3

MOTION IN A PLANE

1. The vector that must be added to the vector $\hat{i}-3\hat{j}+2\hat{k}$ and $3\hat{i}+6\hat{j}+7\hat{k}$ so that the resultant vector is a unit vector along the positive y-axis, is

   (a) $4\hat{i}-2\hat{j}+5\hat{k}$ (b) $-4\hat{i}-2\hat{j}-9\hat{k}$

   (c) $3\hat{i}-4\hat{j}+5\hat{k}$ (d) null vector

2. Two stones are projected from the same point with same speed making angles $(45^\circ+\theta)$ and $(45^\circ-\theta)$ with the horizontal respectively. If $\theta \le 45^\circ$, then the horizontal ranges of the two stones are in the ratio of

   (a) 1 : 1 (b) 1 : 2

   (c) 1 : 3 (d) 1 : 4

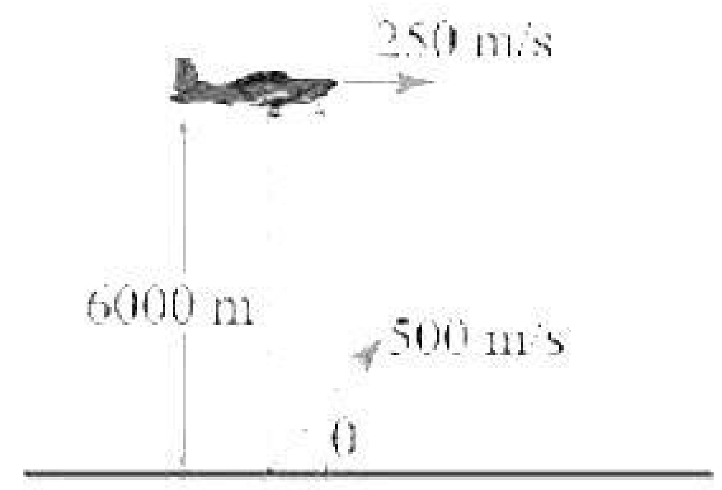
3. The angle between the two vectors

   $\vec{A}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{B}=3\hat{i}+4\hat{j}-5\hat{k}$ will be

   (a) zero (b) 45°

   (c) 90° (d) 180°

4. An aircraft moving with a speed of 250 m/s is at a height of 6000 m, just overhead of an anti aircraft gun. If the muzzle velocity is 500 m/s, the firing angle θ should be:

   (a) 30°

   (b) 45°

   (c) 60°

   (d) 90°.

5. At the height 80 m, an aeroplane is moving with a speed of 150 m/s. A bomb is dropped from it so as to hit a target. At what distance from the target should the bomb be dropped (given $g = 10\,m/s^2$)

   (a) 605.3m (b) 600m

   (c) 80m (d) 230m

6. An object is projected with a velocity of 20 m/s making an angle of 45° with horizontal. The equation for the trajectory is $h = Ax - Bx^2$ where h is height, x is horizontal distance, A and B are constants. The ratio A : B is ($g = 10\,ms^{-2}$)

   (a) 1 : 5 (b) 5 : 1

   (c) 1 : 40 (d) 40 : 1

7. A large number of bullets are fired in all directions with the same speed v from ground. What is the maximum area on the ground on which these bullets will spread?

   (a) $\dfrac{\pi v^2}{g}$ (b) $\dfrac{\pi v^4}{g^2}$

   (c) $\pi^2 \dfrac{v^2}{g^2}$ (d) $\dfrac{\pi^2 v^4}{g^2}$

8. A man can swim in still water with a speed of 2 m/s. If he wants to cross a river of water current speed $\sqrt{3}$ m/s along the shortest possible path, then in which direction should he swim?

(a) At an angle 120° to the water current.

(b) At an angle 150° to the water current.

(c) At an angle 90° to the water current.

