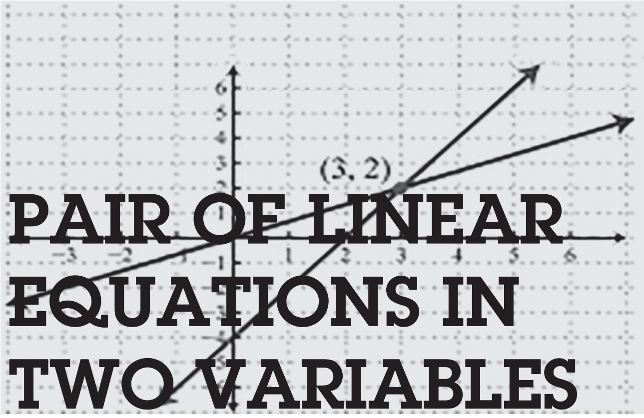
CHAPTER

3 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES



Chapter Objectives

This chapter will help you to understand :

- *Introduction, Pair of linear equations in two variables.*
- *Graphical method of solution of a pair of linear equations.*
- *Algebraic methods of solving a pair of linear equations : Elimination method, Cross-multiplication method.*
- *Equations reducible to a pair of linear equations in two variables.*

Quick Review

❖ Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is :

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$.

❖ **Graphical Method :** The graph of a pair of linear equations in two variables is represented by two lines.

- If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is consistent.
- If the lines coincide, then there are infinitely many solutions, each point on the line being a solution. In this case, the pair of equations is dependent (consistent).
- If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is inconsistent.

❖ There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.

TIPS...

- *Practice some questions by using substitution and elimination methods.*
- *Learn how to implement cross-multiplication method.*

TRICKS...

- *Never confuse with the pair of equations whether it is consistent or inconsistent.*
- *Make an effort to draw the graph neatly and accurately.*

Multiple Choice Questions/True or False

(1 mark each)

Q. 1. Graphically, the pair of equations

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

represents two lines which are

(a) intersecting at exactly one point

(b) intersecting at exactly two points

(c) coincident

(d) parallel [NCERT Exemp. Ex. 3.1, Q. 1, Page 18]

Sol. Correct option : (d)

Explanation :

Here, $\frac{a_1}{a_2} = \frac{6}{2} = 3, \frac{b_1}{b_2} = \frac{-3}{-1} = 3, \frac{c_1}{c_2} = \frac{10}{9}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the system of linear equations is inconsistent (no solution) and graph will be a pair of parallel lines.

Q. 2. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have

(a) a unique solution (b) exactly two solutions

(c) infinitely many solutions (d) no solution

[NCERT Exemp. Ex. 3.1, Q. 2, Page 18]

Sol. Correct option : (d)

Explanation :

Here, $\frac{a_1}{a_2} = \frac{1}{-3} = -\frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the system of linear equations has no solution.

Q. 3. If a pair of linear equations is consistent, then the lines will be

(a) parallel

(b) always coincident

(c) intersecting or coincident

(d) always intersecting

[NCERT Exemp. Ex. 3.1, Q. 3, Page 18]

Sol. Correct option : (c)

Explanation :

Condition for consistency :

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ has unique solution (consistent), *i.e.*, intersecting at one point

or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(Consistent lines, coincident or dependent)

Q. 4. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(a) parallel (b) intersecting at (b, a)

(c) coincident (d) intersecting at (a, b)

[NCERT Exemp. Ex. 3.1, Q. 5, Page 18]

Sol. Correct option : (d)

Explanation : ($x = a$) is the equation of a straight line parallel to the y-axis at a distance 'a' from it. Again, $y = b$ is the equation of a straight line parallel to the x-axis at a distance 'b' from it. So, the pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b).

Q. 5. The pair of equations $y = 0$ and $y = -7$ has

(a) one solution (b) two solutions

(c) infinitely many solutions (d) no solution

[NCERT Exemp. Ex. 3.1, Q. 4, Page 18]

Sol. Correct option : (d)

Explanation : We know that equation of the form $y = a$ is a line parallel to x-axis at a distance 'a' from it. $y = 0$ is the equation of the x-axis and $y = -7$ is the equation of the line parallel to the x-axis. So, these two equations represent two parallel lines. Therefore, there is no solution.

Q. 6. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines ?

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$

(c) 2 (d) –2

[NCERT Exemp. Ex. 3.1, Q. 6, Page 18]

Sol. Correct option : (c)

Explanation : $3x - y = -18$ ...(i)

$6x - ky = -16$ ...(ii)

For coincident lines,

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{6} = \frac{-1}{-k} = \frac{-8}{-16}$

$\Rightarrow \frac{1}{2} = \frac{1}{k} = \frac{1}{2}$

So, $k = 2$.

Q. 7. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(a) $\frac{-5}{4}$ (b) $\frac{2}{5}$

(c) $\frac{15}{4}$ (d) $\frac{3}{2}$

[NCERT Exemp. Ex. 3.1, Q. 7, Page 19]

Sol. Correct option : (c)

Explanation :

For parallel lines (or no solution)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$

$\Rightarrow 4k = 15$

$\Rightarrow \quad k = \frac{15}{4}$

Q. 8. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is

(a) 3 (b) –3

(c) –12 (d) no value

[NCERT Exemp. Ex. 3.1, Q. 8, Page 19]

Sol. Correct option : (d)

Explanation : For infinitely many solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{c}{6} = \frac{-1}{-2} = \frac{2}{3}$

$2c = 6 \Rightarrow c = 3$

$3c = 12 \Rightarrow c = 4$

As from the ratios, values of c are not common. So, there is no common value of c for which lines have infinitely many solutions.

Q. 9. One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be

(a) $10x + 14y + 4 = 0$

(b) $-10x - 14y + 4 = 0$

(c) $-10x + 14y + 4 = 0$

(d) $10x - 14y = -4$

[NCERT Exemp. Ex. 3.1, Q. 9, Page 19]

Sol. Correct option : (d)

Explanation :

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{k}$ ...(i)

Given equation of line is, $-5x + 7y - 2 = 0$
Here, $a_1 = -5, b_1 = 7, c_1 = -2$
From Eq. (i), $-\frac{5}{a_2} = \frac{7}{b_2} = -\frac{2}{c_2} = \frac{1}{k}$
$\Rightarrow a_2 = -5k, b_2 = 7k, c_2 = -2k$
where, k is any arbitrary constant.
Putting $k = 2$, then $a_2 = -10, b_2 = 14$ and $c_2 = -4$
$\therefore$ The required equation of line becomes,
$a_2x + b_2y + c_2 = 0$
$\Rightarrow -10x + 14y - 4 = 0$
$\Rightarrow \quad 10x - 14y + 4 = 0$

Q. 10. A pair of linear equations which has a unique solution $x = 2, y = -3$ is

(a) $x + y = -1$, $2x - 3y = -5$ **(b)** $2x + 5y = -11$, $4x + 10y = -22$
(c) $2x - y = 1$, $3x + 2y = 0$ **(d)** $x - 4y - 14 = 0$, $5x - y - 13 = 0$

[NCERT Exemp. Ex. 3.1, Q. 10, Page 19]

Sol. Correct options : (b) and (d)
Explanation :
(b) $2x + 5y = -11$ and $4x + 10y = -22$
Put $x = 2$ and $y = -3$ in both the equations,
$LHS = 2x + 5y \Rightarrow 2 \times 2 + (-3)$
$\Rightarrow 4 - 15 = -11 = \text{RHS}$
$LHS = 4x + 10y \Rightarrow 4(2) + 10(-3)$
$\Rightarrow 8 - 30 = -22 = RHS$
(d) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$
$x - 4y = 14$ and $5x - y = 13$
Put $x = 2$ and $y = -3$ in both the equations,
$LHS = x - 4y \Rightarrow 2 - 4(-3) \Rightarrow 2 + 12 = 14 = RHS$
$LHS = 5x - y \Rightarrow 5(2) - (-3) \Rightarrow 10 + 3 = 13 = RHS$

Q. 11. If $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

(a) 3 and 5 **(b)** 5 and 3
(c) 3 and 1 **(d)** -1 and 3

[NCERT Exemp. Ex. 3.1, Q. 11, Page 19]

Sol. Correct option : (c)
Explanation : If (a, b) is the solution of the given equations, then it must satisfy the given equations, so,
$a - b = 2$ ...(i)
$30\ a + b = 4$ ...(ii)
$\Rightarrow \quad 2a = 6$ [Adding (i) and (ii)]
$\Rightarrow \quad a = 3$
Now, $3 + b = 4$ [From (ii)]
$\Rightarrow \quad b = 1$
So, $\quad (a, b) = (3, 1)$.

Q. 12. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 **(b)** 35 and 20
(c) 15 and 35 **(d)** 25 and 25

[NCERT Exemp. Ex. 3.1, Q. 12, Page 19]

Sol. Correct option : (d)
Explanation : Let the number of ₹ 1 coins $= x$ and the number of ₹ 2 coins $= y$
So, according to the question,
$x + y = 50$ ...(i)
$x + 2y = 75$ ...(ii)
Subtracting equation (i) from (ii)
$y = 25$
Substituting value of y in (i)
$x = 25$
So, $y = 25$ and $x = 25$

Q. 13. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively

(a) 4 and 24 **(b)** 5 and 30
(c) 6 and 36 **(d)** 3 and 24

[NCERT Exemp. Ex. 3.1, Q. 13, Page 19]

Sol. Correct option : (c)
Explanation : Let the present age of father be x years and the present age of son be y years.
$\therefore$ According to the question,
$x = 6y$ ...(i)
Age of the father after four years $= (x + 4)$ years
Age of son after four years $= (y + 4)$ years
Now, according to the question,
$x + 4 = 4(y + 4)$ ...(ii)
$\Rightarrow \quad x + 4 = 4y + 16$
$\Rightarrow 6y - 4y = 16 - 4$ [From (i), $x = 6y$]
$\Rightarrow \quad 2y = 12$
$\Rightarrow \quad y = 6$
$\therefore \quad x = 6 \times 6 = 36$ years
and $\quad y = 6$ years
So, the present ages of the son and the father are 6 years and 36 years, respectively.

Q. 14. For all real values of c, the pair of equations
$x - 2y = 8$
$5x - 10y = c$
have a unique solution. Justify whether it is true or false. [NCERT Exemp. Ex. 3.2, Q. 5, Page 22]

Sol. False,
$x - 2y = 8$ ...(i)
$5x - 10 = c$ ...(ii)
$\therefore \frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$ and $\frac{c_1}{c_2} = \frac{-8}{-c} = \frac{8}{c}$
As $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, so system of linear equations can never have unique solution.

Very Short Answer Type Questions **(1 or 2 marks each)**

Q. 1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$, $3x - 9y - 2 = 0$ **(ii)** $2x + y = 5$, $3x + 2y = 8$
(iii) $3x - 5y = 20$, $6x - 10y = 40$ **(iv)** $x - 3y - 7 = 0$, $3x - 3y - 15 = 0$

[NCERT Ex. 3.5, Q. 1, Page 62]

Sol. **(i)** $x-3y-3=0$

$3x-9y-2=0$

$$\frac{a_1}{a_2}=\frac{1}{3},\ \frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\ \frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations. **[2]**

(ii) $2x+y=5$

$3x+2y=8$

$$\frac{a_1}{a_2}=\frac{2}{3},\ \frac{b_1}{b_2}=\frac{1}{2},\ \frac{c_1}{c_2}=\frac{-5}{-8}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method, we have

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\frac{x}{-8-(-10)}=\frac{y}{-15+16}=\frac{1}{4-3}$$

$$\frac{x}{2}=\frac{y}{1}=1$$

$$\frac{x}{2}=1,\ \frac{y}{1}=1$$

$$x=2,\ y=1$$

$\therefore$ $x=2,\ y=1$ **[2]**

(iii) $3x-5y=20$

$6x-10y=40$

$$\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{-5}{-10}=\frac{1}{2},\ \frac{c_1}{c_2}=\frac{-20}{-40}=\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Therefore, the given sets of lines will be overlapping each other, *i.e.*, the lines will be coincident to each other and thus, there are infinite solutions possible for these equations. **[2]**

(iv) $x-3y-7=0$

$3x-3y-15=0$

$$\frac{a_1}{a_2}=\frac{1}{3},\ \frac{b_1}{b_2}=\frac{-3}{-3}=1,\ \frac{c_1}{c_2}=\frac{-7}{-15}=\frac{7}{15}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

By cross-multiplication, we have

$$\frac{x}{45-(21)}=\frac{y}{-21-(-15)}=\frac{1}{-3-(-9)}$$

$$\frac{x}{24}=\frac{y}{-6}=\frac{1}{6}$$

$$\frac{x}{24}=\frac{1}{6}\text{ and }\frac{y}{-6}=\frac{1}{6}$$

$$x=4\text{ and }y=-1$$

$\therefore x=4,\ y=-1$ **[2]**

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

Q. 2. The line represented by $x = 7$ is parallel to the x-axis. Justify whether the statement is true or not.
[NCERT Exemp. Ex. 3.2, Q. 6, Page 22]

Sol. The given statement is not true as the line represented by $x = 7$ is of the from $x = a$. The graph of the equation is a line parallel to the y-axis. **[2]**

Q. 3. For the pair of equations
$\lambda x + 3y = -7$
$2x + 6y = 14$
to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.
[NCERT Exemp. Ex. 3.2, Q. 4, Page 21]

Sol. $\lambda x + 3y + 7 = 0$ **(i)**

$2x + 6y - 14 = 0$

For infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{a_1}{a_2}=\frac{\lambda}{2},\ \frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2},\ \frac{c_1}{c_2}=\frac{7}{-14}=\frac{1}{-2}$$

So, $\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, for any value of λ.

Hence, the given statement is not true. **[2]**

Q. 4. The cost of 4 pens and 4 pencil boxes is ₹ 100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.
[NCERT Exemp. Ex. 3.4, Q. 4, Page 33]

Sol. Let the cost of a pen = ₹ x

Let the cost of a pencil box = ₹ y

$\therefore$ The cost of 4 pens and 4 pencil boxes = ₹ 1,00 [Given]

$4x + 4y = 100$ ...(i)

$x + y = 25$ [By dividing (i) by 4] ...(ii)

According to the second condition, we have

$3x = y + 15$

$3x - y = 15$ ...(iii)

By adding (ii) and (iii),

$$\begin{array}{l} x+y=25 \\ 3x-y=15 \\ \hline 4x\quad\ =40 \end{array}$$

$$\Rightarrow\quad x=\frac{40}{4}=10$$

Now, $x + y = 25$ [From (ii)]

$\Rightarrow 10 + y = 25$ [Putting $x = 10$]

$\Rightarrow y = 25 - 10 =$ ₹15

So, $x =$ ₹ 10 and $y =$ ₹ 15

Hence, the cost of a pen and a pencil box are ₹ 10 and ₹ 15, respectively. **[2]**

Q. 5. Are the following pair of linear equations consistent? Justify your answer.

(i) $-3x-4y=12$

$4y+3x=12$

(ii) $\frac{3}{5}x-y=\frac{1}{2}$

$\frac{1}{5}x-3y=\frac{1}{6}$

(iii) $2ax+by=a$

$4ax+2by-2a=0; a, b \neq 0$

(iv) $x+3y=11$

$2(2x+6y)=22$ **[NCERT Exemp. Ex. 3.2, Q. 3, Page 21]**

Sol. Conditions for pair of linear equations are consistent.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

and $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ [Infinitely many solutions]

(i) **No**, the given pair of linear equations

$-3x-4y=12$ and $3x+4y=12$

Here, $a_1=-3, b_1=-4, c_1=-12;$

$a_2=3, b_2=4, c_2=-12$

Now, $\frac{a_1}{a_2}=-\frac{3}{3}=-1, \frac{b_1}{b_2}=-\frac{4}{4}=-1, \frac{c_1}{c_2}=\frac{-12}{-12}=1$

$\because \quad \frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of linear equations has no solution, *i.e.*, inconsistent. **[2]**

(ii) **Yes**, the given pair of linear equations

$\frac{3}{5}x-y=\frac{1}{2}$ and $\frac{1}{5}x-3y=\frac{1}{6}$

Here, $a_1=\frac{3}{5}, b_1=-1, c_1=-\frac{1}{2}$

and $a_2=\frac{1}{5}, b_2=-3, c_2=-\frac{1}{6}$

Now, $\frac{a_1}{a_2}=\frac{3}{1}, \frac{b_1}{b_2}=\frac{-1}{-3}=\frac{1}{3}, \frac{c_1}{c_2}=\frac{3}{1}$

$\left[\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right]$

Hence, the given pair of linear equations has unique solution, *i.e.*, consistent. **[2]**

(iii) *Yes*, the given pair of linear equations

$2ax+by-a=0$

and $4ax+2by-2a=0; a, b \neq 0$

Here, $a_1=2a, b_1=b, c_1=-a;$

$a_2=4a, b_2=2b, c_2=-2a$

Now, $\frac{a_1}{a_2}=\frac{2a}{4a}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{b}{2b}=\frac{1}{2}, \frac{c_1}{c_2}=\frac{-a}{-2a}=\frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{2}$

Hence, the given pair of linear equations has infinitely many solutions, *i.e.*, consistent or dependent. **[2]**

(iv) **No**, the given pair of linear equations

$x+3y=11$ and $2x+6y=11$

Here, $a_1=1, b_1=3, c_1=-11$ **(i)**

$a_2=2, b_2=6, c_2=-11$

Now, $\frac{a_1}{a_2}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}, \frac{c_1}{c_2}=\frac{-11}{-11}=1$

$\therefore \quad \frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of linear equations have no solution, *i.e.*, inconsistent. **[2]**

Q. 6. For which value(s) of k will the pair of equations

$kx+3y=k-3$

$12x+ky=k$

have no solution?

