Chapter 2

Inverse Trigonometric Functions

1. If $\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0$, then x is equal to : [NCERT Exemplar]

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) 0 (d) 1

2. Which of the following corresponds to the principal value branch of $\tan^{-1}x$?

(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(c) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)-\{0\}$ (d) $(0,\pi)$

3. The principal value branch of $\sec^{-1}x$ is:

(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ (b) $[0,-\pi]-\left\{\frac{\pi}{2}\right\}$

(c) $(0,\pi)$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

4. The principal value of the expression $\cos^{-1}[\cos(-680°)]$ is :

(a) $\frac{2\pi}{9}$ (b) $\frac{-2\pi}{9}$

(c) $\frac{34\pi}{9}$ (d) $\frac{\pi}{9}$

5. The value of $\cot(\sin^{-1}x)$ is:

(a) $\frac{\sqrt{1+x^2}}{x}$ (b) $\frac{x}{\sqrt{1+x^2}}$

(c) $\frac{1}{x}$ (d) $\frac{\sqrt{1-x^2}}{x}$

6. The domain of $\sin^{-1}2x$ is:

(a) $[0,1]$ (b) $[-1,1]$

(c) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (d) $[-2,2]$

7. The principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is:

(a) $-\frac{2\pi}{3}$ (b) $-\frac{\pi}{3}$

(c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$

8. The domain of $y=\cos^{-1}(x^2-4)$ is:

(a) $[3,5]$

(b) $[0,\pi]$

(c) $[-\sqrt{5},-\sqrt{3}]\cap[-\sqrt{5},\sqrt{3}]$

(d) $[-\sqrt{5},-\sqrt{3}]\cup[\sqrt{3},\sqrt{5}]$

9. The domain of the function defined by $f(x)=\sin^{-1}x+\cos x$ is:

(a) $[-1,1]$ (b) $[-1,\pi+1]$

(c) $(-\infty,\infty)$ (d) ϕ

10. The value of $\sin[2\sin^{-1}(0.6)]$ is:

(a) ·48 (b) ·96

(c) 1·2 (d) sin 1·2

11. The value of $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right\}$ is :

(a) $\frac{\sqrt{29}}{3}$ (b) $\frac{29}{3}$

(c) $\frac{\sqrt{3}}{29}$ (d) $\frac{3}{29}$

12. If $\cot^{-1}\left(-\frac{1}{5}\right)=\theta$, the value of $\sin\theta$ is :

(a) $\frac{\sqrt{26}}{5}$ (b) $\frac{-5}{\sqrt{26}}$

(c) $\frac{\sqrt{5}}{\sqrt{26}}$ (d) $\frac{5}{\sqrt{26}}$

13. If $\alpha\le 2\sin^{-1}x+\cos^{-1}x\le\beta$, then:

(a) $\alpha=\frac{-\pi}{2},\beta=\frac{\pi}{2}$

(b) $\alpha=0,\beta=\pi$

(c) $\alpha=\frac{-\pi}{2},\beta=\frac{3\pi}{2}$

(d) $\alpha=0,\beta=2\pi$

14. The value of $\tan^2(\sec^{-1}2)+\cot^2(\text{cosec}^{-1}3)$ is:

(a) 5 (b) 11

(c) 13 (d) 15

15. The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is :

(a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{3-\sqrt{5}}{2}$

(c) $\frac{-3+\sqrt{5}}{2}$ (d) $\frac{-3-\sqrt{5}}{2}$

16. The principal value of $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **is:**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) π

17. The principal value of $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ **is :**

(a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$ (d) None of these

18. The inverse of cosine function is defined in the intervals :

(a) $[-\pi, 0]$ (b) $\left[\frac{-\pi}{2}, 0\right]$

(c) $\left[0, \frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}, \pi\right]$

19. If $\sin^{-1} x = y$, **then :**

(a) $0 \le y \le x$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

(c) $0 < y < \pi$ (d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

20. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ **is equal to :**

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1

21. The value of

$$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$ **is :**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$