(d) None of these

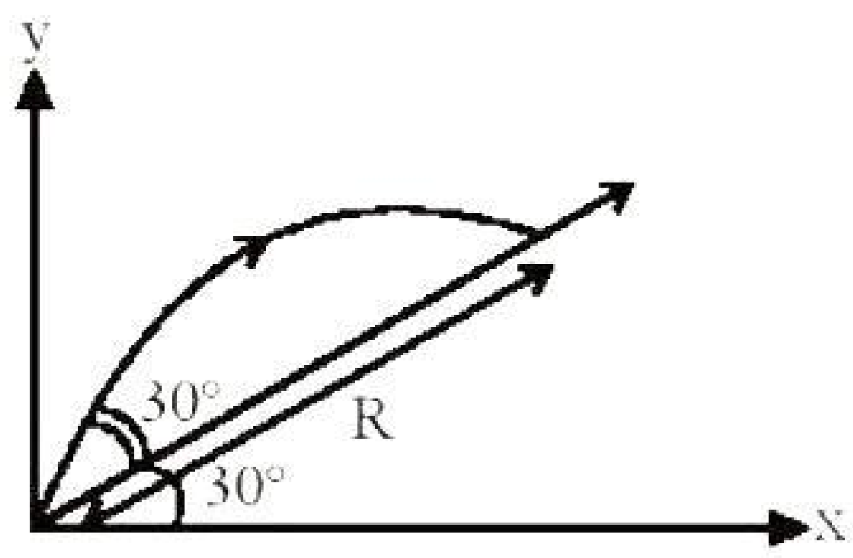
9. Initial velocity with which a body is projected is 10 m/sec and angle of projection is 60° with horizontal, find the range R

