# 3. Algebra

## Exercise 3.1

### 1. Question

State whether the following expressions are polynomials in one variable or not. Give reasons for your answer.

i. $2x^5 - x^3 + x - 6$

ii. $3x^2 - 2x + 1$

iii. $y^3 + 2\sqrt{3}$

iv. $x - \frac{1}{x}$

v. $\sqrt[3]{t} + 2t$

vi. $x^3 + y^3 + z^6$

**Answer**

i. $2x^5 - x^3 + x - 6$

There is only one variable 'x' with whole number power. So, this is polynomial in one variable.

ii. $3x^2 - 2x + 1$

There is only one variable 'x' with whole number power. So, this is polynomial in one variable.

iii. $y^3 + 2\sqrt{3}$

There is only one variable 'y' with whole number power. So, this is polynomial in one variable.

iv. $x - \frac{1}{x}$

$\Rightarrow \frac{x^2 - 1}{x}$

There is only one variable 'x' with whole number power. So, this is polynomial in one variable.

v. $3\sqrt{t} + 2t$

There is only one variable 't' but in $3\sqrt{t}$, power of t is $\frac{1}{2}$ which is not a whole number. So, this is not a polynomial in one variable.

vi. $x^3 + y^3 + z^3$

There are three variables x, y and z, but power is whole number. So, this is not a polynomial in one variable.

**2. Question**

Write the coefficient of $x^2$ and x in each of the following

i. $2 + 3x - 4x^2 + x^3$

ii. $\sqrt{3}\,x + 1$

iii. $x^3 + \sqrt{2}\,x^2 + 4x - 1$

iv. $\frac{1}{3}x^2 + x + 6$

**Answer**

i $2 + 3x - 4x^2 + x^3$

Co-efficient of $x^2$ = -4

Co-efficient of x = 3

ii $\sqrt{3}x + 1$

Co-efficient of $x^2$ = 0

Co-efficient of x = $\sqrt{3}$

iii $x^3 + \sqrt{2}x^2 + 4x - 1$

Co-efficient of $x^2$ = $\sqrt{2}$

Co-efficient of x = 4

iv $\frac{1}{3}x^2 + x + 6$

$$= \frac{(x^2 + 3x + 18)}{3} = 0$$

= $x^2 + 3x + 6 = 0$

Co-efficient of $x^2$ = 1

Co-efficient of x = 3

**3. Question**

Write the degree of each of the following polynomials.

i. $4 - 3x^2$

ii. $5y + \sqrt{2}$

iii. $12 - x + 4x^3$

iv. 5

**Answer**

i. $4 - 3x^2$

Degree of the polynomial = 2

ii. $5y + \sqrt{2}$

Degree of the polynomial = 1

iii. $12 - x + 4x^3$

Degree of the polynomial = 3

iv. 5

Degree of the polynomial = 0

**4. Question**

Classify the following polynomials based on their degree.

i. $3x^2 + 2x + 1$

ii. $4x^3 - 1$

iii. $y + 3$

iv. $y^2 - 4$

v. $4x^3$

vi. $2x$

**Answer**

i. $3x^2 + 2x +1$

Since, the highest degree of polynomial is 2

It is a quadratic polynomial.

ii. $4x^3 - 1$

Since, the highest degree of polynomial is 3.

It is a cubic polynomial.

iii. $y + 3$

Since, the highest degree of polynomial is 1

It is a linear polynomial

iv. $y^2 - 4$

Since, the highest degree of polynomial is 2

It is a quadratic polynomial.

v. $4x^3$

Since, the highest degree of polynomial is 3.

It is a cubic polynomial.

vi. $2x$

Since, the highest degree of polynomial is 1

It is a linear polynomial

**5. Question**

Give one example of a binomial of degree 27 and monomial of degree 49 and trinomial of degree 36.

**Answer**

Binomial means having two terms. So binomial of degree 27 is $x^{27} + y$.

Monomial means having one term. So, monomial of degree is $x^{49}$.

Trinomial means having three term. So, trinomial of degree is $x^{36} + y + 2$.

