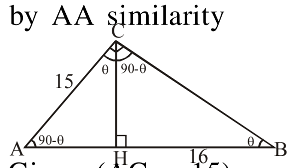
$\sin 45^\circ = \frac{BQ}{BC} \Rightarrow BQ = 3$

In ΔADT AT = BQ = 3

$\tan 60^\circ = \frac{AT}{DT} \Rightarrow DT(x) = \sqrt{3}$

$\therefore$ Length of DC = x + 5 + y = 8 + $\sqrt{3}$

5. $\angle H = \angle C = 90^\circ$
$\angle A = \angle A$ (common)
$\Delta AHC \sim \Delta ACB$ by AA similarity

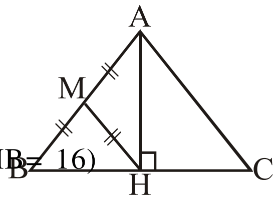


$\frac{AH}{AC} = \frac{AC}{AB}$

$\frac{AH}{AC} = \frac{AC}{AH+16}$ Given (AC = 15)

$\frac{AH}{15} = \frac{AC}{AH+16} \Rightarrow AH = 9$

$\Delta AHC \sim \Delta CHB$



$\therefore \frac{AH}{HC} = \frac{HC}{HB}$ Given (HB = 16)

$\frac{9}{HC} = \frac{HC}{16} \Rightarrow HC = 12$

Area of $\Delta ABC = \frac{1}{2} \times 25 \times 12$

= 150

6. r = 1 cm (given) $\theta = \frac{\pi}{3}$ (given)

$\because \theta = \frac{\ell}{r} \Rightarrow \frac{\pi}{3} = \frac{\ell}{1} \Rightarrow \ell = \frac{\pi}{3}$

Total perimeter = $2\pi r = 2\pi(1) = 2\pi$
Now Perimeter of the monster = $(2\pi - \ell) + 2r$

$= 2\pi - \frac{\pi}{3} + 2$

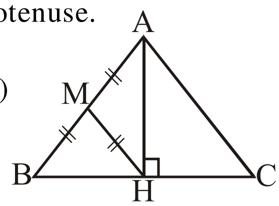
$= \frac{5\pi}{3} + 2$

7. $\angle C = 90^\circ$ & $\angle A = 20^\circ$ (Given)

$\therefore \angle B = 180^\circ - (90^\circ + 20^\circ) = 70^\circ$

Since BD is bisector of $\angle ABC$
$\therefore \angle ABD = \angle CBD = 35^\circ$
Now $\angle BDC = 180^\circ - (\angle BCD + \angle CBD)$
$= 180^\circ - (90^\circ + 35^\circ)$
$= 55^\circ$

8. Since ΔAHB is right angle Δ & AB = 13 (Given)
Length of median from right angle Δ is half the length of hypotenuse.



So $HM = \frac{1}{2}(AB)$

$\therefore HM = \frac{13}{2} = 6.5$

9. $\Delta PAB \sim \Delta PCA$

$$\frac{PC}{PA} = \frac{6}{8} = \frac{PA}{PC+7}$$

$$PA = \frac{4}{3}PC \qquad \ldots(1)$$

$$\frac{PA}{PC+7} = \frac{3}{4} \qquad \ldots(2)$$

From (1) & (2)

$$\frac{\frac{4}{3}PC}{PC+7} = \frac{3}{4}$$

$$\frac{16}{3}PC = 3PC + 21 \quad \Rightarrow \boxed{PC = 9}$$

10. $\Delta DBE \sim \Delta ABC$

Let DE = x   AB = BC = CA = 3 (Given)

$$\frac{1}{3} = \frac{x}{3}$$

$\therefore$ DE = x = 1

CE = CB – EB = 3 – 1 = 2

AD = AB – DB = 3 – 1 = 2

Perimeter of quadrilateral ADEC is

= 3 + 2 + 2 + 1

= 8

11. Using Pythagoras theorem

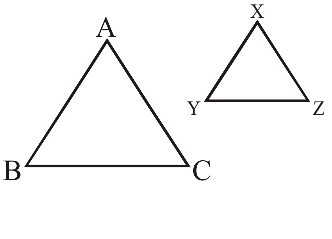
$$BD = \sqrt{3^2 + 4^2} = 5$$

$$AD = \sqrt{(13-3)^2 + 4^2} = \sqrt{116}$$

Thus the sum is AD + BD = 5 + $\sqrt{116}$ and as $100 < 116 < 121 \Rightarrow 10 < \sqrt{116} < 11$ so that the sum is between 15 and 16.

12. $\Delta XYZ \sim \Delta ABC$

$$\therefore \frac{AB}{XY} = \frac{BC}{YZ}$$



$$\frac{3}{5} = \frac{4}{YZ}$$

$$YZ = \frac{20}{3} = 6\frac{2}{3}$$

13. We first observe that the paper strips cover up part of the others. Since the Width of the overlap is 1 and the length of overlap is 1 and the area of the each of strips with the overlap is (10 × 1) – 1 = 9 Since there are 4 strips so area of table covered is = 4 × 9 = 36

14. The definition of an equiangular parallelogram is that all angles are equal and that pairs of sides are parallel. It may be a rectangle, because all the angles are equal and it is a parallelogram. It is not necessarily a regular polygon, because if the polygon is a pentagon, it is not a parallelogram. It is not necessarily a rhombus, because all the angles are not necessarily equal. It may be a square, since it is a parallelogram and all the angles are equal. It means it could be a square or a rectangle. but A square is a rectangle. A rectangle is not necessarily a square. So the most accurate answer is rectangle.

15. $\because$ AD = AB = 2 (given)

AF = 1

$\therefore$ BF = $\sqrt{2^2 + 1^2} = \sqrt{5}$

$$\text{Now Area of } \Delta ABF = \frac{1}{2} \times BF \times AG = \frac{1}{2} \times AB \times AF$$

$$\sqrt{5} \times AG = 2 \times 1$$

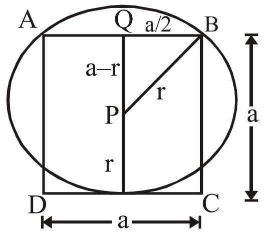
$$AG = \frac{2}{\sqrt{5}} = YZ$$

Now the rectangle must have the same area as the square, as the pieces were put together without gaps so its area is $2^2 = 4$

$$\text{Thus the vertical side WZ} = \frac{\text{Area}}{YZ} = \frac{4}{2/\sqrt{5}} = 2\sqrt{5} = XY$$

$$\text{Then the required ratio} = \frac{2\sqrt{5}}{2/\sqrt{5}} = 5$$

16. In ΔBQP



$$r^2 = (a - r)^2 + \left(\frac{a}{2}\right)^2$$

$$r^2 = a^2 + r^2 - 2ar + \frac{a^2}{4}$$

$$2ar = \frac{5a^2}{4}$$

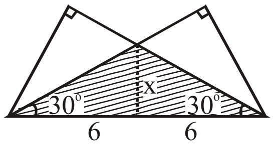
$$a\left(2r - \frac{5a}{4}\right) = 0$$

$$\because a \neq 0 \quad 2r - \frac{5a}{4} = 0$$

$$\Rightarrow \frac{5a}{4} = 2r \Rightarrow a = \frac{8}{5}r$$

So area of square (a^2) = $\frac{64}{25}r^2$

17.



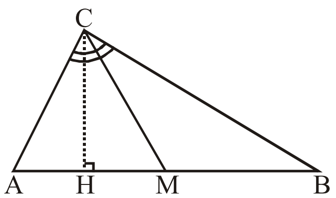
We observe that the altitude of the shaded region bisects the hypotenuse of the original two right triangles.

Now by using 30°–60°–90° triangles the altitude length is $2\sqrt{3}$ $\left(\tan 30^\circ = \frac{x}{6}\right)$

The area of shaded region is $\frac{1}{2} \times 12 \times 2\sqrt{3}$

$$= 12\sqrt{3}$$

18.



Since CM is a medium

So AM = BM

Also By ASA congruency

$$\Delta CHA \cong \Delta CHM$$

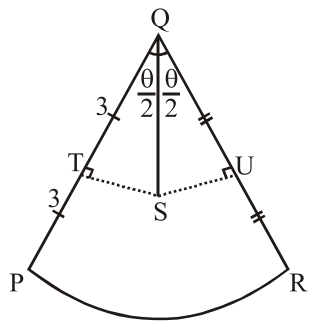
So AH = HM

$$\therefore \quad HM = \frac{1}{4}(AB) \qquad \left(\because AM = \frac{1}{2}AB\right)$$

Since ΔCHM and ΔABC has same altitude

$\therefore$ Area of ΔABC = 4 (Area of ΔCHM) = 4K

19. Let Q be the centre of the circle and P, R be two points on the circle such that $\angle PQR = \theta$. If the circle circumscribes the sector, then the circle must circumscribe ΔPQR.



Drawn the perpendicular bisectors of QP and QR and mark the intersection as point S and draw a line S to Q. By congruency and CPCT.

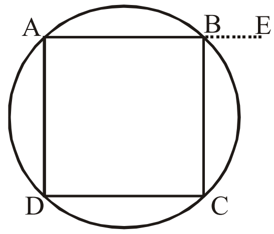
$$\angle PQS = \angle RQS = \theta/2$$

Let R be the circumradius of triangle

$$\cos \theta/2 = \frac{3}{R}$$

$$R = \frac{3}{\cos\theta/2} = 3\sec(\theta/2)$$

20. Since ABCD is cyclic, sum of opposite angle must be 180°



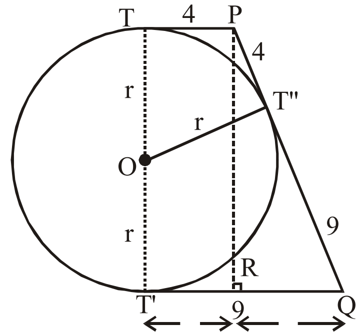
Therefore $\angle ADC + \angle ABC = 180^\circ$

$$\angle ABC = 180^\circ - \angle ADC$$
$$= 180^\circ - 68^\circ$$
$$= 112^\circ$$

Also $\angle ABC + \angle EBC = 180^\circ$

$$\angle EBC = 180^\circ - \angle ABC$$
$$\angle EBC = 180^\circ - 112^\circ$$
$$\angle EBC = 68^\circ$$

21.



PT = PT'' = 4

QT' = QT'' = 9

In right angle ΔPRQ

$$PR = \sqrt{(13)^2 - 5^2} = 12$$

$\therefore$ TT' = PR = 12

TT' = 2r = 12 $\Rightarrow$ r = 6

22. Area of triangle QPO = Area of ΔQAM + Area of ΔPBN + Area of AMONB

$$= \underbrace{\text{Area of AMONB} + \text{Area of MDC}} + \underbrace{\text{Area of NOC}} + \text{Area of DOC}$$

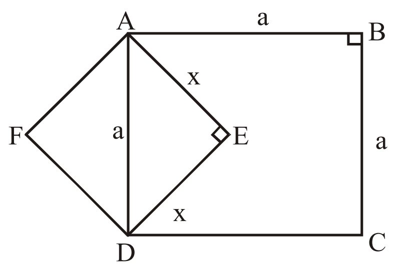
$$= \text{Area of ABCD} + \text{Area of DOC}$$

$$= K + \frac{1}{4}(\text{Area of DCMN})$$

$$= K + \frac{1}{4}\left(\frac{1}{2}K\right)$$

$$= \frac{9K}{8}$$

23.