[NCERT Exemp. Ex. 3.3, Q. 2, Page 25]

Sol. For no solution of the pair of linear equations,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k}{12}=\frac{3}{k} \neq \frac{-(k-3)}{-k}$$

Taking first two parts, we get

$$\Rightarrow \quad \frac{k}{12}=\frac{3}{k}$$

$$\Rightarrow \quad k^2=36$$

$$\Rightarrow \quad k=\pm 6$$

Taking last two parts, we get

$$\frac{3}{k} \neq \frac{k-3}{k}$$

$$\Rightarrow \quad 3k \neq k(k-3)$$

$$\Rightarrow \quad 3k-k(k-3) \neq 0$$

$$\Rightarrow \quad k(3-k+3) \neq 0$$

$$\Rightarrow \quad k(6-k) \neq 0$$

$$\Rightarrow \quad k \neq 0 \text{ and } k \neq 6$$

[2]

Hence the value of $k=\pm 6$

Q. 7. For which values of a and b, will the following pair of linear equations have infinitely many solutions?

$x+2y=1$

$(a-b)x+(a+b)y=a+b-2$

[NCERT Exemp. Ex. 3.3, Q. 3, Page 25]

Sol. $a_1=1, b_1=2$ and $c_1=-1$ [From Eq. (i)]

$a_2=(a-b), b_2=(a+b)$ [From Eq. (ii)]

and $c_2=-(a+b-2)$

For infinitely many solutions of the pairs of linear equations,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{a-b}=\frac{2}{a+b}=\frac{-1}{-(a+b-2)}$$

Taking first two parts,

$$\frac{1}{a-b}=\frac{2}{a+b}$$

$$\Rightarrow \quad a+b=2a-2b$$

$$\Rightarrow 2a-a=2b+b$$

$\Rightarrow \quad a = 3b$ ...(iii)

Taking last two parts,

$$\frac{2}{a+b} = \frac{1}{(a+b-2)}$$

$\Rightarrow 2a + 2b - 4 = a + b$

$\Rightarrow \quad a + b = 4$ ...(iv)

Now, put the value of a from Eq. (iii) in Eq. (iv), we get

$3b + b = 4$

$\Rightarrow \quad 4b = 4$

$\Rightarrow \quad b = 1$

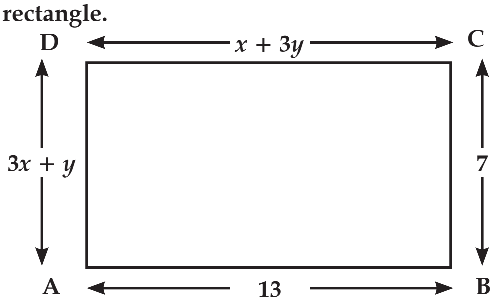
Put the value of b in Eq. (iii), we get

$a = 3 \times 1$

$\Rightarrow \quad a = 3$

So, the values $(a, b) = (3, 1)$ satisfies all the parts. Hence, required values of a and b are 3 and 1, respectively, for which the given pair of linear equations has infinitely many solutions. **[2]**

Q. 8. Find the values of x and y in the following rectangle.



[NCERT Exemp. Ex. 3.3, Q. 8, Page 26]

Sol. By property of rectangle,

Lengths are equal, *i.e.*, $CD = AB$

$\Rightarrow \quad x + 3y = 13$ ...(i)

Breadth are equal, *i.e.*, $AD = BC$

$\Rightarrow \quad 3x + y = 7$ ...(ii)

On multiplying Eq. (ii) by 3 and then subtracting Eq. (i), we get

$$\begin{array}{r} 9x + 3y = 21 \\ \underset{-}{x} + \underset{-}{3y} = \underset{-}{13} \\ \hline 8x \qquad = 8 \\ x = 1 \end{array}$$

On putting $x = 1$ in Eq. (i), we get

$3y = 12 \Rightarrow y = 4$

Hence, the required values of x and y are 1 and 4, respectively. **[2]**

Q. 9. Write a pair of linear equations which has the unique solution $x = -1$, $y = 3$. How many such pairs can you write?

[NCERT Exemp. Ex. 3.3, Q. 6, Page 26]

Sol. For unique solutions, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Let the equations are,

$a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$

Since, $x = -1$ and $y = 3$ is the unique solution of these two equations, then

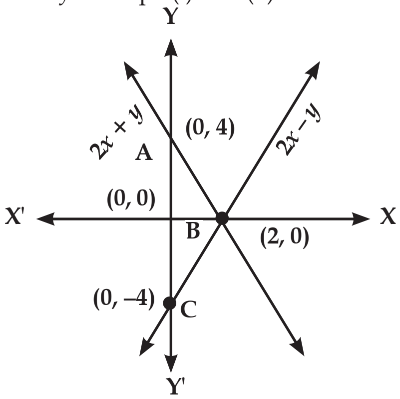
$a_1(-1) + b_1(3) + c_1 = 0$

$\Rightarrow \quad -a_1 + 3b_1 + c_1 = 0$ ...(i)

And $a_2(-1) + b_2(3) + c_2 = 0$

$\Rightarrow \quad -a_2 + 3b_2 + c_2 = 0$ ...(ii)

So, the different values of a_1, a_2, b_1, b_2, c_1 and c_2 satisfy the Eqn. (i) and (ii).



Hence, infinitely many pairs of linear equation are possible. **[2]**

Q. 10. If $2x + y = 23$ and $4x - y = 19$, find the values of $5y - 2x$ and $\frac{y}{x} - 2$.

[NCERT Exemp. Ex. 3.3, Q. 7, Page 26]

Sol. Given equations are :

$2x + y = 23$ ...(i)

and $4x - y = 19$ ...(ii)

On adding both equations, we get

$6x = 42 \Rightarrow x = 7$

Put the value of x in Eq. (i), we get

$2(7) + y = 23$

$\Rightarrow 14 + y = 23$

$\Rightarrow \quad y = 23 - 14$

$\Rightarrow \quad y = 9$

We have,

$5y - 2x = 5 \times 9 - 2 \times 7 = 45 - 14 = 31$

and $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9-14}{7} = -\frac{5}{7}$

Hence, the values of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$ are 31 and $\frac{-5}{7}$, respectively. **[2]**

Q. 11. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1,500 km away in time, it had to increase its speed by 100 km/hr from the usual speed. Find its usual speed. [CBSE Board, All India Region, 2018]

Sol. Let the usual speed of the plane be x km/hr.

Increased speed of the plane $= (x + 250)$ km/hr

Time taken to reach the destination at usual speed,

$t_1 = \frac{1500}{x}$ hr

Time taken to reach the destination at increased speed,

$t_2 = \dfrac{1500}{x+250}$ hr

Given, $t_1 - t_2 = 30$ min

$\therefore \dfrac{1,500}{x} - \dfrac{1,500}{x+250} = \dfrac{30}{60}$hr

$\Rightarrow \dfrac{1,500x + 3,75,000 - 1,500x}{x(x+250)} = \dfrac{1}{2}$

$\Rightarrow \quad x^2 + 250x = 7,50,000$

$\Rightarrow \quad x^2 + 250x - 7,50,000 = 0$

$\Rightarrow x^2 + 1,000x - 750x - 7,50,000 = 0$

$\Rightarrow \quad x(x+1,000) - 750(x+1,000) = 0$

$\Rightarrow \quad (x+1,000)(x-750) = 0$

$\Rightarrow \quad x+1,000 = 0$ or $x - 750 = 0$

$\Rightarrow \quad x = -1,000$ or $x = 750$

$\therefore x = 750$ km/hr $\quad$ ($\because$ Speed cannot be negative) **[2]**

Q. 12. A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km to go upstream than to return downstream to the same spot. Find the speed of the stream.

[CBSE Board, All India Region, 2018]

Sol. Let the speed of the stream x km/h.

Speed of the boat upstream

= Speed of boat in still water – Speed of the stream

$\therefore$ Speed of the boat upstream = $(18 - x)$ km/h

Speed of the boat downstream

= Speed of boat in still water + Speed of the stream

$\therefore$ Speed of the boat downstream = $(18 + x)$ km/h

Time of upstream journey

= Time for downstream journey + 1 h.

$$\therefore \frac{\text{Distance coverd upstream}}{\text{Speed of the boat upstream}} = \frac{\text{Distance coverd downstream}}{\text{Speed of the boat downstream}} + 1 \text{ h}$$

$\Rightarrow \dfrac{24 \text{ km}}{(18-x) \text{ km/hr}} = \dfrac{24 \text{ km}}{(18+x) \text{ km/hr}} + 1$ hr

$\Rightarrow \quad \dfrac{24}{18-x} - \dfrac{24}{18+x} = 1$

$\Rightarrow \dfrac{432 + 24x - 432 + 24x}{(18-x)(18+x)} = 1$

$\Rightarrow \quad x^2 + 48x - 324 = 0$

$\Rightarrow \quad x^2 + 54x - 6x - 324 = 0$

$\Rightarrow x(x+54) - 6(x+54) = 0$

$\Rightarrow \quad (x+54)(x-6) = 0$

$\Rightarrow \quad x + 54 = 0$ or $x - 6 = 0$

$\Rightarrow \quad x = -54$ or $x = 6$

x will never be negative

Hence, x = 54 will be rejected

Hence, speed of stream x = 6 km/hr **[2]**

Q. 13. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not.

[NCERT Exemp. Ex. 3.3, Q. 5, Page 26]

Sol. $x - 3y - 2 = 0$ ...(i)

and $-2x + 6y - 5 = 0$ ...(ii)

On comparing both the equations with $ax + by + c = 0$, we get

$a_1 = 1,\ b_1 = -3$

and $c_1 = -2$ [From Eq. (i)]

$a_2 = -2,\ b_2 = 6$

and $c_2 = -5$ [From Eq. (ii)]

Here, $\dfrac{a_1}{a_2} = \dfrac{1}{-2}$

$\dfrac{b_1}{b_2} = \dfrac{-3}{6} = -\dfrac{1}{2}$ and $\dfrac{c_1}{c_2} = \dfrac{-2}{-5} = \dfrac{2}{5}$

i.e., $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ [Parallel lines]

Hence, two straight paths represented by the given equations never cross each other because they are parallel to each other. **[2]**

Q. 14. A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

[CBSE Board, Foreign, 2016]

Sol. Let the digit at tens place be x and the digit at ones place be y.

Required number = $10x + y$

According to question,

$10x + y = 4(x + y)$

$10x + y = 4x + 4y$

$6x = 3y$

$y = 2x$ ...(i)

$10x + y = 3xy$

$10x + 2x = 3x(2x)$ [From (i)]

$12x = 6x^2$

$x = 2$

Putting the value of x in equation (i)

$y = 2x = 4$

Hence the number = 10(2)+4 = 20+4 = 24 **[2]**

Q. 15. Solve the following pair of equations :

(i) $x + y = 3.3$

$\dfrac{0.6}{3x - 2y} = -1, \quad 3x - 2y \neq 0$

(ii) $\dfrac{x}{3} + \dfrac{y}{4} = 4$

$\dfrac{5x}{6} - \dfrac{y}{8} = 4$

(iii) $4x + \dfrac{6}{y} = 15$

$6x - \dfrac{8}{y} = 14, y \neq 0$

(iv) $\dfrac{1}{2x} - \dfrac{1}{y} = -1$

$\dfrac{1}{x} + \dfrac{1}{2y} = 8, \quad x, y \neq 0$

(v) $43x + 67y = -24$

$67x + 43y = 24$

(vi) $\dfrac{x}{a}+\dfrac{y}{b}=a+b$

$\dfrac{x}{a^2}+\dfrac{y}{b^2}=2, \quad a,b\neq 0$

(vii) $\dfrac{2xy}{x+y}=\dfrac{3}{2}$

$\dfrac{xy}{2x-y}=\dfrac{-3}{10}, \quad x+y\neq 0, 2x-y\neq 0$

[NCERT Exemp. Ex. 3.3, Q. 9, Page 26]

Sol. **(i)** Given pair of linear equations is :

$x+y=3.3$ ...(i)

and $\dfrac{0.6}{3x-2y}=-1$

$\Rightarrow 0.6=-3x+2y$

$\Rightarrow 3x-2y=-0.6$ ...(ii)

Now, multiplying Eq. (i) by 2 and then adding with Eq. (ii), we get

$\Rightarrow \quad 2x+2y=6.6$

$\Rightarrow \quad 3x-2y=-0.6$

$$5x=6 \Rightarrow x=\frac{6}{5}=1.2$$

Now, put the value of x in Eq. (i), we get

$1.2+y=3.3$

$\Rightarrow \quad y=3.3-1.2$

$\Rightarrow \quad y=2.1$

Hence, the required values of x and y are 1.2 and 2.1, respectively. **[2]**

(ii) Given, pair of linear equations is :

$\dfrac{x}{3}+\dfrac{y}{4}=4$

On multiplying both sides by LCM (3, 4) = 12, we get

$4x+3y=48$ ...(i)

and $\dfrac{5x}{6}-\dfrac{y}{8}=4$

On multiplying both sides by LCM (6, 8) = 24, we get

$20x-3y=96$ ...(ii)

Now, adding Eqs. (i) and (ii), we get

$24x=144$

$\Rightarrow x=6$

Now, put the value of x in Eq. (i), we get

$4\times 6+3y=48$

$\Rightarrow \quad 3y=48-24$

$\Rightarrow \quad 3y=24$

$\therefore \quad y=8$

Hence, the required values of x and y are 6 and 8, respectively. **[2]**

(iii) Given pair of linear equations is :

$4x+\dfrac{6}{y}=15$ ...(i)

and $6x-\dfrac{8}{y}=14, \; y\neq 0$ ...(ii)

Let $u=\dfrac{1}{y}$, then above equation becomes,

$4x+6u=15$ ...(iii)

and $6x-8u=14$ ...(iv)

On multiplying Eq. (iii) by 8 and Eq. (iv) by 6 and then adding both of them, we get

$32x+48u=120$

$\Rightarrow 36x-48u=84$

$\Rightarrow \quad 68x=204$

$\therefore \quad x=3$

Now, put the value of x in Eq. (iii), we get

$4\times 3+6u=15$

$\Rightarrow \quad 6u=15-12$

$\Rightarrow \quad 6u=3$

$\Rightarrow \quad u=\dfrac{1}{2}$

$\Rightarrow \quad \dfrac{1}{y}=\dfrac{1}{2}$ $\quad \left[\because u=\dfrac{1}{y}\right]$

$\therefore \quad y=2$

Hence, the required values of x and y are 3 and 2, respectively. **[2]**

(iv) Given pair of linear equations is :

$\dfrac{1}{2x}-\dfrac{1}{y}=-1$ ...(i)

and $\dfrac{1}{x}+\dfrac{1}{2y}=8, \; x, \; y\neq 0$ ...(ii)

Let $u=\dfrac{1}{x}$ and $v=\dfrac{1}{y}$, then the above equations becomes,

$\dfrac{u}{2}-v=-1$

$\Rightarrow u-2v=-2$ ...(iii)

and $u+\dfrac{v}{2}=8$

$\Rightarrow 2u+v=16$ ...(iv)

On, multiplying Eq. (iv) by 2 and then adding with Eq. (iii), we get

$$\begin{array}{r} 4u+2v=32 \\ u-2v=-2 \\ \hline 5u=30 \end{array}$$

$\Rightarrow u=6$

Now, put the value of u in Eq. (iv), we get

$2\times 6+v=16$

$\Rightarrow \quad v=16-12=4$

$\Rightarrow \quad v=4$

$\therefore \quad x=\dfrac{1}{u}=\dfrac{1}{6}$ and $y=\dfrac{1}{v}=\dfrac{1}{4}$

Hence, the required values of x and y are $\dfrac{1}{6}$ and $\dfrac{1}{4}$, respectively. **[2]**

(v) Given pair of linear equations is :

$43x+67y=-24$ ...(i)

and $67x+43y=24$ ...(ii)

On multiplying Eq. (i) by 43 and Eq. (ii) by 67 and then subtracting both of them, we get

$$\begin{array}{l} (67)^2 x + 43\times 67y = 24\times 67 \\ (43)^2 x + 43\times 67y = -24\times 43 \\ - \quad\quad - \quad\quad\quad + \\ \hline \left[(67)^2 - (43)^2\right]x = 24(67+43) \end{array}$$

$$\Rightarrow (67+43)(67-43)x = 24\times 110$$

$$[(a^2-b^2) = (a-b)(a+b)]$$

$$\Rightarrow \quad 110\times 24x = 24\times 110$$

$$\Rightarrow \quad x = 1$$

Now, put the value of x in Eq. (i), we get

$$43\times 1 + 67y = -24$$

$$\Rightarrow \quad 67y = -24-43$$

$$\Rightarrow \quad 67y = -67$$

$$\Rightarrow \quad y = -1$$

Hence, the required values of x and y are 1 and -1, respectively. **[2]**

(vi) Given pair of linear equations is :

$$\frac{x}{a}+\frac{y}{b} = a+b \quad \text{...(i)}$$

and $\frac{x}{a^2}+\frac{y}{b^2} = 2 \quad (a, b \neq 0)$ ...(ii)

On multiplying Eq. (i) by $\frac{1}{a}$ and then subtracting from eq. (ii), we get

$$\begin{array}{l} \frac{x}{a^2}+\frac{y}{b^2} = 2 \\ \frac{x}{a^2}+\frac{y}{ab} = 1+\frac{b}{a} \\ - \quad - \quad - \\ \hline y\left(\frac{1}{b^2}-\frac{1}{ab}\right) = 2-1-\frac{b}{a} \end{array}$$

$$\Rightarrow y\left(\frac{a-b}{ab^2}\right) = 1-\frac{b}{a} = \left(\frac{a-b}{a}\right)$$

$$\Rightarrow \quad y = \frac{ab^2}{a} \Rightarrow y = b^2$$

Now, put the value of y in Eq. (ii), we get

$$\frac{x}{a^2}+\frac{b^2}{b^2} = 2$$

$$\Rightarrow \frac{x}{a^2} = 2-1 = 1$$

$$\Rightarrow x = a^2$$

Hence, the required values of x and y are a^2 and b^2, respectively. **[2]**

(vii) Given pair of equations is :

$$\frac{2xy}{x+y} = \frac{3}{2}, \text{ where } x+y \neq 0$$

$$\Rightarrow \quad \frac{x+y}{2xy} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{x}{xy}+\frac{y}{xy} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{1}{y}+\frac{1}{x} = \frac{4}{3} \quad \text{...(i)}$$

and $\frac{xy}{2x-y} = \frac{-3}{10}$, where $2x-y \neq 0$

$$\Rightarrow \quad \frac{2x-y}{xy} = \frac{-10}{3}$$

$$\Rightarrow \quad \frac{2x}{xy}-\frac{y}{xy} = \frac{-10}{3}$$

$$\Rightarrow \quad \frac{2}{y}-\frac{1}{x} = \frac{-10}{3} \quad \text{...(ii)}$$

Now, put $\frac{1}{x} = u$ and $\frac{1}{y} = v$, then the pair of equations becomes,

$$v+u = \frac{4}{3} \quad \text{...(iii)}$$

and $2v-u = \frac{-10}{3}$ ...(iv)

On adding both equations, we get

$$3v = \frac{4}{3}-\frac{10}{3} = \frac{-6}{3}$$

$$\Rightarrow \quad 3v = -2$$

$$\Rightarrow \quad v = \frac{-2}{3}$$

Now, put the value of v in Eq. (iii), we get

$$\frac{-2}{3}+u = \frac{4}{3}$$

$$\Rightarrow \quad u = \frac{4}{3}+\frac{2}{3} = \frac{6}{3} = 2$$

$$\therefore \quad x = \frac{1}{u} = \frac{1}{2}$$

and $y = \frac{1}{v} = \frac{1}{(-2/3)} = \frac{-3}{2}$

Hence, the required values of x and y are $\frac{1}{2}$ and $\frac{-3}{2}$, respectively. **[2]**

Short Answer Type Questions (3 or 4 marks each)

Q. 1. Do the following pair of linear equations have no solution? Justify your answer.

(i) $2x + 4y = 3$

$12y + 6x = 6$

(ii) $x = 2y$

$y = 2x$

(iii) $3x + y - 3 = 0$

$2x + \frac{2}{3}y = 2$

[NCERT Exemp. Ex. 3.2, Q. 1, Page 21]