## Exercise 3.2

**1. Question**

Find the zeros of the following polynomials.

i. $p(x)= 4x - 1$

ii. $p(x) = 3x + 5$

iii. $p(x) = 2x$

v. p(x) = x + 9

**Answer**

i. p(x) = 4x – 1

$= 4\left(x - \frac{1}{4}\right)$

$\Rightarrow p\left(\frac{1}{4}\right) = 4\left(\frac{1}{4} - \frac{1}{4}\right)$

$\Rightarrow p\left(\frac{1}{4}\right) = 4(0) = 0$

Hence, $\frac{1}{4}$ is the zero of p(x).

ii. p(x) = 3x + 5

$= 3\left(x - \frac{5}{3}\right)$

$\Rightarrow p\left(-\frac{5}{3}\right) = 3\left(-\frac{5}{3} + \frac{5}{3}\right)$

$\Rightarrow p\left(-\frac{5}{3}\right) = 3(0) = 0$

Hence, $-\frac{5}{3}$ is the zero of p(x).

iii. p(x) = 2x

p(0) = 2(0) = 0

Hence, 0 is the zero of p(x).

iv. p(x) = x + 9

p(-9) = -9 + 9 = 0

Hence, -9 is the zero of p(x).

**2. Question**

Find the roots of the following polynomial equations.

i. x – 3 = 0

ii. 5x – 6 = 0

iii. 11x + 1 = 0

iv. −9x = 0

**Answer**

i. x – 5 = 0

⇒ x = 5

∴ x = 5 is a root of x – 5 = 0

ii. 5x – 6 = 0

⇒ 5x = 6

⇒ $x = \frac{6}{5}$

∴ $x = \frac{6}{5}$ is a root of 5x – 6 = 0

iii. 11x + 1 = 0

⇒ 11x = -1

⇒ $x = -\frac{1}{11}$

∴ $x = -\frac{1}{11}$ is a root of 11x + 1 = 0.

iv. -9x = 0

⇒ $-x = \frac{0}{9}$

⇒ x = 0

∴ x = 0 is a root of -9x = 0.

**3 A. Question**

Verify Whether the following are roots of the polynomial equations indicated against them.

$x^2$ – 5x + 6 = 0; x = 2, 3

**Answer**

$x^2$ – 5x + 6 = 0

Let p(x) = $x^2$ – 5x + 6

p(2) = $(2)^2$ – 5(2) + 6

= 4 – 10 + 6

= 10 – 10 = 0

∴ x = 2 is a root of $x^2$ – 5x + 6 = 0

$p(x) = x^2 - 5x + 6$

$p(3) = (3)^2 - 5(3) + 6$

$= 9 - 15 + 6$

$= 15 - 15 = 0$

$\therefore x = 3$ is a root of $x^2 - 5x + 6 = 0$

**3 B. Question**

Verify Whether the following are roots of the polynomial equations indicated against them.

$x^2 + 4x + 3 = 0; x = -1, 2$

**Answer**

$x^2 + 4x + 3 = 0$

let $p(x) = x^2 + 4x + 3$

$p(-1) = (-1)^2 + 4(-1) + 3$

$= 1 - 4 + 3$

$= 4 - 4 = 0$

$\therefore x = -1$ is a root of $x^2 + 4x + 3 = 0$

$p(x) = x^2 + 4x + 3 = 0$

$p(2) = (2)^2 + 4(2) + 3$

$= 4 + 8 + 3$

$= 11 + 4 = 15 \neq 0$

$\therefore x = 2$ is not a root of $x^2 + 4x + 3 = 0$.

**3 C. Question**

Verify Whether the following are roots of the polynomial equations indicated against them.

$x^3 - 2x^2 - 5x + 6 = 0; x = 1, -2, 3$

**Answer**

$x^3 - 2x^2 - 5x + 6 = 0$

let $p(x) = x^3 - 2x^2 - 5x + 6$

$p(1) = (1)^3 – 2(1)^2 – 5(1) + 6$

$= 1 – 2 \times 1 – 5 + 6$

$= 1 – 2 – 5 + 6$

$= 7 – 7 = 0$

$\therefore$ x = 1 is a root of $x^3 – 2x^2 – 5x + 6 = 0$.

$p(x) = x^3 – 2x^2 – 5x + 6$

$p(-2) = (-2)^3 – 2(-2)^2 – 5(-2) + 6$

$= -8 – 2 \times 4 – 5 \times 2 + 6$

$= -8 – 8 + 10 + 6$

$= -16 + 16 = 0$

$\therefore$ x = -2 is a root of $x^3 – 2x^2 – 5x + 6 = 0$.

$p(x) = x^3 – 2x^2 – 5x + 6 = 0$

$p(3) = (3)^3 – 2(3)^2 – 5(3) + 6$

$= 27 – 2 \times 9 – 5 \times 3 + 6$

$= 27 – 18 – 15 + 6$

$= 33 – 33 = 0$

$\therefore$ x = 3 is a root of $x^3 – 2x^2 – 5x + 6 = 0$.

**3 D. Question**

Verify Whether the following are roots of the polynomial equations indicated against them.