Area of first square = a^2

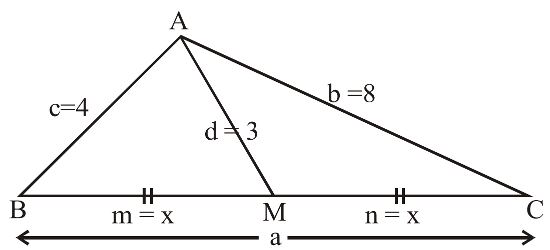
In ΔAED $x^2 + x^2 = a^2$

$$x = \frac{a}{\sqrt{2}}$$

Area of second square = $\frac{a^2}{2}$

$$\text{Ratio of } \frac{\text{Area of first square}}{\text{Area of second square}} = \frac{a^2}{a^2/2} = 2$$

24. Use Stewart Theorem



$b^2m + c^2n = a(mn + d^2)$

$8^2x + 4^2x = 2x(x.x + 3^2)$

$64x + 16x = 2x(x^2 + 9)$

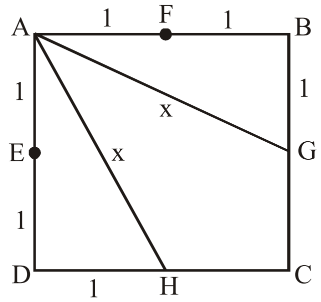
$x^2 = 31$

$x = \sqrt{31}$

So length BC = 2x

$= 2\sqrt{31}$

25.



In ΔABG $2^2 + 1^2 = x^2$

$\therefore$ $x = \sqrt{5}$

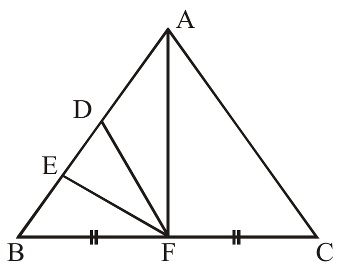
Sum of distances from one vertex to midpoints of each sides of square is

= AF + AE + AH + AG

$= 1 + 1 + \sqrt{5} + \sqrt{5}$

$= 2 + 2\sqrt{5}$

26.



Area of ΔABC = 96 (Given)

Since F is midpoint of BC

Area of ΔABF = Area of ΔACF = 48

Similarly D is midpoint of AB

$\therefore$ Area of ΔADF = Area of ΔDBF = 24

Similarly E is midpoint of DB

$\therefore$ Area of ΔDEF = Area of ΔBEF = 12

$\therefore$ Area of ΔAEF = Area of Δ ADF + Area of ΔDEF

= 24 + 12 = 36

27. Smallest angle (a) = 100^o

largest angle (ℓ) = 140^o

Because Interior angle form A.P. so

$\ell = a + (n-1)d$

$140 = 100 + (n-1)d$

$(n-1)d = 400 \rightarrow (1)$

Sum of interior angles of a polygon is $(n-2)\pi$

$$(n-2)\pi = \frac{n}{2}[2a+(n-1)d]$$

$(n-2)180 = \frac{n}{2}[2\times100+40]$ put the value of $(n-1)d$ from equation (1)

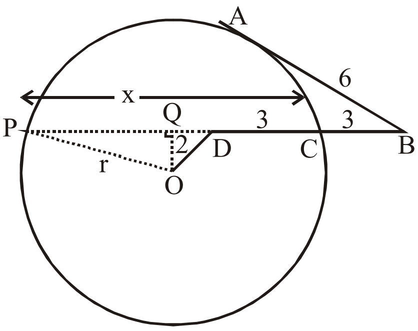
$$(n-2)180 = \frac{n}{2}[2\times100+40]$$

$180n - 360 = n(120)$

$60n = 360$

$n = 6$

28.



$(AB)^2 = (BC)(BP)$

$36 = 3.(3 + x)$

$\boxed{CP = x = 9}$

O is centre so $PQ = CQ = \frac{9}{2} = 4.5$

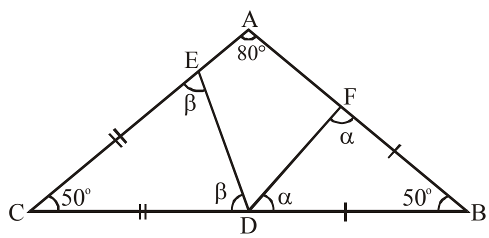
$OQ = r^2 - (4.5)^2 = (2)^2 - (1.5)^2$

$r^2 = 22$

$\boxed{r = \sqrt{22}}$

29. Equilateral triangles can be is different sizes therefore they are not congruent to each other.

30.



$\angle A = 80^o$, AB = AC (Given)

$\therefore \angle B = \angle C = 50^o$

Since CD = CE (Given)

$\angle CED = \angle CDE = \beta$

BF = BD (Given)

$\angle BDF = \angle DFB = \alpha$

In ΔCED $50^o + \beta + \beta = 180^o \Rightarrow \beta = 65^o$

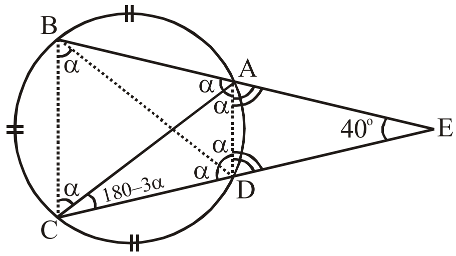
In ΔDFB $50^o + \alpha + \alpha = 180^o \Rightarrow \alpha = 65^o$

Now $\angle CDE + \angle EDF + \angle FDB = 180^o$

$\beta + \angle EDF + \alpha = 180^o$

$\angle EDF = 50^o$

31.



Since arc AB = arc BC = arc CD (Given)

Angle made by chord AB

$\angle BCA = \angle BDA = \alpha$

Similarly by chord BC $\angle BAC = \angle BDC = \alpha$

Similarly by chord CD $\angle CBD = \angle CAD = \alpha$

$\therefore \angle EAD = \angle EDA = 180^o - 2\alpha$

$\angle E = 40^o$(Given)

$$\therefore \angle EAD = \frac{180^o - 40^o}{2} = 70^o$$

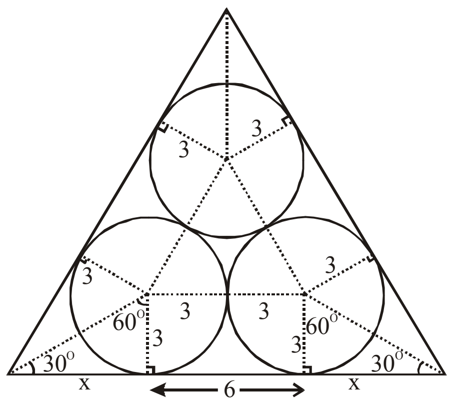
$\angle EAD = 180^o - 2\alpha = 70^o \Rightarrow \alpha = 55^o$

In ΔACD

$\therefore \angle ACD = 180^o - 3\alpha = 180^o - 3(55^o)$

$= 180^o - 165^o = 15^o$

32.



Since triangle is equilateral

Now $\tan 30^o = \frac{3}{x}$

$x = 3\sqrt{3}$

Side length of triangle $= 2x + 6$

$= 2\left(3\sqrt{3}\right)+6$

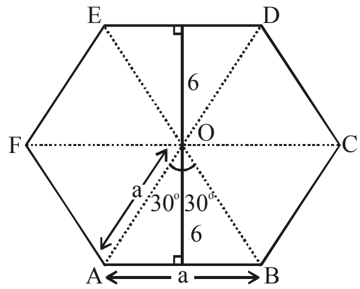
$= 6\sqrt{3}+6$

Now perimeter of triangle is = 3(side length)

$= 3\left(6\sqrt{3}+6\right)$

$= 18 + 18\sqrt{3}$

33.



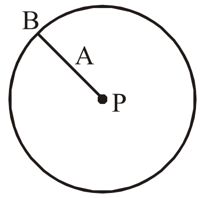
In ΔOAP

$$\cos 30^{o} = \frac{6}{a}$$

$$\frac{\sqrt{3}}{2} = \frac{6}{a} \Rightarrow \sqrt{3}a = 12$$

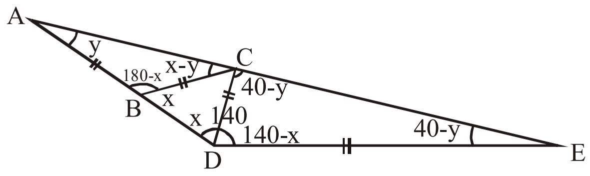
$$a = 4\sqrt{3}$$

34.



For each point A, other than P, the point of intersection of circle(C) with the ray beginning at P and passing through A is the point on circle (C) closest to A. Therefore the ray beginning at P and passing through B is set of all point A such that B is the point on circle(C) which is closest to point A.

35.



$\because \angle ADE = 140^{o}$ (Given)

let $\angle CDB = x$ So $\angle CDE = 140 - x$

Assume $\angle DAE = y \Rightarrow \angle AED = 40 - y$

In ΔCDE $40 - y + 140 - x + 40 - y = 180$

$x + 2y = 40 \quad ...(1)$

In ΔABC $\angle BAC = y$

$\angle CBA = 180 - x \Rightarrow ACB = x - y$

$\because AB = BC$

$\therefore \angle ACB - \angle BAC$

$x - y = y \Rightarrow x = 2y \quad ...(2)$

from (1) & (2) $y = 10^{o}$

36. Since the dimensions of DEFG are half of the dimension of ABCD

Area of DEFG $= \frac{1}{2}.\frac{1}{2}$(Area of ABCD)

$= \frac{1}{4}.(72)$

$= 18$

37. Since ΔABE is equilateral AB = AE

Each angle is 60^{o}

ABCD is square AD = AB

$\therefore$ AD = AE

$\angle ADE = \angle AED = \theta$

In ΔADE $\theta + 150^{o} + \theta = 180^{o}$

$2\theta = 30^{o}$

$\theta = 15^{o}$

38. The area of the large circle is $A_1 + A_2 = 9\pi$

Then A_1, $9\pi - A_1$, 9π are in arithmetic progression

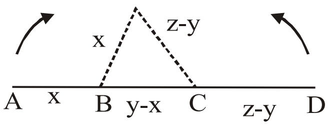
$9\pi - (9\pi - A_1) = (9\pi - A_1) - A_1$

$A_1 = 9\pi - 2A_1$

$3A_1 = 9\pi \Rightarrow A_1 = 3\pi$

The the radius of smaller circle is $\sqrt{3}$

39. Using Triangle Inequality



$x + y - x > z - y \Rightarrow 2y > z$

$y - x + z - y > x \Rightarrow z > 2x$

$x + z - y > y - x \Rightarrow 2x + z > 2y$

So obviously I and II statements are correct.