(c) $-\frac{\pi}{12}$ (d) $\frac{-}{1}$

22. The value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ **is :**

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{6}$

23. The value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ **is :**

(a) 0 (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{3}$

27. The domain of the function $\cos^{-1}(2x-1)$ **is :** **[NCERT Exemplar]**

(a) $[0, 1]$ (b) $[-1, 1]$

(c) $(-1, 1)$ (d) $[0, \pi]$

28. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ **is :** **[NCERT Exemplar]**

(a) $[1, 2]$ (b) $[-1, 1]$

(c) $[0, 1]$ (d) none of these

29. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ **is equal to :** **[NCERT Exemplar]**

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$

(c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$

30. Solve $\sin(\tan^{-1} x)$, $|x| < 1$ **is equal to :** **[NCERT]**

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

Choose the correct option :

(a) Both (A) and (R) are true and R is the correct explanation A.

(b) Both (A) and (R) are true but R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

31. Assertion (A) : $\sin^{-1}(\sin 3) = 3$

Reason (R) : For principal values $\sin^{-1}(\sin x) = +x$

32. Assertion (A) : The solution of system of equations

$$\cos^{-1} x + (\sin^{-1} y)^2 = \frac{p\pi^2}{4}$$

and $(\cos^{-1} x)(\sin^{-1} y)^2 = \frac{\pi^4}{16}$ is $x = \cos\frac{\pi^2}{4}$ and $y = \pm 1$, $\forall p \in I$.

Reason (R) : AM $\ge$ GM

33. Assertion (A) : If $\sum_{i=1}^{2n} \sin^{-1} x_i = n\pi, n \in N$

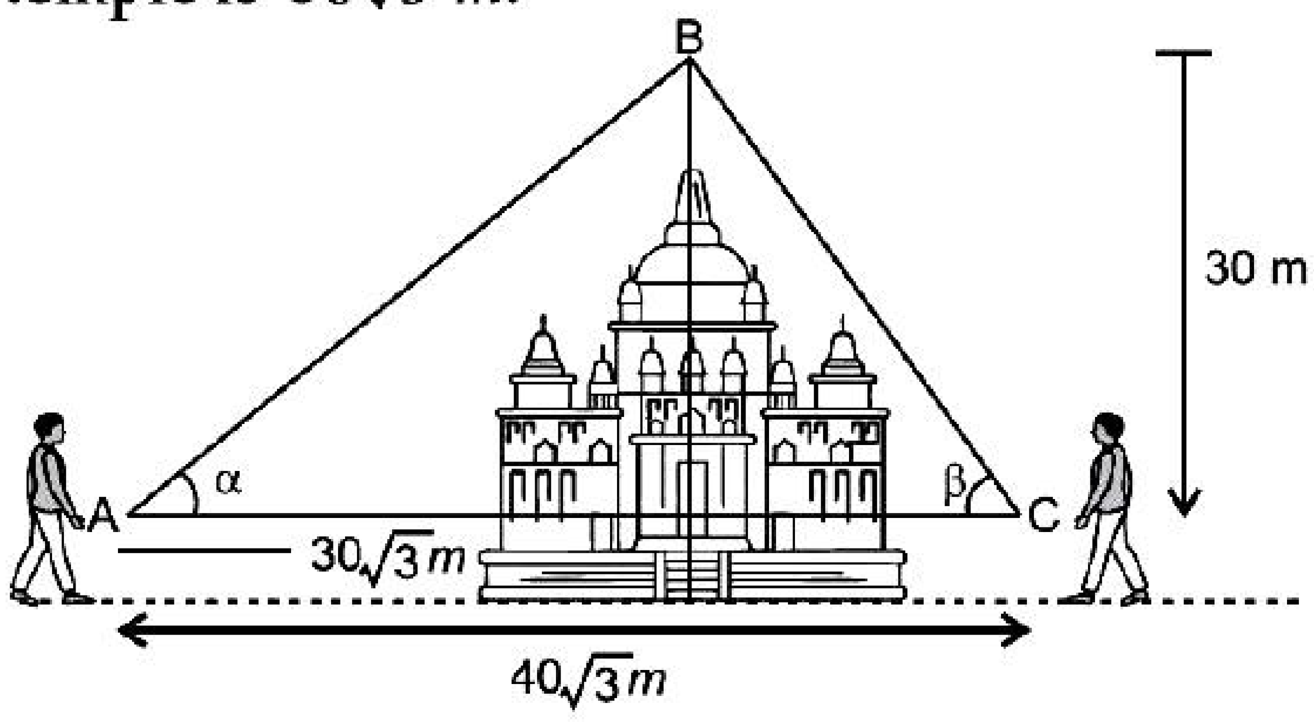
Then, $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i^3$