$x^3 – 2x^2 – x + 2 = 0$; $x = −1, 2, 3$

**Answer**

$x^3 – 2x^2 – x + 2 = 0$

$p(x) = x^3 – 2x^2 – x + 2 = 0$

$p(-1) = (-1)^3 – 2(-1)^2 - (-1) + 2$

$= -1 – 2 \times 1 + 1 + 2$

$= -1 – 2 + 1 + 2$

$= -3 + 3 = 0$

∴ $x = -1$ is a root of $x^3 - 2x^2 - x + 2 = 0$

$p(x) = x^3 - 2x^2 - x + 2 = 0$

$p(2) = (2)^3 - 2(2)^2 - (2) + 2$

$= 8 - 2 \times 4 - 2 + 2$

$= 8 - 8 - 2 + 2$

$= 10 - 10 = 0$

∴ $x = 2$ is a root of $x^3 - 2x^2 - x + 2 = 0$.

$p(x) = x^3 - 2x^2 - x + 2 = 0$

$p(3) = (3)^3 - 2(3)^2 - (3) + 2$

$= 27 - 2 \times 9 - 3 + 2$

$= 27 - 18 - 3 + 2$

$= 29 - 21 = 8 \neq 0$

∴ $x = 3$ is not a root of $x^3 - 2x^2 - x + 2 = 0$.

## Exercise 3.3

### 1 A. Question

Find the quotient the and remainder of the following division.

$(5x^3 - 8x^2 + 5x - 7) \div (x - 1)$

**Answer**

$(5x^3 - 8x^2 + 5x - 7) \div (x - 1)$

We see that the equation is already arranged in descending order.

Now we need to divide $(5x^3 - 8x^2 + 5x - 7)$ by $(x - 1)$.

Now we need to find out by how much should we multiple "x" to get a value as much as $5x^3$.

To get $x^3$, we need to multiply $x \times x^2$.

Therefore, we need to multiply with $5x^2 \times (x - 1)$ and we get $(5x^3 - 5x^2)$ now subtract $(5x^3 - 5x^2)$ from $5x^3 - 8x^2 + 5x - 7$ so we get $- 3x^2$.

Now we carry $5x - 7$ along with $- 3x^2$, as shown below

So, in same way we have keep dividing till we get rid of x as shown below.

$$\begin{array}{l}
\qquad\quad \underline{5x^2 - 3x + 2} \\
x - 1)\; 5x^3 - 8x^2 + 5x - 7 \\
\qquad\quad 5x^3 - 5x^2 \downarrow \\
\qquad\quad \underline{-\qquad +\qquad} \\
\qquad\qquad\quad -3x^2 + 5x - 7 \\
\qquad\qquad\quad -3x^2 + 3x \downarrow \\
\qquad\qquad\quad \underline{+\qquad -\qquad} \\
\qquad\qquad\qquad\qquad 2x - 7 \\
\qquad\qquad\qquad\qquad 2x - 2 \\
\qquad\qquad\qquad\qquad \underline{-\quad +\quad} \\
\qquad\qquad\qquad\qquad\quad -5
\end{array}$$

here (x – 1) × (– 3x)

$= -3x^2 + 3x$

here (x – 1) × 2

= 2x – 2

Therefore, we got the quotient = $5x^2 - 3x + 2$ and

Remainder = –5

**1 B. Question**

Find the quotient the and remainder of the following division.

$(2x^2 - 3x - 14) \div (x + 2)$

**Answer**

$(2x^2 - 3x - 14) \div (x + 2)$

We see that the equation is already arranged in descending order.

Now we need to divide $(2x^2 - 3x - 14)$ by (x + 2).

Now we need to find out by how much should we multiple "x" to get a value as much as $2x^2$.

To get $x^2$, we need to multiply x×x.

Therefore, we need to multiply with 2x × (x + 2) and we get $(2x^2 + 4x)$ now subtract $(2x^2 + 4x)$ from $2x^2 - 3x - 14$ so we get –7x.

Now we carry 14 along with –7x, as shown below

So, in same way we have keep dividing till we get rid of x as shown below.

$$\begin{array}{r|l} & 2x - 7 \\ \hline x + 2 & 2x^2 - 3x - 14 \\ & 2x^2 + 4x \\ & - \quad - \\ \hline & \quad - 7x - 14 \\ & \quad - 7x - 14 \\ & \quad + \quad + \\ \hline & \qquad 0 \end{array}$$

here (x + 2) × (– 7)

= – 7x – 14

Therefore, we got the quotient = 2x – 7 and

Remainder = 0

**1 C. Question**

Find the quotient the and remainder of the following division.

$(9 + 4x + 5x^2 + 3x^3) \div (x+ 1)$

**Answer**

$(9 + 4x + 5x^2 + 3x^3) \div (x + 1)$

We see that the equation is not arranged in descending order, so we need first arrange it in descending order of the power of x.