Sol. **(i)** $2x+4y = 3$ and $12y+6x = 6$

Here, $a_1 = 2, b_1 = 4, c_1 = -3$

$a_2 = 6, b_2 = 12$, and $c_2 = -6$

$\therefore \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations has no solution. **[1]**

(ii) $x = 2y$ and $y = 2x$

or $x - 2y = 0$ and $2x - y = 0$

Here, $a_1 = 1, b_1 = -2$, and $c_1 = 0$;

$a_2 = 2, b_2 = -1, c_2 = 0$

$\therefore \frac{a_1}{a_2} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{2}{1}$

$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of linear equations has unique solution. **[1]**

(iii) $3x + y - 3 = 0$ and $2x + \frac{2}{3}y - 2 = 0$

Here, $a_1 = 3, b_1 = 1$, and $c_1 = -3$,

$a_2 = 2, b_2 = \frac{2}{3}, c_2 = -2$

$\therefore \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{3}{2}$

Hence, the given pair of linear equations is coincident and having infinitely many solutions. **[1]**

Q. 2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $\mathbf{3x + \frac{1}{7}y = 3}$

$\mathbf{7x + 3y = 7}$

(ii) $\mathbf{-2x - 3y = 1}$

$\mathbf{6y + 4x = -2}$

(iii) $\mathbf{\frac{x}{2} + y + \frac{2}{5} = 0}$

$\mathbf{4x + 8y + \frac{5}{16} = 0}$

[NCERT Exemp. Ex. 3.2, Q. 2, Page 21]

Sol. Condition for coincident lines,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(i) **No,** given pair of linear equations is :

$$3x + \frac{y}{7} - 3 = 0$$

and $7x + 3y - 7 = 0$,

where, $a_1 = 3, b_1 = \frac{1}{7}, c_1 = -3$;

$a_2 = 7, b_2 = 3$, and $c_2 = -7$

Now, $\frac{a_1}{a_2} = \frac{3}{7}, \frac{b_1}{b_2} = \frac{1}{21}, \frac{c_1}{c_2} = \frac{3}{7}$ $\left[\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right]$

Hence, the given pair of linear equations has unique solution. **[1]**

(ii) **Yes,** given pair of linear equations,

$-2x - 3y - 1 = 0$ and $6y + 4x + 2 = 0$

Where, $a_1 = -2, b_1 = -3, c_1 = -1$;

$a_2 = 4, b_2 = 6, c_2 = 2$

Now, $\frac{a_1}{a_2} = -\frac{2}{4} = -\frac{1}{2}$

$\frac{b_1}{b_2} = -\frac{3}{6} = -\frac{1}{2}, \frac{c_1}{c_2} = -\frac{1}{2}$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = -\frac{1}{2}$

Hence, the given pair of linear equations is coincident. **[1]**

(iii) **No,** the given pair of linear equations are :

$\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$

Here,

$a_1 = \frac{1}{2}, b_1 = 1, c_1 = \frac{2}{5}$

$a_2 = 4, b_2 = 8, c_2 = \frac{5}{16}$

Now,

$\frac{a_1}{a_2} = \frac{1}{8}, \frac{b_1}{b_2} = \frac{1}{8}, \frac{c_1}{c_2} = \frac{32}{25}$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations has no solution. **[1]**

Q. 3. On comparing the ratios $\mathbf{\frac{a_1}{a_2}}$, $\mathbf{\frac{b_1}{b_2}}$ and $\mathbf{\frac{c_1}{c_2}}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident :

(i) $\mathbf{5x - 4y + 8 = 0}$ **(ii)** $\mathbf{9x + 3y + 12 = 0}$

$\mathbf{7x + 6y - 9 = 0}$ $\mathbf{18x + 6y + 24 = 0}$

(iii) $\mathbf{6x - 3y + 10 = 0}$

$\mathbf{2x - y + 9 = 0}$ **[NCERT Ex. 3.2, Q. 2, Page 49]**

Sol. **(i)** $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 5, b_1 = -4$, and $c_1 = 8$

$a_2 = 7, b_2 = 6$, and $c_2 = -9$

$\frac{a_1}{a_2} = \frac{5}{7}$

$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the lines representing the given pair of equations have a unique solution and the pair of lines intersects at exactly one point. **[1]**

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 9, b_1 = 3$, and $c_1 = 12$

$a_2 = 18, b_2 = 6$, and $c_2 = 24$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the lines representing the given pair of equations are coincident and there are infinite possible solutions for the given pair of equations. **[1]**

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 6, b_1 = -3$, and $c_1 = 10$

$a_2 = 2, b_2 = -1$, and $c_2 = 9$

$$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}$$

$$\frac{c_1}{c_2} = \frac{10}{9}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the lines representing the given pair of equations are parallel to each other and hence, these lines will never intersect each other at any point or there is no possible solution for the given pair of equations. **[1]**

Q. 4. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3, 800. Later, she buys 3 bats and 5 balls for ₹ 1,750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages? [NCERT Ex. 3.3, Q. 3, Page 53]

Sol. (i) Let the first number be x and the other number be y such that $y > x$.

According to the given information,

$y = 3x$ ...(i)

$y - x = 26$ ...(ii)

On substituting the value of y from equation (i) into equation (ii), we obtain

$3x - x = 26$

$x = 13$ ...(iii)

Substituting this in equation (i), we obtain

$y = 39$

Hence, the numbers are 13 and 39. **[3]**

(ii) Let the larger angle be x and smaller angle be y.

We know that the sum of the measures of angles of a supplementary pair is always 180°.

According to the given information,

$x + y = 180°$ ...(i)

$x - y = 180°$ ...(ii)

From Eq. (i), we obtain

$x = 180 - y$ ...(iii)

Substituting this in equation (ii), we obtain

$180° - y - y = 18°$

$162° = 2y$

$y = 18°$ ...(iv)

Putting this in equation (iii), we obtain

$x = 180° - 81° = 99°$

Hence, the angles are 99° and 81°. **[3]**

(iii) Let the cost of a bat and a ball be x and y respectively.

According to the given information,

$7x + 6y = 3{,}800$ ...(i)

$3x + 5y = 1{,}750$ ...(ii)

From Eq. (i), we obtain

$$y = \frac{3{,}800 - 7x}{6} \quad ...(iii)$$

Substituting this value in equation (ii), we obtain

$$3x + 5\left(\frac{3{,}800 - 7x}{6}\right) = 1{,}750$$

$$\Rightarrow 3x + \frac{9{,}500}{3} - \frac{35x}{6} = 1{,}750$$

$$\Rightarrow \quad 3x - \frac{35x}{6} = 1{,}750 - \frac{9{,}500}{3}$$

$$\Rightarrow \quad \frac{18x - 35x}{6} = \frac{5{,}250 - 9{,}500}{3}$$

$$\Rightarrow \quad \frac{-17x}{6} = \frac{-4{,}250}{3}$$

$$\Rightarrow \quad -17x = -8{,}500$$

$$\therefore \quad x = 500 \quad ...(iv)$$

Substituting this in equation (iii), we obtain

$$y = \frac{3{,}800 - 7 \times 500}{6} = \frac{300}{6} = 50$$

Hence, the cost of a bat is ₹ 500 and that of a ball is ₹ 50. **[3]**

(iv) Let the fixed charge be ₹ x and per km charge be ₹ y.

According to the given information,

$x + 10y = 105$ ...(i)

$x + 15y = 155$ ...(ii)

From Eq. (i), we obtain

$x = 105 - 10y$ ...(iii)

Substituting this in equation (ii), we obtain

$105 - 10y + 15y = 155$

$5y = 50$

$y = 10$ ...(iv)

Putting this in equation (iii), we obtain

$x = 105 - 10 \times 10$

$x = 5$

Hence, fixed charge = ₹ 5

And per km charge = ₹ 10

Charge for 25 km = $x + 25y = 5 + 250$ = ₹ 255 **[3]**

(v) Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$11x + 22 = 9y + 18$

$11x - 9y = -4$ ...(i)

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$6x + 18 = 5y + 15$

$6x - 5y = -3$ ...(i)

From equation (i), we obtain $x = \frac{-4+9y}{11}$ ...(iii)

Substituting this in equation (ii), we obtain

$$6\left(\frac{-4+9y}{11}\right) - 5y = -3$$

$-24 + 54y - 55y = -33$

$-y = -9$

$y = 9$ ...(iv)

Substituting this in equation (iii), we obtain

$$x = \frac{-4+81}{11} = 7$$

Hence, the fraction is $\frac{7}{9}$. **[3]**

(vi) Let the age of Jacob be x and the age of his son be y.

According to the given information,

$(x + 5) = 3(y + 5)$

$x - 3y = 10$ ...(i)

$(x - 5) = 7(y - 5)$

$x - 7y = -30$ ...(ii)

From Eq. (i), we obtain

$x = 3y + 10$ ...(iii)

Substituting this value in equation (ii), we obtain

$3y + 10 - 7y = -30$

$-4y = -40$

$y = 10$ ...(iv)

Substituting this value in equation (iii), we obtain

$x = 3 \times 10 + 10 = 40$

Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years. **[3]**

Q. 5. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is

(i) intersecting lines (ii) parallel lines

(iii) coincident lines [NCERT Ex. 3.2, Q. 6, Page 50]

Sol. **(i)** For intersecting lines :

For this condition,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$ **[1]**

The second line such that it is interesting the given line is $2x + 4y - 6 = 0$ as $\frac{a_1}{a_2} = \frac{2}{2} = 1$, $\frac{b_1}{b_2} = \frac{3}{4}$ and $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

(ii) For parallel lines :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the second line can be,

$4x + 6y - 8 = 0$

as $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-8}{-8} = 1$ **[1]**

and clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) For coincident lines :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the second line can be,

$6x + 9y - 24 = 0$

as $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$ **[1]**

and clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Q. 6. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the x- axis, and shade the triangular region.

[NCERT Ex. 3.2, Q. 7, Page 50]

Sol. $x - y + 1 = 0$

$x = y - 1$

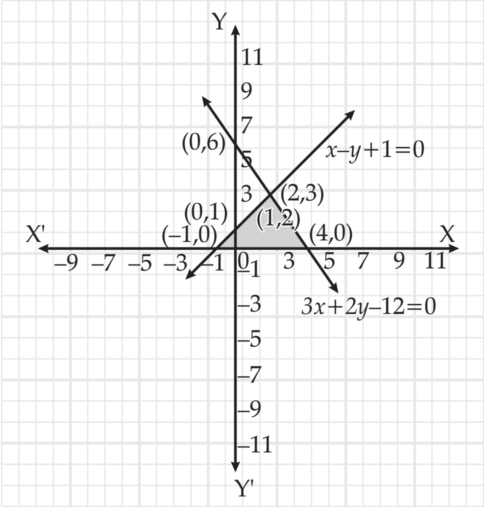
x	0	1	2
y	1	2	3

$3x + 2y - 12 = 0$

$$x = \frac{12 - 2y}{3}$$

x	4	2	0
y	0	3	6

Hence, the graphical representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and x-axis at $(-1, 0)$ and $(4, 0)$. Therefore, the vertices of the triangle are $(2, 3)$, $(-1, 0)$ and $(4, 0)$. **[3]**

Q. 7. For which value(s) of λ, do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

[NCERT Exemp. Ex. 3.3, Q. 1, Page 25]

Sol. **(i)** For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{-\lambda^2}{-1}$$

$$\Rightarrow \quad \lambda^2 - 1 = 0$$

$$\Rightarrow (\lambda - 1)(\lambda + 1) = 0$$

$$\Rightarrow \quad \lambda = 1, -1$$

Here, we take only $\lambda = -1$ because at $\lambda = 1$ the system of linear equations has infinitely many solutions. **[1]**

(ii) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\Rightarrow \quad \frac{\lambda}{1} = \frac{\lambda^2}{1}$$

$$\Rightarrow \lambda(\lambda - 1) = 0$$

When $\lambda \neq 0$, then $\lambda = 1$. **[1]**

(iii) For a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$$\Rightarrow \quad \lambda^2 \neq 1 \Rightarrow \lambda \neq \pm 1$$

So, all real values of λ except ± 1. **[1]**

Q. 8. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

[NCERT Ex. 3.2, Q. 5, Page 50]

Sol. Let the breadth of the rectangular garden = x m

Length of the garden = $(x + 4)$ m

Perimeter = $2(l + b) = 2(x + x + 4) = 4x + 8$

According to question,

$$\frac{1}{2}(4x+8) = 36$$

$$4x + 8 = 72$$

$$4x = 72 - 8$$

$$4x = 64$$

$$x = \frac{64}{4} = 16 \text{ m}$$

Breadth of the rectangle = 16 m

Length of the rectangle = 16+4 = 20 m **[3]**

Q. 9. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

[NCERT Ex. 3.3, Q. 2, Page 53]

Sol. $2x + 3y = 11$ ...(i)

$2x - 4y = -24$ ...(ii)

From equation (i), we obtain

$$x = \frac{11 - 3y}{2} \quad \text{...(iii)}$$

Substituting this value in equation (ii), we obtain

$$2\left(\frac{11-3y}{2}\right) - 4y = -24$$

$$11 - 3y - 4y = -24$$

$$-7y = -35$$

$$y = 5 \quad \text{...(iv)}$$

Putting this value in equation (iii), we obtain

$$x = \frac{11 - 3\times 5}{2} = -\frac{4}{2} = -2$$

Hence, $x = -2$, $y = 5$

Also,

$$y = mx + 3$$

$$5 = -2m + 3$$

$$-2m = 2$$

$$m = -1$$

[3]

Q. 10. Solve the following pair of linear equations by the substitution method :

(i) $x + y = 14$, $x - y = 4$

(ii) $s - t = 3$, $\frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3$, $9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3$, $0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$, $\sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$, $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

[NCERT Ex. 3.3, Q. 1, Page 53]

Sol. **(i)** $x + y = 14$ ...(i)

$x - y = 4$ ...(ii)

From Eq. (i), we obtain

$x = 14 - y$ ...(iii)

Substituting this value in equation (ii), we obtain

$(14-y)-y=4$

$14-2y=4$

$10=2y$

$y=5$ ...(iv)

Substituting this in equation (iii), we obtain

$x=9$

$\therefore x=9, y=5$ **[3]**

(ii) $s-t=3$ ...(i)

$\frac{s}{3}+\frac{t}{2}=6$ ...(ii)

From Eq. (i), we obtain

$s=t+3$ ...(iii)

Substituting this value in equation (ii), we obtain

$\frac{t+3}{3}+\frac{t}{2}=6$

$2t+6+3t=36$

$5t=30$

$t=6$ ...(iv)

Substituting in equation (iii), we obtain

$s=9$

$\therefore s=9,\ t=6$ **[3]**

(iii) $3x-y=3$ ...(i)

$9x-3y=9$ ...(ii)

From Eq. (i), we obtain

$y=3x-3$ ...(iii)

Substituting this value in equation (ii), we obtain

$9x-3(3x-3)=9$

$9x-9x+9=9$

$9=9$

this is always true.

Hence, the given pair of equations has infinite possible solutions and the relations between these variables can be given by,

$y=3x-3$ **[3]**

Therefore, one of its possible solutions is $x = 1$, $y = 0$.

(iv) $0.2x+0.3y=1.3$ ...(i)

$0.4x+0.5y=2.3$ ...(ii)

From equation (i), we obtain

$x=\frac{1.3-0.3y}{0.2}$ ...(iii)

Substituting this value in equation (ii), we obtain

$0.4\left(\frac{1.3-0.3y}{0.2}\right)+0.5y=2.3$

$2.6-0.6y+0.5y=2.3$

$2.6-2.3=0.1y$

$y=3$ ...(iv)

Substituting this value in equation (iii), we obtain

$x=\frac{1.3-0.3\times3}{0.2}$

$=\frac{1.3-0.9}{0.2}=\frac{0.4}{0.2}=2$

$\therefore x=2,\ y=3$ **[3]**

(v) $\sqrt{2}x+\sqrt{3}y=0$ ...(i)

$\sqrt{3}x-\sqrt{8}y=0$ ...(ii)

From equation (i), we obtain

$x=\frac{-\sqrt{3}y}{\sqrt{2}}$ ...(iii)

Substituting this value in equation (ii), we obtain

$\sqrt{3}\left(-\frac{\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$

$-\frac{3y}{\sqrt{2}}-2\sqrt{2}y=0$

$y\left(-\frac{3}{\sqrt{2}}-2\sqrt{2}\right)=0$

$y=0$ ...(iv)

Substituting this value in equation (iii), we obtain

$x=0$

$\therefore x=0,\ y=0$ **[3]**

(vi) $\frac{3}{2}x-\frac{5}{3}y=-2$ ...(i)

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$ ...(ii)

From equation (i), we obtain

$9x-10y=-12$

$x=\frac{-12+10y}{9}$ ...(iii)

Substituting this value in equation (ii), we obtain

$\frac{\frac{-12+10y}{9}}{3}+\frac{y}{2}=\frac{13}{6}$

$\frac{-12+10y}{27}+\frac{y}{2}=\frac{13}{6}$

$\frac{-24+20y+27y}{54}=\frac{13}{6}$

$47y=117+24$

$47y=141$

$y=3$ ...(iv)

Substituting this value in equation (iii), we obtain

$x=\frac{-12+10\times3}{9}=\frac{18}{9}=2$

Hence, $x=2,\ y=3$ **[3]**

Q. 11. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically :

(i) $x+y=5,\ 2x+2y=10$

(ii) $x-y=8,\ 3x-3y=16$

(iii) $2x+y-6=0,\ 4x-2y-4=0$

(iv) $2x-2y-2=0,\ 4x-4y-5=0$

[NCERT Ex. 3.2, Q. 4, Page 49]

Sol. **(i)** $x+y=5$

$2x+2y=10$

$\frac{a_1}{a_2}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{1}{2},\ \frac{c_1}{c_2}=\frac{5}{10}=\frac{1}{2}$

$\because \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Therefore, these linear equations are coincident pair of lines and thus have infinite numbers of possible solutions. Hence, the pair of linear equations is consistent.

$x + y = 5$

$x = 5 - y$

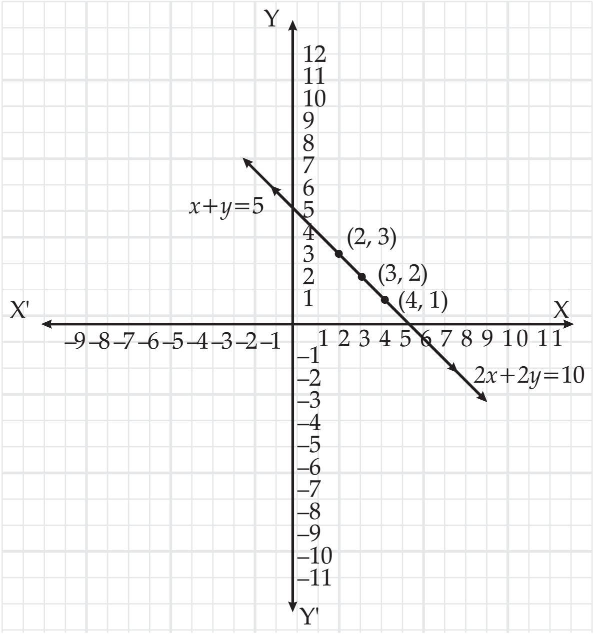
x	4	3	2
y	1	2	3

and, $2x + 2y = 10$

$$x = \frac{10 - 2y}{2}$$

x	4	3	2
y	1	2	3

Hence, the graphical representation is as follows.



