Chapter 3

Stability Analysis

CHAPTER HIGHLIGHTS

- *Introduction to Stability*
- *Routh Hurwitz Criterion*
- *Auxiliary Equation*
- *Root Locus*
- *Frequency Response Analysis*
- *Frequency – Domine Specifications*
- *Band Width*
- *Gain Margin*
- *Phase Margin*
- *Bode Plot*
- *Polar Plot*
- *Nyquist Criterion*

Introduction to Stability

Stability: A linear time-invariant system is stable if the output of the system is bounded for a bounded input and the output of the system tends towards zero in the absence of the input

Stability is classified as follows:

1. Absolute stability
2. Conditional stability
3. Marginal stability
4. Unstable

Absolute stability: A system is absolutely stable with respect to a parameter, if the system is stable for all values of that parameter.

Conditional stability: A system in conditionally stable with respect to a parameter, if the system is stable for only certain bounded ranges of values of this parameter.

Marginal stability: A system is marginally stable if the natural response of the system neither decays nor grows but remains constant or oscillates as time approaches infinity.

Unstable: A system is unstable if its response is unbounded with a bounded input applied.

Stability and poles

The system poles that are in the left half plane yield either pure exponential decay or damped sinusoidal natural response, which is the necessary condition for a system to be stable.

NOTES

1. Stable systems have closed–loop transfer function with poles only in the left half plane.
2. Unstable systems have loop transfer function with at least one pole in the right half plane or poles of multiplicity greater than one on the imaginary axis.
3. Marginally stable systems have closed–loop transfer function with only imaginary poles of multiplicity one and poles in the left half plane.

Necessary Conditions for Stability

1. Positiveness of the coefficients of characteristic equation is a necessary and a sufficient condition for stability of first- and second-order system.
2. Positiveness and existence of the all coefficients of the characteristic equation is a necessary condition for stability of the system.

NOTE

Roots with negative real part indicate all positive coefficients in characteristic equation but all positive coefficients do not indicate proofs with negative real part in the characteristic equation.

Routh–Hurwitz Criterion

Routh–Hurwitz criterion gives the necessary and sufficient condition for all roots of polynomial to lie in the left half of the s-plane, without actually solving for the roots of the equation.

The characteristic equation of the *n*th-order system is

$$D(s) = a_0 s^{n} + a_1 s^{n-1} + a_2 s^{n-2} + \ldots.. + a_{n-1} s + a_n = 0$$

Routh array

$$
\begin{array}{l|lllllll}
S^n & a_0 & a_2 & a_4 & a_6 & \cdot & \cdot & \cdot \\
S^{n-1} & a_1 & a_3 & a_5 & a_7 & \cdot & \cdot & \cdot \\
S^{n-2} & b_1 & b_2 & b_3 & b_4 \\
S^{n-3} & c_1 & c_2 & c_3 \\
S^{n-4} & d_1 & d_2 \\
\cdot \\
\cdot \\
S^2 & e_1 & a_n \\
S^1 & f_1 \\
S^0 & a_n
\end{array}
$$

The coefficients b_1, b_2 …. are evaluated as follows:

$$b_1 = (a_1a_2 - a_0a_3)/a_1;$$
$$b_2 = (a_1a_4 - a_0a_5)/a_1;$$
$$b_3 = (a_1a_6 - a_0a_7)/a_1 \ldots.$$

This process will continue till we get a zero as the least coefficient in the third row. Similarly, the coefficients of the other rows are also evaluated.

The roots of the characteristic equation are all in the left half of s-plane if all the coefficients of the first column of the Routh's tabulation are of the same sign.

The number of changes of signs in the elements of the first column equals the number of roots with positive real parts or in the right half of s–plane.

Special case 1: This happens when the first term in any row of the Routh array is zero while rest of the row has at least one non–zero term.

In this case, if zero appears as the first element of a row, the elements in the next row will all becomes infinite. To overcome this problem, we replace the zero element by an arbitrary small positive number ε and proceed with Routh's tabulation.

Finally, substitute the value of $\varepsilon = 0$ and find the values of the elements of the array which are functions of ε. The resultant Routh's array is analysed as usual.

NOTE

If there is a single element zero in s row, it is considered as row of all zeros.

Special case 2: When all the elements in one row of Routh's tabulation are zeros before the tabulation is properly terminated, it indicates the following:

1. There are symmetrically located roots in s–plane
2. Pair of real roots with opposite signs and/or pair of conjugate roots on the imaginary axis and/or complex conjugate roots forming quadrates in the s-plane.

Auxiliary equation

The polynomial formed by the coefficients of the row just above the row of zeros in the Routh array is called auxiliary equation $[A(s) = 0]$

NOTES

1. The order of the auxiliary equation is always even.
2. The roots of the auxiliary equation also satisfy the original characteristic equation.
3. Break down in the Routh table due to zero row is overcome by replacing the row of zeros with first derivative of auxiliary equation $\left(\frac{dA(s)}{ds}\right)$ with respect to s.

Solved Examples

Example 1

A system transfer function has some poles lying on the imaginary axis and it is

(A) unconditionally stable (B) conditionally stable
(C) unstable (D) marginally stable

Solution

When the poles are on imaginary axis, the system is marginally stable.

Example 2

A system has some roots with real parts equal to zero, but none with positive real part is

(A) absolutely unstable (B) absolutely stable
(C) relatively stable (D) marginally stable

Solution

Marginally stable

Example 3

Closed–loop stability implies that $1 + G(s)H(s)$ has only __________ in the left half of the s-plane.

(A) poles (B) zeroes
(C) poles and zeros (D) poles or zeros

Solution

Zeroes of characteristic equation are poles of the transfer function.

Example 4

None of the poles of a linear control system lie in the right half of s-plane. For a bounded input, the output of this system

(A) could be bounded (B) always tends to zero
(C) is always bounded (D) None of these

Solution

Poles are not on the right half and indicate they can be on imaginary axis. Therefore, stability cannot be justified.

Example 5

For the equation $s^3 - 4s^2 + s + 5 = 0$, the number of roots in the left half of s-plane will be

(A) zero (B) one (C) two (D) three

Solution

Routh array for $s^3 - 4s^2 + s + 5 = 0$

$$\begin{array}{c|l} s^3 & 1 \ 1 \\ s^2 & -4 \ 5 \\ s^1 & 2.25 \\ s^0 & 5 \end{array}$$

Sign changes in first column of Routh array are 2($1 \to -4 \to 1$) poles on left half $= 3 - 2 = 1$

Example 6

The number of roots of the equation $2s^4 + s^3 + 5s + 6 = 0$ that lie in the right half of s-plane is

(A) zero (B) one (C) two (D) four

Solution

Routh array for $2s^4 + s^3 + 3s^2 + 5s + 6 = 0$

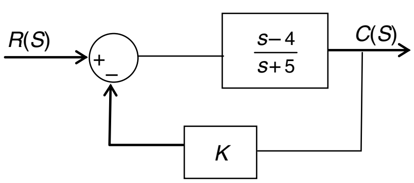
$$\begin{array}{c|lll} s^4 & 2 & 3 & 6 \\ s^3 & 1 & 5 & \\ s^2 & -7 & & 6 \\ s^1 & \frac{41}{7} & & \\ s^0 & 6 & & \end{array}$$

Number of sign changes in Routh array are 2 $\left(1 \to -7 \to \frac{41}{7}\right)$

Number of poles on the right half = 2

Example 7

For what range of K is the following system is asymptotically stable; assume $K \geq 0$?



(A) $0 \leq K < \frac{5}{4}$ (B) $0 \leq K < \frac{4}{5}$

(C) $K > -\frac{4}{5}$ (D) $K > -\frac{5}{4}$

Solution

Given system transfer function

$$= \frac{K(s-4)}{(1+K)s+(5-4K)}$$

Characteristic equation of the system is

$$(1 + K)s + (5 - 4K) = 0$$

For the system to be stable, all the coefficients of 's' in the characteristic equation must be positive.

$1 + K > 0$ $\quad$ $5 - 4K > 0$

$K > -1$ $\quad$ $-4K > -5$

$$K < \frac{5}{4}$$

Actual ranges of K is $-1 < K < \frac{5}{4}$

Given $K \geq 0$; $0 \leq K < \frac{5}{4}$

Example 8

The open-loop transfer function of a unity feedback system is given below.

$$G(s) = \frac{K(s+4)}{(s+1)(s+2)}$$

The range of positive values of K for which the closed–loop system will remain stable is

(A) $2 < K < 3$ (B) $\frac{2}{4} < K < 3$

(C) $0 < K < \infty$ (D) $\frac{2}{4} < K < \infty$

Solution

Closed–loop transfer function $= \frac{G(s)}{1+G(s)}$

$$= \frac{K(s+4)}{s^2+(3+K)s+(2+4K)}$$

Characteristic equation of the system $s^2 + (3 + K)s + (2 + 4K) = 0$

Condition for stability is that all coefficients of s must be grater than zero in characteristic equation

$3 + K > 0$ $\quad$ $2 + 4K > 0$

$K > -3$ $\quad$ $4K > -2$

$$K > -\frac{2}{4}$$

Therefore, the system is stable for all values of $K > -\frac{2}{4}$

Therefore, range of positive values of k for stability is $0 < k < \infty$.

Example 9

A certain closed–loop system with unity feedback has the following transfer function given by $G(s) = \frac{k}{s(s+2)(s+4)}$ with the gain set at the ultimate value. The system will oscillate at an angular frequency of

(A) 2 rad/s (B) 4 rad/s

(C) 8 rad/s (D) $2\sqrt{2}$ rad/s

Solution

Characteristic equation of the system is

$$s^3 + 6s^2 + 8s + K = 0$$

System will oscillate when it is marginally stable or from Routh array.

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 6 & K \\ s^1 & \frac{48-K}{6} & \\ s^0 & K & \end{array}$$

System is marginally stable if $48 - K = 0 \Rightarrow K = 48$

Then, auxiliary equation is $6s^2 + 48 = 0$

$$s^2 = -8 \Rightarrow s = j\,2\sqrt{2}$$

Oscillation frequency = $2\sqrt{2}$ rad/s

Root Locus

Root locus is basically the technique of finding the locus of roots as a single gain is changed, by solving for the roots of the characteristic equation, at each gain.

The gain that is to be varied will be open-loop gain. Note this does not mean the gain of the open-loop system that is typically fixed; this refers to cascading a controller in the forward path. Using the root locus method, the control system engineer can predict the effect of varying gain on the open-loop poles or what effect will be caused by adding open-loop poles or open-loop zeros.

Angle and Magnitude Conditions

Consider the following general system:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

The characteristic equation of the system is obtained by setting the denominator of the closed-loop system to zero as follows:

$$1 + G(s)\,H(s) = 0$$

Therefore, $G(s)\,H(s) = -1$

Since complex variable has both an angle and a magnitude, we can split the above equation into two separate equations as follows:

$$\angle G(s)\,H(s) = \pm 180^\circ\,(2K+1)\ (K = 0, 1, 2\ldots\ldots)$$

→ Angle condition

$$|G(s)H(s)| = 1 \rightarrow \text{magnitude condition}$$

The values of s that satisfy the angle and magnitude conditions are the roots of the characteristic equation (closed–loop poles). Only these values will be the roots. As we vary the gain, these values of s that satisfy both conditions will change. The resulting collection of point in s-plane are called root locus.

NOTE

Open-loop gain K corresponding to any point on root locus can be calculated using the equation:

$$K = \frac{\text{Product of lengh of vectors from open loop poles to the point}}{\text{product of lenght of vectors form open loop zeros to the point}}$$

Rules for construction of root locus

1. The root locus is symmetric about origin.
2. The number of branches in a root locus is equal to either the number of poles (n) or the number of zeros (m) whichever is greater. Each branch of root locus starts from open poles (assuming the number of poles is greater than zero) corresponding to $K = 0$ and terminates at either a finite open-loop zero or infinity corresponding to $K = \infty$. 'n' number of branches will terminate to finite open-loop zeros and the remaining branches of root locus ($n - m$) will terminate to infinity.