Therefore it becomes,

$(3x^3 + 5x^2 + 4x + 9) \div (x + 1)$

Now we need to divide $(3x^3 + 5x^2 + 4x + 9)$ by (x + 1).

Now we need to find out by how much should we multiple "x" to get a value as much as $3x^3$.

To get $x^3$, we need to multiply $x \times x^2$.

Therefore, we need to multiply with $3x^2 \times (x + 1)$ and we get $(3x^3 + 3x^2)$ now subtract $(3x^3 + 3x^2)$ from $3x^3 + 5x^2 + 4x + 9$ so we get $2x^2$.

Now we carry 4x + 9 along with $2x^2$, as shown below

So, in same way we have keep dividing till we get rid of x as shown below.

$$\begin{array}{r} 3x^2 + 2x + 2 \\ x + 1 \overline{)\, 3x^3 + 5x^2 + 4x + 9} \\ 3x^3 + 3x^2 \qquad\quad \\ - \quad - \qquad\qquad \\ \hline 2x^2 + 4x + 9 \\ 2x^2 + 2x \quad \\ - \quad - \quad\;\; \\ \hline 2x + 9 \\ 2x + 2 \\ - \quad - \\ \hline 7 \end{array}$$

here $(x + 1) \times (2x)$

$= 2x^2 + 2x$

here $(x + 1) \times 2$

$= 2x + 2$

Therefore, we got the quotient = $3x^2 + 2x + 2$ and

Remainder = 7

**1 D. Question**

Find the quotient the and remainder of the following division.

$(4x^3 – 2x^2 + 6x + 7) \div (3 + 2x)$

**Answer**

$(4x^3 – 2x^2 + 6x + 7) \div (3 + 2x)$

We see that the equation is not arranged in descending order, so we need first arrange it in descending order of the power of x.

Therefore it becomes,

$(4x^3 – 2x^2 + 6x + 7) \div (2x + 3)$

Now we need to divide $(4x^3 – 2x^2 + 6x + 7)$ by $(2x + 3)$

Now we need to find out by how much should we multiple "x" to get a value as much as $4x^3$.

To get $x^3$, we need to multiply $x \times x^2$.

Therefore we need to multiply with $2x^2 \times (2x + 3)$ and we get $(4x^3 + 6x^2)$ now subtract $(4x^3 + 6x^2)$ from $4x^3 – 2x^2 + 6x + 7$so we get $– 8x^2$.

Now we carry $6x + 7$along with $4x^2$, as shown below

So, in same way we have keep dividing till we get rid of x as shown below.

$$
\begin{array}{r}
2x^2 - 4x + 9 \\
2x + 3 \,\big)\, 4x^3 - 2x^2 + 6x + 7 \\
4x^3 + 6x^2 \qquad\qquad \\
- \quad - \qquad\qquad \\
\hline
- 8x^2 + 6x + 7 \\
- 8x^2 - 12x \qquad \\
+ \quad + \qquad \\
\hline
18x + 7 \\
18x + 27 \\
- \quad - \\
\hline
- 20
\end{array}
$$

here (2x + 3) × (−4x)

= $- 8x^2 - 12x$

here (2x + 3) × 9

= 18x + 27

Therefore, we got the quotient = $2x^2 - 4x + 9$ and

Remainder = −20

**1 E. Question**

Find the quotient the and remainder of the following division.

$(-18 - 9x + 7x^2) \div (x - 2)$

**Answer**

$(-18 - 9x + 7x^2) \div (x - 2)$

We see that the equation is not arranged in descending order, so we need first arrange it in descending order of the power of x.

Therefore it becomes,

$(7x^2 - 9x - 18) \div (x - 2)$

Now we need to divide $(7x^2 - 9x - 18)$ by (x − 2).

Now we need to find out by how much should we multiple “x” to get a value as much as $7x^2$.

To get $x^2$, we need to multiply x×x.

Therefore, we need to multiply with 7x × (x − 2) and we get $(7x^2 - 14x)$ now subtract $(7x^2 - 14x)$ from $7x^2 - 9x - 18$ so we get 5x.