[3]

From the figure, it can be observed that these lines are overlapping each other. Therefore, infinite solutions are possible for the given pair of equations.

(ii) $x - y = 8$

$3x - 3y = 16$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, these linear equations are parallel to each other and thus have no possible solutions. Hence, the pair of linear equations is inconsistent. **[3]**

(iii) $2x + y - 6 = 0$

$4x - 2y - 4 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-1}{2}, \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

$$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

$2x + y - 6 = 0$

$y = 6 - 2x$

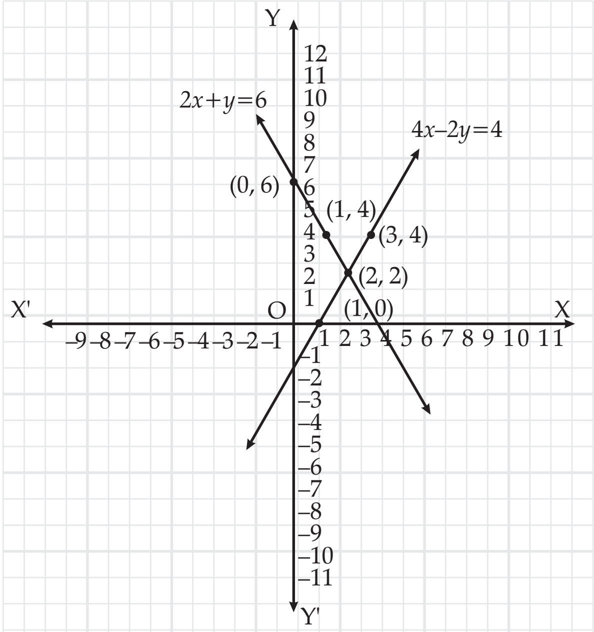
x	0	1	2
y	6	4	2

And $4x - 2y - 4 = 0$

$$y = \frac{4x - 4}{2}$$

x	1	2	3
y	0	2	4

Hence, the graphical representation is as follows :



From the figure, it can be observed that these lines are intersecting each other at the only point, *i.e.*, (2, 2) and it is the solution for the given pair of equations. **[3]**

(iv) $2x - 2y - 2 = 0$

$4x - 4y - 5 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{2}{5}$$

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, these linear equations are parallel to each other and thus have no possible solutions. Hence, the pair of linear equations is inconsistent. **[3]**

Q. 12. Solve the following pair of linear equations by the elimination method and the substitution method.

(i) $x + y = 5$ **and** $2x - 3y = 4$

(ii) $3x + 4y = 10$ **and** $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$ **and** $9x = 2y + 7$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ **and** $x - \frac{y}{3} = 3$

[NCERT Ex. 3.4, Q. 1, Page 56]

Sol. **(i)** By elimination method,

$x + y = 5$ ...(i)

$2x - 3y = 4$ ...(ii)

Multiplying equation (i) by (ii), we obtain

$2x + 2y = 10$ ...(iii)

Subtracting equation (ii) from equation (iii), we obtain

$5y = 6$

$y = \frac{6}{5}$ ...(iv)

Substituting the value in equation (i), we obtain

$x = 5 - \frac{6}{5} = \frac{19}{5}$

$\therefore x = \frac{19}{5}, y = \frac{6}{5}$

By substitution method,

From equation (i), we obtain

$x = 5 - y$ ...(v)

Putting this value in equation (ii), we obtain

$2(5 - y) - 3y = 4$

$-5y = -6$

$y = \frac{6}{5}$

Substituting the value in equation (v), we obtain

$x = 5 - \frac{6}{5} = \frac{19}{5}$

$\therefore x = \frac{19}{5},\ y = \frac{6}{5}$ **[3]**

(ii) By elimination method,

$3x + 4y = 10$ ...(i)

$2x - 2y = 2$ ...(ii)

Multiplying equation (ii) by 2, we obtain

$4x - 4y = 4$ ...(iii)

Adding equations (i) and (iii), we obtain

$7x = 14$

$x = 2$ ...(iv)

Substituting in equation (i), we obtain

$6 + 4y = 10$

$4y = 4$

$y = 1$

Hence, $x = 2, y = 1$

By substitution method,

From equation (ii), we obtain

$x = 1 + y$ ...(v)

Putting this value in equation (i), we obtain

$3(1 + y) + 4y = 10$

$7y = 7$

$y = 1$

Substituting the value in equation (v), we obtain

$x = 1 + 1 = 2$

$\therefore x = 2,\ y = 1$ **[3]**

(iii) By elimination method,

$3x - 5y - 4 = 0$ ...(i)

$9x = 2y + 7$

$9x - 2y - 7 = 0$ ...(ii)

Multiplying equation (i) by 3, we obtain

$9x - 15y - 12 = 0$ ...(iii)

Subtracting equation (iii) from equation (ii), we obtain

$13y = -5$

$y = \frac{-5}{13}$ ...(iv)

Substituting in equation (i), we obtain

$3x + \frac{25}{13} - 4 = 0$

$3x = \frac{27}{13}$

$x = \frac{9}{13}$

$\therefore \quad x = \frac{9}{13},\ y = \frac{-5}{13}$

By substitution method,

From equation (i), we obtain

$x = \frac{5y + 4}{3}$ ...(v)

Putting this value in equation (ii), we obtain

$9\left(\frac{5y + 4}{3}\right) - 2y - 7 = 0$

$13y = -5$

$y = -\frac{5}{13}$

Substituting the value in equation (v), we obtain

$x = \frac{5\left(\frac{-5}{13}\right) + 4}{3}$

$x = \frac{9}{13}$

$\therefore x = \frac{9}{13},\ y = \frac{-5}{13}$ **[3]**

(iv) By elimination method,

$\frac{x}{2} + \frac{2y}{3} = -1$

$3x + 4y = -6$ ...(i)

$x - \frac{y}{3} = 3$

$3x - y = 9$ ...(ii)

Subtracting equation (ii) from equation (i), we obtain

$5y = -15$

$y = -3$ ...(iii)

Substituting this value in equation (i), we obtain

$3x - 12 = -6$

$3x = 6$

$x = 2$

Hence, $x = 2, y = -3$

By substitution method,

From equation (ii), we obtain

$x = \frac{y + 9}{3}$ ...(iv)

Putting this value in equation (i), we obtain

$$3\left(\frac{y+9}{3}\right)+4y=-6$$
$$5y=-15$$
$$y=-3$$

Substituting the value in equation (iv), we obtain

$$x=\frac{-3+9}{3}=2$$

$\therefore x=2,\ y=-3$ **[3]**

Q. 13. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹ 2,000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

[NCERT Ex. 3.4, Q. 2, Page 57]

Sol. **(i)** Let the fraction be $\frac{x}{y}$.

According to the given information,

$\frac{x+1}{y-1}=1 \quad \Rightarrow x-y=-2$ ...(i)

$\frac{x}{y+1}=\frac{1}{2} \quad \Rightarrow 2x-y=1$ ...(ii)

Subtracting equation (i) from equation (ii), we obtain

$x = 3$ ...(iii)

Substituting this value in equation (i), we obtain

$$3-y=-2$$
$$-y=-5$$
$$y=5$$

Hence, the fraction is $\frac{3}{5}$. **[3]**

(ii) Let present age of Nuri = x and present age of Sonu = y

According to the given information,

$(x-5)=3(y-5)$

$x-3y=-10$ ...(i)

$(x+10)=2(y+10)$

$x-2y=10$ ...(ii)

Subtracting equation (i) from equation (ii), we obtain

$y = 20$ ...(iii)

Substituting it in equation (i), we obtain

$x-60=-10$

$x=50$

Hence, age of Nuri = 50 years and age of Sonu = 20 years. **[3]**

(iii) Let the unit digit and tens digits of the number be x and y respectively. Then,

Number = $10y + x$

Number after reversing the digits = $10x + y$

According to the given information,

$x + y = 9$ ...(i)

$9(10y + x) = 2(10x + y)$

$88y - 11x = 0$

$-x + 8y = 0$ ...(ii)

Adding equations (i) and (ii), we obtain

$9y = 9$

$y = 1$ ...(iii)

Substituting the value in equation (i), we obtain

$x = 8$

Hence, the number is $10y + x = 10 \times 1 + 8 = 18$ **[3]**

(iv) Let the number of ₹ 50 notes and ₹ 100 notes be x and y respectively.

According to the given information,

$x+y=25$ ...(i)

$50x+100y=2,000$ ...(ii)

Multiplying equation (i) by 50, we obtain

$50x+50y=1,250$ ...(iii)

Subtracting equation (iii) from equation (ii), we obtain

$$50y=750$$
$$y=15$$

Substituting in equation (i), we have $x = 10$

Hence, Meena has 10 notes of ₹ 50 and 15 notes of ₹ 100. **[3]**

(v) Let the fixed charge for first 3 days and each day charge thereafter be ₹ x and ₹ y respectively.

According to the given information,

$x+4y=27$ ...(i)

$x+2y=21$ ...(ii)

Subtracting equation (ii) from equation (i), we obtain

$2y=6$

$y=3$ ...(iii)

Substituting in equation (i), we obtain

$$x+12=27$$
$$x=15$$

Hence, fixed charge = ₹ 15 and charges per day = ₹ 3 **[3]**

Q. 14. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$

$(a - b)\,x + (a + b)\,y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k - 1)\,x + (k - 1)\,y = 2k + 1$

[NCERT Ex. 3.5, Q. 2, Page 62]

Sol. **(i)** $2x+3y-7=0$

$(a-b)x+(a+b)y-(3a+b-2)=0$

$\frac{a_1}{a_2}=\frac{2}{a-b},\ \frac{b_1}{b_2}=\frac{3}{a+b},$

$\frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{7}{(3a+b-2)}$

For infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{a-b}=\frac{7}{3a+b-2}$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \quad ...(i)$$

$$\frac{2}{a-b}=\frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \quad ...(ii)$$

Subtracting equation (i) from equation (ii), we obtain

$4b=4$

$b=1$

Substituting this in equation (ii), we obtain

$a-5\times1=0$

$a=5$

Hence, $a=5$ and $b=1$ are the values for which the given equations give infinitely many solutions. **[1½]**

(ii) $3x+y-1=0$

$(2k-1)x+(k-1)y-2k-1=0$

$\frac{a_1}{a_2}=\frac{3}{2k-1},\ \frac{b_1}{b_2}=\frac{1}{k-1},\ \frac{c_1}{c_2}=\frac{-1}{-2k-1}=\frac{1}{2k+1}$

For no solution,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{3}{2k-1}=\frac{1}{k-1}\neq\frac{1}{2k+1}$$

$$\frac{3}{2k-1}=\frac{1}{k-1}$$

$$3k-3=2k-1$$

$$k=2$$

Hence, for $k=2$, the given equation has no solution. **[1½]**

Q. 15. Solve the following pair of linear equations by the substitution and cross-multiplication methods :

$8x + 5y = 9$

$3x + 2y = 4$ **[NCERT Ex. 3.5, Q. 3, Page 62]**

Sol. $8x+5y=9 \quad ...(i)$

$3x+2y=4 \quad ...(ii)$

From equation (ii), we obtain

$$x=\frac{4-2y}{3} \quad ...(iii)$$

Substituting this value in equation (i), we obtain

$$8\left(\frac{4-2y}{3}\right)+5y=9$$

$$32-16y+15y=27$$

$$-y=-5$$

$$y=5 \quad ...(iv)$$

Substituting this value in equation (ii), we obtain

$3x+10=4$

$x=-2$

Hence, $x=-2, y=5$

Again, by cross-multiplication method, we obtain

$8x+5y-9=0$

$3x+2y-4=0$

$$\frac{x}{-20-(-18)}=\frac{y}{-27-(-32)}=\frac{1}{16-15}$$

$$\frac{x}{-2}=\frac{y}{5}=\frac{1}{1}$$

$\frac{x}{-2}=1$ and $\frac{y}{5}=1$

$x=-2$ and $y=5$ **[3]**

Q. 16. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method :

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹ 1,000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1,180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle. **[NCERT Ex. 3.5, Q. 4, Page 62]**

Sol. **(i)** Let x be the fixed charge of the food and y be the charge for food per day.

According to the given information,

$x+20y=1,000 \quad ...(i)$

$x+26y=1,180 \quad ...(ii)$

Subtracting equation (i) from equation (ii), we obtain

$6y = 180$

$y = 30$

Substituting this value in equation (i), we obtain

$x + 20 \times 30 = 1{,}000$

$x = 1{,}000 - 600 = 400$

Hence, fixed charge = ₹ 400

And charges per day = ₹ 30 **[3]**

(ii) Let the fraction be $\frac{x}{y}$.

According to the given information,

$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow 3x - y = 3$ ...(i)

$\frac{x}{y+8} = \frac{1}{4} \quad \Rightarrow 4x - y = 8$ ...(ii)

Subtracting equation (i) from equation (ii), we obtain

$x = 5$ ...(iii)

Putting this value in equation (i), we obtain

$15 - y = 3$

$y = 12$

Hence, the fraction is $\frac{5}{12}$. **[3]**

(iii) Let the number of right answers and wrong answers be x and y, respectively.

According to the given information,

$3x - y = 40$ ...(i)

$4x - 2y = 50$

$\Rightarrow 2x - y = 25$ ...(ii)

Subtracting equation (ii) from equation (i), we obtain

$x = 15$ ...(iii)

Substituting this in equation (ii), we obtain

$30 - y = 25$

$y = 5$

Therefore, number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20 **[3]**

(iv) Let the speed of first car and second car be u km/h and v km/h respectively.

Relative speed of both cars while they are travelling in same direction = $(u - v)$ km/h

Relative speed of both cars while they are travelling in opposite directions, *i.e.*, travelling towards each other = $(u + v)$ km/h

According to the given information,

$5(u - v) = 100$

$\Rightarrow u - v = 20$ ...(i)

$(u + v) = 100$ ...(ii)

Adding both the equations, we obtain

$2u = 120$

$u = 60$ km/h ...(iii)

Substituting this value in equation (ii), we obtain

$v = 40$ km/h

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h **[3]**

(v) Let length and breadth of rectangle be x unit and y unit, respectively.

Area = xy

According to the question,

$(x-5)(y+3) = xy - 9$

$\Rightarrow 3x - 5y - 6 = 0$ ...(i)

$(x+3)(y+2) = xy + 67$

$\Rightarrow 2x + 3y - 61 = 0$ ...(ii)

By cross-multiplication method, we obtain

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$x = 17,\ y = 9$

Hence, the length and breadth of the rectangle are 17 units and 9 units, respectively. **[3]**

Q. 17. Solve the following pairs of equations by reducing them to a pair of linear equations :

(i) $\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

(iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1$

(v) $\frac{7x-2y}{xy} = 5$

$\frac{8x+7y}{xy} = 15$

(vi) $6x + 3y = 6xy$

$2x + 4y = 5xy$

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

[NCERT Ex. 3.6, Q. 1, Page 67]

Sol. **(i)** $\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations change as follows :

$\frac{p}{2}+\frac{q}{3}=2 \quad \Rightarrow 3p+2q-12=0$ ...(i)

$\frac{p}{3}+\frac{q}{2}=\frac{13}{6} \quad \Rightarrow 2p+3q-13=0$ ...(ii)

Using cross-multiplication method, we obtain

$$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$\frac{p}{10}=\frac{1}{5}$ and $\frac{q}{15}=\frac{1}{5}$

$p=2$ and $q=3$

$\frac{1}{x}=2$ and $\frac{1}{y}=3$

$x=\frac{1}{2}$ and $y=\frac{1}{3}$

[3]

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

Putting $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$ in the given equations, we obtain

$2p+3q=2$ ...(i)

$4p-9q=-1$ ...(ii)

Multiplying equation (i) by 3, we obtain

$6p+9q=6$ ...(iii)

Adding equations (ii) and (iii), we obtain

$10p=5$

$p=\frac{1}{2}$ ...(iv)

Putting this value in equation (i), we obtain

$2\times\frac{1}{2}+3q=2$

$3q=1$

$q=\frac{1}{3}$

$p=\frac{1}{\sqrt{x}}=\frac{1}{2}$

$\sqrt{x}=2$

$x=4$

and $q=\frac{1}{\sqrt{y}}=\frac{1}{3}$

$\sqrt{y}=3$

$y=9$

Hence, $x=4, y=9$ **[3]**

(iii) $\frac{4}{x}+3y=14$

$\frac{3}{x}-4y=23$

Substituting $\frac{1}{x}=p$ in the given equations, we obtain

$4p+3y=14 \quad \Rightarrow 4p+3y-14=0$ ...(i)

$3p-4y=23 \quad \Rightarrow 3p-4y-23=0$ ...(ii)

By cross-multiplication, we obtain

$$\frac{p}{-69-56}=\frac{y}{-42-(-92)}=\frac{1}{-16-9}$$

$$\frac{p}{-125}=\frac{y}{50}=\frac{-1}{25}$$

$\frac{p}{-125}=\frac{-1}{25}$ and $\frac{y}{50}=\frac{-1}{25}$

$p=5$ and $y=-2$

$p=\frac{1}{x}=5$

$x=\frac{1}{5}$

$y=-2$

[3]

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

Putting $\frac{1}{x-1}=p$ and $\frac{1}{y-2}=q$ in the given equation, we obtain

$5p+q=2$ ...(i)

$6p-3q=1$ ...(ii)

Multiplying equation (i) by 3, we obtain

$15p+3q=6$ ...(iii)

Adding (ii) and (iii), we obtain

$21p=7$

$p=\frac{1}{3}$

Putting this value in equation (i), we obtain

$5\times\frac{1}{3}+q=2$

$q=2-\frac{5}{3}=\frac{1}{3}$

$p=\frac{1}{x-1}=\frac{1}{3}$

$\Rightarrow x-1=3$

$\Rightarrow \quad x=4$

$q=\frac{1}{y-2}=\frac{1}{3}$

$y-2=3$

$y=5$

$\therefore x=4, y=5$

[3]

(v) $\frac{7x-2y}{xy}=5$

$\frac{8x+7y}{xy}=15$

$\frac{7x-2y}{xy}=5$

$\frac{7}{y}-\frac{2}{x}=5$ ...(i)

$\frac{8x+7y}{xy}=15$

$\frac{8}{y}+\frac{7}{x}=15$ ...(ii)

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in the given equation, we obtain