(a) $\frac{15\sqrt{3}\text{m}}{2}$

(b) $\frac{40}{3}$m

(c) $5\sqrt{3}$m

(d) $\frac{20}{3}$m



10. If a_r and a_t represent radial and tangential accelerations, the motion of particle will be uniformly circular, if

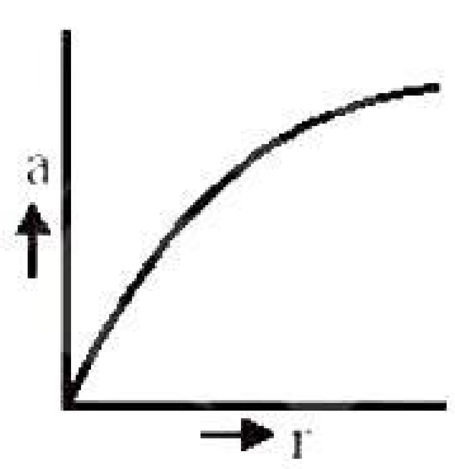
(a) $a_r = 0$ and $a_t = 0$

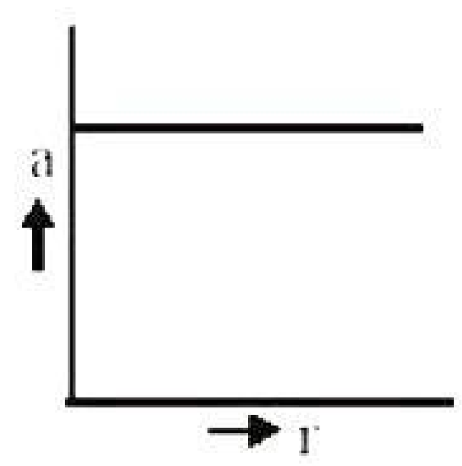
(b) $a_r = 0$ but $a_t \neq 0$

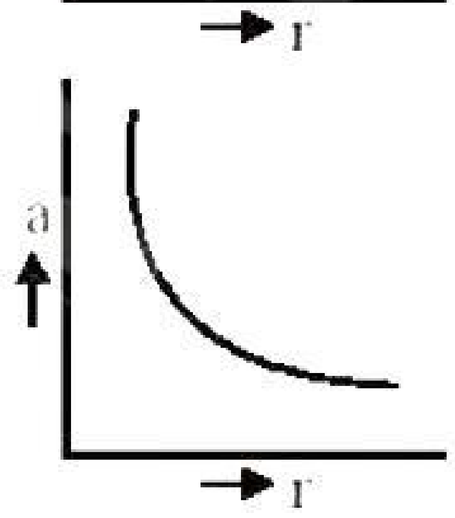
(c) $a_r \neq 0$ and $a_t = 0$

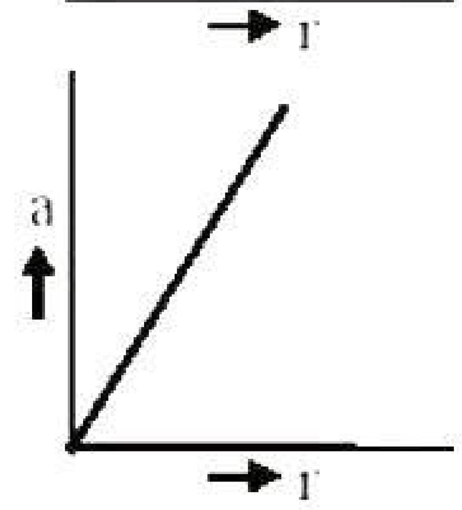
(d) $a_r \neq 0$ and $a_t \neq 0$

11. If a body moving in circular path maintains constant speed of 10 ms^{-1}, then which of the following correctly describes relation between magnitude of acceleration and radius ?

(a)


(b)


(c)


(d)


12. A projectile thrown with velocity v making angle θ with vertical gains maximum height 20m in the time for which the projectile remains in air, the time period is

(a) 15 s (b) 25 s

(c) 9 s (d) 4 s

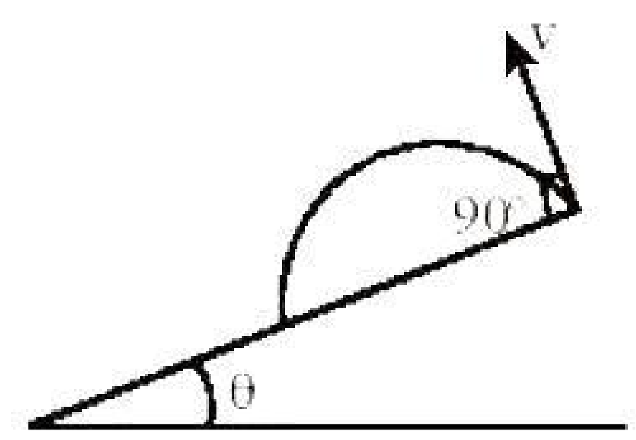
13. A projectile is fired with a velocity v at right angle to the slope which is inclined at an angle θ with the horizontal. The range of the projectile along the inclined plane is:

(a) $\frac{2v^2 \tan\theta}{g}$

(b) $\frac{v^2 \sec\theta}{g}0$

(c) $\frac{2v^2 \tan\theta \sec\theta}{g}$

(d) $\frac{v^2 \sin\theta}{g}$



14. A car is moving along a circular path of radius 500 m with a speed of 30 m/s. If at some instant, its speed increases at the rate of 2 m/s^2, than at that instant the magnitude of resultant acceleration will be

(a) 4.7 m/s^2 (b) 3.8 m/s^2

(c) 3 m/s^2 (d) 2.7 m/s^2

15. A projectile reaches its highest point when it has covered exactly one half of its horizontal range. The corresponding point on the horizontal component of velocity-time (i.e. v_x–t) graph is characterized by