40. $\angle GDA = 360^o - (90^o + 90^o + 60^o)$

$\angle GDA = 120^o$

41. Let PQ = x CQ = AQ = radius

In ΔCPQ $\tan 30^o = \frac{x}{CQ}$

$$CQ = \frac{x}{\tan 30^o} = \sqrt{3}x$$

$\therefore AQ = \sqrt{3}x$

$$\therefore \frac{PQ}{AQ} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

42. Join A and C

Now Δ ABC is right angle Δ

So AC = 5

Now ΔACD is also right angle Δ because side length are 12, 5, 13

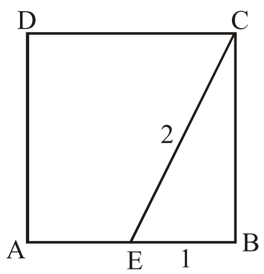
Now area of quadrilateral = Area of ΔABC + Area of ΔACD

$$= \frac{1}{2}\times 3\times 4 + \frac{1}{2}\times 12\times 5$$

$= 6 + 30$

$= 36$

43.



In ΔEBC

$$BC = \sqrt{2^2 - 1^2} = \sqrt{3}$$

$\therefore$ Area of square = $\left(\sqrt{3}\right)^2 = 3$

44. In ΔDCB

$\angle CDB = 180^o - (110^o + 30^o) = 40^o$

Since side AB and CD are parallel

So $\angle CDB = \angle DBA = 40^o$

Diagonal BD = side AD (Given)

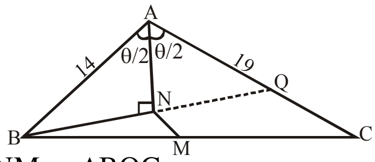
$\therefore$ In ΔADB

$\angle DBA = \angle DAB = 40^o$

$\therefore \angle ADB = 180^o - (40^o + 40^o) = 100^o$

45. $\Delta ANB \cong \Delta ANQ$

Thus AQ = 14



$\Delta BNM \sim \Delta BQC$

$$\frac{NM}{QC} = \frac{BN}{BQ} = \frac{BM}{BC} = \frac{BM}{2BM} = \frac{1}{2}$$

BQ = 2BN

QC = 2MN

$\therefore$ QC = 19 − AQ

2MN = 19 − 14

$$MN = \frac{5}{2}$$

46. We will try to solve for a possible value of the variables. First notice that exchanging a and b is the original equation must also work. Therefore a = b will work.

Now simplifying the original equation

$(2b + c)(2b - c) = 3b^2$

$4b^2 - c^2 = 3b^2$

$b^2 = c^2$

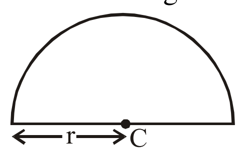
$b = c$

Therefore, it is an equilateral triangle

$\therefore$ Angle opposite the side of length c is 60^o

47. Perimeter of semi-circular region

$= 2r + \pi r$



Area of semi-circular region = $\frac{\pi r^2}{2}$

Perimeter = Area

$2r + \pi r = \frac{\pi r^2}{2}$

$4r + 2\pi r = \pi r^2$

$\pi r^2 - 2\pi r - 4r = 0$

$r(\pi r - 2\pi - 4) = 0$

$r \neq 0$ $r = \frac{2\pi + 4}{\pi}$

$$r = \frac{4}{\pi} + 2$$

48. Sum of interior angles of a polygon = $(n - 2)\pi$

Let remaining angle is x

So $(n - 2)\ 180 = 2570 + x$

$180n - 360 = 2570 + x$

$n = \frac{2930 + x}{180}$ ...(1)

Now $0 < x < 180^o$

When x = 0 then $n \simeq \frac{2930+0}{180} \simeq 16.277$

When x = 180 then $n \simeq \frac{2930+180}{180} \simeq 17.27$

So possible value of n = 17

Now put the value of n in equation (1)

$17 = \frac{2930 + x}{180}$

$x = 180 \times 17 - 2930 = 130^o$

49. Since GP = 15, AP = 75 (Given)

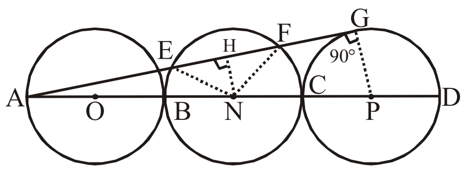
$\angle AGP = 90^o$ $AG = \sqrt{(75)^2 - (15)^2}$

$AG = 15\sqrt{24}$

Now drop an altitude from N to AG at point H

AN = 45

and $\Delta AGP \sim \Delta AHN$



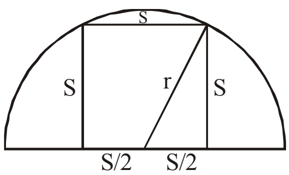
$\frac{45}{75} = \frac{NH}{15} \Rightarrow NH = 9$

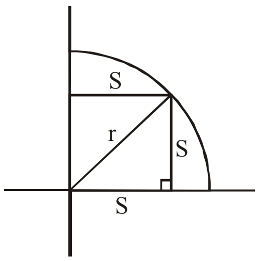
NE = NF = 15

In ΔEHN $EH = \sqrt{EN^2 - HN^2} = \sqrt{15^2 - 9^2} = 12$

Then length of chord EF = 2(EH) = 2(12) = 24

50.





$r^2 = (S)^2 + \left(\frac{S}{2}\right)^2$

$r^2 = S^2 + \frac{S^2}{4}$

$r^2 = \frac{5S^2}{4}$

$\boxed{S^2 = \frac{4}{5}r^2}$

$r^2 = S^2 + S^2$

$r^2 = 2S^2$

$\boxed{S^2 = \frac{r^2}{2}}$

Hence ratio of the area of the square inscribed in a semicircle to that inscribed in a quadrant of the same radius is $\frac{4/5}{1/2} = \frac{8}{5}$

51. Since AZ = AY, BZ = BX & CX = CY

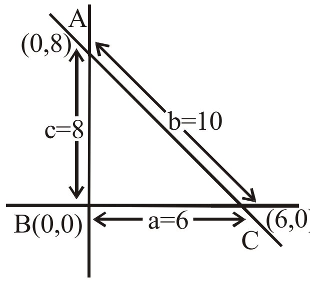
(AZ + ZB)+(BX + XC)+(AY + YC)=AB+BC +CA

(AZ + BX)+(BX + CY)+(CY + AZ) = 4+6+8

2(AZ + BX + CY) = 18

$\therefore$ AZ + BX + CY = 19

52.



Incentre = $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

$= \left(\frac{6\times0 + 10\times0 + 8\times6}{6+10+8}, \frac{6+8+10\times0+8\times0}{6+10+8}\right)$

= (2, 2)

Circum centre is at midpoint of hypotenuse of right angle triangle

Circum centre = $\left(\frac{6+0}{2}, \frac{0+8}{2}\right) = (3,4)$

Distance between incentre and circumcentre

$= \sqrt{(3-2)^2 + (4-2)^2} = \sqrt{1+4} = \sqrt{5}$

53. Integral sides means side length are integer

Let a, b, c are sides lengths a + b + c = 8 ...(1)

We know that :-

→ Sum of two sides of a triangle is greater than third side

→ Absolute value of difference of two sides of a triangle is least than third side

Considering this, only possible values of side length can be

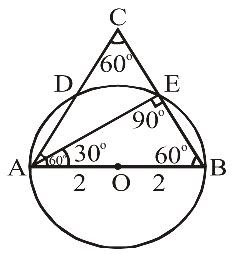
2, 3, 3 $\Rightarrow s = \frac{2+3+3}{2} = 4$

$\therefore$ Area = $\sqrt{4(4-2)(4-3)(4-3)}$

$A = 2\sqrt{2} \in (2,3)$

2 MARK

1. Since AB is a diameter



$\therefore\ \angle AEB = 90^\circ$

AB = 4 (Given)

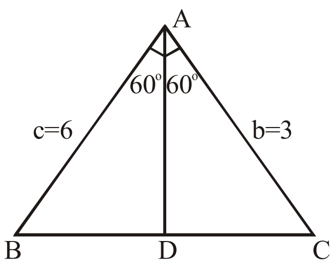
In ΔAEB

$\cos 30^\circ = \frac{AE}{AB}$

$\frac{\sqrt{3}}{2} = \frac{AE}{4}$

$\boxed{AE = 2\sqrt{3}}$

2.

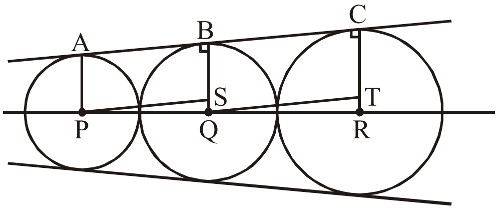


Length of Bisector (AD) $= \frac{2bc}{b+c}\cos\left(\frac{A}{2}\right)$

$= \frac{2(3)(6)}{3+6}\ \cos(60^\circ)$

$= 2$

3.



Consider three consecutive circles, observe their centres P, Q and R are collinear by symmetry. Let A, B and C be the points of tangency and let PS and QT be segments parallel to the upper tangent (L_1). Since PQ is parallel to QR, PS is parallel to QT as both are parallel to L_1, due to tangent being perpendicular to radius

$\Delta PQS \sim \Delta QRT$

Now if we let x, y and z be the radii of the three circles (from smallest to largest) then QS = y – x and RT = z - y.

Thus from the similarity

$$\frac{QS}{PQ} = \frac{RT}{QR} \Rightarrow \frac{y-x}{x+y} = \frac{z-y}{y+z} \Rightarrow y^2 = zx \Rightarrow \frac{y}{x} = \frac{z}{y}$$

So the ratio of consecutive radii is constant forming a geometric sequence. In this case first radius is 8 and last radius is 18 so the constant ratio is $\left(\frac{18}{8}\right)^{1/4}$. Therefore the radius of middle circle is $8\left[\left(\frac{18}{8}\right)^{\frac{1}{4}}\right]^2 = \sqrt{8}.\sqrt{18} = \sqrt{144} = 12$

4. Let the triangle have legs a and b and hypotenuse is $\sqrt{a^2+b^2}$

Perimeter = Area (Given)

$$a + b + \sqrt{a^2+b^2} = \frac{1}{2}ab$$

$$2\sqrt{a^2+b^2} = ab - 2a - 2b$$

$4a^2b + 4ab^2 = a^2b^2 + 8ab$

As a and b are side lengths of a triangle so they must be positive

Now divide by ab

$4a + 4b = ab + 8$

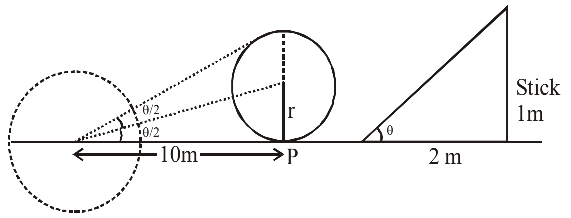
$\Rightarrow\quad b(a - 4) = 4a - 8$

$\Rightarrow\quad b = \frac{4(a-2)}{a+4} = 4 + \frac{8}{a-4}$

So for any value of a other than 4, we can generate a valid corresponding value of b

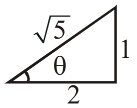
So there are infinitely many non congruent right triangles.