Reason (R) : $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}\ \forall x \in [-1, 1]$

34. Assertion : The equation $2(\sin^{-1} x)^2 - 5(\sin^{-1} x + 2) = 0$.

Reason : $\sin^{-1}(\sin x) = x$ if $x \in [-1.57, 1.57]$.

35. Two men on either side of temple of 30m height observe its top at the angle of elevation α and β respectively. The distance between the two men is $40\sqrt{3}\ m$ and distance between men A and the temple is $30\sqrt{3}\ m$.



Based on above information answer the following questions:

(i) Find $\angle CAB = \alpha =$

(a) $\sin^{\ 1}\frac{1}{2}$ (b) $\sin^{-1}\frac{2}{\sqrt{3}}$

(c) $\sin^{-1}\frac{\sqrt{3}}{2}$ (d) $\sin^{-1} 2$

(ii) $\angle CAB = \alpha = ?$

(a) $\cos^{-1}\frac{1}{5}$ (b) $\cos^{-1}\frac{2}{5}$

(c) $\cos^{-1}\frac{\sqrt{3}}{2}$ (d) $\cos^{-1}\frac{4}{5}$

(iii) $\angle BCA = \beta = ?$

(a) $\tan^{-1}\frac{1}{2}$ (b) $\tan^{-1} 2$

(c) $\tan^{-1}\frac{1}{\sqrt{3}}$ (d) $\tan^{-1}\sqrt{3}$

(iv) $\angle ABC = ?$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(v) Domain and range of $\cos^{-1} x$?

(a) $(-1, 1), (0\ \pi)$ (b) $[-1, 1], (0, \pi)$

(c) $[0, \pi], [-1, 1]$ (d) $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Solutions

1. (b) $\frac{2}{5}$

Explanation :

We have,

$$\cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1} x\right) = 0$$

$$\Rightarrow \quad \sin^{-1}\frac{2}{5} + \cos^{-1} x = \cos^{-1} 0$$

$$\Rightarrow \quad \sin^{-1}\frac{2}{5} + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}\frac{2}{5} + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \quad \cos^{-1} x = \frac{\pi}{2} - \sin^{-1}\frac{2}{5}$$

$$\Rightarrow \quad \cos^{-1} x = \cos^{-1}\frac{2}{5}$$

$$\left(\because \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}\right)$$

$$\therefore \quad x = \frac{2}{5}$$

2. (a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3. (b) $[0, -\pi] - \left\{\frac{\pi}{2}\right\}$

4. (a) $\frac{2\pi}{9}$

Explanation :

$$\cos^{-1}[\cos(-680°)] = \cos^{-1}[\cos(720° - 40°)]$$

$$= \cos^{-1}[\cos(-40°)]$$

$$= \cos^{-1}[\cos(40°)]$$

$$= 40° = \frac{2\pi}{9}.$$

5. (d) $\frac{\sqrt{1-x^2}}{x}$

Explanation :

Let $\sin^{-1} x = \theta$,

then $\sin\theta = x$

$$\Rightarrow \quad \operatorname{cosec}\theta = \frac{1}{x}$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta = \frac{1}{x^2}$$

$$\Rightarrow \quad 1 + \cot^2\theta = \frac{1}{x^2}$$

$$\Rightarrow \quad \cot\theta = \frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow \quad \cot(\sin^{-1} x) = \frac{\sqrt{1-x^2}}{x}$$

6. (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Explanation :

Let $\sin^{-1} 2x = \theta$

So that $2x = \sin\theta$.