   $B = P$ if $P > Z \Rightarrow P - Z$ branches will terminate at ∞
   $B = Z$ if $Z > P \Rightarrow Z - P$ branches will terminate at ∞
   P = Number of poles, Z = number of zeros
   B = Number of branches of root locus
3. A section of real axis lies on root locus if the total number of open-loops poles plus zeros to the right of that section is odd.
4. The angle of asymptotes and centroid:
   If $P > Z$, $P - Z$ number of branches will terminate at ∞ along straight line (asymptotes) making angle with real axis given by

   $$\phi_A = \frac{180(2q+1)}{P-Z};\ (q = 0, 1, 2, 3, \ldots(P-Z-1))$$

   $$\text{If } Z > P \Rightarrow \phi_A = \frac{180(2q+1)}{Z-P};\ (q = 0, 1, 2, 3,..(Z-P-1))$$

   The point of intersection of the asymptotes with the real axis is called centroid and is denoted by σ.
   Centroid (σ) =

   $$\frac{\text{Sum of real part of pole} - \text{Sum of real part of zeros}}{\text{P} - \text{Z}}$$
5. Break away or in point
   A point on root locus where multiple poles or zeros exist is known as break away or in point.
   The break away or break in point is given by the roots of the equation $\frac{dK}{ds} = 0$, where K is obtained form $1 + KG(S)\,H(S) = 0$

NOTES

1. Break away point exists if there is a root locus on real axis between two adjacent poles.
2. Break in point exists if there is a root locus on real axis between two adjacent zeros
3. Break in point exists if there is a zero on real axis and left to that there is no root loci or poles or zeros.

6. The angle of departure or arrival
   The angle of departure or angle of arrival is given by
   Angle of departure = 180 – ϕ
   Angle of arrival = 180 + ϕ
   Where ϕ =

$$\begin{bmatrix} \text{Sum of angles of vectors} & \text{Sum of angles of vectors} \\ \text{to the complex pole / zero A from} - & \text{to the complex pole/} \\ & \text{zero from} \\ \text{other poles} & \text{other zeros.} \end{bmatrix}$$

7. The intersection of root locus with the imaginary axis can be determined by the use of Routh criterion (finding poles on imaginary axis)

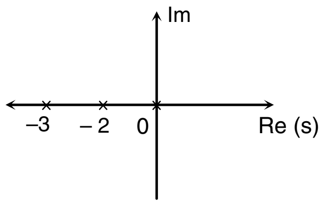
Example 10

Plot the root locus for a transfer function

$$G(s) = \frac{K}{s(s+2)(s+3)}$$

Solution

Number of poles = 3
The poles are at $s = 0$, $s = -2$ and $s = -3$



Break away point

$$\frac{d}{ds}(s^3 + 5s^2 + 6s) = 0$$

$$3s^2 + 10s + 6 = 0$$

$s = -0.784$ and $s = -2.549$

$s = -2.549$ does not lie on the root locus

Asymptotes

$$\theta_1 = \pm\frac{180}{3} = \pm 60°$$

$$\theta_3 = \pm\frac{3\times 180}{3} = \pm 180°$$

Centroid

$$s = \frac{(0-2-3)-(0)}{3-0} = -1.667$$

Imaginary axis cross-over

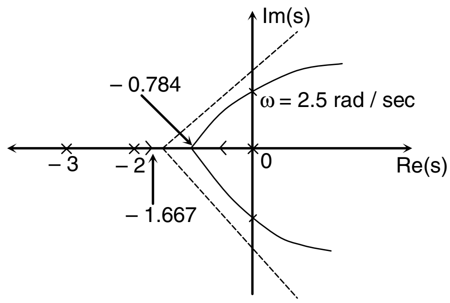
$$G(j\omega) = \frac{K}{j\omega(j\omega+2)(j\omega+3)}$$

$$G(j\omega) = K\left[\frac{-5\omega^2}{25\omega^4 + (6\omega - \omega^3)^2} - j\frac{(6\omega - \omega^3)}{25\omega^4 + (6\omega-\omega^3)^2}\right]$$

Equating the imaginary part to zero

$$\omega = \pm 2.5 \text{ rad/s}$$

The root locus is drawn as shown in the figure:



Example 11

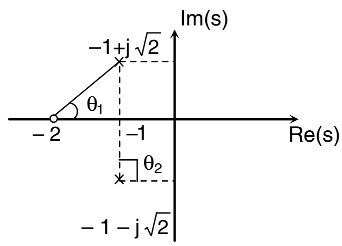
$$G(s) = \frac{K(s+2)}{s^2+2s+3},\ H(s) = 1$$

Sketch the root locus.

Solution

Number of branches of root locus = 2

The poles are at $s = -1 \pm j\sqrt{2}$
The zero is at $s = -2$



The root locus starts from the conjugate poles and break in on the real axis between –2 and – ∞. One root locus ends in $s = -2$, the other ends at $s = -\infty$.

Asymptote

$$\theta_1 = \pm\frac{180}{2-1} = \pm 180$$

Angle of departure

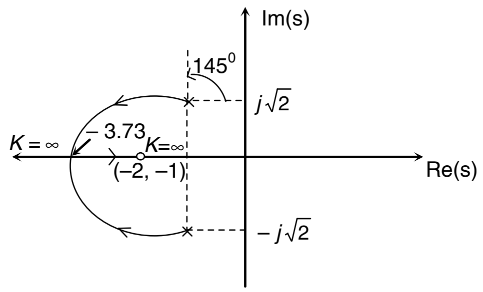
$$= 180 - \left(90 - \tan\frac{\sqrt{2}}{1}\right) = 145°$$

Break away point

$$\frac{d\,G(s)}{ds} = 0$$

$$s = -3.73$$

The root locus is drawn in the figure:



1. N-$P = Z$
   Here, $P = 1$
   If $K > 1$, $N = 1$,
   $Z = 0$, then the closed-loop system is stable.
2. If $K < 1$, $N = 0$
   $Z \neq 0$, then the closed-loop system is unstable.

Example 12

Given $G(s)\,H(s) = \dfrac{K}{s(s+2)(s+5)}$, the point of intersection of the asymptotes of the root locus with the real axis is

(A) 0 (B) –2 (C) –2.3 (D) –3.5

Solution

Number of poles (P) = 3 (0, –2, –5)

Number of zeros (Z) = 0

Number of asymptotes = 3

Centroid (Intersection of the asymptotes)

$$= \frac{\Sigma \text{ real part of all poles} - \Sigma \text{ real part of all zero}}{P - Z}$$

$$= \frac{0-2-5-0}{3} = \frac{-7}{3} = -2.33$$

Example 13

The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K(s+2)}{s(s^2+2s+1)}$$

The centroid and angles of root locus are, respectively,

(A) $-\dfrac{2}{3}$ and + 60°, – 60° (B) – 2 and + 90° , – 90°

(C) zero and + 90° , – 90° (D) – 2 and + 60°, – 60°

Solution

Number of poles = 3 (0, – 1, – 1)

Number of zeros = 1 (– 2)

Number of asymptotes = 2

Angle of asymptotes = + 90° and – 90°

Centroid =

$$\frac{\Sigma \text{ real part of all poles} - \Sigma \text{ real part of all zero}}{2}$$

$$= \frac{(0-1-1)-(-2)}{2} = \frac{-2+2}{2} = 0$$

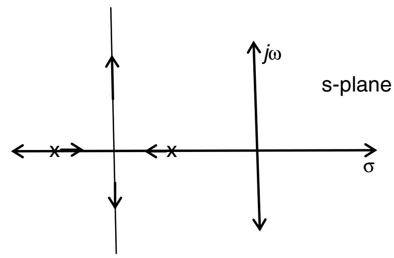
Example 14

The figure shown below gives root locus of the open-loop transfer function $G(s)\,H(s)$ of a system.

Consider the following inference drawn from the figure:

1. It has no zero.
2. It is a stable system.
3. It is a second-order system.

Which of these inferences are correct?



(A) 1, 2, and 3 (B) 1 and 2
(C) 2 and 3 (D) 1 and 3

Solution

Two poles terminated to infinity indicate that there are no zeros.

Two poles indicate the order of the system as 2.

Example 15

The characteristic equation of a unity–feedback control system is given by $S^3 + AS^2 + S + B = 0$

Consider the following statements in this regard:

1. For a given value of B, all the root locus branches will terminate at infinity for the variable A in the positive direction.
2. For a given value of B, only one root locus branch will terminate at infinity for the variable K, in the positive direction.
3. For a given value, of A, all the root locus branches will terminate at infinity for the variable 'B' in the positive direction.

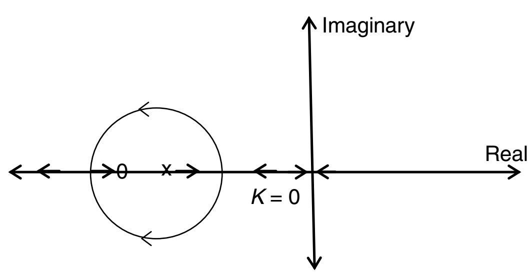
Of these statements,

(A) 1 and 3 are correct. (B) 2 and 3 are correct.
(C) Only 2 is correct. (D) Only 1 is correct.

Solution: (B)

Example 16

The root locus of a unity feedback system is shown in figure. The open-loop transfer function is given by



(A) $\dfrac{K}{s(s+2)(s+4)}$ (B) $\dfrac{K(s+4)}{s(s+3)(s+5)}$

(C) $\dfrac{K(s+4)}{s(s+3)}$ (D) $\dfrac{Ks}{(s+3)(s+5)}$

Solution

The given root locus indicates that the open-loop transfer function has two poles and one zero.

One pole is at the origin and another pole location is on the right side to the zero.

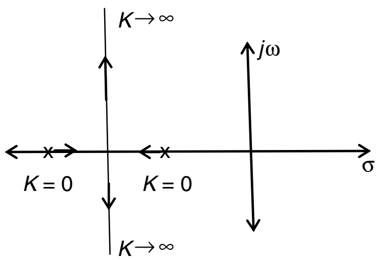
Example 17

The closed-loop transfer function of a feedback system is given by

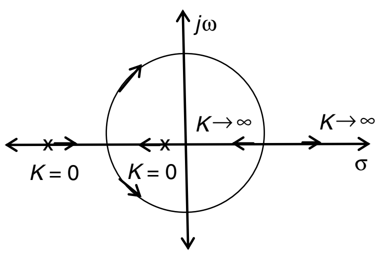
$$\frac{C(S)}{R(S)} = \frac{K}{s^2+(4-K)s+3}$$

Which of the following diagrams represent a root locus of the system for $K > 0$?

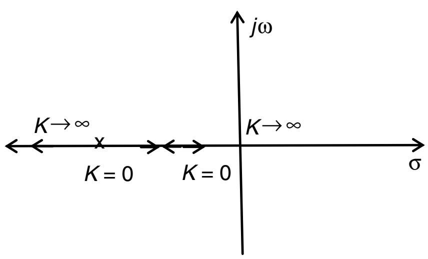
(A)



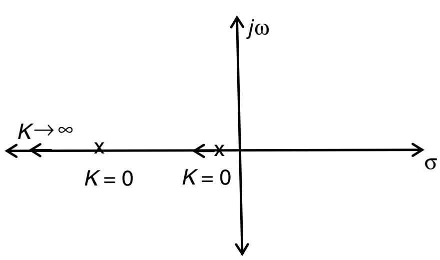
(B)



(C)



(D)



Solution

When the value of K increases, the location of poles will tend to right-hand side of the s-plane when $K > 4$.