5.



Since angle made by stick

$$\tan\theta = \frac{1}{2}$$



$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$$

$$\frac{r}{10} = \frac{1-\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} = \sqrt{5}-2$$

$$r = 10\sqrt{5}-20$$

6. Let G be the intersection point of AE and DF

Area of quad DBEF = Area of triangle ABE (given) Area of quad DBEG = Area of ΔEFG

= Area of quad DBEG + Area of ΔADG

$\therefore$ Now area of ΔEFG = Area of ΔADG

Now Area of ΔADG + Area of ΔAGF = area of ΔEFG + area of ΔAGF

$\therefore$ Area of ΔADF = Area of ΔAFE

Now taking AF as the base of ΔADF and ΔAFE (By using the fact that triangles with the same base and same perpendicular height, have the same area)

So we deduce that perpendicular distance from D to AF is same as the perpendicular distance from E to AF

This implies that

AF || DE (Since A, F and C are collinear)

AC || DE

Thus ΔDBE ~ ΔABC

$$\frac{BE}{BC} = \frac{BD}{BA} = \frac{3}{5}$$

Since ΔABE and ΔABC have the same perpendicular height (Taking AB as the base)

$\therefore$ Area of ΔABE = $\frac{3}{5}$ Area of ΔABC

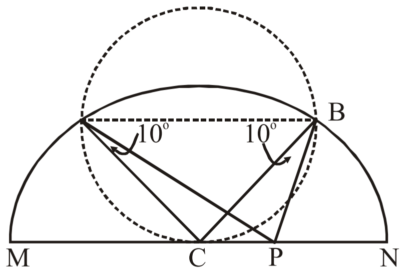
$$= \frac{3}{5}\times 10 = 6$$

7. Since $\angle CAP = \angle CBP = 10^{o}$

quadrilateral ABPC is cyclic & angle inscribed in the same arc are equal.

Since $\angle$ACM = 40°

$\angle$ACP = 140°



So using the fact that sum of opposite angles in a cycle quadrilateral is 180°.

$\therefore$ $\angle$ABP = 40°

$\therefore$ $\angle ABC = \angle ABP - \angle CBP = 40° - 10° = 30°$

Since CA = CB $\therefore$ ΔABC is isosceles

$\angle$BAC = $\angle$ABC = 30°

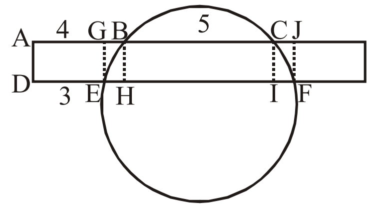
Now $\angle BAP = \angle BAC - \angle CAP = 30° - 10° = 20°$

$\angle$BCP = $\angle$BAP = 20° (Angles inscribed in the same arc are equal)

8. Draw perpendicular from

B to DF, E to AC

C to DF, F to AC



Since AGED is a rectangle since it has 4 right angles.

Therefore AG = DE = 3

$\therefore$ GB = 4 – 3 = 1

Now GBHE is also a rectangle and EH = GB = 1 & BCIH is also a rectangle and BC = HI = 5

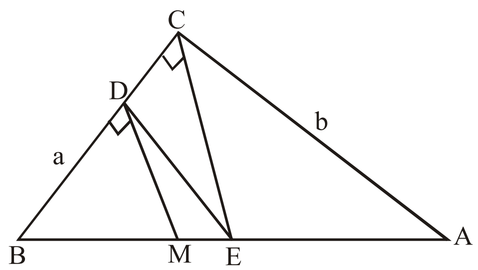
$\because$ BE = CF

So IF = EH = 1

Therefore EF = EH + HI + IF

= 1 + 5 + 1 = 7

9.



Area of $\Delta ABC = 24 = \frac{AB.BC.\sin\theta}{2} = \frac{c.a.\sin\theta}{2}$

$\because$ AM = MB (Given)

$MB = \frac{AB}{2} = \frac{c}{2}$

In ΔBDM BD = BM $\cos\theta$ = $\frac{c}{2}\cos\theta$

In ΔBCE CE = a $\tan\theta$

Now note that CE is the height of triangle BDE originating with vertex E so area of

$\Delta BED = \frac{1}{2}.$ BD.CE

$= \frac{1}{2}.\frac{c}{2}.\ \cos\theta.\ a\tan\theta$

$= \frac{1}{2}\left(\frac{a.c.\sin\theta}{2}\right) = \frac{1}{2}(\text{Area of } \Delta ABC)$

$= \frac{1}{2}(24) = 12$

10. Let AC = x and BF = y

$\Delta AFC \sim \Delta BFA$ so $\frac{AF}{FC} = \frac{BF}{AF} \Rightarrow \frac{AF}{1} = \frac{y}{AF}$

So $AF = \sqrt{y}$

Also AD = x – 1 and using the Pythagoras theorem on ΔABD $AB = \sqrt{2x - x^2}$

Again apply Pythagoras Theorem on ΔAFB

$= (y)^2 + \left(\sqrt{y}\right)^2 = \left(\sqrt{2x - x}\right)^2$

$\Rightarrow y^2 + y = 2x - x^2$

Now substitute $y = x^2 - 1$

$(x^2 - 1)^2 + (x^2 - 1) = 2x - x^2$

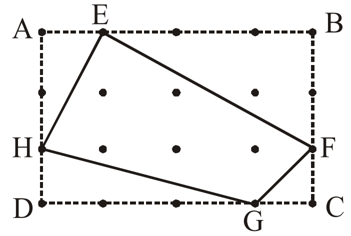
$x^4 - 2x^2 + 1 + x^2 - 1 = 2x - x^2$

$x^4 - 2x = 0$

$x(x^3 - 2) = 0$

$x^3 = 2 \Rightarrow x = \sqrt[3]{2}$

11. First Join the points of perimeter



Now Area of ABCD = 3 × 4 = 12

Area of ΔEBF = $\frac{1}{2} \times 3 \times 2 = 3$

Area of ΔAEH $= \frac{1}{2} \times 1 \times 2 = 1$

Area of ΔFCG $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of ΔGDH = $= \frac{1}{2} \times 3 \times 1 = \frac{3}{2}$

Now Area of EFGH = Area of ABCD – (Area of ΔEBF + Area of ΔAEH + Area of ΔFCG + Area of ΔGDH)

$= 12 - \left(3 + 1 + \frac{1}{2} + \frac{3}{2}\right)$

= 16

12. The sum of the interior angle measures of an n sided polygon is 180°(n – 2).

Let the three obtuse angle be A_1, A_2, A_3 and n-3 acute angle measures be a, a_2, a_3 ....., a_{n-3}

Since $90 < A_1, A_2, A_3 < 180$

$270 < A_1 + A_2 + A_3 < 540$ ...(1)

$0 < a_1, a_2, ...\ a_3 < 90$

Similarly $0 < a_1 + a_2 + a_3 + ... + a_{n-3} < 90(n-3)$ ...(2)

Add (1) and (2)

$270 < 180(n - 2) < 540 + 90n - 270$

$270 < 180n - 260 < 90n + 270$

$\Rightarrow n < 7$

So the largest possible value of n is 6

13. Let B be the intersection of line L and ED because AB is the altitude on the hypotenuse of right angle ΔAED we have

$AB^2 = BD.BE$

$AB^2 = 1.2 \Rightarrow AB = \sqrt{2}$

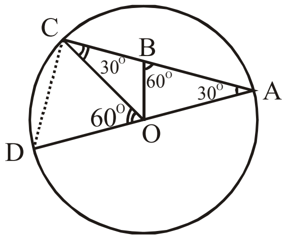
Now area of $\Delta AED = \frac{1}{2}$ ED × AB = $\frac{3}{\sqrt{2}}$

Now area of rectangle ABCD = 2 × Area of ΔAED

$= 2\left(\frac{3}{\sqrt{2}}\right) = 3\sqrt{2} \cong 4.2$

14. $\widehat{CD} = 60^o$

So $\angle COD = 60^o$



$\angle DAC = \frac{1}{2}\angle COD = 60^o$

$\angle DAC = 30^o$

OA = OC

$\angle OCA = \angle OCB = \angle OAC = 30^o$

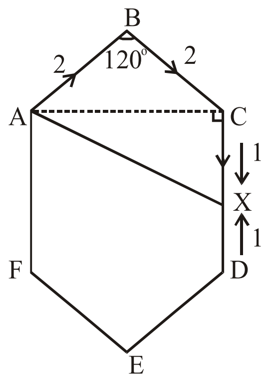
$\angle BOC = 30^o$ ($\angle OBA = \angle BCO + \angle BOC$)

So $\angle BCO = \angle BOC$

ΔBOC is isosceles triangle

So BC = BO = 5

15. Let Alice was on vertex A



$\angle ABC = 120^o$

$\cos 120^o = \frac{2^2 + 2^2 - (AC)^2}{2\times 2\times 2} = \frac{-1}{2}$

$(AC)^2 = 12$

$AC = 2\sqrt{3}$

In ΔACX

$(AC)^2 + (CX)^2 = (AX)^2 = 12 + 1$

$AX = \sqrt{13}$ km

16. Let Radius of circle is R

$\frac{1}{2}(AB)\times AC - \frac{1}{2}\theta R^2 = \frac{1}{2}\theta R^2$

$\frac{(AB)(AC)}{2} = \theta R^2$

AC = R = CD = CE

$\frac{AB}{2} = \theta R$

$\frac{AB}{R} = 2\theta$

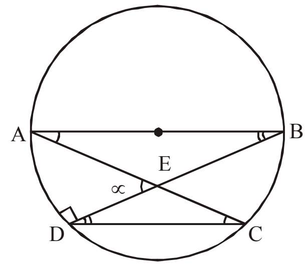
$\frac{AB}{AC} = 2\theta$

$\tan\theta = 2\theta$

17. AB || DC, $\widehat{AD} = \widehat{CB}$ and

$\Delta CDE \sim \Delta ABE$

$\frac{Area\Delta CDE}{Area\Delta ABE} = \left(\frac{DE}{AE}\right)^2$



Draw AD, AB is diameter so

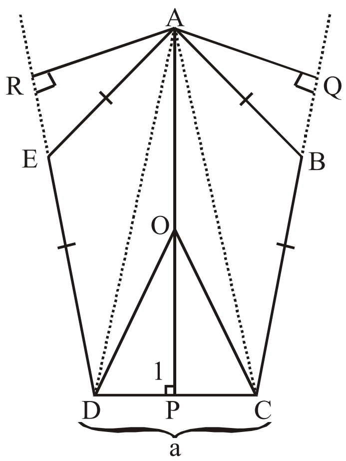
$\angle ADB = 90^o$

Thus in ΔADE

$DE = AE\cos\alpha$

$\left(\frac{DE}{AE}\right)^2 = \cos^2\alpha$

18. Let side of pentagon is a



Area of ($\Delta AED + \Delta ADC + \Delta ABC$)