Now, $-1 \leq \sin\theta \leq 1$, *i.e.*, $-1 \leq 2x \leq 1$ which gives $-\frac{1}{2} \leq x \leq \frac{1}{2}$.

7. (b) $-\frac{\pi}{3}$

Explanation :

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{3}\right)$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

$$= -\frac{\pi}{3}.$$

8. (d) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Explanation :

$$y = \cos^{-1}(x^2 - 4)$$

$\Rightarrow \quad \cos y = x^2 - 4$

i.e., $\quad -1 \leq x^2 - 4 \leq 1$ (since $-1 \leq \cos y \leq 1$)

$\Rightarrow \quad 3 \leq x^2 \leq 5$

$\Rightarrow \quad \sqrt{3} \leq |x| \leq \sqrt{5}$

$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

9. (a) $[-1, 1]$

Explanation :

The domain of cos is R and the domain of $\sin^{-1}$ is $[-1, 1]$.

$\therefore$ The domain of $\cos x + \sin^{-1} x$ is $R \cap [-1, 1]$, *i.e.*, $[-1, 1]$.

10. **(b)** $\cdot 96$

Explanation :

Let $\sin^{-1}(\cdot 6) = \theta$, *i.e.*, $\sin\theta = \cdot 6$.

Now, $\sin(2\theta) = 2\sin\theta\cos\theta = 2(\cdot 6)(\cdot 8) = \cdot 96$.

11. (d) $\frac{3}{29}$

Explanation :

$$\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right\}$$

Let $\quad A = \cos^{-1}\frac{1}{5\sqrt{2}}$

$\therefore \quad \cos A = \frac{1}{5\sqrt{2}}$

$\Rightarrow \quad \cos^2 A = \frac{1}{50}$

$\Rightarrow \quad \sec^2 A = 50$

$\Rightarrow \quad \tan^2 A = 50 - 1 = 49$

and $\quad B = \sin^{-1}\frac{4}{\sqrt{17}}$

$$\sin B = \frac{4}{\sqrt{17}}$$

$$\sin^2 B = \frac{16}{17}$$

$$\text{cosec}^2 B = \frac{17}{16}$$

$$\cot^2 B = \frac{17}{16} - 1 = \frac{1}{16}$$

$\therefore \quad \tan A = 7$ and $\tan B = 4$

Now, $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right\}$

$$= \tan(A - B)$$

$$= \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{7 - 4}{1 + 7 \times 4} = \frac{3}{29}.$$

12. (d) $\frac{5}{\sqrt{26}}$

Explanation :

Given : $\cot^{-1}\left(\frac{-1}{5}\right) = \theta$, where $\theta \in (0, \pi)$

$\therefore \quad \cot\theta = \frac{-1}{5}$

$\because \quad \sin\theta = \frac{1}{\text{cosec}\,\theta} = \frac{1}{\sqrt{1 + \cot^2\theta}}$

$$= \frac{1}{\sqrt{1 + \left(-\frac{1}{5}\right)^2}} = \frac{1}{\sqrt{1 + \frac{1}{25}}} = \frac{1}{\sqrt{\frac{26}{25}}}$$

$$= \frac{5}{\sqrt{26}}$$

13. (d) $\alpha = 0, \beta = \pi$

Explanation :

We have $\quad \frac{-\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

$\Rightarrow \quad \frac{-\pi}{2} + \frac{\pi}{2} \leq \sin^{-1} x + \frac{\pi}{2} \leq \frac{\pi}{2} + \frac{\pi}{2}$

$\Rightarrow \quad 0 \leq \sin^{-1} x + (\sin^{-1} x + \cos^{-1} x) \leq \pi$

$\Rightarrow \quad 0 \leq 2\sin^{-1} x + \cos^{-1} x \leq \pi.$

14. (b) 11

Explanation :

$\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3)$

$$= \sec^2(\sec^{-1} 2) - 1 + \text{cosec}^2(\text{cosec}^{-1} 3) - 1$$

$$= 2^2 \times 1 + 3^2 - 2 = 11.$$

15. (b) $\frac{3-\sqrt{5}}{2}$

Explanation :