(a) negative slope

(b) negative slope and negative curvature

(c) zero slope

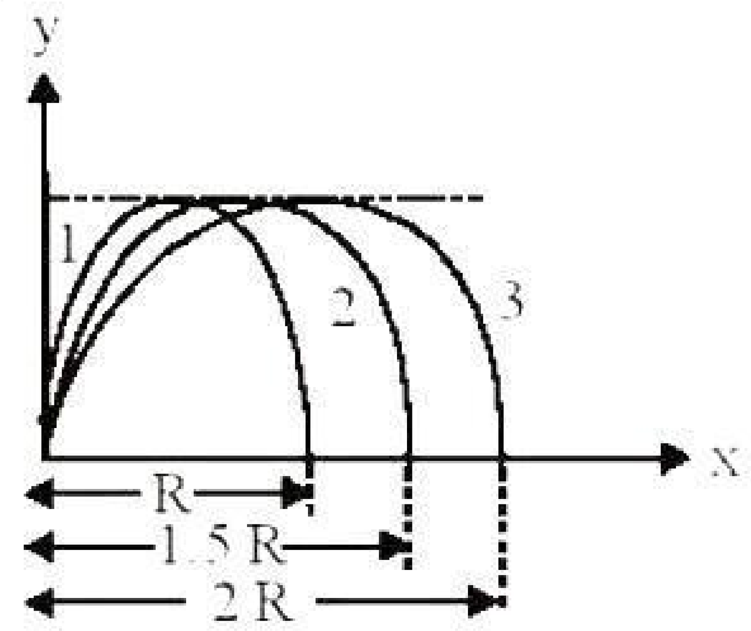
(d) positive slope

16. Two bodies are thrown up at angles of 45° and 60° respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is

(a) $\sqrt{\frac{2}{3}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$

17. Trajectories are shown in figure are for three kicked footballs, ignoring the effect of the air on the footballs. If T_1, T_2 and T_3 are their respective time of flights then:



(a) $T_1 > T_3$

(b) $T_1 < T_3$

(c) $T_2 = \frac{T_3}{2}$

(d) $T_1 = T_2 = T_3$

18. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is :

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

19. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of:

(a) 10^{-3} (b) 10^{-4}

(c) 10^{-2} (d) 10^{-1}

20. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is :

(a) $7\sqrt{2}$ units (b) 7 units

(c) 8.5 units (d) 10 units

21. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(a) $10\sqrt{3}$m/s (b) zero

(c) $10\sqrt{2}$m/s (d) 10 m/s

22. A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 ms^{-1}. Then the time after which its inclination with horizontal is 45°, is

(a) 15 s (b) 10.98 s

(c) 5.49 s (d) 2.745 s

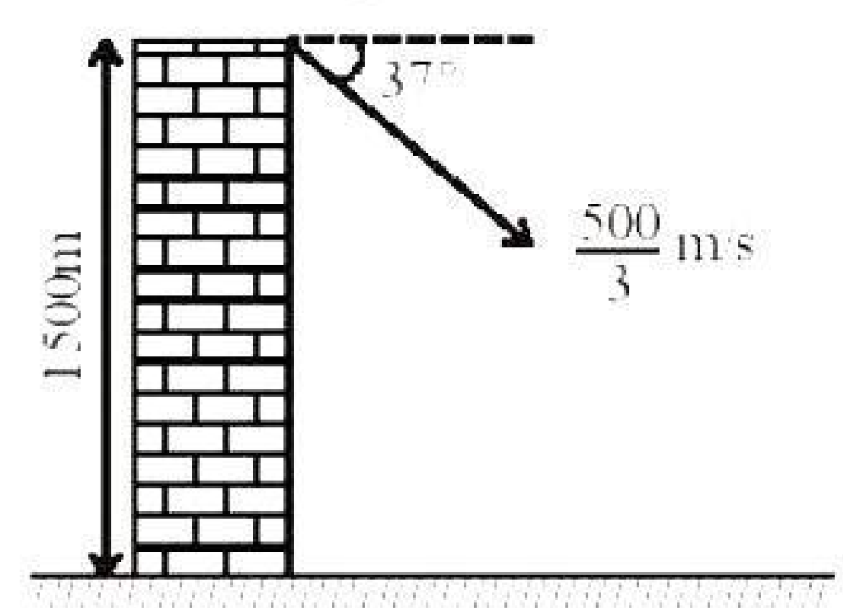
23. A boy playing on the roof of a 20 m high tower throws a ball with a speed of 20m/s at an angle of 45° with the horizontal. How far from the throwing point will the ball be at the height of 20 m from the ground ?

$[g = 10 \text{m}/\text{s}^2]$

(a) 20m (b) 4.33m

(c) 2.60m (d) 40m

24. A particle is projected from a tower as shown in figure, then the distance from the foot of the tower where it will strike the ground will be



(a) 4000/3 m

(b) 2000/m

(c) 1000/3 m

(d) 2500/3 m

25. An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acceleration of a point on the tip of the blade is

(a) 1600 ms^{-2} (b) 47.4 ms^{-2}

(c) 23.7 ms^{-2} (d) 50.55 ms^{-2}

26. A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is :

(a) 4.4m (b) 2.4m

(c) 3.6m (d) 1.6m

27. A particle is projected with a certain velocity at an angle α above the horizontal from the foot of an inclined plane of inclination 30°. If the particle strikes the plane normally then α is

(a) $30° + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) $30° + \tan^{-1}\frac{1}{2}$

(c) $30° + \tan^{-1}1$ (d) 60°

28. A ball is thrown from a point with a speed $'v_0'$ at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ?

(a) No (b) Yes, 30°

(c) Yes, 60° (d) Yes, 45°

29. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is :

(a) R/4g (b) R/g

(c) R/2g (d) 2R/g

30. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ?