$-2p+7q=5 \Rightarrow -2p+7q-5=0$ ...(iii)

$7p+8q=15 \Rightarrow 7p+8q-15=0$ ...(iv)

By cross-multiplication method, we obtain

$$\frac{p}{-105-(-40)}=\frac{q}{-35-30}=\frac{1}{-16-49}$$

$$\frac{p}{-65}=\frac{q}{-65}=\frac{1}{-65}$$

$$\frac{p}{-65}=\frac{1}{-65} \text{ and } \frac{q}{-65}=\frac{1}{-65}$$

$$p=1 \text{ and } q=1$$

$p=\frac{1}{x}=1 \quad q=\frac{1}{y}=1$

$x=1 \quad y=1$ **[3]**

(vi) $6x+3y=6xy$

$2x+4y=5xy$

$6x+3y=6xy$

$\Rightarrow \frac{6}{y}+\frac{3}{x}=6$ ...(i)

$2x+4y=5xy$

$\frac{2}{y}+\frac{4}{x}=5$ ...(ii)

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in these equations, we obtain

$3p+6q-6=0$

$4p+2q-5=0$

By cross-multiplication method, we obtain

$$\frac{p}{-30-(-12)}=\frac{q}{-24-(-15)}=\frac{1}{6-24}$$

$$\frac{p}{-18}=\frac{q}{-9}=\frac{1}{-18}$$

$$\frac{p}{-18}=\frac{1}{-18} \text{ and } \frac{q}{-9}=\frac{1}{-18}$$

$$p=1 \text{ and } q=\frac{1}{2}$$

$$p=\frac{1}{x}=1 \quad q=\frac{1}{y}=\frac{1}{2}$$

$$x=1 \quad y=2$$

Hence, $x=1, y=2$ **[3]**

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

Putting $\frac{1}{x+y}=p$ and $\frac{1}{x-y}$ in the given equations, we obtain

$10p+2q=4 \Rightarrow 10p+2q-4=0$ ...(i)

$15p-5q=-2 \Rightarrow 15p-5q+2=0$ ...(ii)

Using cross-multiplication method, we obtain

$$\frac{p}{4-20}=\frac{q}{-60-(20)}=\frac{1}{-50-30}$$

$$\frac{p}{-16}=\frac{q}{-80}=\frac{1}{-80}$$

$$\frac{p}{-16}=\frac{1}{-80} \text{ and } \frac{q}{-80}=\frac{1}{-80}$$

$$p=\frac{1}{5} \text{ and } q=1$$

$$p=\frac{1}{x+y}=\frac{1}{5} \text{ and } q=\frac{1}{x-y}=1$$

$x+y=5$ ...(iii)

And $x-y=1$ ...(iv)

Adding equations (iii) and (iv), we obtain

$2x=6$

$x=3$ ...(v)

Substituting in equation (iii), we obtain

$y=2$

Hence, $x=3, y=2$ **[3]**

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Putting $\frac{1}{3x+y}=p$ and $\frac{1}{3x-y}=q$ in these equations, we obtain

$p+q=\frac{3}{4}$ ...(i)

$\frac{p}{2}-\frac{q}{2}=\frac{-1}{8}$

$p-q=\frac{-1}{4}$ ...(ii)

Adding equations (i) and (ii), we obtain

$$2p = \frac{3}{4} - \frac{1}{4}$$
$$2p = \frac{1}{2}$$
$$p = \frac{1}{4}$$

Substituting in Eq. (ii), we obtain

$$\frac{1}{4} - q = \frac{-1}{4}$$
$$q = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$
$$p = \frac{1}{3x+y} = \frac{1}{4}$$
$$3x + y = 4 \quad ...(iii)$$
$$q = \frac{1}{3x-y} = \frac{1}{2}$$
$$3x - y = 2 \quad \textbf{(iv)}$$

Adding equations (iii) and (iv), we obtain

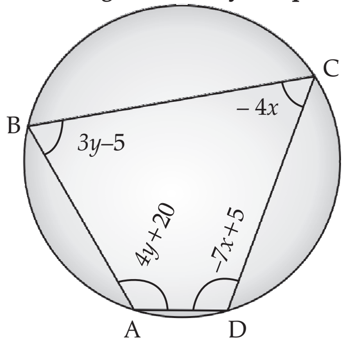
$$6x = 6$$
$$x = 1 \quad \textbf{(v)}$$

Substituting in Eq. (iii), we obtain

$$3(1) + y = 4$$
$$y = 1$$

Hence, $x = 1, y = 1$ **[3]**

Q. 18. ABCD is a cyclic quadrilateral as shown in fig. Find the angles of the cyclic quadrilateral.



[NCERT Ex. 3.7, Q. 8, Page 68]

Sol. We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.

Therefore, $\angle A + \angle C = 180°$

$$4y + 20 - 4x = 180°$$
$$-4x + 4y = 160°$$
$$x - y = -40° \quad ...(i)$$

Also, $\angle B + \angle D = 180°$

$$3y - 5 - 7x + 5 = 180°$$
$$-7x + 3y = 180° \quad ...(ii)$$

Multiplying equation (i) by 3, we obtain

$$3x - 3y = -120° \quad ...(iii)$$

Adding equations (ii) and (iii), we obtain

$$-7x + 3x = 180° - 120°$$
$$-4x = 60°$$
$$x = -15°$$

By using equation (i), we obtain

$$x - y = -40°$$
$$-15 - y = -40°$$
$$y = -15 + 40 = 25°$$
$$\angle A = 4y + 20 = 4(25) + 20 = 120°$$
$$\angle B = 3y - 5 = 3(25) - 5 = 70°$$
$$\angle C = -4x = -4(-15) = 60°$$
$$\angle D = -7x + 5 = -7(-15) + 5 = 110°$$

[3]

Q. 19. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

[NCERT Ex. 3.7, Q. 1, Page 68]

Sol. The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Let the age of Ani and Biju be x and y years, respectively.

Therefore,

age of Ani's father, Dharam = $2 \times x = 2x$ years

And age of Biju's sister, Cathy $= \frac{y}{2}$ years

By using the information given in the question,

Case I : When Ani is older than Biju by 3 years,

$$x - y = 3 \quad ...(i)$$
$$2x - \frac{y}{2} = 30$$
$$4x - y = 60 \quad ...(ii)$$

Subtracting (i) from (ii), we obtain

$$3x = 60 - 3 = 57$$
$$x = \frac{57}{3} = 19$$

Therefore, age of Ani = 19 years

And age of Biju = 19 − 3 = 16 years

Case II : When Biju is older than Ani,

$$y - x = 3 \quad ...(i)$$
$$2x - \frac{y}{2} = 30$$
$$4x - y = 60 \quad ...(ii)$$

Adding (i) and (ii), we obtain

$$3x = 63$$
$$x = 21$$

Therefore, age of Ani = 21 years

And age of Biju = 21 + 3 = 24 years **[3]**

Q. 20. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [NCERT Ex. 3.7, Q. 2, Page 68]

Sol. Let those friends were having ₹ x and y with them.

Using the information given in the question, we obtain

$$x + 100 = 2(y - 100)$$
$$x + 100 = 2y - 200$$
$$x - 2y = -300 \quad ...(i)$$

And, $6(x - 10) = (y + 10)$

$$6x - 60 = y + 10$$
$$6x - y = 70 \quad ...(ii)$$

Multiplying equation (ii) by 2, we obtain

$$12x - 2y = 140 \quad ...(iii)$$

Subtracting equation (i) from equation (iii), we obtain

$11x = 140 + 300$
$11x = 440$
$x = 40$

Using this in equation (i), we obtain

$40 - 2y = -300$
$40 + 300 = 2y$
$2y = 340$
$y = 170$

Therefore, those friends had ₹ 40 and ₹ 170 with them respectively. **[3]**

Q. 21. In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles. [NCERT Ex. 3.7, Q. 5, Page 68]

Sol. Given that,

$\angle C = 3\angle B = 2(\angle A + \angle B)$
$3\angle B = 2(\angle A + \angle B)$
$3\angle B = 2\angle A + 2\angle B$
$\angle B = 2\angle A$
$2\angle A - \angle B = 0$ ...(i)

We know that the sum of the measures of all angles of a triangle is 180°. Therefore,

$\angle A + \angle B + \angle C = 180°$
$\angle A + \angle B + 3\angle B = 180°$
$\angle A + 4\angle B = 180°$ ...(ii)

Multiplying equation (i) by 4, we obtain

$8\angle A - 4\angle B = 0$ ...(iii)

Adding equations (ii) and (iii), we obtain

$9\angle A = 180°$
$\angle A = 20°$

From equation (ii), we obtain

$20° + 4\angle B = 180°$
$4\angle B = 160°$
$\angle B = 40°$
$\angle C = 3\angle B = 3 \times 40° = 120°$

Therefore, $\angle A$, $\angle B$ and $\angle C$ are 20°, 40° and 120°, respectively. **[3]**

Q. 22. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class. [NCERT Ex. 3.7, Q. 4, Page 68]

Sol. Let the number of rows be x and number of students in a row be y.

Total students of the class = Number of rows × Number of students in a row = xy

Using the information given in the question,

Condition I :

Total number of students = $(x - 1)(y + 3)$

$xy = (x - 1)(y + 3) = xy - y + 3x - 3$
$3x - y - 3 = 0$
$3x - y = 3$ **(i)**

Condition II :

Total number of students = $(x + 2)(y - 3)$

$xy = xy + 2y - 3x - 6$
$3x - 2y = -6$ **(ii)**

Subtracting equation (ii) from equation (i), we obtain

$(3x - y) - (3x - 2y) = 3 - (-6)$
$-y + 2y = 3 + 6$
$y = 9$

By using equation (i), we obtain

$3x - 9 = 3$
$3x = 9 + 3 = 12$
$x = 4$

Number of rows = $x = 4$

Number of students in a row = $y = 9$

Number of total students in a class = xy
$= 4 \times 9$
$= 36$ **[3]**

Q. 23. Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y- axis. [NCERT Ex. 3.7, Q. 6, Page 68]

Sol. $5x - y = 5$

Or, $y = 5x - 5$

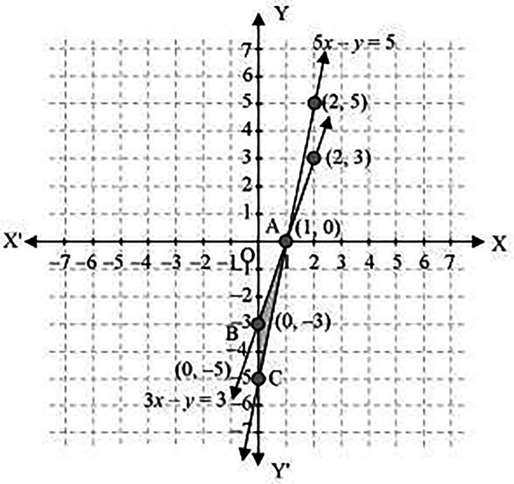
x	0	1	2
y	−5	0	5

$3x - y = 3$

Or, $y = 3x - 3$

x	0	1	2
y	− 3	0	3

The graphical representation of these lines will be as follows :



It can be observed that the required triangle is ΔABC formed by these lines and y-axis.

The co-ordinates of vertices are A (1, 0), B (0, − 3), C (0, − 5). **[3]**

Q. 24. Solve the following pair of linear equations :

(i) $px + qy = p - q$
$qx - py = p + q$

(ii) $ax + by = c$
$bx + ay = 1 + c$

(iii) $\dfrac{x}{a} - \dfrac{y}{b} =$
$ax + by = a^2 + b^2$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$
$(a + b)(x + y) = a^2 + b^2$

(v) $152x - 378y = -74$
$-378x + 152y = -604$

[NCERT Ex. 3.7, Q. 7, Page 68]

Sol. **(i)** $px + qy = p - q$ ...(i)
$qx - py = p + q$ ...(ii)

Multiplying equation (i) by p and equation (ii) by q, we obtain

$p^2x + pqy = p^2 - pq$ ...(iii)

$q^2x - pqy = pq + q^2$ ...(iv)

Adding equations (iii) and (iv), we obtain

$p^2x + q^2x = p^2 + q^2$

$(p^2 + q^2)x = p^2 + q^2$

$$x = \frac{p^2+q^2}{p^2+q^2} = 1$$

From equation (i), we obtain

$p(1) + qy = p - q$

$qy = -q$

$y = -1$ **[3]**

(ii) $ax + by = c$ ...(i)

$bx + ay = 1 + c$ ...(ii)

Multiplying equation (i) by a and equation (ii) by b, we obtain

$a^2x + aby = ac$ ...(iii)

$b^2x + aby = b + bc$ ...(iv)

Subtracting equation (iv) from equation (iii),

$(a^2 - b^2)x = ac - bc - b$

$$x = \frac{c(a-b)-b}{a^2-b^2}$$

From equation (i), we obtain

$ax + by = c$

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\} + by = c$$

$$\frac{ac(a-b)-ab}{a^2-b^2} + by = c$$

$$by = c - \frac{ac(a-b)-ab}{a^2-b^2}$$

$$by = \frac{a^2c - b^2c - a^2c + abc + ab}{a^2-b^2}$$

$$by = \frac{abc - b^2c + ab}{a^2-b^2}$$

$$by = \frac{bc(a-b)+ab}{a^2+b^2}$$

$$y = \frac{c(a-b)+a}{a^2-b^2}$$ **[3]**

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

or, $bx - ay = 0$ ...(i)

$ax + by = a^2 + b^2$ ...(ii)

Multiplying equation (i) and (ii) by b and a, respectively, we obtain

$b^2x - aby = 0$ ...(iii)

$a^2x + aby = a^3 + ab^2$ ...(iv)

Adding equations (iii) and (iv), we obtain

$b^2x + a^2x = a^3 + ab^2$

$x(b^2 + a^2) = a(a^2 + b^2)$

$x = a$

By using (i), we obtain

$b(a) - ay = 0$

$ab - ay = 0$

$ay = ab$

$y = b$ **[3]**

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ ...(i)

$(a + b)(x + y) = a^2 + b^2$

$(a + b)x + (a + b)y = a^2 + b^2$ ...(ii)

Subtracting equation (ii) from equation (i), we obtain

$(a - b)x - (a + b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$

$(a - b - a - b)x = -2ab - 2b^2$

$-2bx = -2b(a + b)$

$x = a + b$

Using equation (i), we obtain

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$(a + b)y = -2ab$

$$y = \frac{-2ab}{a+b}$$ **[3]**

(v) $152x - 378y = -74$

$76x - 189y = -37$

$$x = \frac{189y-37}{76}$$ ...(i)

$-378x + 152y = -604$

$-189x + 76y = -302$ ...(ii)

Substituting the value of x in equation (ii), we obtain

$$-189\left(\frac{189y-37}{76}\right) + 76y = -302$$

$-(189)^2y + 189 \times 37 + (76)^2y = -302 \times 76$

$189 \times 37 + 302 \times 76 = (189)^2y - (76)^2y$

$6993 + 22952 = (189 - 76)(189 + 76)y$

$29945 = (113)(265)y$

$y = 1$

From equation (i), we obtain

$$x = \frac{189(1)-37}{76}$$

$$x = \frac{189-37}{76} = \frac{152}{76}$$

$x = 2$ **[3]**

Q. 25. A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

[NCERT Exemp. Ex. 3.4, Q. 7, Page 34]

Sol. Let the speed of the stream be x km/h.

And, the speed of the boat in still water = 5 km/h

Speed of the boat upstream = $(5 - x)$ km/h

Speed of the boat downstream = $(5 + x)$ km/h

Time taken in rowing 40 km upstream = $\frac{40}{5-x}$ hrs

Time taken in rowing 40 km downstream = $\frac{40}{5+x}$ hrs

According to the question,

Time taken in 40 km upstream = 3 × Time taken in 40 km downstream

$$\therefore \quad \frac{40}{5-x} = \frac{3\times 40}{5+x}$$

$$\Rightarrow \quad \frac{1}{5-x} = \frac{3}{5+x}$$

$$\Rightarrow -3x + 15 = x + 5$$

$$\Rightarrow -3x - x = 5 - 15$$

$$\Rightarrow -4x = -10$$

$$\Rightarrow x = \frac{10}{4}$$

$$\Rightarrow x = 2.5 \text{ km/h}$$

Hence, the speed of stream is 2.5 km/h. **[3]**

Q. 26. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.
[NCERT Exemp. Ex. 3.4, Q. 8, Page 34]

Sol. Let $u \neq v$ be the speed of motor boat in still water and stream respectively.

Time taken by motor boat to go 30 km in upstream.

$$t_1 = \frac{30}{u-v} \text{ h}$$

And motorboat has taken time to travel 28 km downstream,

$$t_2 = \frac{28}{u+v} \text{ h}$$

By first condition, a motorboat can travel 30 km upstream and 28 km downstream in 7 hour, *i.e.,*

$$t_1 + t_2 = 7 \text{ h}$$

$$\Rightarrow \frac{30}{u-v} + \frac{28}{u+v} = 7 \quad \text{...(i)}$$

Now, motorboat has taken time to travel 21 km upstream and return are,

$$t_3 = \frac{21}{u-v} \quad \text{[For upstream]}$$

And $t_4 = \dfrac{21}{u+v}$ [For downstream]

By second condition,

$$t_4 + t_3 = 5 \text{ h}$$

$$\Rightarrow \frac{21}{u+v} + \frac{21}{u-v} = 5 \quad \text{...(ii)}$$

Let $y = \dfrac{1}{u+v}$ and $x = \dfrac{1}{u-v}$

Equations (i) and (ii) becomes,

$$30x + 28y = 7 \quad \text{...(iii)}$$

And $21x + 21y = 5$

$$\Rightarrow x + y = \frac{5}{21} \quad \text{...(iv)}$$

Now, multiplying in Eq. (iv) by 28 and then subtracting from eq. (iii), we get

$$30x + 28y = 7$$

$$28x + 28y = \frac{140}{21}$$

$$- \quad - \quad -$$

$$2x = 7 - \frac{20}{3} = \frac{21-20}{3}$$

$$\Rightarrow 2x = \frac{1}{3}$$

$$\therefore \quad x = \frac{1}{6}$$

On putting the value of x in Eq. (iv), we get

$$\frac{1}{6} + y = \frac{5}{21}$$

$$\Rightarrow \quad y = \frac{5}{21} - \frac{1}{6} = \frac{10-7}{42} = \frac{3}{42}$$

$$\therefore \quad y = \frac{1}{14}$$

$$\therefore x = \frac{1}{u-v} = \frac{1}{6}$$

So, $u - v = 6$ ...(v)

Again, $y = \dfrac{1}{u+v} = \dfrac{1}{14}$

So, $u + v = 14$ ...(vi)

Now, adding equations (v) and (vi), we get

$$2u = 20$$

$$\therefore u = 10 \text{ km/h}$$

Putting the value of u in Eq. (vi), we get

$$u + v = 14$$

$$v = 14 - u$$

$$v = 14 - 10$$

$$\therefore \quad v = -4 \text{ km/h}$$

[3]

Hence, the speed of motor boat in still water in 10 km/h and speed of stream in 4 km/h

Q. 27. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number. [NCERT Exemp. Ex. 3.4, Q. 9, Page 34]

Sol. Let the two-digit number = $10x + y$

Case I : Multiplying the sum of the digits by 8 and then subtracting 5 = two-digit number

$$\Rightarrow 8 \times (x+y) - 5 = 10x + y$$

$$\Rightarrow \quad 8x + 8y - 5 = 10x + y$$

$$\Rightarrow \quad 2x - 7y = -5 \quad \text{...(i)}$$

Case II : Multiplying the difference of the digits by 16 and then adding 3 = two-digit number

$$\Rightarrow 16 \times (x-y) + 3 = 10x + y$$

$$\Rightarrow \quad 16x - 16y + 3 = 10x + y$$

$$\Rightarrow \quad 6x - 17y = -3 \quad \text{...(ii)}$$

Now, multiplying in Eq. (i) by 3 and then subtracting from Eq. (ii), we get

$$6x - 17y = -3$$

$$6x - 21y = -15$$

$$- \quad + \quad +$$

$$4y = 12$$

$$y = \frac{12}{4} = 3$$

$$\therefore \quad y = 3$$

Now, put the value of y in eq. (i), we get

$$2x - 7 \times 3 = -5$$

$$\Rightarrow \quad 2x = 21 - 5 = 16$$

$$\therefore \quad x = 8$$

Hence, the required two-digit number

$$= 10x + y = 10 \times 8 + 3 = 80 + 3 = 83$$

[3]

Q. 28. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw, and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

[NCERT Exemp. Ex. 3.4, Q. 6, Page 33]

Sol. Let the speed of the rickshaw and the bus are x and y km/h, respectively.