Let $\quad y = \tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$

Putting, $\quad x = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$

$\Rightarrow \quad \cos x = \frac{\sqrt{5}}{3} \quad$ ...(i)

Now, $\quad y = \tan\left(\frac{1}{2}\times x\right)$

$y = \tan\left(\frac{x}{2}\right)$

$\therefore \quad y = \sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$

$\therefore \quad y = \sqrt{\frac{1-\sqrt{5}/3}{1+\sqrt{5}/3}} \quad$ [From (i)]

$\therefore \quad y = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}}$

$\therefore \quad y = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}} = \frac{3-\sqrt{5}}{\sqrt{4}} = \frac{3-\sqrt{5}}{2}$

$\therefore \quad y = \frac{3-\sqrt{5}}{2}$

i.e., $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right] = \frac{3-\sqrt{5}}{2}$.

16. (b) $\frac{\pi}{6}$

17. (a) $\frac{5\pi}{6}$

Explanation :

$$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

18. (a) $[-\pi, 0]$

Explanation :

Cosine functions respected to any interval $[-\pi, 0]$, $[0, \pi]$, $[\pi, 2\pi]$ etc., is bijective with range $[-1, 1]$.

19. (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

Explanation :

Range of $\sin^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \quad \frac{-\pi}{2} \le y \le \frac{\pi}{2}$

20. (d) 1

Explanation :

$$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$$

$$= \sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right] \quad \left[\because \sin^{-1}\left(\frac{-1}{2}\right) = \frac{-\pi}{6}\right]$$

$$= \sin\frac{\pi}{2} = 1.$$

21. (c) $\frac{-\pi}{12}$

Explanation :

$$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

$$= \frac{-\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{-2\pi + 4\pi - 3\pi}{12} = \frac{-\pi}{12}$$

$$\left[\because \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}, \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}, \sin\left(\frac{-\pi}{2}\right) = -1\right]$$

22. (a) $\frac{\pi}{3}$

Explanation :

Given, $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$$= \tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right) = \tan^{-1}\left(2\sin\frac{\pi}{3}\right)\right]$$

$$= \tan^{-1}\left(2\times\frac{\sqrt{3}}{2}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}.$$

23. (a) 0

Explanation :

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) = \tan^{-1}\tan\left(\pi - \frac{\pi}{6}\right)$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) = \frac{-\pi}{6}$$

and $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$$= \cos^{-1}\cos\left(2\pi + \frac{\pi}{6}\right) = \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$$

$$\frac{-\pi}{6} + \frac{\pi}{6} = 0.$$

27. (a) $[0, 1]$

Explanation :

We know that $\cos^{-1} x$ is defined for $x \in [-1, 1]$

$\therefore f(x) = \cos^{-1}(2x-1)$ is defined if

$-1 \le 2x - 1 \le 1$

$\Rightarrow \quad 0 \le 2x \le 2$

$\Rightarrow \quad 0 \le x \le 1.$

28. (a) [1, 2]

Explanation :

We know that $\sin^{-1} x$ is defined for

$$x \in [-1, 1]$$

$\therefore f(x) = \sin^{-1} \sqrt{x-1}$ is defined if

$$\Rightarrow \quad 0 \le \sqrt{x-1} \le 1$$

$$\Rightarrow \quad 0 \le x-1 \le 1$$

$$[\because \sqrt{x-1} \ge 0 \text{ and } -1 \le \sqrt{x-1} \le 1]$$

$$\Rightarrow \quad 1 \le x \le 2$$

$$\therefore \quad x \in [1, 2]$$

29. (a) $\frac{\pi}{2}$

Explanation :

$$\cos^{-1}\left(\cos\frac{3\pi}{2}\right) \ne \frac{3\pi}{2} \text{ as } \frac{3\pi}{2} \notin [0, \pi]$$

$$\therefore \quad \cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1} 0 = \frac{\pi}{2}$$

30. (d) $\frac{x}{\sqrt{1+x^2}}$

Explanation :

$$\tan y = x$$

$$\Rightarrow \quad \sin y = \frac{x}{\sqrt{1+x^2}}$$

Let $\tan^{-1} x = y$. Then,

$$\therefore \quad y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad \tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\therefore \quad \sin(\tan^{-1} x) = \sin\left(\sin^{-1}\frac{x}{1+x^2}\right) = \frac{x}{\sqrt{1+x^2}}$$

31. (d) A is false but R is true.