→ When $K = 4$, characteristic equation $s^2 + 3 = 0$

$$s = \pm j\sqrt{3} =$$

(Poles are on imaginary axis)

→ When $K = 10$, characteristic equation $s^2 - 6s + 3 = 0$

$$(s-3)^2 = 0$$

$$s = +3, +3$$

(Poles are on right-hand side and equal)

→ When $K > 10$, poles are on right-hand side but not equal

Example 18

A control system has

$$G(s)\,\underline{H}(s) = \frac{K(s+5)}{(s+2)(s+3)}$$

The break away and break in points are located respectively at

(A) – 2 and – 1 (B) – 1.589 and – 7.5
(C) – 2.55 and – 7.5 (D) – 1.5 and – 6.89

Solution

Characteristic equation

$$1 + G(s)\,H(s) = 0$$

$$\Rightarrow K = \frac{-(s+2)(s+3)}{(s+5)} = \frac{s^2+5s+6}{s+5}$$

Break away or break in points are roots of $\dfrac{dK}{ds} = 0$

$$\frac{dK}{ds} = \frac{(2s+5)(s+5)-(s^2+5s+6)}{(s+5)^2} = 0$$

$$2s^2 + 15s + 25 - s^2 - 5s - 6 = 0$$

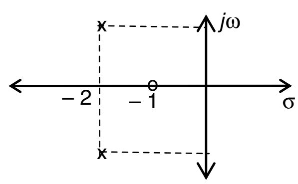
$$s^2 + 10s - 19 = 0$$

$$s = -2.55, -7.449$$

Break away point is –2.55, and break in point is –7.449.

Example 19

A transfer function $G(s)$ has type pole zero plot as shown in the figure. Given that the steady state gain is 3, the transfer function $G(s)$ will be



(A) $\dfrac{2(s+1)}{s^2+4s+5}$ (B) $\dfrac{5(s+1)}{s^2+4s+5}$

(C) $\dfrac{15(s+1)}{s^2+4s+5}$ (D) $\dfrac{15(s+1)}{(s+2)^2}$

Solution

From the given pole zero plot, transfer function has

1. zero is at (–1, 0)
2. complex poles
   Only for option '*C*' steady-state gain is '3'.

$$Lt_{s\to 0}G(s) = Lt_{s\to 0}\frac{15(s+1)}{s^2+4s+5} = \frac{15}{5} = 3.$$

Frequency Response Analysis

The magnitude and phase relationship between the sinusoidal input and the steady state output of a system is termed a frequency response. In linear time-invariant systems, the frequency response is independent of the amplitude and phase of the input signal**,** when the input of a linear time invariant system is sinusoidal with amplitude A and frequency ω_o.

$$r(t) = A \sin \omega_o t.$$

The steady-state output of a system $y(t)$ will be a sinusoidal with the same frequency ω_o but possibly with different amplitude and phase.

$$y(t) = \text{B} \sin(\omega_o t + \phi)$$

$$\frac{B}{A} = |\text{Transfer function}| = \frac{|G(j\omega)|}{1+G(j\omega)H(j\omega)} = |M(j\omega)|$$

$$\phi = \angle\text{Transfer function} = \angle G(j\omega) - \angle[1+G(j\omega)H(j\omega)] = \angle M(j\omega)$$

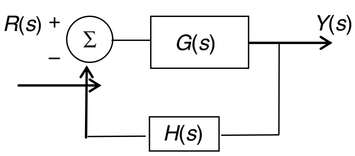
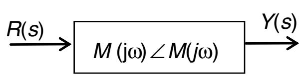


Fig: closed loop control system



The ease and accuracy of measurements are some of the advantages of the frequency response method. Extraction of transfer function is easy from frequency response test than step response test (time response). The design and parameter adjustment of the open-loop transfer function of a system for specific. Closed-loop performance is carried out more easily in frequency domain than in time domain. The effect of noise disturbance and parameter variation are relatively easy to visualize and access through frequency response. Nyquist criterion is a powerful frequency domain method of extracting. The information regarding stability as well as relative stability of a system without the need to evaluate roots of the characteristic equation.

Frequency–domine Specifications

Resonant Peak (M_r)

The resonant peak M_r is the maximum value of $|M(j\omega)|$. The magnitude M_r gives indication on the relative stability of a stable closed-loop system.

For second-order system,

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \text{ For } \xi \le \frac{1}{\sqrt{2}}$$

$$M_r = 1 \text{ For } \xi > \frac{1}{\sqrt{2}}$$

NOTE

A large M_r corresponds to a large maximum over short of the step response.

Resonant frequency (ϖ_r)

The resonant frequency ω_r is the frequency at which the peak resonance M_r occurs.

For second-order system, $\omega_r = \omega_n\sqrt{1-2\xi^2}$ for $\xi \le \dfrac{1}{\sqrt{2}}$

$$\omega_r = 0 \text{ for } \xi > \frac{1}{\sqrt{2}}$$

Band width (BW)

The band width Bω is the frequency at which $|M(j\omega)|$ drops to 70.7% of or 3 dB down from its zero frequency value.

For second-order system,

$$\text{BW} = \omega_n\left[(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}\right]^{1/2}$$

Bandwidth gives an indication of the transient response of a control system, noise filtering characteristics, and robustness of the system.

Gain Margin (GM)

Gain margin is the amount of gain in decibel (dB) that can be added to the open-loop before the closed-loop system becomes unstable.

$$\text{Gain margin} = \text{GM} = 20\log_{10}\frac{1}{[M(j\omega_{pc})]}$$

$$= -20\log|M(j\omega_{pc})|\,\text{dB}$$

The phase cross over frequency (ω_{pc}) is the frequency at which phase angle becomes – 180°

Phase Margin

Phase margin (PM) is defined as the angle m degrees through which the M(jω) plot must be rotated about the origin so that the gain cross over passes through the (–1. j0) point.

$$\text{Phase margin} = \text{PM} = \angle M(j\omega_{gc}) - 180°$$

Gain cross over frequency (ωgc) is the frequency at which $M(j\omega)$ becomes 1 or decibel magnitude of $M(j\omega)$ becomes zero.

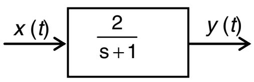
Relation Between Time Domain and Frequency Domain Characteristics

1. The resonant peak M_r of the closed-loop frequency response depends on ξ only. when $\xi = 0$, $M_r = \infty$. when ξ is negative, the system is unstable, and the value of M_r ceases to have any meaning. As ξ increases, M_r decreases.
   In comparison to time–response, maximum peak over shoot also depends only on 'ξ'. The peak overshoots in zero if $\xi \geq 1$.
2. Bandwidth is directly proportional to ω_n.
   $\rightarrow$ Bω increases linearly with ω_n.
   $\rightarrow$Bω decreases with increase in ξ for a fixed ω_n.
   For time response, rise time increases as ω_n decreases.
   Therefore, Bandwidth $\alpha \dfrac{1}{\text{Rise time}}$.
3. Bandwidth (BW) and M_r are proportional to each other for $0 \leq \xi \leq \dfrac{1}{\sqrt{2}}$.

Example 20

For the system shown in figure, the input $x(t) = \sin t$.

In the steady state, the response $y(t)$ will be



(A) $\dfrac{1}{\sqrt{2}}\sin(t - 45°)$ (B) $\sqrt{2}\sin(t - 45°)$

(C) $\dfrac{1}{\sqrt{2}}\sin(t - 45°)$ (D) $\sqrt{2}\sin(t - 45°)$

Solution

Transfer function $(T) = \dfrac{2}{s+1} = \dfrac{2}{j\omega + 1}$

Input = sint $[\therefore\ \omega = 1]$

$$|T|\angle\theta = \frac{2}{\sqrt{1+1}}\angle -\tan^{-1}\left(\frac{1}{1}\right)$$

$$= \sqrt{2}\ \angle -45°$$

$$y(t) = 1 \times \sqrt{2}\ \sin(t - 45°)$$

$$y(t) = \sqrt{2}\ \sin(t - 45°)$$

Example 21

A system with zero initial condition has the closed-loop transfer function.

$$T(s) = \frac{s^2+16}{(s+2)(s+3)}$$

The system output is zero at the frequency.

(A) 1 rad/s (B) 2 rad/s

(C) 3 rad/s (D) 4 rad/s

Solution

Magnitude of transfer function $= \dfrac{|-\omega^2+16|}{|(j\omega+2)(j\omega+3)|}$

The magnitude of transfer function will affect the magnitude of the system output. The output becomes zero when transfer function magnitude is zero.

$$\frac{|-\omega^2+16|}{|(j\omega+2)(j\omega+3)|} = 0$$

$$-\omega^2 + 16 = 0$$

$$\omega = 4 \text{ rad/s}$$

Example 22

The gain margin of a unity feedback control system with the open–loop transfer function $G(s) = \dfrac{s+4}{s^2}$ is

(A) 0 (B) $\dfrac{1}{\sqrt{4}}$ (C) $\sqrt{4}$ (D) ∞

Solution

Phase crossover frequency $\angle G(s) = -180°$

$$\tan^{-1}\left(\frac{\omega_{pc}}{4}\right) - 180° = -180°$$

$$\tan^{-1}\left(\frac{\omega_{pc}}{4}\right) = 0$$

$$\omega_{pc} = 0$$

Magnitude of transfer function at phase crossover frequency

$$\left[\frac{j\omega_{pc}+4}{(j\omega_{pc})^2}\right]\omega_{pc=0} = \infty$$

$$\text{Gain margin} = \frac{1}{|G(j\omega_{pc})|} = \frac{1}{\infty} = 0$$

Example 23

The open-loop transfer function of a unit feedback control system B is given as $G(s) = \dfrac{sx+1}{s^2}$. The value of x to give a phase margin of $\dfrac{\pi}{4}$ is equal to

(A) 0.441 (B) 0.141

(C) 1.141 (D) 0.841

Solution

Phase margin = 180° $\angle G(j\omega) = 45°$

$$180° + \tan^{-1}\left(\frac{x\omega_{gc}}{1}\right) - 180° = 45°$$

$$x\omega_{gc} = 1$$

Magnitude phase margin $|G(j\omega_{pc})| = 1$

$$\left|\frac{j\omega_{gc}+1}{(j\omega_{gc})^2}\right| = 1$$

$$_{gc} = 12^{1/4.}$$

$$x\,\omega_{gc} = 1$$

$$x = \frac{1}{2^{1/4}} = 0.841$$

Example 24

In the $G(s)\,H(s)$ plane, the Nyquist plot of the loop transfer function $G(s)\,H(s) = \dfrac{\pi . e^{-s}}{s}$ passes through the negative real axis at the point

(C) $(-3, j0)$ (D) $(-2, j0)$
(A) $(-1, j0)$ (B) $(-0.5, j0)$

Solution

At the point of intersection with negative real axis,

$$\angle G(s)\,H(s) = -\pi$$

$$-\left(\omega_{pc} + \frac{\pi}{2}\right) = -\pi$$

$$\omega_{pc} = \frac{\pi}{2}$$

Magnitude of the $G(s)H(s)$ at $\omega = \omega_{pc}$ in the intersection point with negative real axis

$$|G\,(s)\;H\,(s)|\,\omega = \omega_{pc} = \left|\frac{\pi e^{-s}}{s}\right| = \left|\frac{\pi e^{-j\omega_{pc}}}{j\omega_{pc}}\right|$$

$$= \frac{\pi}{\omega_{pc}} = \frac{\pi}{\pi/2} = 2$$

Therefore, Nyquist plot passes through (–2,0)

Bode Plot

Bode plot is a graph of the transfer function of a linear time-invariant system frequency plotted with a log-frequency axis to show the system's frequency response. It is usually a combination of a Bode magnitude plot, expressing the magnitude of the frequency response gain, and a Bode phase plot expressing the frequency response phase shift.

The standard logarithmic magnitude of open–loop transfer function of $G(j\omega)$ is $20 \log_{10} |G(j\omega)|$. The unit used in this representation of the magnitude is the decibel, usually denoted as dB.

Generally, a transfer function can be expressed in terms of factors of its poles and zeros. The advantage of the logarithmic plot is the conversion of these multiplicative factors to additive terms.