Now we carry 18 along with 5x, as shown below

So, in same way we have keep dividing till we get rid of x as shown below.

```
         7x + 5
x - 2) 7x² – 9x – 18
       7x² - 14x
      -     +
         5x - 18
         5x - 10
        -    +
            - 8
```

here (x – 2) × 5

= 5x – 10

Therefore, we got the quotient = 7x + 5 and

Remainder = – 8

## Exercise 3.4

### 1 A. Question

Find the remainder using remainder theorem, when

$3x^3 + 4x^2 - 5x + 8$ is divided by x – 1

**Answer**

$3x^3 + 4x^2 - 5x + 8$ is divided by x – 1

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let $p(x) = 3x^3 + 4x^2 - 5x + 8$ and we have (x – 1)

The zero of (x – 1) is 1

Now using Remainder theorem,

$p(x) = 3x^3 + 4x^2 - 5x + 8$ is divided by x – 1 then, p(1) is the remainder

$p(1) = 3(1)^3 + 4(1)^2 - 5(1) + 8$

= 3 + 4 – 5 + 8

= 10

Remainder = 10

### 1 B. Question

Find the remainder using remainder theorem, when

$5x^3 + 2x^2 - 6x + 12$ is divided by $x + 2$

**Answer**

$5x^3 + 2x^2 - 6x + 12$ is divided by $x + 2$

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let $p(x) = 5x^3 + 2x^2 - 6x + 12$ and we have $(x + 2)$

The zero of $(x + 2)$ is $-2$

Now using Remainder theorem,

$p(x) = 5x^3 + 2x^2 - 6x + 12$ is divided by $x + 2$ then, $p(-2)$ is the remainder

$p(-2) = 5(-2)^3 + 2(-2)^2 - 6(-2) + 12$

$= 5\times(-8) + 2\times4 - (-12) + 12$

$= -40 + 8 + 12 + 12$

$= -40 + 32$

$= -8$

Remainder $= -8$

**1 C. Question**

Find the remainder using remainder theorem, when

$2x^3 - 4x^2 + 7x + 6$ is divided by $x - 2$

**Answer**

$2x^3 - 4x^2 + 7x + 6$ is divided by $x - 2$

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let $p(x) = 2x^3 - 4x^2 + 7x + 6$ and we have $(x - 2)$

The zero of $(x - 2)$ is 2

Now using Remainder theorem,

$p(x) = 2x^3 - 4x^2 + 7x + 6$ is divided by $x - 2$ then, $p(2)$ is the remainder

$p(2) = 2(2)^3 - 4(2)^2 + 7(2) + 6$

$= 16 - 16 + 14 + 6$

$= 20$

Remainder = 20

**1 D. Question**

Find the remainder using remainder theorem, when

$4x^3 - 3x^2 + 2x - 4$ is divided by $x + 3$

**Answer**

$4x^3 - 3x^2 + 2x - 4$ is divided by $x + 3$

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let $p(x) = 4x^3 - 3x^2 + 2x - 4$ and we have $(x + 3)$

The zero of $(x + 3)$ is $-3$

Now using Remainder theorem,

$p(x) = 4x^3 - 3x^2 + 2x - 4$ is divided by $x + 3$ then, $p(-3)$ is the remainder

$p(-3) = 4(-3)^3 - 3(-3)^2 + 2(-3) - 4$

$= -108 - 27 - 6 - 4$

$= -145$

Remainder = $-145$

**1 E. Question**

Find the remainder using remainder theorem, when

$4x^3 - 12x^2 + 11x - 5$ is divided by $2x - 1$

**Answer**

$4x^3 - 12x^2 + 11x - 5$ is divided by $2x - 1$

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let $p(x) = 4x^3 - 12x^2 + 11x - 5$ and we have $(2x - 1)$

The zero of (2x – 1) is $\frac{1}{2}$

Now using Remainder theorem,

p(x) = $4x^3 - 12x^2 + 11x - 5$ is divided by 2x – 1 then, $p\left(\frac{1}{2}\right)$ is the remainder

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) - 5$$

$$p\left(\frac{1}{2}\right) = \frac{4}{8} - \frac{12}{4} + \frac{11}{2} - 5$$

$$p\left(\frac{1}{2}\right) = \frac{1}{2} - 3 + \frac{11}{2} - 5$$

$$p\left(\frac{1}{2}\right) = \frac{1 - 6 + 11 - 10}{2}$$

$$p\left(\frac{1}{2}\right) = \frac{12 - 16}{2}$$

$$p\left(\frac{1}{2}\right) = -\frac{4}{2}$$

$$p\left(\frac{1}{2}\right) = -2$$

Remainder = –2

**1 F. Question**

Find the remainder using remainder theorem, when

$8x^4 + 12x^3 - 2x^2 - 18x + 14$ is divided by x + 1

**Answer**

$8x^4 + 12x^3 - 2x^2 - 18x + 14$ is divided by x + 1

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let p(x) = $8x^4 + 12x^3 - 2x^2 - 18x + 14$ and we have (x + 1)

The zero of (x + 1) is – 1

Now using Remainder theorem,

p(x) = $8x^4 + 12x^3 - 2x^2 - 18x + 14$ is divided by x + 1 then, p(– 1) is the remainder