Explanation :

$\because 3 \approx 171°$ (lies in II quadrant)

$$\therefore \quad \sin^{-1} \sin 3 = 3 - \pi \ne 3$$

But $\sin^{-1} \sin x = x$ for principal values.

32. (a) Both (A) and (R) are true and R is the correct explanation A.

Explanation :

$$\because \quad AM \ge GM$$

$$\therefore \quad \frac{\cos^{-1} x + (\sin^{-1} y)^2}{2} \ge \sqrt{(\cos^{-1} x)(\sin^{-1} y)^2}$$

$$\Rightarrow \quad \frac{p\pi^2}{8} \ge \frac{p\pi^2}{8}$$

$$\Rightarrow \quad p \ge 2$$

Thus, we conclude that the only value of p that satisfies all conditions is $p = 2$.

Then, $\quad \cos^{-1} x = (\sin^{-1} y)^2$

$$\Rightarrow \quad (\cos^{-1} x)^2 = \frac{\pi^4}{16}$$

$$\Rightarrow \quad \cos^{-1} x = \pm\frac{\pi^2}{4}$$

$$\Rightarrow \quad x = \cos\left(\pm\frac{\pi^2}{4}\right)$$

$$\therefore \quad x = \cos\left(\frac{\pi^2}{4}\right)$$

Also, $\quad (\sin^{-1} y)^4 = \frac{\pi^4}{16}$

$$\Rightarrow \quad \sin^{-1} y = \pm\frac{\pi}{2}$$

$$\therefore \quad y = \sin\left(\pm\frac{\pi}{2}\right)$$

$$= \pm 1$$

33. (a) Both (A) and (R) are true and R is the correct explanation A.

Explanation :

Since, maximum value of $\sin^{-1} x_i$ is $\frac{\pi}{2}$

$\therefore \sum_{i=1}^{2n} \sin^{-1} x_i = n\pi$ is possible, if

$$x_1 = x_2 = x_3 = ... = x_{2n} = 1$$

$$\therefore \sum_{i=1}^{n} x_i = 1 + 1 + 1 + ... \text{ upto } n \text{ times} = n$$

$$\therefore \sum_{i=1}^{n} x_i^2 = 1^2 + 1^2 + 1^2 + 1^2 + ... \text{ upto } n \text{ times} = n$$

$$\text{and } \sum_{i=1}^{n} x_i^3 = 1^3 + 1^3 + 1^3 + ... \text{ upto } n \text{ times} = n$$

$$\text{Hence, } \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i^3 = n$$

34. (d) A is false but R is true.

Explanation :

$$2(\sin^{-1} x)^2 - 5(\sin^{-1} x) + 2 = 0$$

$$\Rightarrow \quad \sin^{-1} x = \frac{5 \pm \sqrt{25-16}}{4} = 2, \frac{1}{2}$$