= Area of pentagon

$\frac{1}{2}(AR)a + \frac{1}{2}(AP)a + \frac{1}{2}(AQ)\times a$

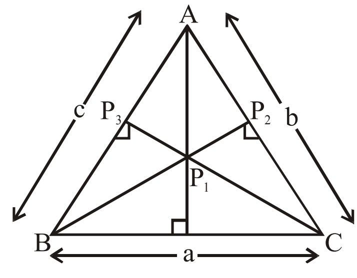
$= 5 \times \Delta ODC$

$\frac{a}{2}(AR + AP + AQ) = 5\times\frac{1}{2}\times 1\times a$

AP + AQ + AR = 5

19. Let $P_1 = 4$

$P_2 = 12$



$\frac{1}{\ }P_1 a = \frac{1}{\ }P_2 b = \frac{1}{\ }P_2 c = \Delta$

$$a = \frac{2\Delta}{P_1}, b = \frac{2\Delta}{P_2}, c = \frac{2\Delta}{P_3}$$

$$a + b > c$$

$$\frac{2\Delta}{P_1} + \frac{2\Delta}{P_2} > \frac{2\Delta}{P_3}$$

$$\frac{1}{4} + \frac{1}{12} > \frac{1}{P_3} \Rightarrow P_3 > 3$$

$$b + c > a \Rightarrow \frac{1}{P_3} > \frac{1}{4} - \frac{1}{12} > \frac{1}{6}$$

$$\boxed{P_3 < 6}$$

So $P_3 \in (3, 6)$

biggest possible integral Value of $P_3 = 5$

20. Let d_1 , d_2, d_3 d_{600} are the diameters of concentric circles. These d's form an AP with $d_1 = 2$ cm and $d_{600} = 10$ cm. If L is total length of Tape then

$L = \pi d_1 + \pi d_2 + \ldots + \pi d_{600}$

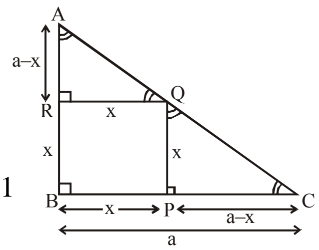
$= \pi(d_1 + d_2 + \ldots + d_{600})$

$$= \pi . 600 \left(\frac{d_1 + d_{600}}{2}\right)$$

$$= \pi \,.\, 600 . \frac{12}{2}$$

$= 3600\ \pi$ cm $= 36\pi$ metre

21. ΔPCQ is isosceles



$x = a - x$

$2x = a$

$4x^2 = a^2$

given that $x^2 = 441$

$x = 21$

$$\boxed{a = 42}$$

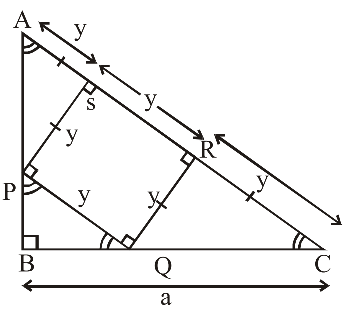
ΔQRC and ΔASP are isosceles right triangle

So AS = RC = y

AC = 3y

$(AC)^2 = 9y^2$

$2a^2 = 9y^2$



$$y^2 = \frac{2}{9}a^2$$

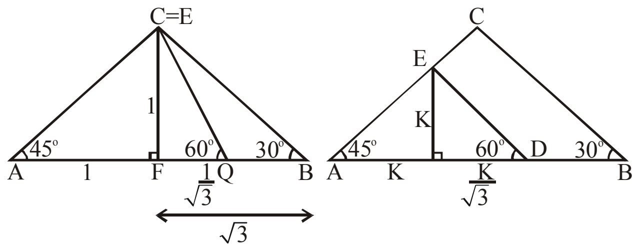
$$= \frac{2}{9} \times 42 \times 42$$

$y^2 = 392$ AC $= \sqrt{2}a$

22. Let E = C and CF = 1

$$\frac{\text{Area of } \Delta EAD}{\text{Area of } \Delta EAB} = \frac{AD}{AB} = \frac{1 + 1/\sqrt{3}}{1 + \sqrt{3}} = \frac{1}{\sqrt{3}} > \frac{1}{2}$$

Thus we must move DE to left and shrinking the dimension of ΔEAD by a factor K so that



Area of EAD $= \frac{1}{2}$ Area of CAB

$$\frac{1}{2}k^2\left(1 + \frac{1}{\sqrt{3}}\right) = \frac{1}{4}(1 + \sqrt{3})$$

$$K = \sqrt[4]{\frac{3}{4}}$$

$$\text{Thus } \frac{AD}{AB} = K\left(\frac{1 + \frac{1}{\sqrt{3}}}{1 + \sqrt{3}}\right) = \frac{K}{\sqrt{3}} = \frac{1}{\sqrt[4]{12}}$$

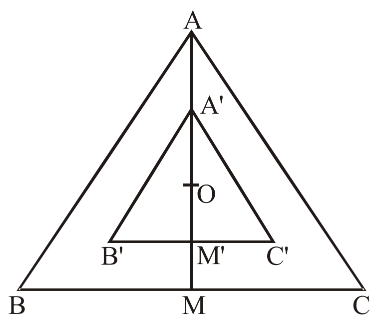
23. Let ΔABC & $\Delta A'B'C'$ have height h and h'.

Thus required ratio is $\left(\frac{h'}{h}\right)^2$

Let O is common centre and M & M' be intersection of BC and B'C' with common altitude from A

So OM $= \frac{h}{3}$ and OM' $= \frac{h'}{3}$

MM' = h/6 so

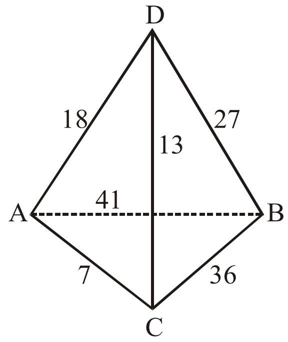


$$\frac{h}{3} = \frac{h'}{3} + \frac{h}{6}$$

$$\text{So } \frac{h'}{h} = \frac{1}{2}$$

$$\left(\frac{h'}{h}\right)^2 = \frac{1}{4}$$

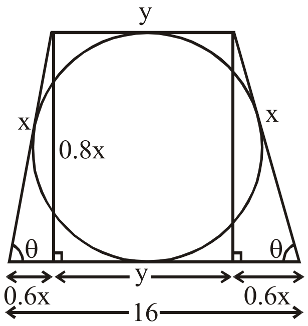
24. Taking care of Triangle inequalities



only above arrangement is constructible

So CD = 13

25. Sum of opposite sides is equal



$\sin\theta = 0.8$

$\cos\theta = 0.6$

$2y + 1.2x = 2x$

$y + 1.2\ x = 16$

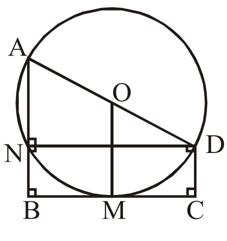
So $y = 4$

$x = 10$

Area $= \frac{1}{2}(4+16)8$

Area = 80

26. Area $= \frac{1}{2}BC(AB + CD)$



BCDN is rectangle

BN = CD

$(BM)^2 = BN.BA$

$BC = 2\sqrt{(AB)(CD)}$

So Area $= (AB + CD)\sqrt{(CD)(BA)}$

for area to be integer AB. CD must be perfect square since AB and CD are integer in all cases.

27. Let n denote the number of sides in convex polygon. let the excepted angle is x then

$$180^\circ(n - 2) = 2190^\circ + x$$

$$n - 2 = \frac{2190}{180} + \frac{x}{180}$$

for convex polygon

$$0^\circ < x < 180^\circ$$

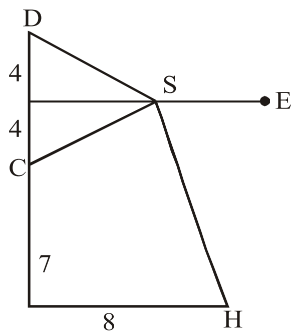
$$\frac{2190}{180} < n - 2 < \frac{2190}{180} + 1$$

$$12\frac{1}{6} < n - 2 < 13\frac{1}{6}$$

$$14\frac{1}{6} < n < 15\frac{1}{6}$$

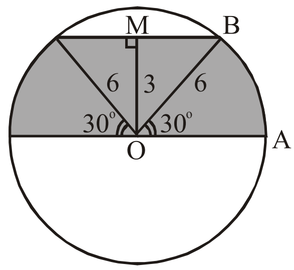
so $\boxed{n = 15}$

28. S denotes an arbitrary point on the stream SE and C, H and D denotes position of cow boy, his cabin and the point 8 miles north of C, respectively. CS + SH = DS + SH which is least when DSH is a straight line.



CSH = DSH = $\sqrt{8^2 + 15^2} = 17$ miles

29. O is centre of plot and M is midpoint of the side of walk not passing through O. If AB denotes arc connecting opposite sides of walk. The OMBA consist of 30° sector OAB and 30°-60°–90° ΔOMB



area of walk =

$$2\left[\frac{1}{2}.\frac{\pi}{6}.6^2\right]+\frac{1}{2}\times 6^2\times\left(\frac{\sqrt{3}}{2}\right)=6\pi+9\sqrt{3}$$

the required area is

$$\pi 6^2-\left(6\pi+9\sqrt{3}\right)=30\pi-9\sqrt{3}$$

30. each chord of length s substends an angle 36^o at centre 0 and MN of length d substends $3\times 36^o = 108^o$

$s = 2\sin 18^o$ and $d = 2\sin 54^o$

$d = 2\cos 36^o = 2(1-2\sin^2 18^o) = 2-s^2$

$s = 2\sin 18^o = 2\cos 76^o = 2(2\cos^2 36^o-1)$

$s = d^2 - 2$ ...(1)

and $d = 2 - s^2$ ...(2)

(1) + (2) $d + s = (d + s)(d - s)$

$d - s = 1$

$d = s + 1$

Substituting in (1)