Now, she has taken time to travel 2 km by rickshaw,

$t_1 = \frac{2}{x}$ h $\left[\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right]$

And she has taken time to travel remaining distance, *i.e.*,

$(14 - 2) = 12$ km by bus $= t_2 = \frac{12}{y}$ h.

By first condition,

$$t_1 + t_2 = \frac{1}{2}$$

$$\Rightarrow \frac{2}{x} + \frac{12}{y} = \frac{1}{2} \quad \text{...(i)}$$

Now, she has taken time to travel 4 km by rickshaw,

$t_3 = \frac{4}{x}\ h$

And she has taken time to travel remaining distance, *i.e.*, $(14 - 4) = 10$ km by bus $t_4 = \frac{10}{y}\ h$.

By second condition,

$$t_3 + t_4 = \frac{1}{2} + \frac{9}{60} = \frac{1}{2} + \frac{3}{20}$$

$$\Rightarrow \frac{4}{x} + \frac{10}{y} = \frac{13}{20} \quad \text{...(ii)}$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$, then Eqs. (i) and (ii) becomes,

$$2u + 12v = \frac{1}{2} \quad \text{...(iii)}$$

And $4u + 10v = \frac{13}{20}$

On multiplying in Eq. (iii) by 2 and then subtracting, we get

$$\begin{array}{r} 4u + 24v = 1 \\ 4u + 10v = \frac{13}{20} \\ - \quad - \quad - \\ \hline 14v = 1 - \frac{13}{20} \end{array}$$

$$\Rightarrow 14v = \frac{7}{20}$$

$$\therefore \quad v = \frac{7}{20 \times 14} = \frac{1}{40}$$

Now, put the value of v in Eq. (iii), we get

$$2u + 12\left(\frac{1}{40}\right) = \frac{1}{2}$$

$$\Rightarrow 2u = \frac{1}{2} - \frac{3}{10} = \frac{5-3}{10}$$

$$\Rightarrow 2u = \frac{2}{10}$$

$$\Rightarrow u = \frac{1}{10}$$

$$\because \frac{1}{x} = u$$

$$\Rightarrow \frac{1}{x} = \frac{1}{10}$$

$$\therefore x = 10 \text{ km/h}$$

And

$$\frac{1}{y} = v$$

$$\Rightarrow \frac{1}{y} = \frac{1}{40}$$

$$\therefore y = 40 \text{ km/h}$$

Hence, the speed of rickshaw and the bus are 10 km/h and 40 km/h, respectively. **[3]**

Q. 29. Determine, algebraically, the vertices of the triangle formed by the lines

$$3x - y = 3$$
$$2x - 3y = 2$$
$$x + 2y = 8$$

[NCERT Exemp. Ex. 3.4, Q. 5, Page 33]

Sol. $3x - y = 3$ ...(i)

$2x - 3y = 2$ ...(ii)

$x + 2y = 8$ ...(iii)

$$\begin{array}{r} 9x - 3y = 9 \\ 2x - 3y = 2 \\ - \quad + \quad - \\ \hline 7x = 7 \end{array}$$

$$x = \frac{7}{7} = 1$$

$$\therefore x = 1$$

Fig.

On putting the value of x in Eq. (i), we get

$$3 \times 1 - y = 3$$

$$\therefore y = 0$$

So, the co-ordinate of point or vertex B is (1, 0).

On solving lines (ii) and (iii), we will get the intersecting point C.

On multiplying Eq. (iii) by 2 and then subtracting, we get

$$\begin{array}{r} 2x + 4y = 16 \\ 2x - 3y = 2 \\ - \quad + \quad - \\ \hline 7y = 14 \end{array}$$

$$y = \frac{14}{7} = 2$$

$$\therefore y = 2$$

On putting the value of y in Eq. (iii), we get

$$x+2\times 2=8$$
$$\Rightarrow \quad x=8-4=4$$
$$\therefore \quad x=4$$

Hence, the co-ordinate of point or vertex C is (4, 2).
On solving lines (iii) and (i), we will get the intersecting point A.
On multiplying in Eq. (i) by 2 and then adding Eq. (iii), we get

$$\begin{array}{r} 6x-2y=6 \\ x+2y=8 \\ \hline 7x=14 \end{array}$$

$$x=\frac{14}{7}=2$$
$$\therefore \quad x=2$$

On putting the value of x in Eq. (i), we get
$$3\times 2-y=3$$
$$\Rightarrow \quad y=6-3$$
$$\Rightarrow \quad y=3$$ **[3]**

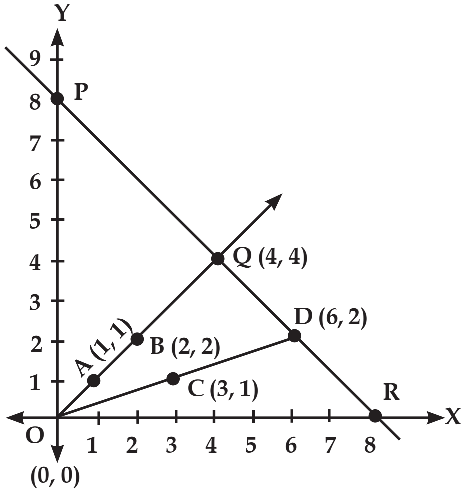
Hence, the coordination of A vertex will be (2, 3).

Q. 30. Determine, graphically, the vertices of the triangle formed by the lines $y = x$, $3y = x$, and $x + y = 8$.
[NCERT Exemp. Ex. 3.4, Q. 2, Page 33]

Sol. **Given :** Equations of lines

$y = x$ ...(i)
$3y = x$ ...(ii)
$x + y = 8$ ...(iii)

For lines $y = x$
If $x = 0$, then $y = 0$
If $x = 1$, then $y = 1$
So, two points will be $A(0, 0)$, B (1, 1)
For lines $3y = x$
If $x = 0$, then $y = 0$
If $y = 1$, then $x = 3$
So, two points will be $A(0, 0)$, C (3 – 1)
For lines $x + y = 8$
If $x = 0$, then $y = 8$
If $y = 0$, then $x = 8$
So, two points will be D (0, 8) and E (8, 0)



Hence, the required vertices of ΔAQR will be (0, 0), (4, 4) and (6, 2)

Q. 31. The angles of a cyclic quadrilateral $ABCD$ are $\angle A = (6x + 10)°$, $\angle B = (5x)°$, $\angle C = (x + y)°$ and $\angle D = (3y - 10)°$. Find x and y, and hence the values of the four angles.
[NCERT Exemp. Ex. 3.3, Q. 22, Page 28]

Sol. **Since,** $\angle A + \angle C = 180°$

$$7x+y=170 \quad \text{...(i)}$$
$[\because \angle A=(6x+10)°, \angle C=(x+y)°$, given]
And $\angle B+\angle D=(5x)°+(3y-10)°=180°$
$[\because \angle B=(5x)°,\ \angle D=(3y-10)°$, given]
$$5x+3y=190° \quad \text{...(ii)}$$

On multiplying Eq. (i) by 3 and then subtracting, we get
$$3\times(7x+y)-(5x+3y)=510°-190°$$
$$\Rightarrow \quad 21x+3y-5x-3y=320°$$
$$\Rightarrow \quad 16x=320°$$
$$\therefore \quad x=20°$$

On putting $x = 20°$ in Eq. (i), we get
$$7\times 20+y=170°$$
$$\Rightarrow \quad y=170°-140°$$
$$\Rightarrow \quad y=30°$$

Therefore,
$$\angle A=(6x+10)°=6\times 20°+10°$$
$$=120°+10°=130°$$
$$\angle B=(5x)°=5\times 20°=100°$$
$$\angle C=(x+y)°=20°+30°=50°$$
$$\angle D=(3y-10)°=3\times 30°-10°$$
$$=90°-10°=80°$$ **[3]**

Q. 32. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.
[NCERT Exemp. Ex. 3.3, Q. 18, Page 28]

Sol. Let the two numbers be x and y.
Then, by first condition, ratio of these two numbers = 5 : 6
$$x:y=5:6$$
$$\Rightarrow \frac{x}{y}=\frac{5}{6}$$
$$\therefore \quad y=\frac{6x}{5} \quad \text{...(i)}$$

And by second condition, when, 8 is subtracted from each of the numbers, then ratio becomes 4 : 5.
$$\frac{x-8}{y-8}=\frac{4}{5}$$
$$\Rightarrow 5x-40=4y-32$$
$$\Rightarrow 5x-4y=8 \quad \text{...(ii)}$$

Now, put the value of y in Eq. (ii), we get
$$5x-4\left(\frac{6x}{5}\right)=8$$
$$\Rightarrow 25x-24x=40$$
$$\therefore \quad x=40$$

Put the value of x in Eq. (i), we get

$$\therefore y = \frac{6}{5} \times 40 = 6 \times 8 = 48$$

Hence, the required numbers are 40 and 48. **[3]**

Q. 33. There are some students in the two examination halls *A* and *B*. To make the number of students equal in each hall, 10 students are sent from *A* to *B*. But if 20 students are sent from *B* to *A*, the number of students in *A* becomes double the number of students in *B*. Find the number of students in the two halls. [NCERT Exemp. Ex. 3.3, Q. 19, Page 28]

Sol. Let the number of students in halls A and B are x and y, respectively.

Now, by given condition,

$$x - 10 = y + 10$$

$$x - y = 20 \quad \text{...(i)}$$

And $(x + 20) = 2(y - 20)$

$$x - 2y = -60 \quad \text{...(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$(x - y) - (x - 2y) = 20 + 60$$

$$x - y - x + 2y = 80$$

$$\therefore \quad y = 80$$

On putting $y = 80$ in Eq. (i), we get

$x - 80 = 20 \Rightarrow x = 100$ and $y = 80$

Hence, 100 students are there in hall A and 80 students are in hall B. **[3]**

Q. 34. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now? [NCERT Exemp. Ex. 3.3, Q. 16, Page 28]

Sol. Let Salim and his daughter's age be x and y, respectively.

Now, by first condition,

Two years ago, Salim was thrice as old as his daughter.

$$x - 2 = 3(y - 2)$$

$$\Rightarrow \quad x - 2 = 3y - 6$$

$$\Rightarrow x - 3y = -4 \quad \text{...(i)}$$

And by second condition, 6 years later, Salim will be 4 years older than twice her age.

$$x + 6 = 2(y + 6) + 4$$

$$\Rightarrow \quad x + 6 = 2y + 12 + 4$$

$$\Rightarrow \quad x - 2y = 16 - 6$$

$$\Rightarrow \quad x - 2y = 10 \quad \text{...(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$\begin{array}{l} x - 2y = 10 \\ x - 3y = -4 \\ - \quad + \quad + \\ \hline \quad y = 14 \end{array}$$

Put the value of y in Eq. (ii), we get

$$\begin{array}{l} x - 2y = 10 \\ x - 3y = -4 \\ - \quad + \quad + \\ \hline \quad y = 14 \end{array}$$

Hence, Salim and his daughter's ages are 38 years and 14 years, respectively. **[3]**

Q. 35. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father. [NCERT Exemp. Ex. 3.3, Q. 17, Page 28]

Sol. Let the present age (in years) of father and his two children be x, y and z years respectively.

Now by given condition,

$$x = 2(y + z) \quad \text{...(i)}$$

And after 20 years,

$$(x + 20) = (y + 20) + (z + 20)$$

$$\Rightarrow y + z + 40 = x + 20$$

$$\Rightarrow \quad y + z = x - 20$$

On putting the value of $(y + z)$ in Eq. (i) and get the present age of father,

$$x = 2(x - 20)$$

$$x = 2x - 40$$

$$2x - x = 40$$

$$\therefore \quad x = 40$$

Hence, the father's present age is 40 years. **[3]**

Q. 36. The angles of a triangle are *x*, *y* and 40°. The difference between the two angles *x* and *y* is 30°. Find *x* and *y*. [NCERT Exemp. Ex. 3.3, Q. 15, Page 27]

Sol. Given that, x, y and 40° are the angels of a triangle.

$$x + y + 40° = 180°$$

[Since the sum of all the angels of a triangle is 180°.]

$$\Rightarrow \quad x + y = 140° \quad \text{...(i)}$$

Also, $x - y = 30°$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2x = 170°$$

$$\Rightarrow \quad x = \frac{170°}{2} = 85°$$

$$\therefore \quad x = 85°$$

On putting $x = 85°$ in Eq. (i), we get

$$85° + y = 140°$$

$$y = 140° - 85° = 55°$$

$$\therefore \quad y = 55°$$

Hence, the required values of x and y are 85° and 55°, respectively. **[3]**

Q. 37. If $x+1$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that $2a - 3b = 4$. [NCERT Exemp. Ex. 3.3, Q. 14, Page 27]

Sol. $\Rightarrow$ Put $x = -1$ in given expression

$$\Rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$$

$$\Rightarrow \quad -2 + a - 2b + 1 = 0$$

$$\Rightarrow \quad a - 2b - 1 = 0 \quad \text{...(i)}$$

Also, $2a - 3b = 4$

$$\Rightarrow \quad 3b = 2a - 4$$

$$\Rightarrow \quad b = \left(\frac{2a - 4}{3}\right) \quad \text{...(ii)}$$

Now, put the value of b in Eq. (i), we get

$$a-2\left(\frac{2a-4}{3}\right)-1=0$$

$$\Rightarrow \quad 3a-2(2a-4)-3=0$$

$$\Rightarrow \quad 3a-4a+8-3=0$$

$$\Rightarrow \quad -a+5=0$$

$$\Rightarrow \quad a=5$$

Now, put the value of a in Eq. (ii), we get

$$b=\frac{2\times-4}{3}$$

$b = 2$

Hence, the required value of ab will be 5 and 2 respectively. **[3]**

Q. 38. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.

[NCERT Exemp. Ex. 3.3, Q. 12, Page 27]

Sol. The given pair of linear equations :

$2x + y = 4$

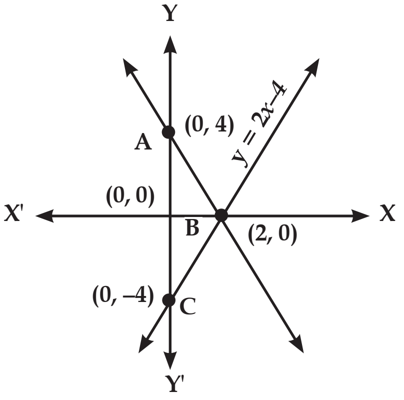
and $2x - y = 4$

Table for line $2x + y = 4$,

x	0	2
$y = 4 - 2x$	4	0
Points	A	B

And table for line $2x - y = 4$,

x	0	2
$y = 2x - 4$	-4	0
Points	C	B



Here, both lines and y-axis form ΔABC.

Hence, the vertices of ΔABC are $A(0, 4)$ $B(2, 0)$ and $C(0, -4)$.

$\therefore$ Required area of $\Delta ABC = 2\times$ Area of ΔAOB

$$=2\times\frac{1}{2}\times4\times2=8 \text{ sq. units}$$

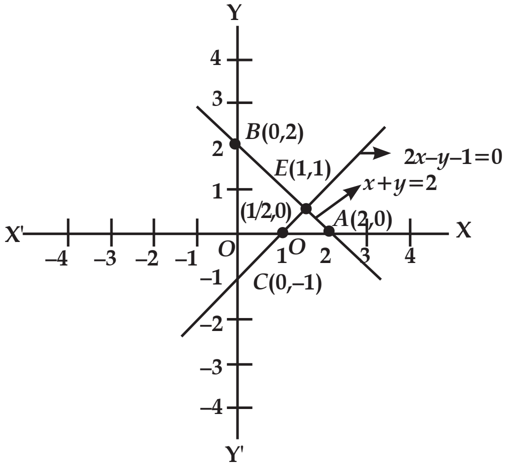
Hence, the required area of the triangle is 8 sq. units. **[3]**

Q. 39. Write an equation of a line passing through the point representing solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How many such lines can we find?

[NCERT Exemp. Ex. 3.3, Q. 13, Page 27]

Sol. If $x = 0$, then $y = -1$; if $x=\frac{1}{2}$, then $y = 0$ and if $x = 1$, then $y = 1$

x	0	½	1
y	-1	0	1
Points	C	D	E



[3]

Q. 40. Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream. [CBSE Board, Delhi Region, 2017]

Sol. Let the speed of stream be x km/hr.

$\therefore$ Speed of boat upstream $= (15 - x)$ km/hr.

Speed of boat downstream $= (15 + x)$ km/hr.

According to question,

$$\frac{30}{15-x}+\frac{30}{15+x}=4\frac{1}{2}$$

$$\frac{30}{15-x}+\frac{30}{15+x}=\frac{9}{2}$$

$$\Rightarrow \quad \frac{30(15+x+15-x)}{(15-x)(15+x)}=\frac{9}{2}$$

$$\Rightarrow \quad 200=225-x^2$$

$$\Rightarrow \quad x^2=225-200=25$$

$$\Rightarrow \quad x^2=5^2$$

$$\therefore \quad x=5$$

Thus, speed of stream = 5 km/h. **[3]**

Q. 41. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

[CBSE Board, All India Region, 2016]

Sol. Let the speed of the stream be s km/h.

Speed of the motor boat 24 km/h

Speed of the motor boat upstream $24 - s$ km/h.

Speed of the motor boat downstream $24 + s$ km/h.