Consider the general open–loop transfer function.

$$G(s) = \frac{K(1+sT_{z1})(1+sT_{z2}).....(1+sT_{zm})}{s^P(1+sT_{P1})(1+sT_{p2}).....(1+sT_{pn})}$$

In this example, the transfer function includes m number of zeros, p number of poles at origin, and in the mentioned part, n number of poles. Let $m = 1$, $n = 2$, $p = 1$.

$$\Rightarrow G(s) = \frac{K(1+sT_{z1})}{s(1+sT_{p1})(1+sT_{p2})}$$

$$G(j\omega) = \frac{K(1+j\omega T_{z1})}{j\omega(1+j\omega T_{p1})(1+j\omega T_{p2})}$$

Magnitude of $G(j\omega)$ =

$$\frac{K\sqrt{1+\omega^2 T_{z1}{}^2}}{\omega\sqrt{1+\omega^2\,T_{P_1}{}^2}\,\sqrt{1+\omega^2\;T_{P_2}{}^2}}$$

Magnitude of $G(j\omega)$ is decibels is

$$|G(j\omega)| \text{ in dB} = 20\log|G(j\omega)|$$

$$= 20\log K + 20\log\,\sqrt{1+\omega^2\,T_{p1}^2}\; - 20\,\log\omega$$

$$- 20\log\,\sqrt{1+\omega^2 T_{pl}^2} - 20\log\sqrt{1+\omega^2 T_{p2}^2}$$

The phase angle of $G(j\omega) = \angle G(j\omega) = \tan^{-1}\omega T_{z1} - 90° - \tan^{-1}\,\omega T_{P1} - \tan^{-1}\,\omega T_{P2}$

NOTE

From the above analysis, it is clear that when the magnitude is expressed in dB, the multiplication is converted to addition.

Therefore, to sketch the magnitude plot, knowledge of the magnitude variation of individual factors of the open-loop transfer function is essential. The various factors of open-loop transfer function are as follows:

1. Constant gain, K
2. Poles (or zeros) at origin, $\dfrac{1}{(j\omega)^n}$ *or* $(j\omega)^m$
3. First-order factor, $\dfrac{1}{1+j\omega T_p}$ *or* $1+j\omega T_z$
4. Quadratic factor, $\dfrac{1}{1+2\xi(j\omega/\omega_n)+\left(\dfrac{j\omega}{\omega_n}\right)^2}$ or

$$1+2\xi\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2 K$$

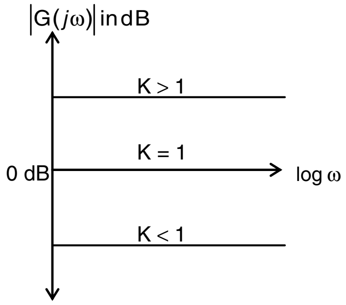
Constant gain: *K*

Let $G(s) = K$

Therefore, $G(j\omega) = K\,\angle 0°$

$$|G(j\omega)| \text{ in dB} = 20\log K$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left(\frac{0}{k}\right) = 0°$$



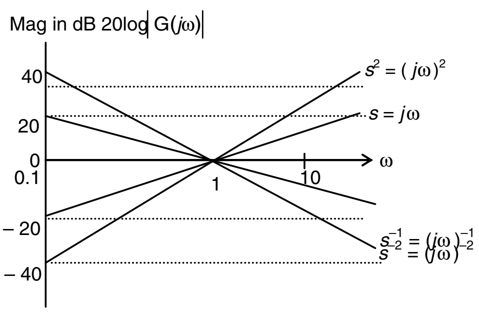
NOTES

1. The magnitude plot and phase plot of a constant K is independent of frequency and straight line.
2. A constant (K) greater than unity has a positive value in decibels, while a number smaller than unity has a negative value in decibels.
3. The change in the value of gain (K) of transfer function is increase or decrease in the magnitude plot.

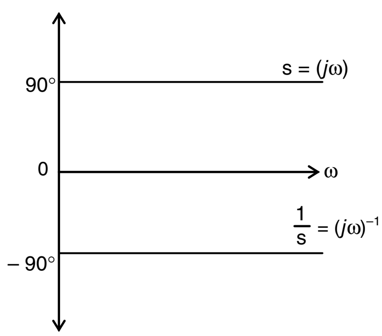
Poles (zeros) at origin $(j\omega)^{\pm n}$

Pole: Open–loop transfer function $[G(s)] = \dfrac{1}{s^n}$

$$\text{Phase angle } \phi = -n.tan^{-1}\left(\frac{\omega}{0}\right) = -n \times 90°$$



$\phi(j\omega)$ in degrees



Zeros: open-loop transfer function $[G(S)] = s^n$

$$\text{Log magnitude} = 20 \log|(j\omega)^n) = 20n \log \omega$$

$$\text{Phase angle } (\phi) = n \times \tan^{-1}\left(\frac{\omega}{0}\right) = n \times 90°$$

NOTES

1. Magnitude plot of $S^{\pm n}$ is a straight line with slop of $\pm 20 \times n$ dB / decade that passes through the point [0 dB, 1 rad/s]
2. Phase angle plot of $S^{\pm n}$ is independent of frequency and it is constant angle of value $\pm 90n$ degrees.

First-order factor $(1 + j\omega T)^{\pm 1}$:

Pole: open-loop transfer function $G(s) = \dfrac{1}{1 + sTp}$

Log magnitude =

$$20 \log\left|\frac{1}{1 + j\omega T_p}\right| = -20 \log\sqrt{1 + \omega^2 T_p^{\,2}}$$

For $\omega << \dfrac{1}{T_p}$; the asymptote is 20 log1 = 0 dB

For $\omega >> \dfrac{1}{T_p}$; The asymptote is $-20 \log \omega T_p$: It is a straight line with slope of – 20dB/decade. This asymptote intersect 0 dB at the break frequency $\omega_c = 1/T_p$ which is known as corner frequency.

$$\text{Phase angle } \phi = -\tan^{-1}\left(\frac{\omega T_p}{1}\right) = -\tan^{-1}(\omega T_p)$$

At corner frequency $\phi = -\tan^{-1}(\omega T_p)$

$$= -\tan^{-1} 1 = 45°$$

The phase angle of the factor $(1 + sT_p)^{-1}$ varies form 0 to –90° as 'ω' is varied form 0 to infinity. The phase angle plot crosses –45° at $\omega = \omega_c = \dfrac{1}{T_p}$

Zero: open-loop transfer function

$$G(s) = (1 + sT_z)$$

Log magnitude $= 20 \log\sqrt{1 + \omega^2 T_z^{\,2}}$

For $\omega << \dfrac{1}{T_z}$; the asymptote is 20 log 1 = 0 dB

For $\omega >> \dfrac{1}{T_z}$; the asymptote is $20 \log \omega T_z$: it is a straight line with slop of + 20 dB/decade. This asymptote intersects 0 dB at the break frequency $\omega_c = \dfrac{1}{T_z}$. This is known as corner frequency.

$$\text{Phase angle } \phi = \tan^{-1} \omega T_z$$

The phase angle of the factor $(1 + sT_z)$ varies from 0 to 90° as 'ω' is varied from zero to infinity. The phase angle plot crosses 45° at $\omega = \omega_c = \dfrac{1}{T_z}$

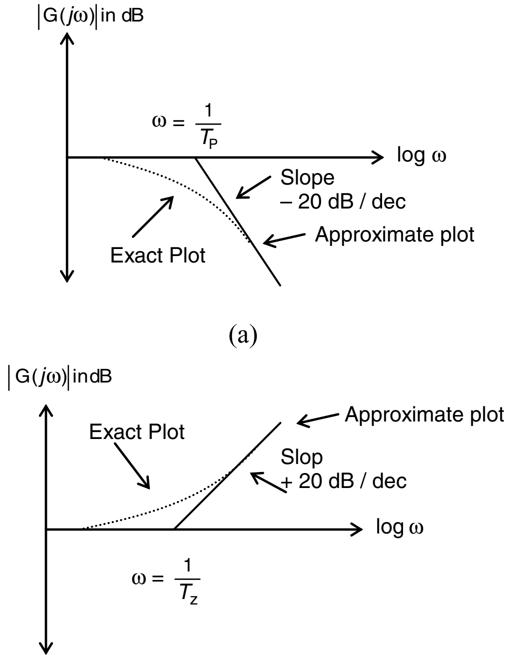


Figure 1 Magnitude plot for first-order pole (a) and first-order zero (b).

Quadratic factor $\left[1+2\xi\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2\right]^{\pm 1} =$

Open-loop transfer function =

$$\left\{1+2\xi\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2\right\}^{\pm 1}$$

The magnitude in decibels is =

$$\pm 20\log\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}$$

For $\omega \ll \omega_n$, the log magnitude is asymptotic to a straight line of constant gain 0 dB and phase angle approaches 0 degree.

For $\omega \gg \omega_n$, the log magnitude approaches

$\pm 40\log\left(\frac{\omega}{\omega_n}\right)$; a straight line with slop of ± 40 dB/dec

Asymptote intersect 0 dB at corner frequency $\omega = \omega_n$

NOTE

The resonant frequency is given by

$$\omega_r = \omega_n\sqrt{1-2\xi^2} \quad \text{for } \xi < \frac{1}{\sqrt{2}}$$

The maximum magnitude is

$$Mp = |G(j\omega_r)| = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad \text{for } \xi < \frac{1}{\sqrt{2}}$$

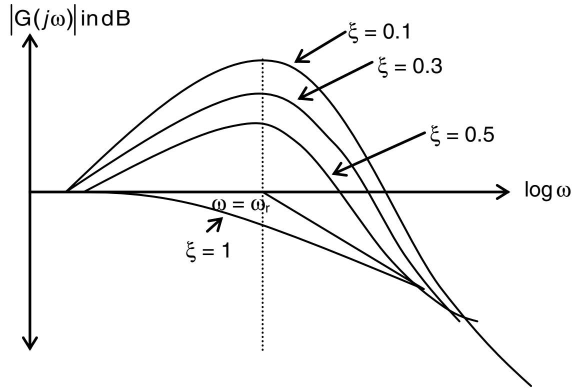


Figure Bode plot for quadratic factor in denominator

Example 25

Draw the Bode plot for a system having

$$G(s)H(s) = \frac{100}{s(s+1)(s+2)}$$

Find
(A) Gain margin
(B) Phase margin
(C) Gain cross-over frequency
(D) Phase cross-over frequency

Solution

$$G(j\omega)H(j\omega) = \frac{50}{j\omega(1+j\omega)(1+0.5j\omega)}$$

The corner frequencies are

$\omega = 1$ rad/s and $\omega = 2$ rad/s

For $\omega \le 1$ rad/s

$$G(j\omega)H(j\omega) = \frac{50}{j\omega}$$

slope = – 20 dB/decade

$|G(j\omega)H(j\omega)|_{dB} = 20\log 50 - 20\log\omega$

at $\omega = 0.1$

$|G(j\omega)H(j\omega)|_{dB} = 20\log 50 - 20\log(0.1)$

$= 53.98$dB

At $\omega = 1$

$|G(j\omega)H(j\omega)| = 20\log 50 = 33.98$ dB

For $1 < \omega \le 2$

$$G(j\omega)H(j\omega) = \frac{50}{j\omega(1+j\omega)}$$

slope = –20 – 20

= – 40 dB / decade

As ω increases from 1 to 2, the reduction in gain

$$= 40\log\left(\frac{2}{1}\right) = 12.04 \text{ dB}$$

At $\omega = 2$

$|G(j\omega)H(j\omega)|_{dB} = 21.94$ dB

For $\omega > 2$

$$G(j\omega)H(j\omega) = \frac{50}{j\omega(1 + j\omega)\,(1 + 0.5\,j\omega)}$$

$$\text{slope} = -40 - 20 = -60 \text{ dB/decade}$$

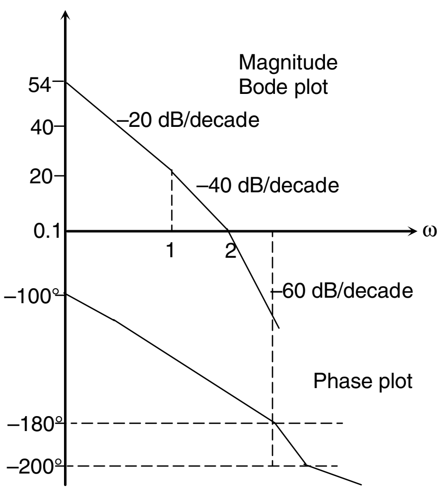
As ω increases from 2 to 10, the reduction in gain = 60 log $\left(\frac{10}{2}\right)$ = 41.94 dB

At $\omega = 10$,

$|G(j\omega)H(j\omega)|_{\text{dB}} = -\,19.99$ dB

$\angle G(j\omega)H(j\omega) = -90 - \tan^{-1}\,\omega - \tan^{-1}(0.5\omega)$

ω	$\angle G(j\omega)H(j\omega)$
0	–90
0.1	–98.6
0.2	–107
0.5	–130.6
1	–161.6
1.3	–175.5
1.4	–179.5
1.5	–183.2
2	–198.4



Gain cross over frequency = 4.45 rad/s

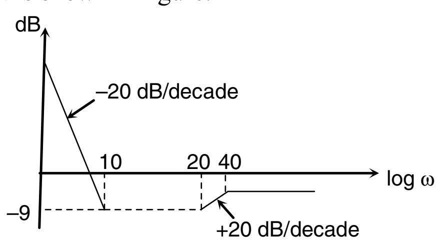
Phase cross over frequency = 1.40 rad/s

Gain margin = 27 dB

Phase margin = 53°

Example 26

Find the transfer function of the system whose asymptotic Bode plot is shown in figure.