$s = (s + 1)^2 - 2$

$s^2 + s - 1 = 0$

$$s=\frac{\sqrt{5}-1}{2}$$

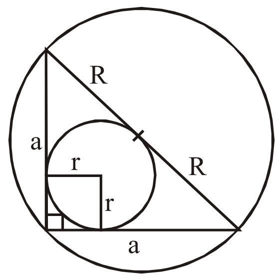
$$d=\frac{\sqrt{5}+1}{2}$$

$d^2 - s^2 = (d + s)(d - s) = \sqrt{5}$

$ds = 1$ & $d^2 - s^2 = \sqrt{5}$

31. $2R=\sqrt{2}a$

$R=a/\sqrt{2}$



$$r=\frac{\Delta}{s}=\frac{\frac{1}{2}.a.a}{2a+\sqrt{2}\,a}$$

$$r=\frac{(2-\sqrt{2})a}{2}$$

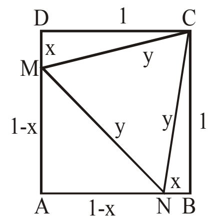
$$\frac{R}{r}=\frac{\frac{a}{\sqrt{2}}}{\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)a}=\frac{1}{\sqrt{2}-1}=\sqrt{2}+1$$

32. Let DM = NB = x then

AM = AN = 1 – x

Area of ΔCMN = area ABCD – area ΔANM)

–Area ΔNBC – Area ΔCDB



$$=1-\frac{1}{2}(1-x)^2-\frac{x}{2}-\frac{x}{2}$$

$$=\frac{1}{2}\left(1-x^2\right)$$

Let side of equilateral ΔCMN is y

$x^2 + 1^2 = y^2$ and $(1-x)^2 + (1-x)^2 = y^2$

$2(1-x)^2 = x^2 + 1$

$x^2 - 4x + 1 = 0$

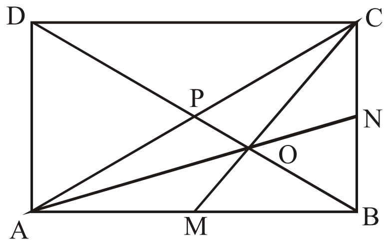
$x = 2-\sqrt{3}$

area of ΔCMN = $\frac{\sqrt{3}}{4}\left[(2-\sqrt{3})^2+1\right]$

$= 2\sqrt{3}-3$

33. Diagonal AC and DB are drawn O is inter sections of medians of ΔABC. Altitude of ΔAOB from O is $\frac{1}{3}$ of altitude of ΔABC from C area of ΔAOB

$= \frac{1}{3}$(Area of ΔABC)



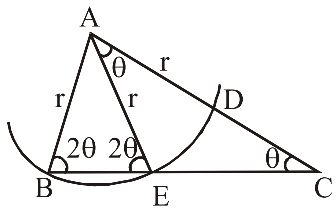
$$= \frac{1}{3}\left(\frac{1}{2}s^2\right) = \frac{1}{6}s^2$$

Similarly area of $\Delta COB = \frac{1}{6}s^2$

Area of AOCD $= s^2 - \frac{1}{3}s^2 = \frac{2}{3}s^2$

ratio $= \frac{2}{3}$

34. (1) If $0^o < \theta < 45^o$



$2\theta = \angle EAC + \theta$

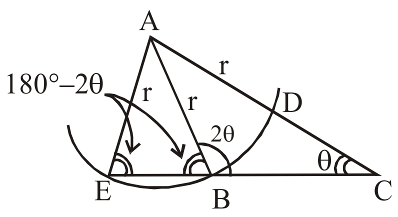
$= \angle EAC = \theta$

ΔEAC is Isosceles

hence EC = AE = AD

(2) If $\theta = 45^o$ then ΔABC is a $45^o - 45^o - 90^o$ triangle. $\angle E = \angle B$ then EC = BC = AB = AD

(3) If $45^o < \theta < 60^o$



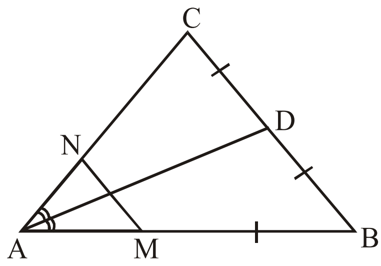
$\angle EAC = 180^o - \angle AEC - \angle C$

$= 180 - (180 - 2\theta) - \theta$

$= \theta$

Thus ΔEAC is isosceles and EC = EA = AD

35. Angular bisector theorem



$$\frac{BD}{CD} = \frac{AB}{AC}$$

Since CN = CD

and BM = BD

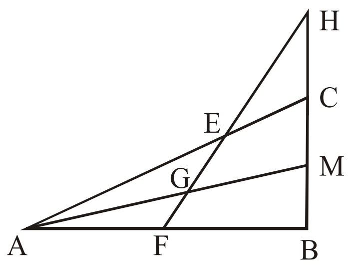
We have $\frac{BM}{CN} = \frac{AB}{AC}$

Which implies that **MN || BC**

Since only one choice is correct it must therefore be option 'C'. It is easy to verify that A, B, D are false if $\angle A = 90° - \sigma$, $\angle B = 60°$ and $\angle C = 30° + \sigma$. Where σ is any sufficiently small positive angle.

36. Extend BC and FE until they intersect at point H. The collinear points A, G, M lies on sides BF, FH, HB of ΔFBH. They also lies on extension of sides CE, EH, HC of ΔECH.

Now apply Menelaus's theorem



$$\frac{HG}{FG} \cdot \frac{FA}{BA} \cdot \frac{BM}{HM} = 1 \quad ...(1)$$

$$\frac{HG}{EG} \cdot \frac{EA}{CA} \cdot \frac{CM}{HM} = 1 \quad ...(2)$$

CM = BM and EA = 2FA

$$\frac{(1)}{(2)} \qquad \frac{EG}{FG} = 2\frac{BA}{CA} = 2.\frac{12}{16} = \frac{3}{2}$$

37. Let G and H be the points at which altitudes from C and F intersect AB and DE

$\Delta AGC \cong \Delta FHD$ Since AG = FH and AC = DF

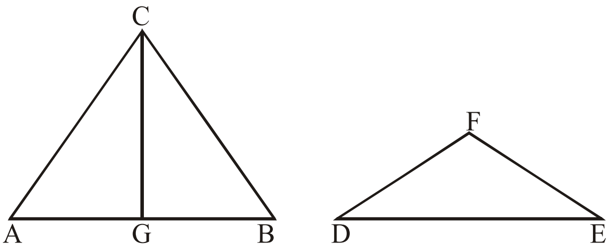
So $\angle GAC = \angle DFH$

$\angle ACG + \angle GAC = \angle ACG + \angle DFH = 90°$

So $\angle ACB + \angle DFE = 2\angle ACG + 2\angle DFH = 180°$

and area ΔABC = 2 (Area ΔACG) = 2 (Area ΔDFH)

= Area (ΔDEF)



38. Let coordinate of P(x,y) in which the vertices of equilateral triangle are (0,0) (s,0) $\left(\frac{s}{2}, \frac{s\sqrt{3}}{2}\right)$ then P belongs to the locus if and only if

$$a = x^2 + y^2 + (x - s)^2 + y^2 + \left(x - \frac{s}{2}\right)^2 + \left(y - \frac{s\sqrt{3}}{2}\right)^2$$

$$a = (3x^2 - 3sx) + (3y^2 - s\sqrt{3}\,y) + 2s^2$$

$$\frac{a - 2s^2}{3} = \left(x - \frac{s}{2}\right)^2 + \left(y - \frac{s\sqrt{3}}{6}\right)^2 - \frac{s^2}{3}$$

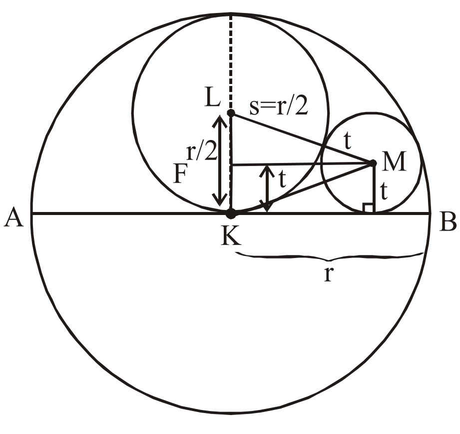
$$\frac{a - s^2}{3} = \left(x - \frac{s}{2}\right)^2 + \left(y - \frac{s\sqrt{3}}{6}\right)^2$$

Thus locus is empty set if $a < s^2$

Locus is a single point if $a = s^2$

Locus is a circle if $a > s^2$

39.



MF || AB

Let r, s, t be the radius of circles with centres K, L and M respectively.

In ΔFLM and ΔFKM

$$(MF)^2 = \left(\frac{r}{2} + t\right)^2 - \left(\frac{r}{2} + -t\right)^2$$

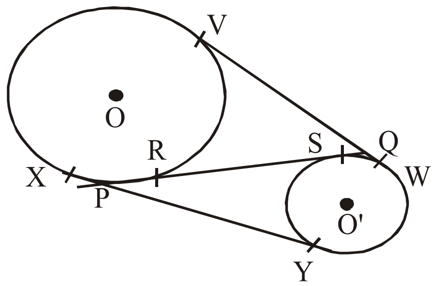
$$(MF)^2 = (r - t)^2 - t^2$$

After solving RHS

$$\frac{r}{t} = 4$$

Therefore the ratio is 16

40. X, Y, V, W are point of tangency of external tangents and R, S are point of tangency of internal tangent



PR = PX, PS = PY

QS = QW, QR = QV

So

PR + PS + QS + QR = PX + PY + QW + QV

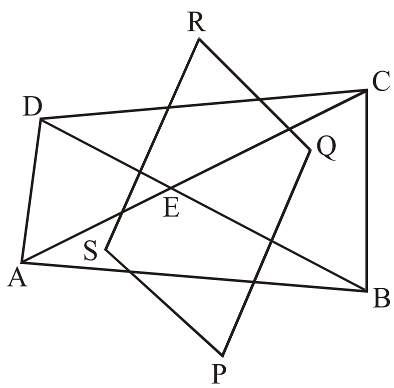
Thus

2PQ = XY + VW

Since XY = VW

PQ = XY = VW

41.



The centre of a circle circumscribing a triangle is the point of intersection of the perpendicular bisector of the sides of triangle. So P, Q, R, S are the intersection of the perpendicular Bisector of line segments AE, BE, CE and DE since line segments perpendicular to the same line are parallel so PQRS is a **parallelogram**.