According to the given condition,

$$\frac{32}{24-s} - \frac{32}{24+s} = 1$$

$$\therefore 32\left(\frac{1}{24-s} - \frac{1}{24+s}\right) = 1$$

$$\therefore\ 32\left(\frac{24+s-24+s}{576-s^2}\right) = 1$$

$$\therefore \qquad 32 \times 2s = 576 - s^2$$

$$\therefore \qquad s^2 + 64s - 576 = 0$$

$$\therefore \qquad (s+72)(s-8) = 0$$

Therefore,

$s = -72$ or $s = 8$

Since, speed of the stream cannot be negative, the speed of the stream is 8 km/h. **[3]**

Q. 42. Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$ and $\frac{x}{8} + \frac{y}{6} = 15$. Hence, find λ, if $y = \lambda x + 5$.

[NCERT Exemp. Ex. 3.3, Q. 10, Page 27]

Sol. Given pair of equations is :

$$\frac{x}{10} + \frac{y}{5} - 1 = 0 \qquad \text{...(i)}$$

And $\frac{x}{8} + \frac{y}{6} = 15$ ...(ii)

Now, multiplying both sides of Eq. (i) by *LCM* (10, 5) = 10, we get

$$x + 2y - 10 = 0$$

$$\Rightarrow x + 2y = 10 \qquad \text{...(iii)}$$

Again, multiplying both sides of Eq. (iv) by *LCM* (8, 6) = 24, we get

$$3x + 4y = 360 \qquad \text{...(iv)}$$

On, multiplying Eq. (iii) by 2 and then subtracting from Eq. (iv), we get

$$3x + 4y = 360$$
$$2x + 4y = 20$$
$$- \quad - \qquad -$$
$$x = 340$$

Put the value of x in Eq. (iii), we get

$$340 + 2y = 10$$

$$\Rightarrow \qquad 2y = 10 - 340 = -330$$

$$\Rightarrow \qquad y = -165$$

Given that, the linear relation between x, y and λ is $y = \lambda x + 5$

Now, put the values of x and y in above relation, we get

$$-165 = \lambda(340) + 5$$

$$\Rightarrow 340\lambda = -170$$

$$\therefore \qquad \lambda = -\frac{1}{2}$$

Hence, the solution of the pair of equations is $x = 340$, $y = -165$ and the required value of λ is $-\frac{1}{2}$. **[3]**

Q. 43. By the graphical method, find whether the following pairs of equations are consistent or not. If consistent, solve them.

(i) $3x + y + 4 = 0$

$6x - 2y + 4 = 0$

(ii) $x - 2y = 6s$

$3x - 6y = 0$

(iii) $x + y = 3$

$3x + 3y = 9$ **[NCERT Exemp. Ex. 3.3, Q. 11, Page 27]**

Sol. **(i)** Given pair of equations is :

$$3x + y + 4 = 0 \qquad \text{...(i)}$$

And $6x - 2y + 4 = 0$ ...(ii)

On comparing with $ax + by + c = 0$, we get

$a_1 = 3,\ b_1 = 1$

And $c_1 = 4$ [from Eq. (i)]

$a_2 = 6,\ b_2 = -2$

And $c_2 = 4$ [From Eq. (ii)]

Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2};\ \frac{b_1}{b_2} = -\frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{4}{4} = \frac{1}{1}$

$$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the given pair of linear equations intersects at one point, therefore these lines have unique solution.

Hence, given pair of linear equations is consistent.

We have, $3x + y + 4 = 0$

$\Rightarrow y = -4 - 3x$

x	0	−1	−2
y	−4	−1	2
Points	B	C	A

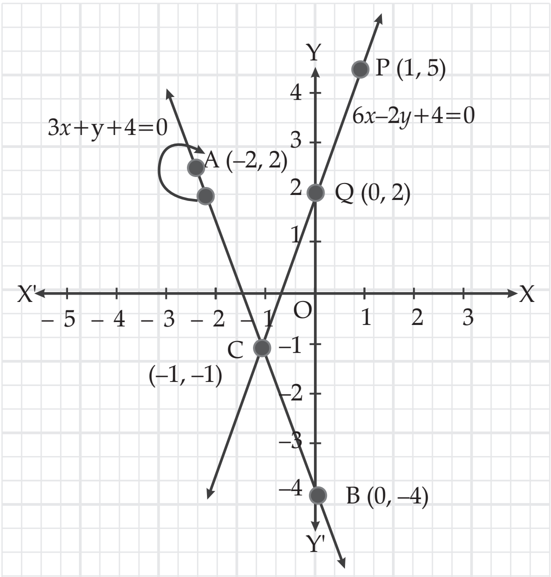
And $6x - 2y + 4 = 0$

$\Rightarrow 2y = 6x + 4$

$\Rightarrow \quad y = 3x + 2$

x	−1	0	1
y	−1	2	5
Points	C	Q	P

Plotting the points $B(0, -4)$ and $A(-2, 2)$, we get the straight tine AB. Plotting the points $Q(0, 2)$ and $P(1, 5)$, we get the straight line PQ. The lines AB and PQ intersect at $C(-1, -1)$.



[3]

(ii) Given pair of equations is $x - 2y = 6$ ...(i)

And $3x - 6y = 0$ ...(ii)

On comparing with $ax + by + c = 0$, we get

$a_1 = 1, b_1 = -2$ and $c_1 = -6$ [From Eq. (i)]

$a_2 = 3, b_2 = -6$ and $c_2 = 0$ [From Eq. (ii)]

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{-6} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-6}{0}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines represented by the given equations are parallel. Therefore, it has no solution. So, the given pair of equations is inconsistent. **[3]**

(iii) Given pair of equations is $x + y = 3$ ...(i)

And $3x + 3y = 9$ ...(ii)

On comparing with $ax + by + c = 0$, we get

$a_1 = 1, b_1 = 1$ and $c_1 = -3$ [From Eq. (i)]

$a_2 = 3, b_2 = 3$ and $c_2 = -9$ [From Eq. (ii)]

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-3}{-9} = \frac{1}{3}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given pair of equations is consistent. Therefore, these lines have infinitely many solutions.

Now, $x + y = 3$

$\Rightarrow y = 3 - x$

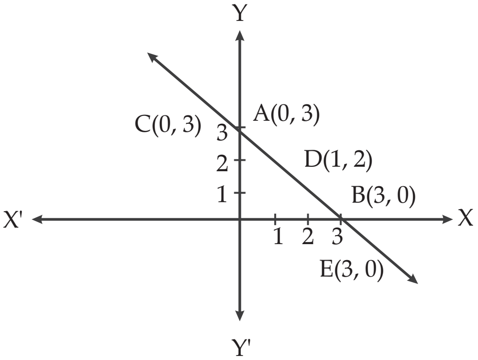
x	0	3
y	3	0
Points	A	B

and $3x + 3y = 9$

$\Rightarrow 3y = 9 - 3x$

$\therefore y = \frac{9 - 3x}{3}$

x	0	1	3
y	3	2	0
Points	C	D	E



Plotting the points $A(0, 3)$ and $B(3, 0)$, we get the line AB. Again, plotting the points $C(0, 3)$, $D(1, 2)$ and $E(3, 0)$, we get the line CDE.

We observe that the lines represented by Eqs. (i) and (ii) are coincident. **[3]**

Long Answer Type Questions **(5 marks each)**

Q. 1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

[NCERT Ex. 3.2, Q. 1, Page 49]

Sol. **(i)** Let the number of girls be x and the number of boys be y.

According to the question, the algebraic representation is

$x + y = 10$

$x - y = 4$

For $x + y = 10$,

$x = 10 - y$

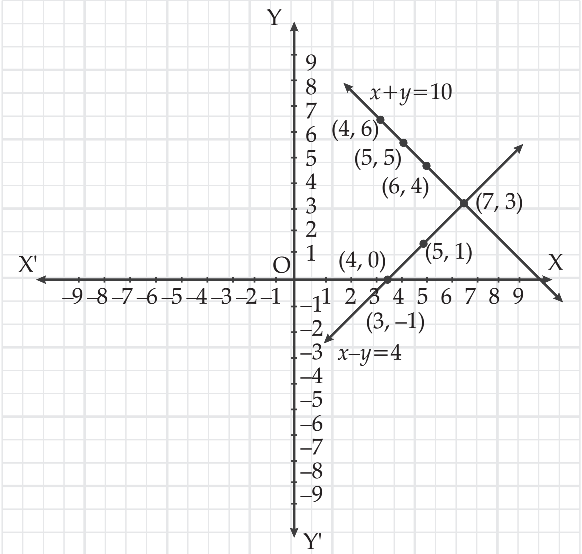
x	5	4	6
y	5	6	4

For $x - y = 4$,

$x = 4 + y$

x	5	4	3
y	1	0	−1

Hence, the graphical representation is as follows :



From the figure, it can be observed that these lines intersect each other at point (7, 3). **[2½]**

Therefore, the number of girls and boys in the class are 7 and 3, respectively.

(ii) Let the cost of 1 pencil be ₹ x and the cost of 1 pen be ₹ y.

According to the question, the algebraic representation is

$5x + 7y = 50$

$7x + 5y = 46$

For $5x + 7y = 50$,

$$x = \frac{50 - 7y}{5}$$

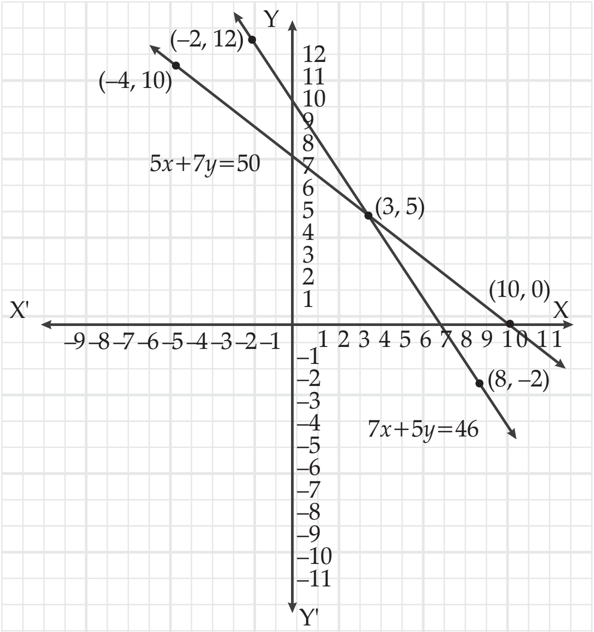
x	3	10	−4
y	5	0	10

$7x + 5y = 46$

$$x = \frac{46 - 5y}{7}$$

x	8	3	−2
y	−2	5	12

Hence, the graphical representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (3, 5). **[2½]**

Therefore, the cost of a pencil and a pen are ₹ 3 and ₹ 5, respectively.

Q. 2. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

[NCERT Ex. 3.1, Q. 1, Page 44]

Sol. Let the present age of Aftab be x.

And, present age of his daughter $= y$

Seven years ago,

Age of Aftab $= x - 7$

Age of his daughter $= y - 7$

According to the question,

$(x-7) = 7(y-7)$

$x - 7 = 7y - 49$

$x - 7y = -42$ ...(i)

Three years hence,

Age of Aftab $= x + 3$

Age of his daughter $= y + 3$

According to the question,

$(x+3) = 3(y+3)$

$x + 3 = 3y + 9$

$x - 3y = 9 - 3$

$x - 3y = 6$ ...(ii)

Therefore, the algebraic representation is

$x - 7y = -42$

$x - 3y = 6$

For $x - 7y = -42$,

$$x = -42 + 7y$$

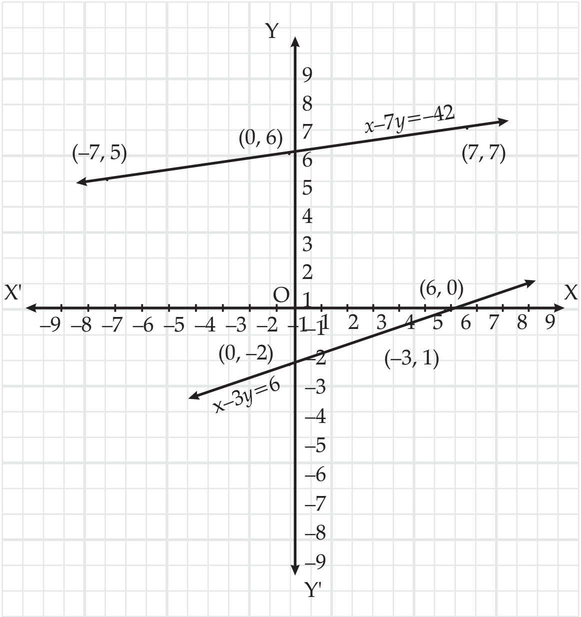
x	− 7	0	7
y	5	6	7

For $x - 3y = 6$,

$$x = 6 + 3y$$

x	6	3	0
y	0	− 1	− 2

The graphical representation is as follows.



[5]

Q. 3. The coach of a cricket team buys 3 bats and 6 balls for ₹ 3,900. Later, she buys another bat and 3 more balls of the same kind for ₹ 1,300. Represent this situation algebraically and geometrically.

[NCERT Ex. 3.1, Q. 2, Page 44]

Sol. Let cost of 1 cricket bat = ₹ x and
and, cost of 1 cricket ball = ₹ y

According to given conditions, we have

$3x + 6y = 3{,}900$

$\Rightarrow \quad x + 2y = 1{,}300$ ...(i)

And $x + 3y = 1{,}300$ ...(ii)

To represent them graphically, we will find 3 sets of points which lie on the lines.

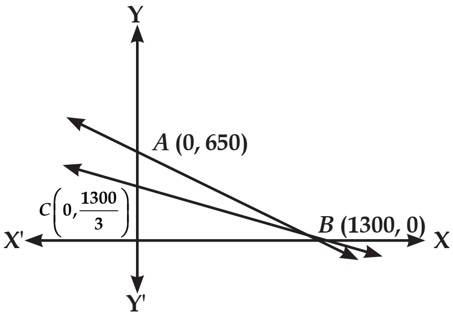
For equation $x + 2y = 1{,}300$, we have following points which lie on the line.

x	0	1,300
y	650	0

For equation $x + 3y = 1{,}300$, we have following points which lie on the line.

x	0	1,300
y	$\frac{1300}{3}$	0

We plot the points for both of the equations and it is the graphical representation of the given situation.



It is clear that these lines intersect at B (1300, 0). **[5]**

Q. 4. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹ 300. Represent the situation algebraically and geometrically. [NCERT Ex. 3.1, Q. 3, Page 44]

Sol. Let the cost of 1 kg of apples be ₹ x.

And, cost of 1 kg of grapes = ₹ y

According to the question, the algebraic representation is :

$2x + y = 160$

$4x + 2y = 300$

For $2x + y = 160$,

$y = 160 - 2x$

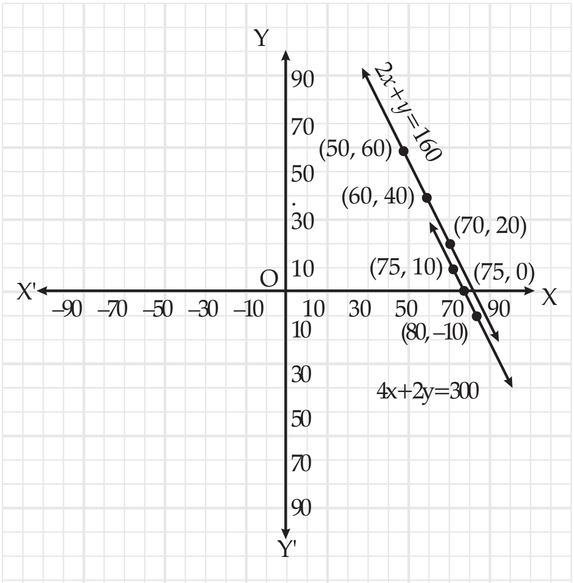
x	50	60	70
y	60	40	20

For $4x + 2y = 300$,

$$y = \frac{300 - 4x}{2}$$

x	70	80	75
y	10	−10	0

The graphical representation is as follows :



[5]

Q. 5. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5;\ 2x - 3y = 7$

(ii) $2x - 3y = 8;\ 4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7;\ 9x - 10y = 14$

(iv) $5x - 3y = 11;\ -10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8;\ 2x + 3y = 12$

[NCERT Ex. 3.2, Q. 3, Page 49]

Sol. (i) $3x + 2y = 5$

$2x - 3y = 7$

$$\frac{a_1}{a_2} = \frac{3}{2},\ \frac{b_1}{b_2} = \frac{2}{-3},\ \frac{c_1}{c_2} = \frac{5}{7}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Since, the condition is satisfied for unique solution.

So, These linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent. **[1]**

(ii) $2x - 3y = 8$

$4x - 6y = 9$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},\ \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2},\ \frac{c_1}{c_2} = \frac{8}{9}$$

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2},$$

Therefore, these linear equations are parallel to each other and thus have no possible solutions. Hence, the pair of linear equations is inconsistent. **[1]**

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

$9x - 10y = 14$

$$\frac{a_1}{a_2} = \frac{3/2}{9} = \frac{3}{2 \times 9} = \frac{1}{6}$$

$$\frac{b_1}{b_2} = \frac{5/3}{-10} = \frac{5}{3 \times (-10)} = \frac{-1}{6}$$

$$\frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$$

$$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Since, the obtained condition satisfied the unique solution condition : **[1]**

(iv) $5x - 3y = 11$

$-10x + 6y = -22$

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2},\ \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2},\ \frac{c_1}{c_2} = \frac{11}{-22} = \frac{-1}{2}$$

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent. **[1]**

(v) $\frac{4}{3}x + 2y = 8$

$2x + 3y = 12$

$$\frac{a_1}{a_2} = \frac{4/3}{2} = \frac{2}{3},\ \frac{b_1}{b_2} = \frac{2}{3},\ \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

$$\because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent. **[1]**

Q. 6. Formulate the following problems as a pair of equations, and hence find their solutions :

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

[NCERT Ex. 3.6, Q. 2, Page 67]

Sol. **(i)** Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h, respectively.

Speed of Ritu while rowing

Upstream = $(x - y)$ km/h

Downstream = $(x + y)$ km/h

According to question,

$2(x + y) = 20$

$\Rightarrow x + y = 10$ ...(i)

$2(x - y) = 4$

$\Rightarrow x - y = 2$ ...(ii)

Adding equations (i) and (ii), we obtain

$2x = 12 \Rightarrow x = 6$

Putting this in equation (i), we obtain

$y = 4$

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h. **[5]**

(ii) Let the number of days taken by a woman and a man be x and y, respectively.