Solution

The line with a slope of –20 dB/decade does not pass through $\omega = 1$ rad/s. There is a term $\frac{K}{s}$

$$20 \log K = -9$$

$$K = 0.35$$

At $\omega = 1$ rad/s, slope changes to 0 dB/dec indicating a zero at $\omega = 1$ rad/s. The term is $(1 + s)$

At $\omega = 20$ rad/s, the slope changes to +20 dB/decade, indicating a term $\left(1 + \frac{s}{20}\right)$ or $(1 + 0.05s)$

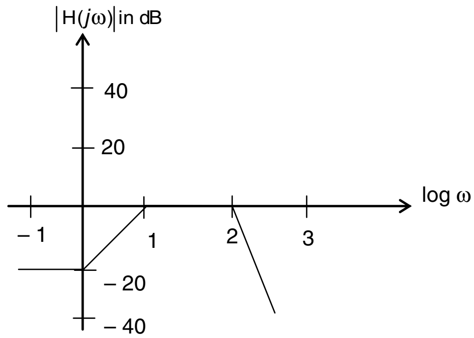
At $\omega = 40$ rad/s, the slope changes to 0dB/dec indicating a term $\left(1 + {}^{s}\!/_{40}\right)$ in the denominator.

$$\text{Therefore, } G(s) = \frac{0.35\,(1+s)(1 + 0.05s)}{s(1 + 0.025\,s)}$$

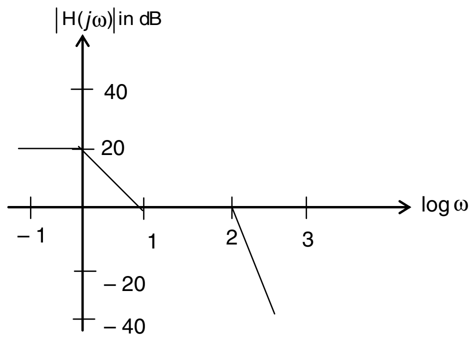
Example 27

The Bode magnitude plot of $H(s) = \dfrac{10^4(1+s)}{(10+s)(100+s)^2}$

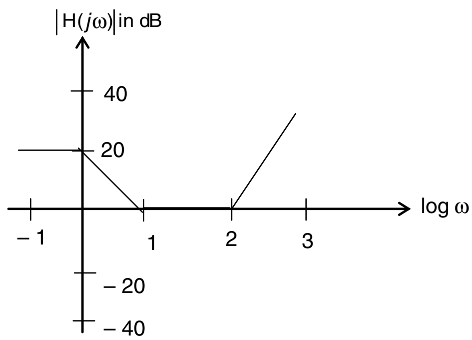
(A)



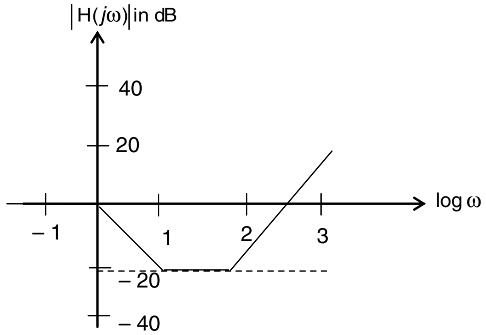
(B)



(C)



(D)



Solution

Given function $H(s) = \dfrac{10^4(1+s)}{(10+s)(10+s)^2}$

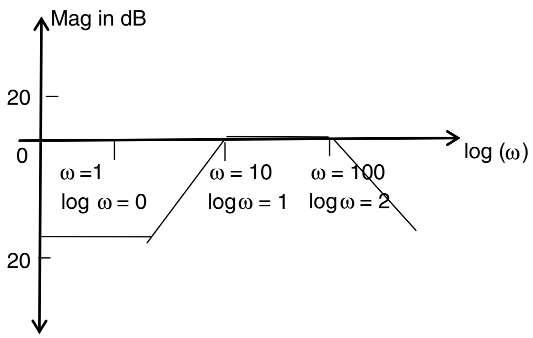
$$H(s) = \frac{10^4(1+s)}{(1+0.1s)(1+0.01s)^2 \times 10 \times 100^2}$$

$$H(s) = \frac{0.1(1+s)}{(1+0.1s)(1+0.01s)^2}$$

Corner frequencies are 1, 10, and 100

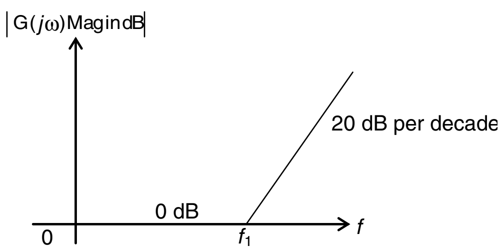
Initial magnitude = 20 log 0.1 = – 20 dB

Magnitude starts increasing with slope of +20 dB/dec at $\omega_c = 1$ rad, constant at $\omega = 10$ rad, and decays with a slop of 20 dB/dec at $\omega = 100$ rad.



Example 28

The function corresponding to the Bode plot of figure is



(A) $G = jf/f_1$

(B) $G = \dfrac{1}{\left(1 + j\dfrac{f}{f_1}\right)}$

(C) $G = \dfrac{1}{\left(1 - j\dfrac{f}{f_1}\right)}$

(D) $G = 1 + jf/f_1$

Solution

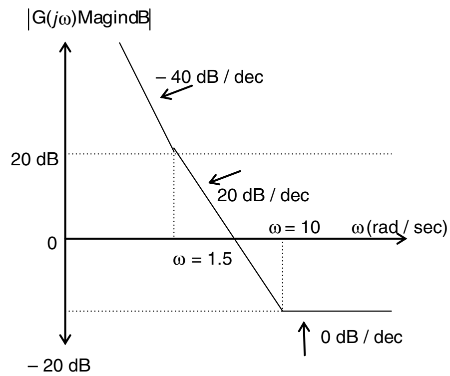
Magnitude plot slope change at frequency f_1 and is increasing. This indicates there is a zero at $f = f_1$.

$$G = (1 + sT_1) = (1 + j\omega T_1) = \left(1 + j\frac{2\pi f}{2\pi f_1}\right)$$

$$\begin{bmatrix} \omega = 2\pi f \\ T_1 = \dfrac{1}{\omega_c} = \dfrac{1}{2\pi f_1} \end{bmatrix} \Rightarrow G = 1 + j\frac{f}{f_1}$$

Example 29

The asymptotic Bode magnitude plot of a minimum phase transfer function is shown in the figure.



This transfer function has

(A) two poles and one zero.
(B) two poles and two zeros.
(C) one pole and two zeros.
(D) three poles and one zero.

Solution

Initial slope of the magnitude plot is –40 dB/dec and indicates two poles of the system are at origin.

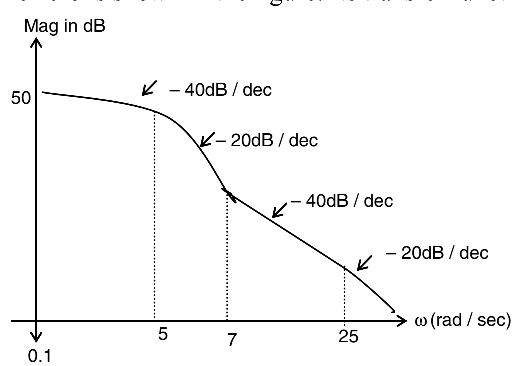
→ Reduction is slope by 20 dB/dec at $\omega = 1.5$ indicates a zero

→ Reduction in slop by 20 dB/dec at $\omega = 10$ indicates another zero.

Therefore, Total two poles and two zeros.

Example 30

The asymptotic approximation of the log– magnitude versus frequency plot of a minimum phase system with real poles and one zero is shown in the figure. Its transfer function is



(A) $\dfrac{10(s+5)(s+25)}{s^2(s+7)}$ (B) $\dfrac{4.4(s+7)(s+5)}{s^2(s+25)}$

(C) $\dfrac{10(s+7)}{s^2(s+5)(s+25)}$ (D) $\dfrac{4.4(s+5)(s+25)}{s^2(s+7)}$

Solution

Transfer function has corner frequencies 5, 7, and 25

Zeros are at $\omega = 5$ and $\omega = 25$

Poles are at $\omega = 7$

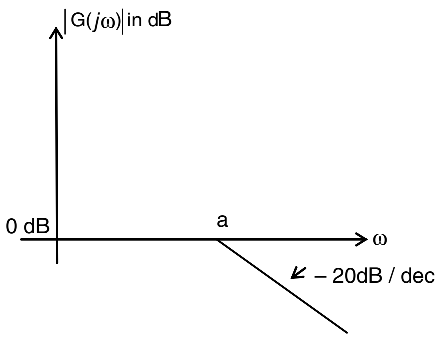
$$\text{T. F} = \frac{K\left(1+\frac{s}{5}\right)\left(1+\frac{s}{25}\right)}{s^2(1+s/7)} = \frac{7\times 5}{25}\frac{K(s+5)(s+25)}{s^2(s+7)}$$

Initial magnitude = 20log K = 10 $\Rightarrow K = 3.16$

$$\text{Transfer function} = \frac{4.4(s+5)(s+25)}{s^2(s+7)}$$

Example 31

The asymptotic Bode magnitude plot of a transfer function $\dfrac{1}{1+s/a}$ is shown in the figure. The error in dB gain at a frequency of $\omega = 0.5a$ is



(A) 3dB (B) 9dB

(C) 0.97dB (D) 5.4dB

Solution

Actual magnitude of given transfer function

$$= -20\log\left(1+\frac{j\omega}{a}\right)$$

At $\omega = 0.5a \Rightarrow |G(j\omega)| = -20 \log\left(1+j\dfrac{0.5a}{a}\right)$

$$= -20\log\sqrt{1+(0.5)^2}$$

$$|G(j\omega)| = -0.969 = -0.97 \text{ dB}$$

Approximated magnitude in given plot at $\omega = 0.5a$ is 0 dB

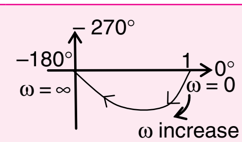
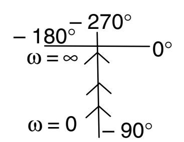
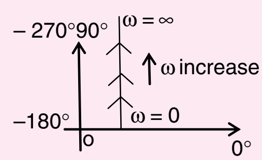
Error = 0 dB – (0.97 dB) = 0.97 dB

Polar Plot

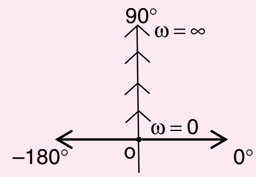
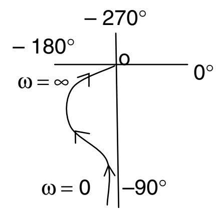
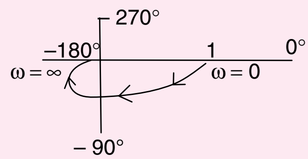
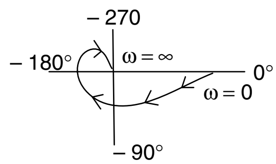
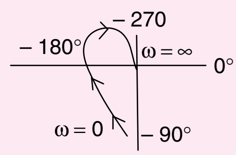
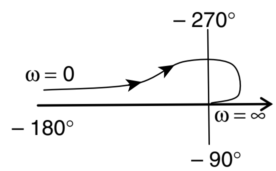
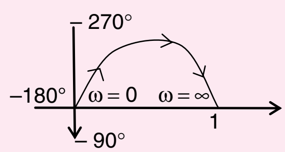
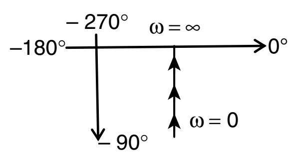
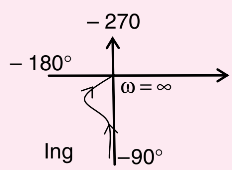
The transfer function $G(s)$ is a complex function and it is given by

$$G(s) = G(j\omega) = |G(j\omega)| \ \angle G(j\omega) \ = M\angle\phi$$

As the input frequency is varied from 0 to ∞, the magnitude M and phase angle ϕ change, the locus traced by the tip of the phasor $G(j\omega)$ is known as polar plot.