(a) $R^2 = 4h_1h_2$ (b) $R^2 = 16h_1h_2$

(c) $R^2 = 2h_1h_2$ (d) $R^2 = h_1h_2$

ANSWER KEY

1	(b)	4	(c)	7	(b)	10	(c)	13	(c)	16	(c)	19	(a)	22	(c)	25	(b)	28	(c)
2	(a)	5	(a)	8	(b)	11	(c)	14	(d)	17	(d)	20	(a)	23	(d)	26	(b)	29	(d)
3	(c)	6	(d)	9	(d)	12	(d)	15	(c)	18	(b)	21	(d)	24	(a)	27	(a)	30	(b)

3 Motion in a Plane

1. **(b)** Unit vector along y axis = j, so the required vector

$= j - [(i - 3j + 2k) + (3i + 6j + 7k)]$

$= -4i - 2j - 9k$

2. **(a)** Note that the given angles of projection add upto 90°. So, the ratio of horizontal ranges is 1 : 1.

3. **(c)** $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$, $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{A} \cdot \vec{B} = (3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 5\hat{k})$

$|\vec{A}|\,|\vec{B}| \cos\theta = 9 + 16 - 25 = 0$

$|\vec{A}| \neq 0$, $|\vec{B}| \neq 0$, hence, $\cos\theta = 0 \Rightarrow \theta = 90°$

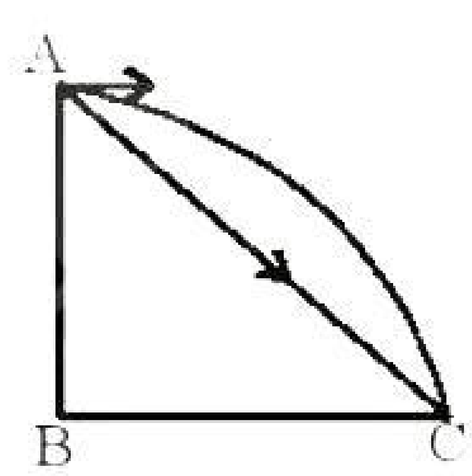
4. **(c)** $500\cos\theta = 250 \Rightarrow \cos\theta = \frac{1}{2}$

or $\theta = 60°$.

5. **(a)** The horizontal distance covered by bomb,

$BC = u_H \times \sqrt{\frac{2h}{g}}$

$= 150\sqrt{\frac{2 \times 80}{10}} = 600\text{m}$



∴ The distance of target from dropping point of bomb.

$AC = \sqrt{AB^2 + BC^2} = \sqrt{(80)^2 + (600)^2}$

$= 605.3\text{m}$

6. **(d)** Standard equation of projectile motion

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Comparing with given equation

$$A = \tan\theta \quad \text{and} \quad B = \frac{g}{2u^2\cos^2\theta}$$

$$\text{So } \frac{A}{B} = \frac{\tan\theta \times 2u^2\cos^2\theta}{g} = 40$$

$(\text{As } \theta = 45°, u = 20\text{ m/s}, g = 10\text{ m/s}^2)$

7. **(b)** Maximum possible horizontal range $= v^2/g$

Maximum possible area of the circle

$$= \pi\left(\frac{v^2}{g}\right)^2 = \frac{\pi v^4}{g^2}$$

8. **(b)**

9. **(d)** $t = \frac{2u\sin 30°}{g\cos 30°} = \frac{2(10)(1/2)}{10(\sqrt{3}/2)} = \frac{2}{\sqrt{3}}\text{sec}$

$R = 10\cos 30°\, t - \frac{1}{2} g\sin 30°\, t^2$

$$= \frac{10\sqrt{3}}{2}\left(\frac{2}{\sqrt{3}}\right) - \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{4}{3}$$

$$= 10 - \frac{10}{3} = \frac{20}{3}\text{m}$$

10. **(c)** When a particle moves on a circular path with a constant speed, then its motion is said to be a uniform circular motion in a plane. This motion has radial acceleration whose magnitude remains constant but whose direction changes continuously, So $a_r \neq 0$ and $a_t = 0$.