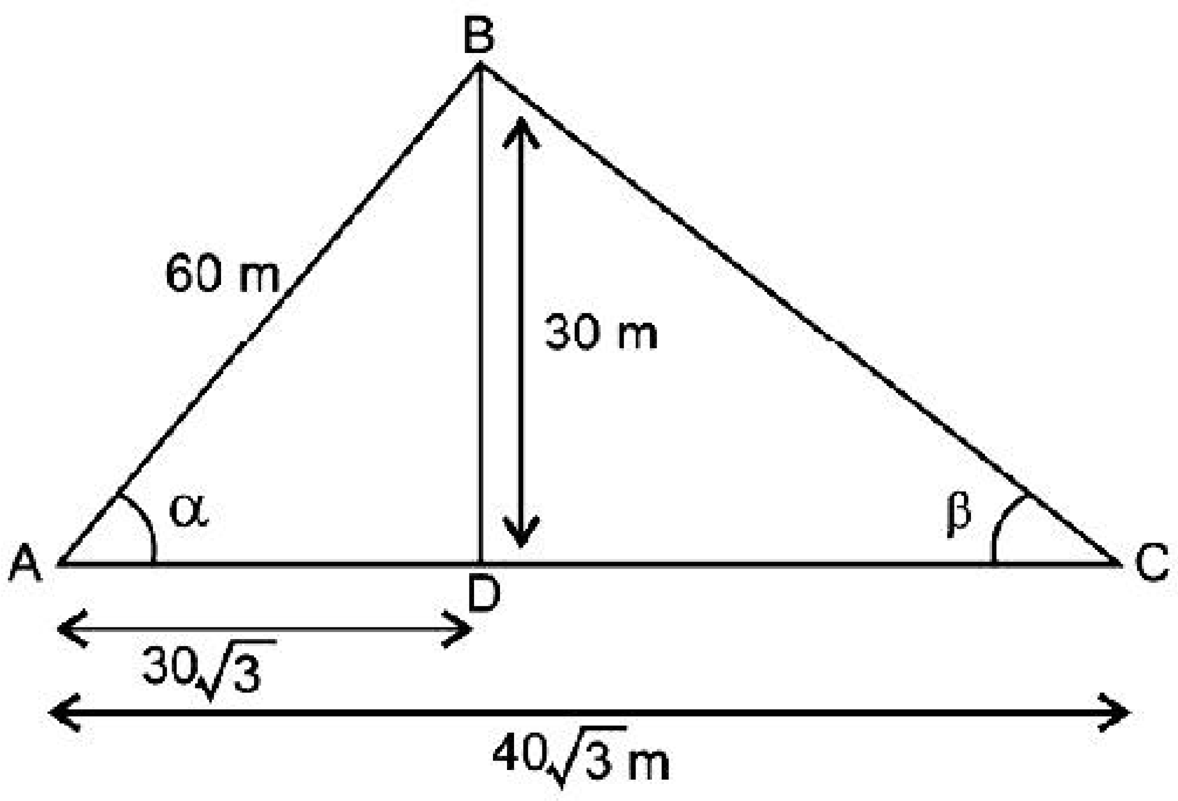
$$\Rightarrow \quad \sin^{-1} x = \frac{1}{2}, \sin^{-1} x = 2$$

$\therefore x = \sin\left(\frac{1}{2}\right)$ and $x = \sin^{-1} 2$ is not possible

$\therefore x = \sin\left(\frac{1}{2}\right)$ is only solution

$\therefore$ Assertion (A) is false

35. (i) (a)



$$\sin\alpha = \frac{BD}{AB}$$

$$AB = \sqrt{(30\sqrt{3})^2 + 30^2}$$

$$= \sqrt{2700 + 900}$$

$$= \sqrt{3600} = 60.$$

$$\sin\alpha = \frac{30}{60} = \frac{1}{2}$$

$$\boxed{\alpha = \sin^{-1}\frac{1}{2}}$$

(ii) (c) $\cos\alpha = \dfrac{AD}{AB} = \dfrac{30\sqrt{3}}{60} = \dfrac{\sqrt{3}}{2}$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

(iii) (d) $\tan\beta = \dfrac{BD}{DC} = \dfrac{30}{10\sqrt{3}} = \dfrac{3}{\sqrt{3}} = \sqrt{3}$

$$\boxed{\beta = \tan^{-1}\sqrt{3}}$$

(iv) (c) $BC = \sqrt{30^2 + (10\sqrt{3})^2}$

$$= \sqrt{900 + 300} = \sqrt{1200} = 20\sqrt{3}\,m$$

$\because$ $AB^2 + BC^2 = AC^2$

$$\sqrt{(60)^2 + 1200} = \sqrt{3600 + 1200}$$

$$AC = \sqrt{4800} = 40\sqrt{3}$$

$\because$ By convex of pythagoras theory

$$\angle ABC = \frac{\pi}{2}$$

(v) (c) $[0, \pi]$, $[-1, 1]$