$p(-1) = 8(-1)^4 + 12(-1)^3 - 2(-1)^2 - 18(-1) + 14$

$= 8 - 12 - 2 + 18 + 14$

$= 26$

Remainder = 26

**1 G. Question**

Find the remainder using remainder theorem, when

$x^3 - ax^2 - 5x + 2a$ is divided by $x - a$

**Answer**

$x^3 - ax^2 - 5x + 2a$ is divided by $x - a$

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let $p(x) = x^3 - ax^2 - 5x + 2a$ and we have $(x - a)$

The zero of $(x - a)$ is a

Now using Remainder theorem,

$p(x) = x^3 - ax^2 - 5x + 2a$ is divided by $x - a$ then, p(a) is the remainder

$p(a) = (a)^3 - a(a)^2 - 5(a) + 2a$

$= a^3 - a^3 - 5a + 2a$

$= -3a$

Remainder $= -3a$

**2. Question**

When the polynomial $2x^3 - 2x^2 + 9x - 8$ is divided by $x - 3$ the remainder is 28. Find the value of a.

**Answer**

$2x^3 - ax^2 + 9x - 8$ is divided by $x - 3$ and remainder = 28

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let $p(x) = 2x^3 - ax^2 + 9x - 8$ and we have $(x - 3)$

The zero of $(x - 3)$ is 3

Now using Remainder theorem,

p(x) = $2x^3 - ax^2 + 9x - 8$ is divided by x – a then, p(3) is the remainder which is 28

p(3) = $2x^3 - ax^2 + 9x - 8 = 28$

= $2(3)^3 - a(3)^2 + 9(3) - 8 = 28$

= 54 – 9a + 27 – 8 = 28

= 73 – 9a = 28

= 9a = 73 –28

= 9a = 45

$$a = \frac{45}{9}$$

a = 5

**3. Question**

Find the value of m if $x^3 - 6x^2 + mx + 60$ leaves the remainder 2 when divided by (x + 2).

**Answer**

$x^3 - 6x^2 + mx + 60$ divided by (x + 2) and remainder = 2

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let p(x) = $x^3 - 6x^2 + mx + 60$ and we have (x + 2)

The zero of (x + 2) is –2

Now using Remainder theorem,

p(x) = $x^3 - 6x^2 + mx + 60$ is divided by x + 2 then, p(–2) is the remainder which is 2

p(–2) = $x^3 - 6x^2 + mx + 60 = 2$

= $(-2)^3 - 6(-2)^2 + m(-2) + 60 = 2$

= – 8 – 24 – 2m + 60 = 2

= – 32 – 2m + 60 = 2

= 28 – 2m = 2

$= 2m = 28 - 2$

$= 2m = 26$

$m = 13$

**4. Question**

If $(x - 1)$ divides $mx^3 - 2x^2 + 25x - 26$ without remainder find the value of m

**Answer**

$mx^3 - 2x^2 + 25x - 26$ is divided by $(x - 1)$ without remainder that means remainder = 0

Remainder theorem states that if p(x) is any polynomial and a is any real number and If p(x) is divided by the linear polynomial (x – a) , then the remainder is p(a).

Let p(x) $mx^3 - 2x^2 + 25x - 26$ and we have $(x - 1)$

The zero of $(x - 1)$ is 1

Now using Remainder theorem,

$p(x) = mx^3 - 2x^2 + 25x - 26$ is divided by $x - 1$ then, p(1) is the remainder which is 0

$p(1) = mx^3 - 2x^2 + 25x - 26 = 0$

$= m(1)^3 - 2(1)^2 + 25(1) - 26 = 0$

$= m - 2 + 25 - 26 = 0$

$= m - 3 = 0$

$m = 3$

**5. Question**

If the polynomials $x^3 + 3x^2 - m$ and $2x^3 - mx + 9$ leaves the same remainder when they are divided by $(x - 2)$, find the value of m. Also find the remainder

**Answer**

$x^3 + 3x^2 - m$ and $2x^3 - mx + 9$ is divided by $(x - 2)$ and the remainder is same.