11. **(c)** Speed, v = constant (from question)

Centripetal acceleration, $a = \frac{V^2}{r}$

ra = constant

Hence graph (c) correctly describes relation between acceleration and radius.

12. **(d)**

13. **(c)** If t is the time of flight, then

$$0 = v\,t - \frac{1}{2}(g\cos\theta)t^2.$$

$$\therefore \quad t = \frac{2v}{g\cos\theta}$$

Range, $R = 0 + \frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}g\sin\theta\left(\frac{2v}{g\cos\theta}\right)^2$

$$= \frac{2v^2\tan\theta\sec\theta}{g}$$

14. **(d)** Centripetal acc,

$$a_c = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8\,m/s^2$$

Tangentical acc, $a_t = 2\ m/s^2$

$\therefore$ Resultant acc, $a = \sqrt{a_t^2 + a_c^2} = 2.7\,m/s^2$

15. **(c)**

16. **(c)** $H_{max} = \dfrac{u^2\sin^2 q}{2g}$

According to question,

$$\frac{u_1^2\sin^2 45^\circ}{2g} = \frac{u_2^2\sin^2 60^\circ}{2g}$$

$$\Rightarrow \frac{u_1^2}{u_2^2} = \frac{\sin^2 60^\circ}{\sin^2 45^\circ} \Rightarrow \frac{u_1}{u_2} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \sqrt{\frac{3}{2}}.$$

17. **(d)** As maximum height attained by each one is same, so u_y is also same. As

$$T = \frac{2u_y}{g},$$

So $T_1 = T_2 = T_3$.

18. **(b)** From equation, $\vec{r} = \hat{i} + 2\hat{j}$

$\Rightarrow x = t$ ...(i)

$y = 2t - \frac{1}{2}(10t^2)$ ...(ii)

From (i) and (ii), $y = 2x - 5x^2$

19. **(a)** Here, $R = 0.1$ m

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105\ \text{rad/s}$$

Acceleration of the tip of the clock second's hand,

$$a = \omega^2 R = (0.105)^2(0.1) = 0.0011$$

$$= 1.1 \times 10^{-3}\ m/s^2$$

Hence, average acceleration is of the order of 10^{-3}.

20. **(a)** Given $\vec{u} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, t=10s

From 1st equatoin of motion.

$$a = \frac{v - u}{t}$$

$\therefore v = at + u$

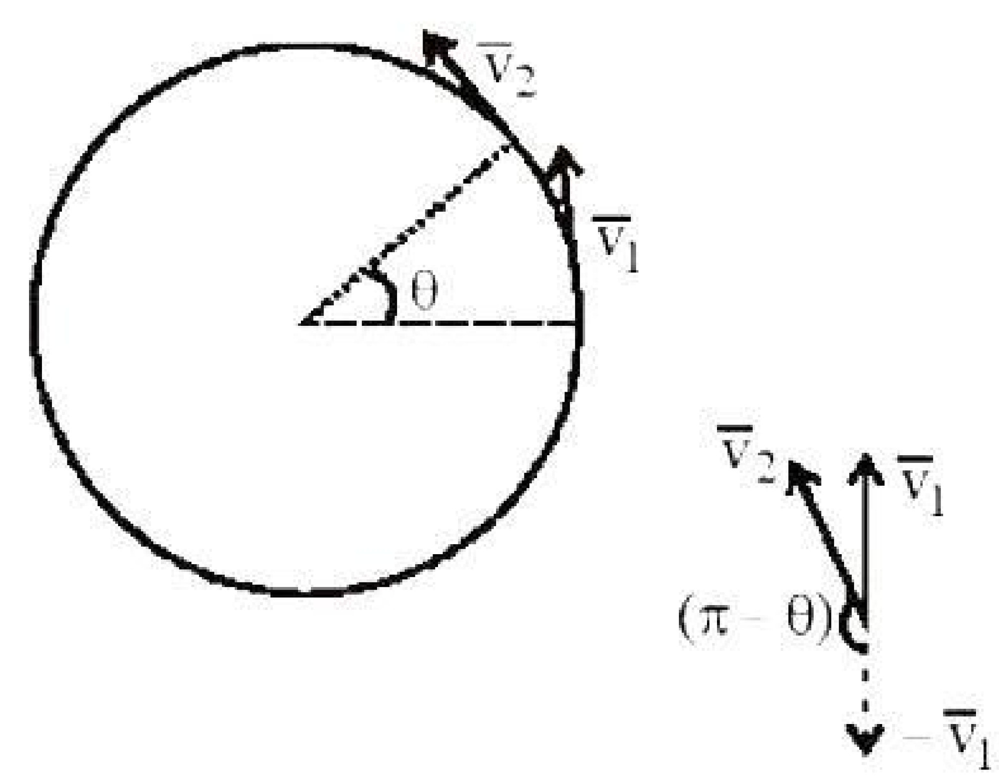
$\Rightarrow v = (0.4\hat{i} + 0.3\hat{j}) \times 10 + (3\hat{i} + 4\hat{j})$