G(s)	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$	$\lvert G(j\omega)\rvert \angle G(j\omega)$ value $\omega \to 0$	$\omega \to \infty$	**Polar plot**
$\dfrac{1}{1+sT}$	$\dfrac{1}{\sqrt{1+\omega^2T^2}}$	$-\tan^{-1}\omega T$	$1\angle 0$	$0\angle -90^\circ$	
$\dfrac{1}{s}$	$\dfrac{1}{\omega}$	-90°	$\infty\angle -90^\circ$	$0\angle -90^\circ$	
1 + sT	$1+\omega T$	90°	$(1+0)\angle 90^\circ$	$(1+\infty)\angle 90^\circ$	

(*Continued*)

$G(s)$	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$	$\lvert G(j\omega)\rvert \angle G(j\omega)$ value $\omega \to 0$	$\omega \to \infty$	Polar plot
s	ω	90°	$0\angle 90^\circ$	$\infty\angle 90^\circ$	
$\dfrac{1}{s(1+sT)}$	$\dfrac{1}{\omega\sqrt{1+\omega^2T^2}}$	$-90^\circ - \tan^{-1} \omega T$	$\infty\angle -90^\circ$	$0\angle -180^\circ$	
$\dfrac{1}{(1+sT_1)(1+sT_2)}$	$\dfrac{1}{\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}}$	$-\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$	$1\angle 0^\circ$	$0\angle -180^\circ$	
$\dfrac{1}{s(1+sT_1)(1+sT_2)}$	$\dfrac{1}{\omega\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}}$	$-90^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$	$\infty\angle -90^\circ$	$0\angle -270^\circ$	
$\dfrac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$	$\dfrac{1}{\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}\sqrt{1+\omega^2T_3^2}}$	$-\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-}\omega T_3$	$1\angle 0^\circ$	$0\angle -270^\circ$	
$\dfrac{1}{s^2(1+sT_1)(1+sT_2)}$	$\dfrac{1}{\omega^2\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}}$	$-180^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$	$\infty\angle -180^\circ$	$0\angle -360^\circ$	
$\dfrac{s}{1+sT}$	$\dfrac{\omega}{\sqrt{1+\omega^2T^2}}$	$90^\circ - \tan^{-1}\omega T$	$0\angle 90^\circ$	$1\angle 0^\circ$	
$\dfrac{1+sT}{s}$	$\dfrac{\sqrt{1+\omega^2T^2}}{\omega}$	$-90^\circ + \tan^{-1}\omega T$	$\infty\angle -90^\circ$	$1\angle 0^\circ$	
$\dfrac{1+sT_1}{s(1+sT_2)(1+sT_3)}$	$\dfrac{\sqrt{1+\omega^2T_1^2}}{\omega\sqrt{1+\omega^2T^2}\sqrt{1+\omega^2T_3^2}}$	$\tan^{-1}\omega T_1 - 90^\circ - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3$	$\infty\angle -90^\circ$	$0\angle -180^\circ$	

NOTES

1. Addition of a non-zero pole to a transfer function results in further rotation of the polar plot through an angle of –90° as $\omega \to \infty$ (head of the polar plot shifts)
2. Addition of a pole at origin to a transfer function results in rotation of the polar plot at zero and infinite frequency (head and tail of polar plot) by further angle of – 90°.
3. The effect of addition of a zero to a transfer function is to rotate the high frequency portion of the polar plot by 90° in counter-clockwise direction.

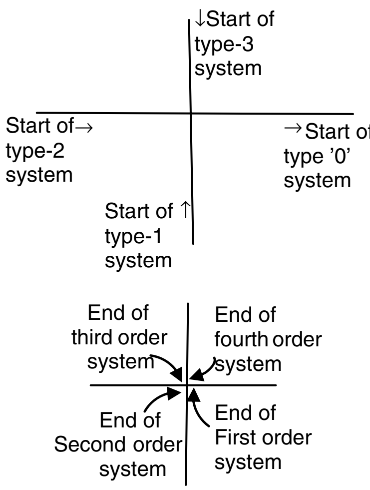


Figure 2 Start point and end point of polar plot for different system types and orders.

Nyquist Criterion

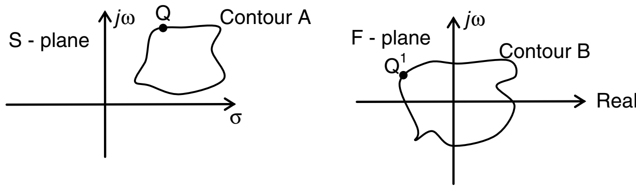
The Nyquist criterion relates the stability of a closed–loop-system to the open-loop frequency response and open-loop pole location. This criterion tells us how many closed-loop poles are in the right half of the s-plane.

The Nyquist criterion used the following concepts for the establishment of criterion.

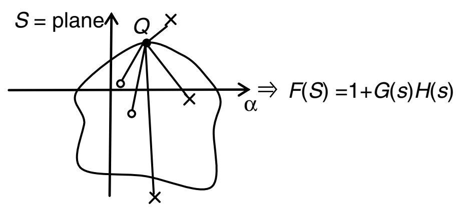
1. The poles of $1 + G(s)H(s)$ and the poles of $G(s)H(s)$ are same.
2. The zero of the $1 + G(s)H(s)$ is the poles of the closed-loop transfer function $T(s)$ of the system.
3. Mapping: Consider a complex number on the s-plane and substitute it into a function $F(s)$, and the result is another complex number. This process is called mapping.
   For example, substituting $s = 4 + j3$ into function $F(s) = s^2 + 2s + 1$ results in $16 + j30$. We say that $4 + j3$ maps into $16 + j30$ through the function $(s^2 + 2s + 1)$.
5. Mapping Contours:
   Consider the collection of points, called a contour, shown in figure as contour A. Assume that

$$F(s) = \frac{(s - Z_1)(s - Z_2)(s - Z_3)........}{(s - P_1)(s - P_2)(s - P_3)........}$$

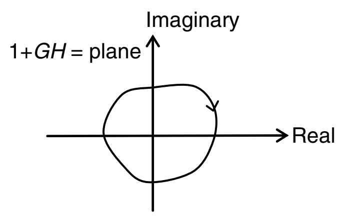
Contour A can be mapped through $F(s)$ into contour B by substituting each point of contour A into the function $F(s)$ and plotting the resulting complex numbers. For example, point Q in s-plane maps into point Q through the function $F(s)$.



Let us first assume that $F(s) = 1 + G(s)H(s)$, with the picture of the poles and zeros of $1 + G(s)H(s)$ as shown in the figure below. As each point Q of contour A is substituted into $1 + G(s)H(s)$, a mapped point results on contour B. As we move around contour A in a clockwise direction, each vector of $F(s)$ that lies inside contour A will appear to undergo a complete rotation or a change in angle of 360°. On the other hand, each vector drawn from the poles and zeros of $1 + G(s)H(s)$ that exists outside contour A will appear to oscillate and return to its previous position, undergoing a net angular change of 0°.



Contour A



Contour B

Number of counter clockwise rotations of contour B about origin $(N) = P - Z$

Where P = Number of poles of $1 + G(s)H(s)$ insider contour A.

Z = Number of zeros of $1 + G(s)H(s)$ inside contour A.

NOTE

Since the poles of $1 + G(s)H(s)$ are the poles of $G(s)H(s)$ and zeros of $1 + G(s)H(s)$ are poles of closed–loop system,

P = Number of open poles enclosed

Z = Number of closed–loop poles enclosed

$N = Z - P$ = Number of closed–loop poles inside the contour.

If we extend the contour to include the entire right half of s-plane, we can count the number of right-half-plane closed-loop poles inside contour A and determine a system's stability.

NOTE

When we map the entire right half of s-plane through $G(s)H(s)$ instead of $1 + G(s)H(s)$, the resulting contour is same as mapping through $1 + G(s)H(s)$, except that it is translated one unit to the left. So we count rotations about $-1 + j.0$ instead of rotations about the origin.

Statement of the Nyquist Stability Criterion

If a contour A that encircles the entire right half-plane is mapped through $G(s)H(s)$, then the number of the closed-loop poles (Z) in the right half-plane equals the number of open-loop poles(P) that are in the right half-plane minus the number of contour clockwise revolutions (N) around -1 of the mapping (i.e., $Z = P - N$). The mapping is called the Nyquist diagram/Nyquist plot of $G(s)\ H(s)$.

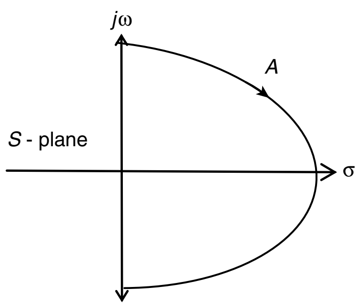


Figure 3 Contour enclosing right half of S-plane of determine stability.

NOTES

1. If contour A of the open–loop transfer function $G(s)H(s)$ corresponding to the Nyquist contour in the s-plane encircles the point $(-1 + j0)$ in the counter-clockwise direction as many times as the number of right half s-plane poles $G(s)H(s)$, the closed-loop system is stable.
2. No encirclement of $-1 + j0$ implies that the system is stable if there are no poles of $G(s)H(s)$ in the right half of s-plane; otherwise the system is unstable.
3. Clockwise encirclements in the Nyquist plane indicate that the system is unstable.

If $G(s)H(s)$ has any poles on $j\omega$ axis, the Nyquist contour defined earlier cannot be used as such. The s-plane contour should not pass through a singularity of $1 + G(s)H(s)$. The stability in such cases is studied with modified Nyquist contour which bypasses any $j\omega$ - axis poles. This is accomplished by indenting the Nyquist contour around the $j\omega$ poles along a semicircle of radius ε, where ε → 0.

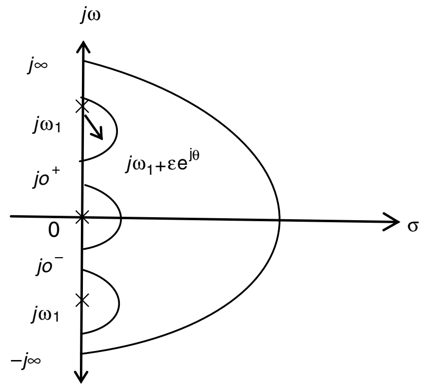


Figure 4 Indented Nyquist Contour for $j\omega$-axis open-loop poles.

Example 32

Consider a system with an open-loop transfer function

$$G(s)H(s) = \frac{(4s+1)}{s^2(s+1)(2s+1)}$$

Find the stability of the system using Nyquist plot.

Solution

The given open-loop transfer function has a double pole at origin. The Nyquist contour is intended to bypass the origin. The mapping of Nyquist contour is obtained as follows.