Now let $p(x) = x^3 + 3x^2 - m$ is divided by $x - 2$ then, p(2) is the remainder

$p(2) = (2)^3 + 3(2)^2 - m$

$= 8 + 12 - m$

$= 20 - m$

Now let $q(x) = 2x^3 - mx + 9$ is divided by $x - 2$ then, $q(2)$ is the remainder

$q(2) = 2(2)^3 - m(2) + 9$

$= 16 - 2m + 9$

$= 25 - 2m$

Now, as the question says that the remainder for $p(x)$ and $q(x)$ is same

Therefore, $p(2) = q(2)$

$20 - m = 25 - 2m$

$2m - m = 25 - 20$

$m = 5$

Remainder $= p(2) = 20 - m$

$= 15$

## Exercise 3.5

### 1 A. Question

Determine whether $(x + 1)$ is a factor of the following polynomials:

$6x^4 + 7x^3 - 5x - 4$

**Answer**

Let $f(x) = 6x^4 + 7x^3 - 5x - 4$

By factor theorem,

$x + 1 = 0 ; x = -1$

If $f(-1) = 0$ then $(x + 1)$ is a factor of $f(x)$

$\therefore f(-1) = 6(-1)^4 + 7(-1)^3 - 5(-1) - 4$

$= 6 - 7 + 5 - 4 = 11 - 11 = 0$

$\therefore (x + 1)$ is a factor of $f(x) = 6x^4 + 7x^3 - 5x - 4$

### 1 B. Question

Determine whether $(x + 1)$ is a factor of the following polynomials:

$2x^4 + 9x^3 + 2x^2 + 10x + 15$

**Answer**

Let $f(x) = 2x^4 + 9x^3 + 2x^2 + 10x + 15$

By factor theorem,

$x + 1 = 0 ; x = -1$

If $f(-1) = 0$ then $(x + 1)$ is a factor of $f(x)$

$\therefore f(-1) = 2(-1)^4 + 9(-1)^3 + 2(-1)^2 + 10(-1) + 15$

$= 2 - 9 + 2 - 10 + 15 = 19 - 19 = 0$

$\therefore (x + 1)$ is a factor of $f(x) = 2x^4 + 9x^3 + 2x^2 + 10x + 15$

**1 C. Question**

Determine whether $(x + 1)$ is a factor of the following polynomials:

$3x^3 + 8x^2 + 6x - 5$

**Answer**

Let $f(x) = 3x^3 + 8x^2 + 6x - 5$

By factor theorem,

$x + 1 = 0 ; x = -1$

If $f(-1) = 0$ then $(x + 1)$ is a factor of $f(x)$

$\therefore f(-1) = 3(-1)^3 + 8(-1)^2 - 6(-1) - 5$

$= -3 + 8 + 6 - 5 = 6$(not equal to 0)

$\therefore (x + 1)$ is not a factor of $f(x) = 3x^3 + 8x^2 + 6x - 5$

**1 D. Question**

Determine whether $(x + 1)$ is a factor of the following polynomials:

$x^3 - 14x^2 + 3x + 12$

**Answer**

Let $f(x) = x^3 - 14x^2 + 3x + 12$

By factor theorem,

$x + 1 = 0 ; x = -1$

If $f(-1) = 0$ then $(x + 1)$ is a factor of $f(x)$

$\therefore f(-1) = (-1)^3 - 14(-1)^2 + 3(-1) + 12$

$= -1 - 14 - 3 + 12 = -6$(not equal to 0)

$\therefore$(x + 1) is not a factor of $f(x) = x^3 - 14x^2 + 3x + 12$

**2. Question**

Determine whether (x + 4) is a factor of $x^3 + 3x^2 - 5x + 36$.

**Answer**

Let $f(x) = x^3 + 3x^2 - 5x + 36$.

By factor theorem,

$x + 4 = 0$: $x = -4$

If $f(-4) = 0$, then (x + 4) is a factor

$\therefore f(-4) = (-4)^3 + 3(-4)^2 - 5(-4) + 36$

$= -64 + 48 + 20 + 36$

$= -64 + 104 = 40$

$\therefore f(-4)$ is not equal to 0

So, (x + 4) is not a factor of f(x).

**3. Question**

Using factor theorem show that (x – 1) is a factor of $4x^3 - 6x^2 + 9x - 7$.

**Answer**

$f(x) = 4x^3 - 6x^2 + 9x - 7$

By factor theorem,

$(x - 1) = 0$ ; $x = 1$

Since, (x – 1) is a factor of f(x)

Therefore, $f(1) = 0$

$f(1) = 4(1)^3 - 6(1)^2 + 9(1) - 7 = 4 - 6 + 9 - 7 = 13 - 13 = 0$

$\therefore$(x – 1) is a factor of f(x)

**4. Question**

Determine whether (2x + 1) is a factor of $4x^3 + 4x^2 - x - 1$.

**Answer**

Let $f(x) = 4x^3 + 4x^2 - x - 1$

By factor Theorem,

2x + 1 = 0 ; x = – 1/2

∴f(– 1/2) = 4(– 1/2)$^3$ + 4(– 1/2)$^2$ – (– 1/2) – 1

= 4(– 1/8) + 4(1/4) + (1/2) – 1

= (– 1/2) + 1 + (1/2) – 1 = 0

∴f(– 1/2) = 0

So, (2x + 1) is a factor of f(x).