$\Rightarrow \quad 4\hat{i} + 3\hat{j} + 3\hat{j} + 4\hat{j}$

$\Rightarrow \quad v = 7\hat{i} + 7\hat{j}$

$\Rightarrow \quad |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ unit.

21. **(d)**



Change in velocity,

$$|\Delta\vec{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\pi - \theta)}$$

$$= 2v\sin\frac{\theta}{2} \qquad (\because |\vec{v}_1| = |\vec{v}_2|) = v$$

$$= (2 \times 10) \times \sin(30^\circ) = 2 \times 10 \times \frac{1}{2}$$

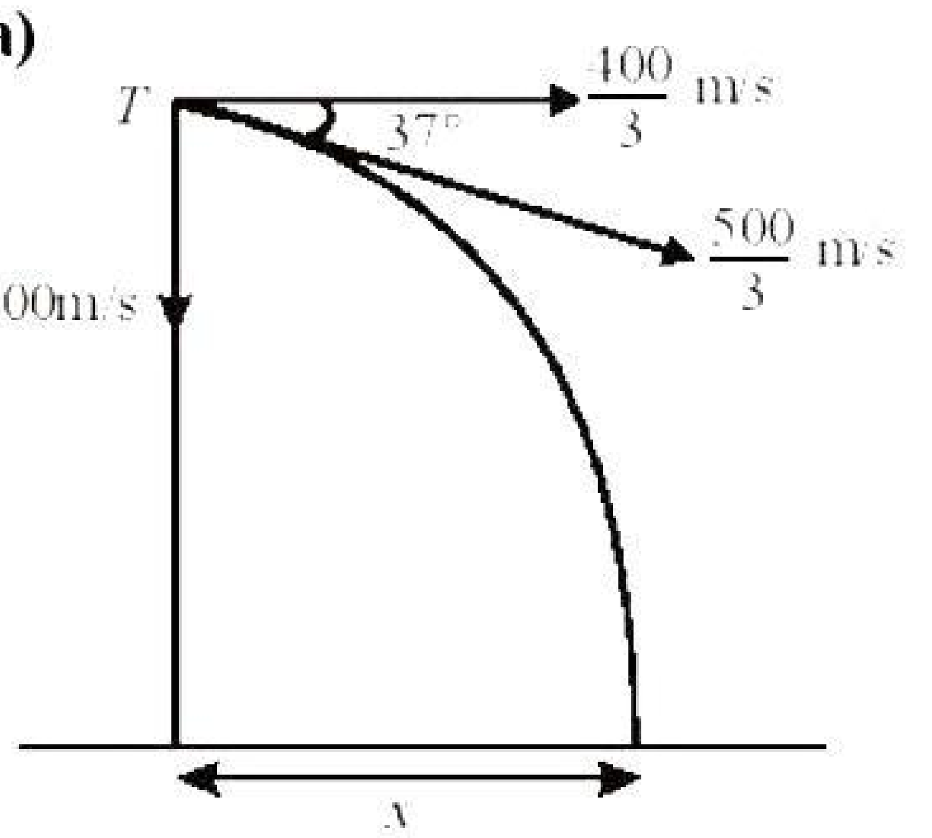
$$= 10\ m/s$$

22. **(c)**

23. **(d)** Tower height and final height is same so, required range is:

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \sin(2 \times 45°)}{10} = 40 \text{ m}$$

24. **(a)**

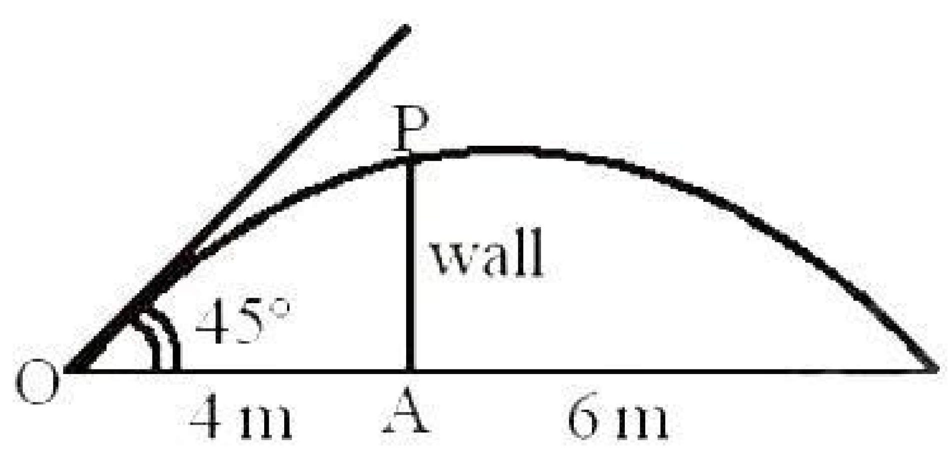


$$1500 = 100t + \frac{1}{2}10t^2 \; ; \; t = 10 \text{ sec.}$$

$$x = \frac{400}{3}t = \frac{4000}{3} \text{m}$$

25. **(b)**

26. **(b)**



As ball is projected at an angle 45° to the horizontal therefore Range = 4H

or $\quad 10 = 4\text{H} \Rightarrow \text{H} = \frac{10}{4} = 2.5 \text{m}$

$(\because \text{Range} = 4\text{ m} + 6\text{ m} = 10\text{m})$

Maximum height, $\text{H} = \frac{\text{u}^2 \sin^2\theta}{2\text{g}}$

$$\therefore \; \text{u}^2 = \frac{\text{H} \times 2\text{g}}{\sin^2\theta} = \frac{2.5 \times 2 \times 10}{\left(\frac{1}{\sqrt{2}}\right)^2} = 100$$

or, $\quad \text{u} = \sqrt{100} = 10 \text{ ms}^{-1}$

Height of wall PA

$$= \text{OA} \tan\theta - \frac{1}{2}\frac{\text{g(OA)}^2}{\text{u}^2 \cos^2\theta}$$

$$= 4 - \frac{1}{2} \times \frac{10 \times 16}{10 \times 10 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = 2.4 \text{m}$$

27. **(a)**

28. **(c)** Yes, Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man.

$$\frac{v_o}{2} = v_o \cos\theta$$

or $\quad \theta = 60°$

29. **(d)** R will be same for θ and $90° - \theta$.

Time of flights:

$$t_1 = \frac{2u\sin\theta}{g} \text{ and}$$

$$t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\text{Now, } t_1 t_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right)$$

$$= \frac{2}{g}\left(\frac{u^2 \sin 2\theta}{g}\right) = \frac{2R}{g}$$

30. **(b)** For same range, the angle of projections are:

θ and $90° - \theta$. So,

$$h_1 = \frac{u^2 \sin^2\theta}{2g} \text{ and}$$

$$h_2 = \frac{u^2 \sin^2(90° - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$$

$$\text{Also, } R = \frac{u^2 \sin 2\theta}{g}$$

$$h_1 h_2 = \frac{u^2 \sin^2\theta}{2g} \times \frac{u^2 \cos^2\theta}{2g}$$

$$= \frac{u^2}{16}\frac{u^2(2\sin\theta\cos\theta)^2}{g^2}$$

$$= \frac{R^2}{16}$$

or $R^2 = 16\, h_1 h_2$