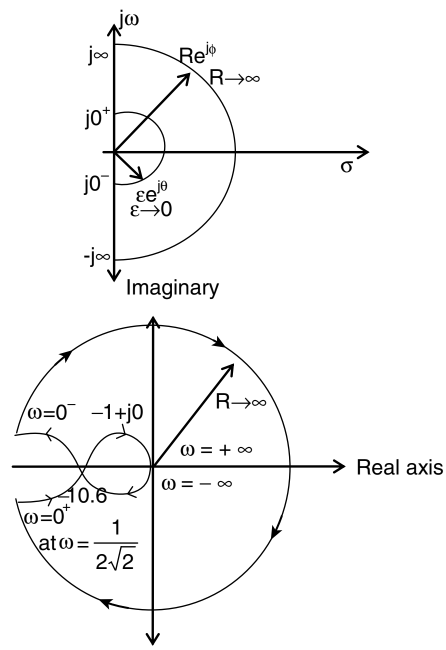


Figure 5 Nyquist contour and corresponding Nyquist plot.

1. Semicircular indent represented by $s = \lim_{\varepsilon \to 0} \varepsilon e^{j\theta}$ (where 'θ' varied from $-90°$ through $0°$ to $90°$) is mapped into

$$\lim_{\varepsilon \to 0}\left[\frac{4\varepsilon e^{j\theta}+1}{\varepsilon^2 e^{j20}(\varepsilon e^{j\theta}+1)(2\varepsilon e^{j\theta}+1)}\right] = \lim_{\varepsilon \to 0}\left(\frac{1}{\varepsilon^2 e^{j2\theta}}\right) = \infty e^{-j2\theta}$$

$$= \infty\ (\angle 180° \to 0° \to \angle -180°).$$

This part of the map is an infinite circle.

2. Mapping of positive imaginary axis

$$G(j\omega)H(j\omega) = \frac{(1+j4\omega)}{(j\omega)^2(1+j\omega)(1+j2\omega)}$$

For various values of ω, $G(j\omega)H(j\omega)$ is calculated and plotted using polar plots.

The $G(j\omega)H(j\omega)$ – locus intersects the real axis at a point where

$$\angle G(j\omega)\,H(j\omega) = -180°$$

$$-180° - \tan^{-1}\omega - \tan^{-1} 2\omega + \tan^{-1} 4\omega = -180°$$

$$\text{Therefore, } \omega = \frac{1}{2\sqrt{2}} = 0.354 \text{ rad/sec}$$

$$\therefore \left|G(j\omega)H(j\omega)\right|_{\omega=\frac{1}{2\sqrt{2}}} = 10.6$$

Further as $\omega \to +j\infty$

$$\Rightarrow \left|G(j\omega)H(j\omega)\right| \angle G(j\omega)H(j\omega) \Rightarrow 0\angle -270°$$

as $\omega \to 0^+ \Rightarrow |G(j\omega)\,H(j\omega)| \Rightarrow \infty\angle -180°$

3. The infinite semicircle of the Nyquist contour represented by $s = \lim_{R\to\infty} Re^{j\phi}$ (ϕ varies from + 90° through 0° to + 90°) is mapped to

$$\lim_{R\to\infty} \frac{(1+4\,Re^{j\phi})}{R^2e^{j2\phi}(1+Re^{j\phi})(1+2\,Re^{j\phi})} = 0e^{-j3\phi}$$

$$= 0(\angle -270° \to \angle 0° \to \angle +270°)$$

Number of counter clockwise encirclements to origin is –2.

Number of right half poles of open–loop is zero.

$$Z = P - N = 0 - (-2) = +2$$

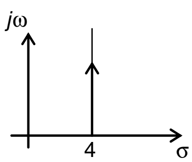
Therefore, number of poles on right half plane for closed-loop transfer function is 2.

Therefore, the system is unstable.

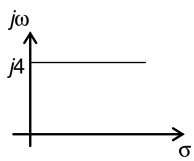
Example 33

Nyquist plot for the transfer function $G(s) = (4 + s)$ for positive frequencies has the form

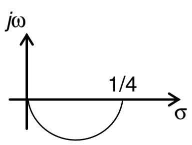
(A)



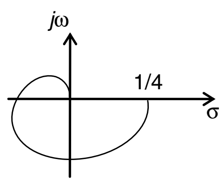
(B)



(C)



(D)



Solution

Given transfer function $G(s) = 4 + s = 4 + j\omega$

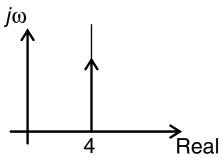
At $\omega = 0 \Rightarrow 4 + j.0$

$$\omega = 10 \Rightarrow 4 + j10$$

$$\omega = 100 \Rightarrow 4 + j100$$

$$\omega = \infty \Rightarrow 4 + j\infty$$

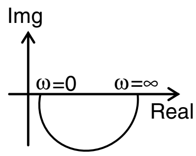
Therefore, Nyquist plot is parallel to imaginary axis.



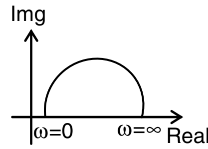
Example 34

Which one of the following polar diagrams correspond to a lag network?

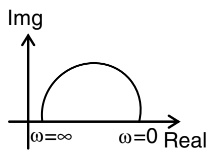
(A)



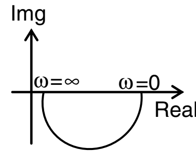
(B)



(C)



(D)



Solution

Lag network offers only negative phase angles and. Let us consider a lag network example.

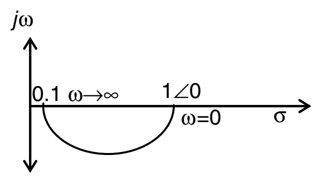
$$G(j\omega) = \frac{s+1}{10s+2}$$

$$|G(j\omega)|\sqrt{\frac{\omega^2+1}{100\omega^2+1}};\ \angle G(j\omega) = \tan^{-1}\frac{\omega}{1} - \tan^{-1} 10\omega\,.$$

At $\omega = 0 \Rightarrow 1\angle 0$

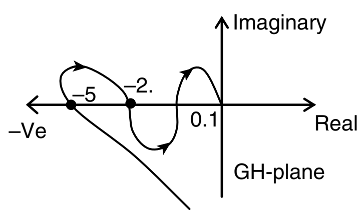
At $\omega = 5 \Rightarrow 0.103\angle -10.16$

At $\omega = \infty \Rightarrow 0.1\ \angle 0°$



Example 35

The polar plot of a conditionally stable system for open-loop gain $K = 1$ is shown in the figure. The open-loop transfer function of the system is known to be stable. The closed loop system is stable for



(A) $0.5 < K < 10$ (B) $K < \frac{1}{5}$

(C) Both (A) and (B) (D) $0.5 > K > 10$

Solution

System gain K should be adjusted such that the point ($-1 + j0$) lies in the 0.1 to 02 region, because the number of encirclements in this case is zero which results in stable operation of the system.

Therefore,

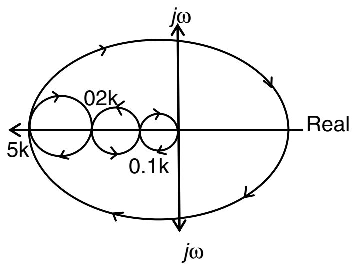
$$0.1k < 1 \Rightarrow K<10$$

$$2K > 1 \Rightarrow K > 0.5$$

Range of K is $\underline{0.5 < K < 10}$

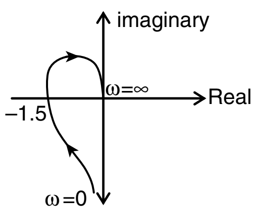
System is also stable if $5K < 1$ [no of encirclements will be zero in this case].

$$K < \frac{1}{5}$$



Example 36

The polar plot of an open-loop stable system as shown below the closed-loop system is



(A) marginally stable
(B) always stable
(C) unstable with one pole on the RH s-plane
(D) unstable with two poles on the RH s-plane

Solution

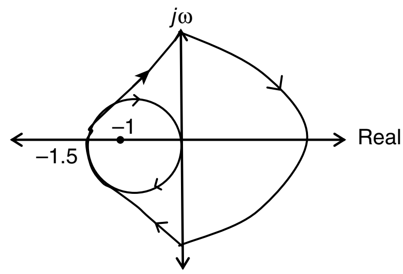
Complete the polar plot of the given system is given in the figure. Number of encirclements of ($-1 + j.0$) are -2.

Therefore, number of open-loop poles the right-hand side = 0

$$N = P - Z \Rightarrow Z = P - N$$

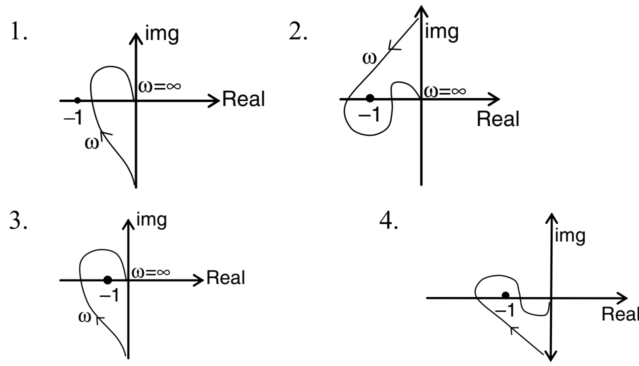
$$Z = 0 - (-2) = 2$$

Therefore, RH s-plane poles of closed-loop system are 2 ⇒ Unstable



Example 37

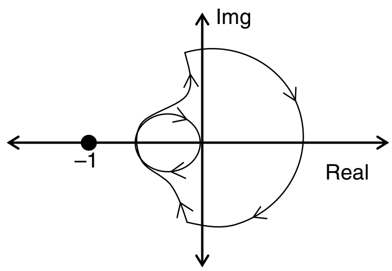
Consider the following Nyquist plots of loop transfer function over positive frequencies. Which of the plots represents an unstable system?



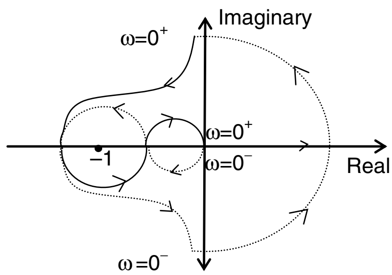
(A) (1), (2), and (3) (B) (2), (3), and (4)
(C) (1), (2), and (4) (D) (3) and (4) only

Solution

Plot (1) Nyquist plot is

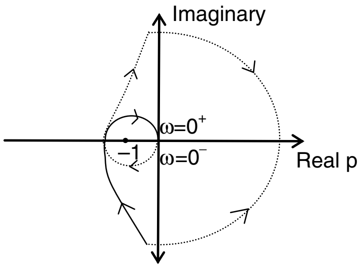


Number of encirclements (N) = 0
If the open–loop poles on RHS = 0, the system is stable
Plot (2) complete Nyquist plot is

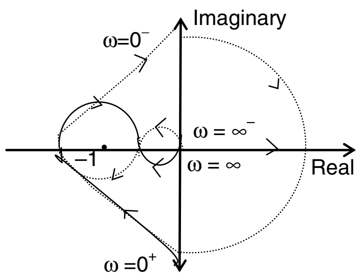


Number of counter clockwise encirclements (N) = 2.
Number of counter-clockwise encirclements (N) = $P - Z = 2$
Number of open–loop poles on RHS side (p) = 0
Number of poles of closed–loop system = 2

Therefore, the system is unstable
Plot (3) complete Nyquist plot is



Number of counter clockwise encirclements (N) = –2
Number of poles of closed–loop system on RHS = 2
Therefore, it is an unstable system.
Option (4) complete Nyquist plot is



Number of counter clockwise encirclements (N) = – 2
Number of RHS poles of closed–loop control system (Z) = P – N = 2
Therefore, the system is unstable.