42. If MNPQ, is convex then A is sum of areas of triangle into which MNPQ is divided by diagonal MP so that

$$A=\frac{1}{2}ab\sin N+\frac{1}{2}cd\sin Q$$

Similarly dividing MNPQ with diagonal NQ.

$$A = \frac{1}{2}\ ad\ \sin\ M + \frac{1}{2}bc\ \sin\ P$$

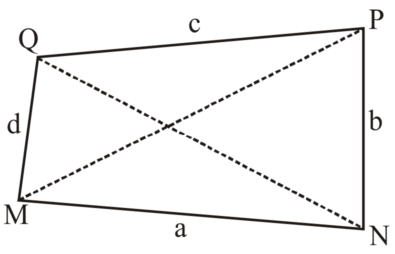
In any case

$$A \le \frac{1}{4}\ (ab + cd + ad + bc) = \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$$

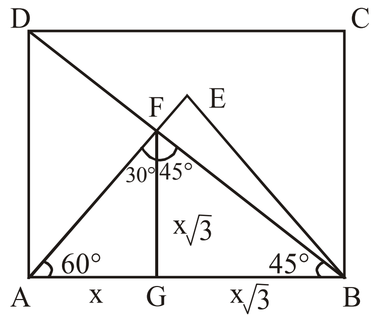
equality holds if

sin M = sin N = sin P = sin Q = 1

If MNPQ is **rectangle**



43. Let FG be an altitude of ΔAFB and let x denotes the length of AG



$$\sqrt{1+\sqrt{3}} = AB = x\left(1+\sqrt{3}\right)$$

$$1+\sqrt{3} = x^2\left(1+\sqrt{3}\right)^2$$

$$x^2\left(1+\sqrt{3}\right) = 1$$

Area of ΔABF is $= \frac{1}{2}AB \cdot FG$

$$= \frac{1}{2}\underbrace{x^2\left(1+\sqrt{3}\right)}_{1}\sqrt{3}$$

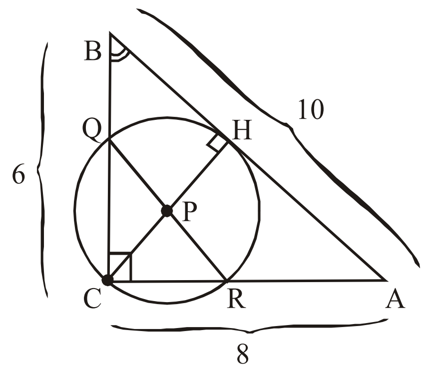
$$= \frac{\sqrt{3}}{2}$$

44. ∠RCQ = 90° thus QR is diameter of P

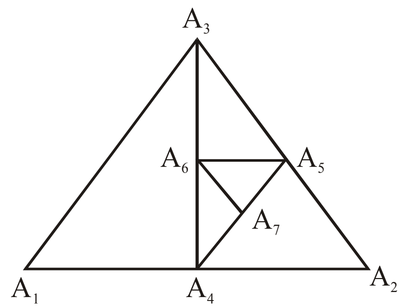
ΔCBH ~ ΔABC

$$\frac{CH}{6}=\frac{8}{10}$$

So QR = CH = **4.8**



45.



$\Delta A_2A_3A_4$ has 60° – 30° – 90°

$\angle A_1A_2A_3 = 60°$

and $A_2A_4 = A_2A_5$ & $\Delta A_2A_4A_5$ is equilateral

$\Delta A_3A_4A_5$ has 30° – 30° – 120°

$\Delta A_4A_5A_6$ has 30° – 60° – 90°

Finally $\angle A_4A_5A_6 = 60°$

$A_5A_6 = A_5A_7$ and $\Delta A_5A_6A_7$ is equilateral

So there is a cycle of Δ repeating in 4.

So $\Delta A_nA_{n+1}A_{n+2} \sim \Delta A_{n+4}A_{n+5}A_{n+6}$ with A_n and A_{n+4} as corresponding vertex

So $\angle A_{44}A_{45}A_{43} = \angle A_4A_5A_3 =$ **120°**

46. AA' = AD and corresponding altitudes of ΔA'A'B' has twice length of corresponding altitudes of ΔABD

So Area ΔAA'B' = 2(area ΔADB)

Let $\angle DAB = \theta$

area of $\Delta AA'B' = \frac{1}{2}$ (AD) (2AB) sin (180° – θ)

$= 2\left(\frac{1}{2}(AD)(AB)\sin\theta\right)$

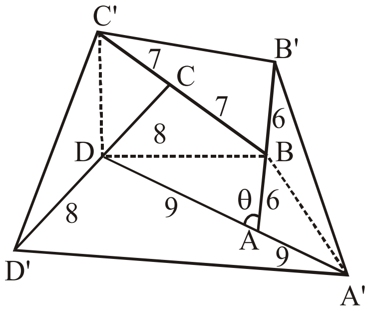
= 2 area of ΔABD

Similarly

area ΔBB'C' = 2 area ΔBAC

area ΔCC'D' = 2 area ΔCBD

area ΔDD'A' = 2 area ΔDCA



So area A'B'C'D' = area (ΔAA'B' + ΔBB'C' + ΔCC'D' + ΔDD'A') + area ABCD

= 2 area (ΔABD + ΔBAC)

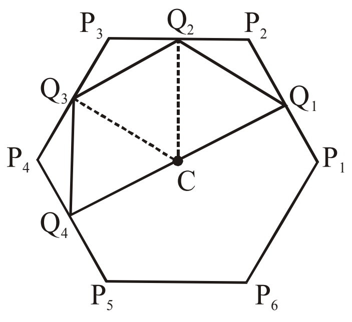
+ 2 (area (ΔCBD + ΔDCA)

+ Area ABCD

= 5 (Area of ABCD)

= 50

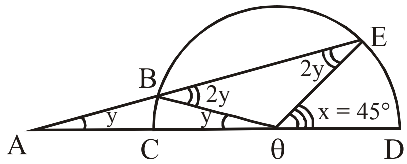
47. Let C is centre of Hexagon then area of $Q_1Q_2Q_3Q_4$ is sum of areas of three equilateral ΔQ_1Q_2C, ΔQ_2Q_3C, ΔQ_3Q_4C each of has side length 2



Area of $Q_1Q_2Q_3Q_4 = 3\left(\frac{\sqrt{3}}{4}\cdot 2^2\right)$

= $\mathbf{3\sqrt{3}}$

48.



Draw line segment BO and

x = $\angle$EOD and y = $\angle$BAO

AB = OD = OE = OB

In Δ ABO

$\angle$EBO = $\angle$BEO = 2y

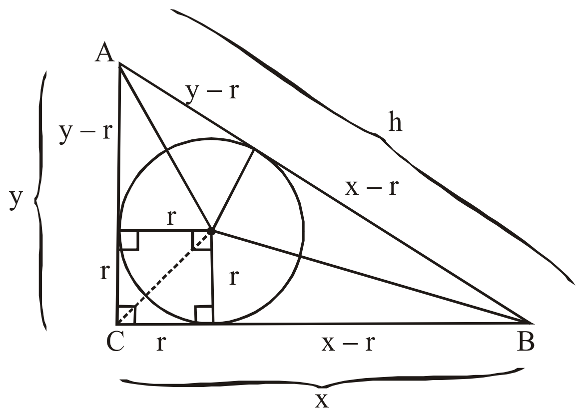
In ΔAEO

x = y + 2y

3y = 45°

y = 15°

49.



y – r + x– r = h

x + y = h + 2r

Area of $\Delta ABC = \frac{1}{2}xy$

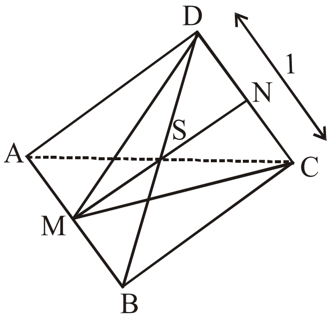
$= \frac{1}{2}\left(\frac{(x+y)^2-(x^2+y^2)}{2}\right)$

$= \frac{1}{4}\left((h+2r)^2-h^2\right)$

= hr + r^2

ratio $= \frac{\pi r^2}{hr+r^2} = \frac{\pi r}{h+r}$

50. Let M, N be the midpoints of AB and CD.



We claim that M, N are unique choices for P and Q. Which minimise the distance PQ. To show this we consider the set S of all points equidistant from A and B. S is the plane perpendicular to AB through M. Since C, D are equidistant from A and B, they lie in S, and so does the line through C and D.

Now M is the foot of perpendicular to AB from any point Q on CD.

Therefore is P is any point on AB

MQ < PQ unless P = M

Similarly the plane through N perpendicular to CD contains AB. MN $\perp$ CD thus MN < MQ unless Q = N.

So MN < PQ unless P = M and Q = N

This proves the claim.

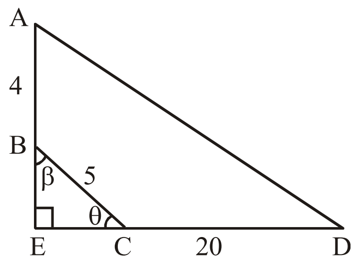
To compute the MN, MN is altitude of isosceles

ΔDMC with side length = $\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 1$

MN = $\sqrt{(MC)^2 - (NC)^2} = \sqrt{\frac{3}{4} - \frac{1}{4}}$

MN = $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ = Minimal distance PQ

51.



Let E is intersection of line AB and CD

Let $\angle$EBC = β and $\angle$ECB = θ

$\cos\beta = -\cos B = \sin C = \sin\theta$

$\beta + \theta = 90°$ so $\angle$BEC is right angle

BE = BC $\sin\theta$ = 3

CE = BC $\sin\beta$ = 4

Therefore AE = 7, DE = 24 and AD which is hypotenuse of ΔADE

AD = $\sqrt{7^2 + 24^2}$

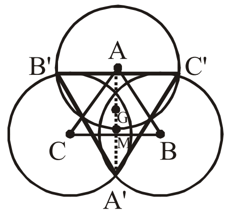
AD = 25

52. By symmetry

ΔABC and ΔA'B'C' are equilateral and have a common centroid (say G) M be the midpoint of BC

In ΔA'MC

$A'M = \sqrt{r^2 - 1}$



Now $\frac{B'C'}{BC} = \frac{A'G}{AG}$

$B'C' = 2\left(\frac{A'G}{AG}\right)$ {BC = 2, AM = $\sqrt{3}$ }

So

$AG = \frac{2}{3}AM = \frac{2}{3}\sqrt{3}$

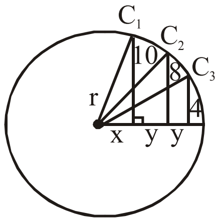
$MG = \frac{1}{3}AM = \frac{1}{3}\sqrt{3}$

A'G = A'M + MG = $\sqrt{r^2 - 1} + \frac{\sqrt{3}}{3}$

Thus

$$B'C' = 2\frac{\sqrt{r^2 - 1} + \frac{1}{\sqrt{3}}}{2\left(\frac{1}{\sqrt{3}}\right)} = \boxed{\sqrt{3(r^2 - 1)} + 1}$$

53. Let r be radius, x is distance from centre to closest chord and y is common distance between chords.



Now

$r^2 = x^2 + 10^2$ ...(1)

$r^2 = (x + y)^2 + 8^2$ ...(2)

$r^2 = (x + 2y)^2 + 4^2$ ...(3)

(1) – (2) $\Rightarrow$ $0 = 2xy + y^2 - 36$ ...(4)

(2) – (3) $\Rightarrow$ $0 = 2xy + 3y^2 - 48$ ...(5)

by (4) & (5) $y^2 = 6 \Rightarrow y = \sqrt{6}$

$$x = \frac{15}{\sqrt{6}}$$

$$r = \sqrt{x^2 + 10^2}$$

$$r = \frac{5\sqrt{22}}{2}$$

54. Line segment from E to G, the midpoint of DC is drawn

Area $\Delta EBG = \frac{2}{3}$ (Ares of ΔEBC)

Area of $\Delta BDF = \frac{1}{4}$ (Area of ΔEBG)

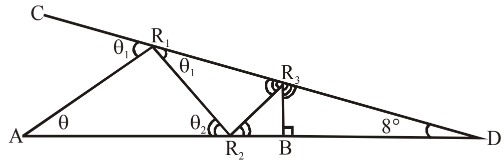
$= \frac{1}{6}$(Area of ΔEBC)

Now, EG||AD (Join of mid points)

Area of FDCE $= \frac{5}{6}$ (Area of ΔEBC)

Area $= \frac{\text{area } \Delta BDF}{\text{area FDCE}} = \frac{1}{5}$

55. Let $\angle DAR_1 = \theta$ and let θ_i be the acute angle. The light beam and the reflecting line form at the i[th] point of reflection. Applying theorem of exterior angle of triangle to $\angle AR_1D$ then $\angle R_1R_2D$ and so on.