**5. Question**

Determine the value of p if (x + 3) is a factor of $x^3 - 3x^2 - px + 24$.

**Answer**

Let f(x) = $x^3 + 3x^2 - px + 24$.

By factor theorem,

x + 3 = 0;x = – 3

∴(x + 3) is a factor of f(x)

So, f(– 3) = 0.

f(– 3) = (– 3)$^3$ – 3(– 3)$^2$ – p(– 3) + 24 = 0

= >27 – 27 + 3p + 24 = 0

= > – 59 + 24 + 3p = 0

∴3p – 30 = 0

⇒ p = 30/3

**⇒ p = 10**

## Exercise 3.6

**1. Question**

The coefficient of $x^2$ & x in $2x^3 - 3x^2 - 2x + 3$ are respectively:

A. 2, 3

B. – 3, – 2

C. – 2, – 3

D. 2, – 3

**Answer**

$2x^3 - 3x^2 - 2x + 3$: Coefficient of $x^2 = -3$

Coefficient of $x = -2$

**2. Question**

The degree of polynomial $4x^2 - 7x^3 + 6x + 1$ is:

A. 2

B. 1

C. 3

D. 0

**Answer**

Degree of polynomial = Highest power of x in the polynomial = 3

**3. Question**

The polynomial $3x - 2$ is a :

A. Linear polynomial

B. Quadratic polynomial

C. Cubic polynomial

D. constant polynomial

**Answer**

Given polynomial has degree = 1

**4. Question**

The polynomial $4x^2 + 2x - 2$ is a :

A. Linear polynomial

B. Quadratic polynomial

C. Cubic polynomial

D. constant polynomial

**Answer**

Given polynomial has degree = 2

**5. Question**

The zero of the polynomial $2x - 5$:

A. 5/2

B. – 5/2

C. 2/5

D. – 2/5

**Answer**

Given : $2x - 5 = 0$

$\therefore x = 5/2$

So, zero of polynomial = 5/2

**6. Question**

The root of polynomial equation $3x - 1$ is:

A. – 1/3

B. 1/3

C. 1

D. 3

**Answer**

Given polynomial equation: $3x - 1 = 0$

$\therefore x = 1/3$

So, root = 1/3

**7. Question**

The root of polynomial equation $x^2 + 2x = 0$:

A. 0, 2

B. 1, 2

C. 1, – 2

D. 0, – 2

**Answer**

Given polynomial equation :

$x^2 + 2x = 0$

$\therefore x(x + 2) = 0$

$x = 0, x + 2 = 0$

$x = 0, x = -2$

So, the roots are 0 and – 2

**8. Question**

If a polynomial p(x) is divided by (ax + b), then the remainder is:

A. p(b/a)

B. p(– b/a)

C. p(a/b)

D. p(– a/b)

**Answer**

$f(x) = ax + b$

Therefore, by Remainder Theorem, $f(x) = 0$

$ax + b = 0$

$x = -b/a$ $\therefore$Remainder = $p(x) = p(-b/a)$

**9. Question**

If a polynomial $x^3 - ax^2 + ax - a$ is divided by $(x - a)$, then the remainder is:

A. $a^3$

B. $a^2$

C. a

D. – a

**Answer**

Given $f(x) = x^3 - ax^2 + ax - 2$

By Remainder Theorem,

$x - a = 0$

$x = a$

$\therefore$Remainder = $f(a) = a^3 - a^3 + 2a - a = a$

**10. Question**

If (ax – b) is a factor of p(x) then,

A. p(b) = 0

B. p(– b/a) = 0

C. p(a) = 0

D. p(b/a) = 0

**Answer**

As (ax – b) is a factor of p(x)

ax – b = 0

∴x = b/a So, p(x) = 0 ;

p(b/a) = 0

**11. Question**

One of the factor of $x^2 - 3x - 10$ is :

A. x – 2

B. x + 5

C. x – 5

D. x – 3

**Answer**

$x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

x(x – 5) + 2(x – 5) = 0

(x – 5)(x + 2) = 0 Hence, (x – 5) is a factor.

**12. Question**

One of the factor of $x^3 - 2x^2 + 2x - 1$ is :

A. x – 1

B. x + 1

C. x – 2

D. x + 2

**Answer**

Given: $f(x) = x^3 - 2x^2 + 2x - 1 = 0$

By hit and trial method,

put $x = 1$

$f(1) = 1 - 2 + 2 - 1 = 0$

$\therefore (x - 1)$ is a factor.