Therefore, work done by a woman in 1 day = $\frac{1}{x}$

Work done by a man in 1 day = $\frac{1}{y}$

According to the question,

$$4\left(\frac{2}{x} + \frac{5}{y}\right) = 1$$

$$\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$$

$$3\left(\frac{3}{x} + \frac{6}{y}\right) = 1$$

$$\frac{3}{x} + \frac{6}{y} = \frac{1}{3}$$

Putting $\frac{1}{x} = p$ and $\frac{1}{y} = q$ in these equations, we obtain

$$2p + 5q = \frac{1}{4}$$

$$\Rightarrow 8p + 20q = 1$$

$$3p + 6q = \frac{1}{3}$$

$$\Rightarrow 9p + 18q = 1$$

By cross-multiplication, we obtain

$$\frac{p}{-20-(-18)} = \frac{q}{-9-(-8)} = \frac{1}{144-180}$$

$$\frac{p}{-2} = \frac{q}{-1} = \frac{1}{-36}$$

$$\frac{p}{-2} = \frac{-1}{36} \text{ and } \frac{q}{-1} = \frac{1}{-36}$$

$$p = \frac{1}{18} \text{ and } q = \frac{1}{36}$$

$$p = \frac{1}{x} = \frac{1}{18} \text{ and } q = \frac{1}{y} = \frac{1}{36}$$

$x = 18 \qquad y = 36$

Hence, number of days taken by the woman = 18

Number of days taken by the man = 36 **[5]**

(iii) Let the speed of train and bus be u km/h and v km/h, respectively.

According to the given information,

$$\frac{60}{u} + \frac{240}{v} = 4 \quad \text{...(i)}$$

$$\frac{100}{u} + \frac{200}{v} = \frac{25}{6} \quad \text{...(ii)}$$

Putting $\frac{1}{u} = p$ and $\frac{1}{v} = q$ in these equations, we obtain

$60p + 240q = 4$ ...(iii)

$100p + 200q = \frac{25}{6}$

$600p + 1,200q = 25$ ...(iv)

Multiplying equation (iii) by 10, we obtain

$600p + 2,400q = 40$ ...(v)

Subtracting equation (iv) from equation (v), we obtain

$1,200q = 15$

$q = \frac{15}{1,200} = \frac{1}{80}$ ...(vi)

Substituting in equation (iii), we obtain

$60p + 3 = 4$

$60p = 1$

$p = \frac{1}{60}$

$p = \frac{1}{u} = \frac{1}{60}$ and $q = \frac{1}{v} = \frac{1}{80}$

u = 60 km/h and v = 80 km/h

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h **[5]**

Q. 7. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train. [NCERT Ex. 3.7, Q. 3, Page 68]

Sol. Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel was d km. We know that,

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$$

$x = \frac{d}{t}$

Or, $d = xt$ ...(i)

Using the information given in the question, we obtain

$(x + 10) = \frac{d}{(t-2)}$

$(x + 10)(t - 2) = d$

$xt + 10t - 2x - 20 = d$

By using equation (i), we obtain

$-2x + 10t = 20$ ...(ii)

$(x - 10) = \frac{d}{(t+3)}$

$(x - 10)(t + 3) = d$

$xt - 10t + 3x - 30 = d$

By using equation (i), we obtain

$3x - 10t = 30$ ...(iii)

Adding equations (ii) and (iii), we obtain

$x = 50$

Using equation (ii), we obtain

$(-2) \times (50) + 10t = 20$

$-100 + 10t = 20$

$10t = 120$

t = 12 hours

From equation (i), we obtain

Distance to travel, $d = xt = 50 \times 12 = 600$ km

Hence, the distance covered by the train is 600 km. **[5]**

Q. 8. Vijay had some bananas, and he divided them into two lots *A* and *B*. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of Re 1 per banana, and got a total of ₹ 400. If he had sold the first lot at the rate of Re 1 per banana, and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

[NCERT Exemp. Ex. 3.4, Q. 13, Page 34]

Sol. **Case I :** Cost of first lot at the rate of ₹ 2 for 3 bananas + Cost of second lot at the rate of ₹ 1 per banana = Amount received

$\Rightarrow \frac{2}{3}x + y = 400$

$\Rightarrow 2x + 3y = 1,200$ **(i)**

Case II : Cost of first lot at the rate of ₹ 1 per banana + Cost of second lot at the rate of ₹ 4 for 5 bananas = Amount received

$\Rightarrow x + \frac{4}{5}y = 460$

$\Rightarrow 5x + 4y = 2,300$

On multiplying in Eq. (i) by 4 and Eq. (ii) by 3 and then subtracting them, we get

$$\begin{array}{r} 8x + 12y = 4,800 \\ \underline{15x + 12y = 6,900} \\ -7x = -2,100 \end{array}$$

$\therefore \quad x = 300$

Now, put the value of x in Eq. (i), we get

$2 \times 300 + 3y = 1,200$

$\Rightarrow \quad 600 + 3y = 1,200$

$\Rightarrow \quad 3y = 1,200 - 600$

$\Rightarrow \quad 3y = 600$

$\therefore \quad y = 200$

$\therefore$ Total number of bananas = Number of bananas in lot A + Number of bananas in lot B

$= x + y = 300 + 200 = 500$

Hence, he had 500 bananas. **[5]**

Q. 9. Susan invested certain amount of money in two schemes *A* and *B*, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received ₹ 1,860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?

[NCERT Exemp. Ex. 3.4, Q. 12, Page 34]

Sol. **Case I :**

$\Rightarrow \frac{x \times 8 \times 1}{100} + \frac{y \times 9 \times 1}{100} =$ ₹1,860

$$\left[\because \text{Simple interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}\right]$$

$\Rightarrow 8x+9y=1,86,000$ ...(i)

Case II : Interest at the rate of 9% per annum on scheme A + interest at the rate of 8% per annum on scheme B = ₹ 20 more as annual interest

$\Rightarrow \frac{x\times9\times1}{100}+\frac{y\times8\times1}{100}$ = ₹ 20 + ₹ 1,860

$\Rightarrow \frac{9x}{100}+\frac{8y}{100}=1,880$

$\Rightarrow 9x+8y=1,88,000$

On multiplying Eq. (i) by 9 and Eq. (ii) by 8 and then subtracting them, we get

$$72x+81y=9\times1,86,000$$
$$72x+64y=8\times1,88,000$$
$$-\quad-\qquad-$$

$$\Rightarrow \quad 17y=1,000[(9\times186)-(8\times188)]$$
$$=1,000(1,674-1,504)=1,000\times170$$
$$17y=1,70,000\Rightarrow y=10,000$$

On putting the value of y in Eq. (i), we get

$$8x+9\times10,000=1,86,000$$
$$\Rightarrow \quad 8x=1,86,000-90,000$$
$$\Rightarrow \quad 8x=96,000$$
$$\therefore \quad x=12,000$$

Hence, she had invested for scheme A ₹ 12,000 and for B scheme ₹ 10,000. **[5]**

Q. 10. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum ₹ 1,008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got ₹ 1,028. Find the cost price of the saree and the list price (price before discount) of the sweater.

[NCERT Exemp. Ex. 3.4, Q. 11, Page 34]

Sol. Let ₹ x be the cost price of saree & ₹ y be the list price of sweater.

Case I : Sells a saree at 8% profit + Sells a sweater at 10% discount = ₹ 1,008

$$\Rightarrow \quad (100+8)\%\text{ of }x+(100-10)\%\text{ of }y=1,008$$
$$\Rightarrow \quad 108\%\text{ of }x+90\%\text{ of }y=1,008$$
$$\Rightarrow \quad 1.08x+0.9y=1,008 \quad ...(i)$$

Case II : Sold the saree at 10% profit + Sold the sweater at 8% discount = ₹ 1,028

$$\Rightarrow \quad (100+10)\%\text{ of }x+(100-8)\%\text{ of }y=1,028$$
$$\Rightarrow \quad 110\%\text{ of }x+92\%\text{ of }y=1,028$$
$$\Rightarrow \quad 1.1x+0.92y=1,028 \quad ...(ii)$$

On putting the value of y from Eq. (i) into Eq. (ii), we get

$$1.1x+0.92\left(\frac{1,008-1.08x}{0.9}\right)=1,028$$
$$\Rightarrow 1.1\times0.9x+927.36-0.9936x=1,028\times0.9$$
$$\Rightarrow \quad 0.99x-0.9936x=9,252-927.36$$
$$\Rightarrow \quad -0.0036x=-2.16$$
$$\therefore \quad x=\frac{2.16}{0.0036}=600$$

On putting the value of x in Eq. (i), we get

$$1.08\times600+0.9y=1,008$$
$$\Rightarrow \quad 108\times6+0.9y=1,008$$
$$\Rightarrow \quad 0.9y=1,008-648$$
$$\Rightarrow \quad 0.9y=360$$
$$\therefore \quad y=\frac{360}{0.9}=400$$

[5]

Hence, x = ₹ 600 and y = ₹ 400

Q. 11. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs ₹ 2,530. Also, one reserved first class ticket and one reserved first class half ticket from A to B costs ₹ 3,810. Find the full first class fare from station A to B, and also the reservation charges for a ticket.

[NCERT Exemp. Ex. 3.4, Q. 10, Page 34]

Sol. **Case I :** The cost of one reserved first class ticket from the stations A to B = ₹ 2,530

$\Rightarrow x+y=2,530$ **(i)**

Case II : The cost of one reserved first class ticket and one reserved first class half ticket from stations A to B = ₹ 3,810

$$\Rightarrow x+y+\frac{x}{2}+y=3,810$$
$$\Rightarrow \quad \frac{3x}{2}+2y=3,810$$
$$\Rightarrow \quad 3x+4y=7,620 \quad \textbf{(ii)}$$

Now, multiplying Eq. (i) by 4 and then subtracting from eq. (ii), we get

$$3x+4y=7,620$$
$$4x+4y=10,120$$
$$-\quad-\qquad-$$
$$-x=-2,500$$
$$\Rightarrow x=2,500$$

On putting the value of x in Eq. (i), we get

$$2,500+y=2,530$$
$$\Rightarrow \quad y=2,530-2,500$$
$$\therefore \quad y=30$$

[5]

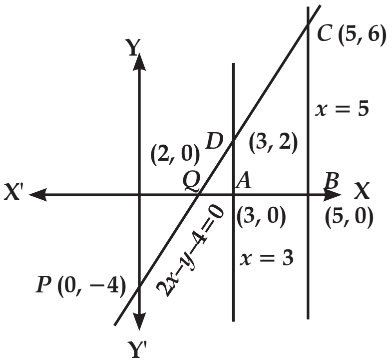
Q. 12. Draw the graphs of the equations x = 3, x = 5 and $2x-y-4=0$. Also find the area of the quadrilateral formed by the lines and the x-axis.

[NCERT Exemp. Ex. 3.4, Q. 3, Page 33]

Sol.

x	0	2
$y = 2x - 4$	−4	0
Points	P	Q

Draw the points P (0, −4) and Q (2, 0) and join these points and form a line PQ also draw the lines x = 3 and x = 5.



$\therefore$ Area of quadrilateral $ABCD = \frac{1}{2} \times$ distance between parallel lines $(AB) \times (AD + BC)$

[Since, quadrilateral $ABCD$ is a trapezium.]

$= \frac{1}{2} \times 2 \times (6+2)$

$[\because AB = OB - OA = 5 - 3 = 2, AD = 2 \text{ and } BC = 6]$

$= 8$ sq. units **[5]**

Q. 13. Graphically, solve the following pair of equations :

$2x + y = 6$

$2x - y + 2 = 0$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

[NCERT Exemp. Ex. 3.4, Q. 1, Page 33]

Sol. Given equations are $2x + y = 6$ and $2x - y + 2 = 0$

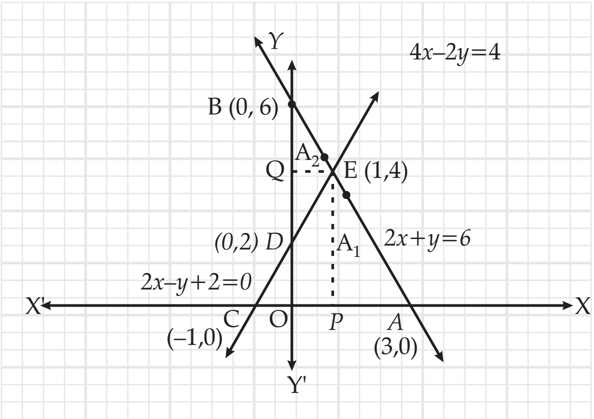
Table for equation $2x + y = 6$,

x	0	3
y	6	0
Points	B	A

Table for equation $2x - y + 2 = 0$,

x	0	−1
y	2	0
Points	D	C

Let A_1 and A_2 represent the areas of ΔACE and ΔBDE, respectively.



$$A_1 = \text{Area of } \Delta ACE = \frac{1}{2} \times AC \times PE = \frac{1}{2} \times 4 \times 4 = 8$$

$$A_2 = \text{Area of } \Delta BDE = \frac{1}{2} \times BD \times QE = \frac{1}{2} \times 4 \times 1 = 2$$

$\therefore A_1 : A_2 = 8 : 2 = 4 : 1$

Hence, the pair of equations intersect graphically at point $E(1, 4)$, *i.e.*, $x = 1$ and $y = 4$. **[5]**

Q. 14. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for six days, while Anand paid ₹ 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

[NCERT Exemp. Ex. 3.3, Q. 20, Page 25]

Sol. Let Latika takes a fixed charge for the first 2 days is ₹ x and additional charge for each day thereafter ₹ y.

Now by first condition,

Latika paid ₹ 22 for a book kept for 6 days,

$x + 4y = 22$ ...(i)

And by second condition,

Anand paid ₹ 16 for a book kept for 4 days,

$x + 2y = 16$ ...(ii)

Now, subtracting Eq. (ii) from Eq. (i), we get

$2y = 6 \Rightarrow y = 3$

On putting the value of y in Eq. (ii), we get

$x + 2 \times 3 = 16$

$x = 16 - 6 = 10$

Hence, the fixed charge = ₹ 10

And the charge for each extra day = ₹ 3 **[5]**

Q. 15. In a competitive examination, one mark is awarded for each correct answer while $\frac{1}{2}$ mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

[NCERT Exemp. Ex. 3.3, Q. 21, Page 28]

Sol. Let x be the number of correct answers of the questions, then $(120 - x)$ be the number of wrong answers of the questions.

Then, by given condition,

$$x \times 1 - (120 - x) \times \frac{1}{2} = 90$$

$$\Rightarrow \quad x - 60 + \frac{x}{2} = 90$$

$$\Rightarrow \quad \frac{3x}{2} = 150$$

$$\therefore \quad x = \frac{150 \times 2}{3} = 50 \times 2 = 100$$

Hence, Jayanti answered 100 questions correctly. **[5]**

Q. 16. Find the value(s) of p in (i) to (iv) and p and q in (v) for the following pair of equations :

(i) **$3x - y - 5 = 0$ and $6x - 2y - p = 0$, if the lines represented by these equations are parallel.**

(ii) **$-x + py = 1$ and $px - y = 1$, if the pair of equations has no solution.**

(iii) **$-3x + 5y = 7$ and $2px - 3y = 1$, if the lines represented by these equations are intersecting at a unique point.**

(iv) **$2x + 3y - 5 = 0$ and $px - 6y - 8 = 0$, if the pair of equations has a unique solution.**

(v) **$2x + 3y = 7$ and $2px + py = 28 - qy$, if the pair of equations have infinitely many solutions. Answer right** **[NCERT Exemp. Ex. 3.3, Q. 4, Page 25]**

Sol. **(i)** Given equations are

$3x - y - 5 = 0$ ...(i)

$6x - 2y - p = 0$ ...(ii)

$\therefore \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-p} = \frac{5}{p}$

The lines will be parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{2} = \frac{1}{2} \neq \frac{5}{p}$

$\Rightarrow p \Rightarrow 10$

So, the given linear are parallel for all real values of expect 10. **[1]**

(ii) Given pair of equations is

$-x + py = 1$ ...(i)

$px - y = 1$ ...(ii)

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \underset{\text{I}}{-\frac{1}{p}} = \underset{\text{II}}{\frac{p}{-1}} \neq \underset{\text{III}}{\frac{1}{1}}$

From ratio I and II, $p^2 = 1$ or $p = \pm 1$

Using ratios II and III, $p \neq -1$

$\therefore$ For $p = 1$, the given equations have no solution. **[1]**

(iii) $-3x + 5y = 7$ ...(i)

$2px - 3y = 1$ ...(ii)

For unique solution, we have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{-3}{2p} \neq \frac{5}{-3}$

$\Rightarrow 10p \neq +9$

$\Rightarrow \quad p \neq \frac{9}{10}$

Hence, the given equations have unique solution for all real values of p, except $\frac{9}{10}$. **[1]**

(iv) $2x + 3y - 5 = 0$ ...(i)

$px - 6y - 8 = 0$ ...(ii)

Pair of equations have unique solution if

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{2}{p} \neq \frac{3}{-6}$

$\Rightarrow 3p \neq -2 \times 6$

$\Rightarrow \quad p \neq -\frac{12}{3}$

$\Rightarrow \quad p \neq -4$

Hence, the system of linear equations has unique solution for all real values of p, except – 4. **[1]**

(v) $2px + py = 28 - qy$

$2px + (p + q)y = 28$ ...(i)

$2x + 3y = 7$ ...(ii)

The system of equations will have infinitely many solutions if,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \underset{\text{I}}{\frac{2p}{2}} = \underset{\text{II}}{\frac{p+q}{3}} = \underset{\text{III}}{\frac{28}{7}}$

Using ratios I and II we get,

$\frac{2p}{2} = \frac{p+q}{3}$

$\Rightarrow \quad 3p = p + q$

$\Rightarrow 2p - q = 0$

$\Rightarrow \quad q = 2p$ ...(iii)

Using ratios I and III, we get

$\frac{2p}{2} = \frac{28}{7} \Rightarrow p = 4$

$\therefore q = 2p = 2 \times 4 = 8$ [From (iii)]

$\therefore q = 8$, and $p = 4$

Now, $\frac{2p}{2} = \frac{p+q}{3} = \frac{28}{7}$

$\frac{p}{1} = \frac{p+q}{3} = \frac{4}{1}$

By substituting the values of p and q, we have

$4 = \frac{4+8}{3} = 4$

$\Rightarrow 4 = \frac{12}{3} = 4$

$\Rightarrow 4 = 4 = 4$

Hence, the given system of equations has infinitely many solutions when $p = 4$ and $q = 8$. **[1]**

Some Commonly Made Errors

- Students generally make a mistake while copying the terms.
- Errors in transformation and process skill in solving the word problems.
- Students make mistake in plotting the graph of the solution of a pair of linear equations.
- Students get confused with the application of different methods of solving the linear equations.
- They make computational and algebraic errors.
- Errors in taking the variable and substituting the wrong value.

EXPERT ADVICE

- ☞ *Learn different methods of solving equations through practicing.*
- ☞ *Study the difference between elimination and substitution methods.*
- ☞ *Try to learn using alternative algebraic strategies when solving problems.*
- ☞ *Remember that a system of linear equations is not completely solved until values for both x and y are found. To avoid this mistake, write all answers as an ordered pair.*
- ☞ *Learn how to interpret the information given in the equation graphically and algebraically.*