We get

$\theta_1 = \theta + 8°$

$\theta_2 = \theta_1 + 8° = \theta + 16°$

$\theta_3 = \theta_2 + 8° = \theta + 24°$

$\theta_n = \theta_{n-1} + 8° = \theta + (8n)°$

$90° = \theta_n + 8° = \theta + 8n + 8°$

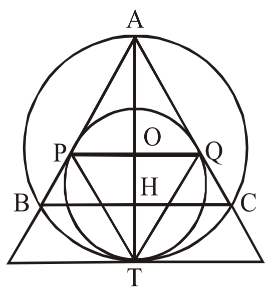
$0 \le \theta = 90° - (8n + 8)°$

$n \le \frac{82}{8}$

$n < 11$

The maximum value of n = 10 occur when $\theta = 2°$.

56. ΔPQT is also an equilateral triangle



$\Delta APQ \cong \Delta PQT$

AO = OT So

O is centre of larger circle

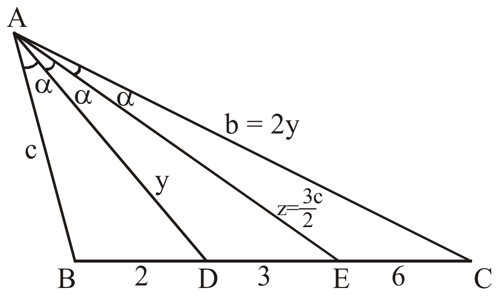
$AO = \frac{2}{3}AH$ (O is centroid)

$\Delta APQ \sim \Delta ABC$

So $PQ = \frac{2}{3}(BC)$ [BC = 12]

PQ = 8

57. $\angle BAC = 3\alpha$



$c = AB$

$y = AD$

$z = AE$

$b = AC$

By angle bisector theorem

$$\frac{c}{z}=\frac{2}{3} \quad \text{and} \quad \frac{y}{b}=\frac{1}{2}$$

So $z = \frac{3c}{2}$, $b = 2y$

Cosine law in ΔADB, ΔAED and ΔACE

$$\cos\alpha = \frac{c^2+y^2-4}{2cy} = \frac{\frac{9}{4}c^2+y^2-9}{3cy} = \frac{\frac{9}{4}c^2+4y^2-36}{6cy}$$

(i) = (ii) = (iii)

From (i) and (ii)

$3c^2 - 2y^2 = 12$ ......(iv)

From (i) and (iii)

$\Rightarrow 3c^2 - 4y^2 = -96$ ......(v)

From (iv) & (v)

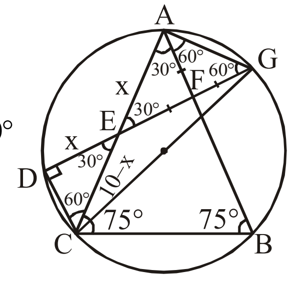
$\Rightarrow c^2 = 40,\ y^2 = 54$

So $AB = c = 2\sqrt{10}$

$AC = b = 2y = 2\sqrt{54} = 6\sqrt{6}$

$BC = 11$

58. DC is drawn



$\overset{\frown}{AC} = 150°$

$\overset{\frown}{AD} = \overset{\frown}{AC} - \overset{\frown}{DC}$

$\overset{\frown}{AD} = 150° - 30° = 120°$

So $\angle ACD = 60°$

Since $AC = DG$

$\overset{\frown}{GA} = \overset{\frown}{GD} - \overset{\frown}{AD}$

$= \overset{\frown}{AC} - 120° = 30°$

So $\overset{\frown}{CG} = 180°$ and $\angle CDG = 90°$

$AC = AB = DG = 10$

and $AE = DE$ $(\Delta ECD \cong \Delta EFA)$

Let x be their common length

then $CE = 10 - x = \frac{2x}{\sqrt{3}}$

we get $AE = x = 10\left(2\sqrt{3}-3\right)$

Now $FG = FA$ and ΔFAE is isosceles $EF = FA$

$EF = FG = \frac{1}{2}(10 - x)$

Area of $\Delta AFE = \frac{1}{2} AE \cdot AF \sin 30°$

$$= \frac{1}{2}\cdot x\left(\frac{10-x}{2}\right)\cdot\frac{1}{2}$$

$$= \frac{x}{8}\cdot\frac{2x}{\sqrt{3}} = \frac{x^2}{4\sqrt{3}} = 100 \times 3\frac{\left(7-4\sqrt{3}\right)}{4\sqrt{3}}$$

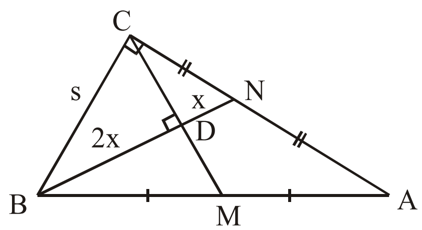
$$= 25\sqrt{3}\left(7-4\sqrt{3}\right) = 25\left(7\sqrt{3}-12\right)$$

Area of $\Delta ABC = \frac{1}{2} \times AB \cdot AC \cdot \sin 30°$

$$= \frac{1}{2} \times 10^2 \times \frac{1}{2} = 25$$

$$\frac{\text{Area of } \Delta AFE}{\text{Area of } \Delta ABC} = 7\sqrt{3} - 12$$

59. Let $x = DN$ and $2x = BD$



Right triangle $\Delta BCN \sim \Delta BDC$

So $\frac{s}{3x} = \frac{2x}{s}$

$s^2 = 6x^2$

$x = \frac{s}{\sqrt{6}}$

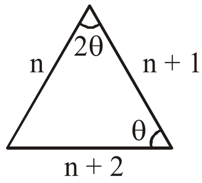
$BN = 3x$

$= \frac{3s}{\sqrt{6}}$

$BN = \frac{s\sqrt{6}}{2}$

60. Apply sine rule

$$\frac{\sin\theta}{n}=\frac{\sin 2\theta}{n+2}=\frac{2\sin\theta\cos\theta}{n+2}$$



$\cos\theta = \dfrac{n+2}{2n}$

Now applying cosine rule

$$\cos\theta = \frac{(n+1)^2+(n+2)^2-n^2}{2(n+1)(n+2)}$$

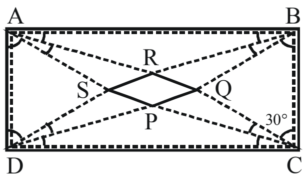
$$=\frac{(n+1)(n+5)}{2(n+1)(n+2)}=\frac{n+5}{2(n+2)}$$

$$\frac{n+2}{2n}=\frac{n+5}{2(n+2)}$$

$n = 4$

$$\boxed{\cos\theta=\frac{n+2}{2n}=\frac{3}{4}}$$

61. By the result of trisectors.



Isosceles $\Delta ASD \cong$ isosceles ΔBQC and
Isosceles $\Delta ARB \cong$ isosceles ΔDPC
Since AS = SD = QB = QC and
AR = RB = DP = CP and
$\angle SAR = \angle RBQ = \angle QCP = \angle SDP = 30°$
$\Delta SAR \cong \Delta RBQ \cong \Delta QCP \cong \Delta SDP$ (SAS)
Therefore SR = RQ = QP = PS

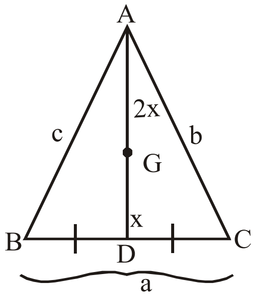
$\boxed{\text{So PQRS is a rhombus}}$

62. We know that in a ΔABC

$$m_a^2+m_b^2+m_c^2=\frac{3}{4}(a^2+b^2+c^2)$$

$$\frac{9}{4}\left(GA^2+GB^2+GC^2\right)=\frac{3}{4}\left(5^2+8^2+7^2\right)$$

$$\boxed{GA^2+GB^2+GC^2=46}$$



$m_a = AD = 3x$
$2x = AG$

$m_a = AD = \dfrac{3}{2}(GA)$

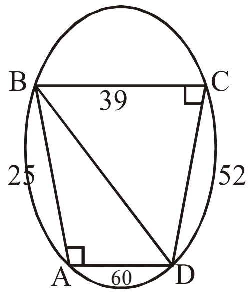
63. We can observe

$25^2 + 60^2 = 52^2 + 39^2$
$= 4225$

ΔABD and ΔBCD
are Right angle Δ

$(BD)^2 = 4225$

$\boxed{BD = 65}$



* Other wise we can use A + C = 180°
So cosA + cosC = 0 and Apply cosine Rule

64. In ΔO_1O_2X

$\sin\theta = \dfrac{6}{12}=\dfrac{1}{2}$

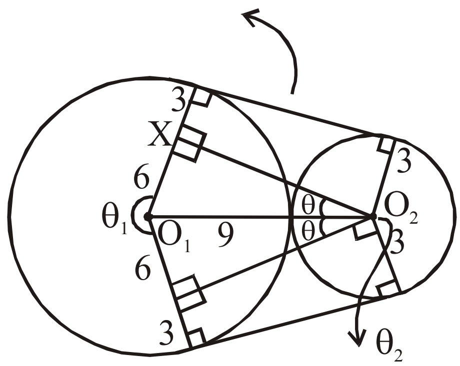
$\theta = 30° = \dfrac{\pi}{6}$

Length of common

tangent $= \sqrt{12^2-6^2}$

$=\sqrt{108}$

$=6\sqrt{3}$



$\theta_1 = \dfrac{4\pi}{3}$ and $\theta_2 = \dfrac{2\pi}{3}$

Length of shortest rope

$= 9\theta_1 + 3\theta_2 + 2(6\sqrt{3})$

$= 9.\dfrac{4\pi}{3}+3\left(\dfrac{2\pi}{3}\right)+12\sqrt{3}$

$$\boxed{=14\pi+12\sqrt{3}}$$