Example 38

A unity feedback system has the open–loop transfer function

$$G(s) = \frac{1}{(s-1)(s+2)(s+3)}$$

The Nyquist plot of G encircles the origin
(A) Once (B) Twice (C) Thrice (D) Never

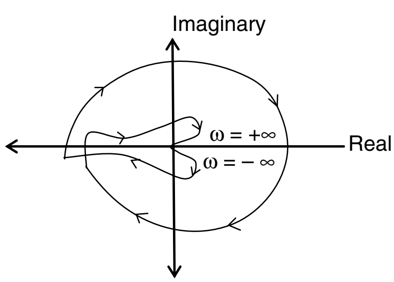
Solution

Number of encirclements equals the difference between no. of right-hand side poles of $G(s)$ and zeros.

$$N = P_{\text{OLTF}} - Z_{\text{OLTF}} \qquad P_{\text{OLTF}} = 1 \;\&\; Z_{\text{OLTF}} = 0 \qquad N = 1$$

Example 39

Which of the following is the transfer function of a system having the Nyquist plot shown in the figure?



(A) $\dfrac{K}{s^2(s+4)(s+8)}$ (B) $\dfrac{K}{s(s+4)^2(s+5)}$
(C) $\dfrac{K(s+2)}{s^2(s+4)(s+8)}$ (D) $\dfrac{K(s+2)(s+3)}{s^2(s+4)(s+5)}$

Solution

Nyquist plot started at –180° angle. It indicates that the open-loop system has two poles at origin.

Magnitude and phase angle at $\omega \to 0$

$$\Rightarrow \infty\angle -180^\circ$$

Magnitude and phase angle at $\omega \to \infty$

$$\Rightarrow 0\angle -360^\circ$$

Angle at the termination of NP is – 360°

Angle of termination −360° indicated system order is 4.

Therefore, the system is type 2 and order 4 system with no zeros.

Example 40

In the GH-plane, the Nyquist plot of the loop transfer function G(s)H(s) $= \dfrac{2\pi e^{-5s}}{s}$ pass through the negative real axis at the, point
(A) (–5, j0) (B) (–2, j0)
(C) (–10, j0) (D) (–20, j0

Solution

At the point of intersection of Nyquist plot with real axis phase angle $\angle G(s)\,H(s) = -180^\circ = -\pi$

$$\angle \frac{2\pi e^{-5j\omega}}{j\omega} = -\pi$$

$$-5\omega - \tan^{-1}\frac{\omega}{0} = -\pi$$

$$-5\omega - \frac{\pi}{2} = -\pi$$

$$5\omega = \frac{\pi}{2} \Rightarrow \omega = \frac{\pi}{10}$$

$$|G(s)H(s)|_{\omega=\frac{\pi}{10}} = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/10} = 20$$

Therefore, Nyquist plot passes through (–20, j0)

Exercises

Practice Problems I

Direction for questions 1 to 22: Select the correct alternative from the given choices.

1. The characteristic equation of a system is given by $s^6 + 3s^5 + 8s^4 + 18s^3 + 37s^2 + 75s + 50 = 0$. The system is

(A) stable. (B) unstable.
(C) marginally stable. (D) conditionally stable.

2. How many roots of the characteristic equation $s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4 = 0$ lie in the left half of s-plane?

(A) 4 (B) 5 (C) 1 (D) 6

3. A system described by the transfer function

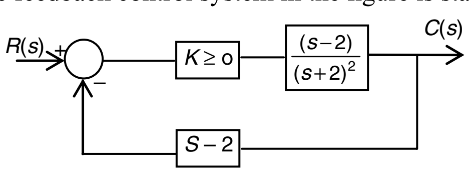
$H(s) = \dfrac{1}{s^3 + \alpha s^2 + ks + 2}$ is stable. The constraints on α and k are

(A) $\alpha > 0,\ \alpha k > 2$ (B) $\alpha > 0,\ \alpha k < 2$
(C) $\alpha > 0,\ \alpha k > 0$ (D) $\alpha < 0,\ \alpha k < 0$

4. The characteristic equation of a system is given by $s(s^2 + 2s + 2) + K(s + 3) = 0$. The range of k for which the system is stable is

(A) $0 < k < 30$. (B) $K > 3$.
(C) $0 < k < 4$. (D) $3 < K < 30$.

5. The feedback control system in the figure is stable



(A) for all $K \geq 0$ (B) only if $K \geq 1$
(C) only if $0 \leq k < 1$ (D) only if $0 \leq k \leq 1$

6. Consider the points $S_1 = -3 + j4$ and $S_2 = -3 - j2$ in the s–plane. For a system with the open-loop transfer function, $G(s)H(s) = \dfrac{k}{(s+1)^4}$ is

(A) S_1 is on the root locus, but not S_2.
(B) Both S_1 and S_2 are on the root locus.
(C) S_2 is on the root locus, but not S_1.
(D) Neither S_1 nor S_2 on the root locus.

7. The gain margin (in dB) of a system having the open–loop transfer function.

$G(s)\,H(s) = \dfrac{\sqrt{2}}{s(s+1)}$ is

(A) 0 (B) 3.01 (C) –3.01 (D) ∞

8. The characteristic equation of a feedback control system is given by $s^3 + 5s^2 + (K + 6)s + K = 0$ In the root loci diagram, the asymptotes of the root loci for large 'K' meet at a point in the s–plane whose coordinates are

(A) (2, 0) (B) (–1, 0)
(C) (–2, 0) (D) (–3, 0)

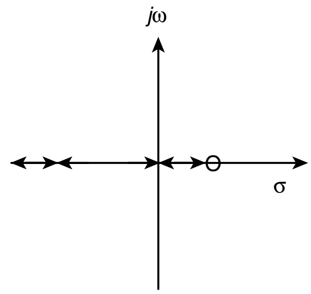
9. The open-loop transfer function of a system is given by $G(s) = \dfrac{k}{s(s+1)(s+2)}$ the value of k which will cause sustained oscillations in the closed–loop unity feed book system is

(A) 4 (B) 6 (C) 5 (D) 3

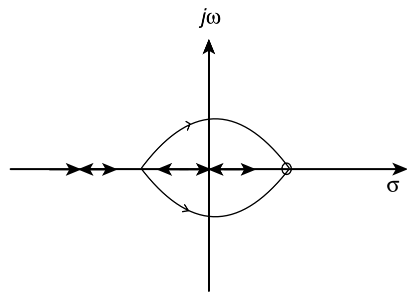
10. A unity feedback system is given as

$G(s) = \dfrac{k(1-s)}{s(s+3)}$. Indicate the correct root locus diagram.

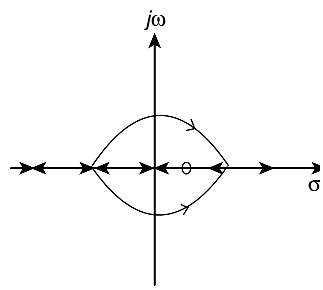
(A)



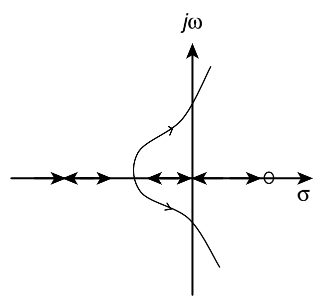
(B)



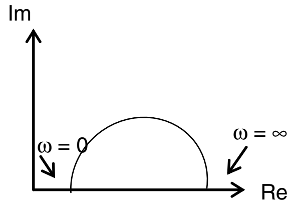
(C)

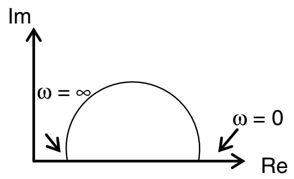


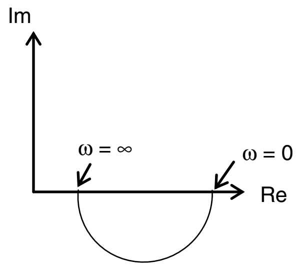
(D)

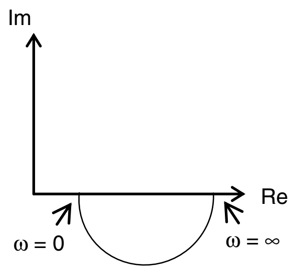


11. Which one of the following polar diagrams corresponds to a lag network?

(A) 

(B) 

(C) 

(D) 

Direction for questions 12 and 13:

The open-loop transfer function of a unity feedback system is given by $G(S) = \dfrac{3e^{-2s}}{s(s+2)}$

12. The gain and phase cross–over frequencies in rad/s, respectively, are

(A) 0.485 and 0.632. (B) 1.26 and 0.632.
(C) 0.632 and 1.26. (D) 0.632 and 0.485.

13. Based on the above results, the gain and phase margins of the system will be

(A) –7.09 dB and 87.5°. (B) 7.09 dB and 87.5°.
(C) 7.09 dB and –87.5°. (D) –7.09 dB and –87.5°.

14. The loop transfer function of a closed-loop control system is given as

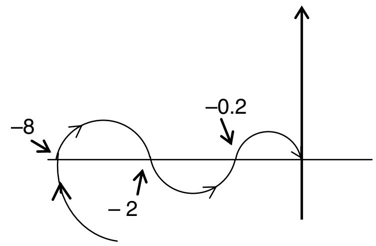
$G(s)H(s) = \dfrac{k(s+1)}{s(s+2)(s+3)}$. The centroid of the asymptotes is

(A) (–4, 0) B) (–1, 0) (C) (–2, 0) (D) (–3, 0)

15. A system has 10 poles and 2 zeroes. The slope of its highest frequency asymptote in its magnitude plot is

(A) –100 dB/dec. (B) –120 dB/dec.
(C) –160 dB/dec (D) –240 dB/dec.

16. The polar diagram of a conditionally stable system for open-loop gain $K = 1$ is shown in figure. The open-loop transfer function of the system is known to be stable. The closed-loop system is stable for



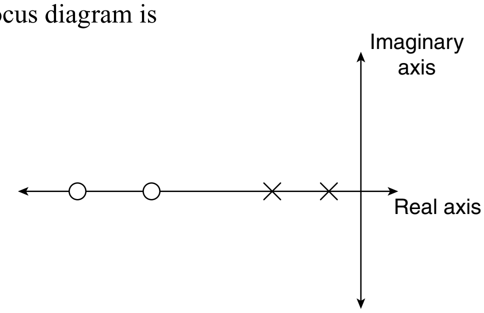
(A) $K < 5$ and $\frac{1}{2} < K < \frac{1}{8}$

(B) $K < \frac{1}{8}$ and $\frac{1}{2} < K < 5$

(C) $K < \frac{1}{8}$ and $5 < K$

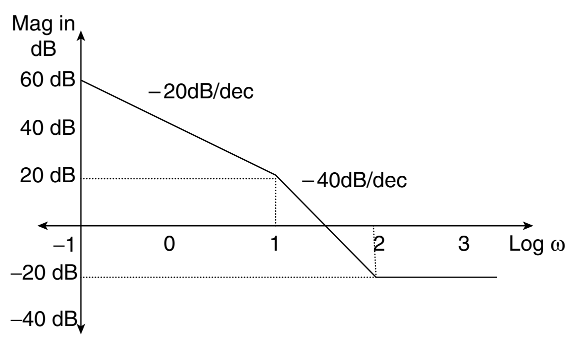
(D) $K > \frac{1}{8}$ and $K < 5$

17. Pole zero plot of a loop transfer function is shown in the figure below. The breakaway/break in points in the root locus diagram is



(A) 1 (B) 2 (C) 0 (D) 3

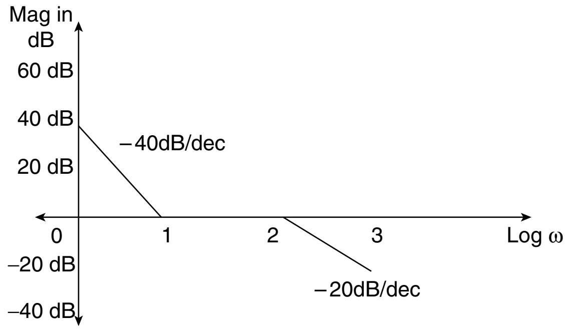
18. Loop transfer function $G(s)H(s)$ of the magnitude plot shown in the figure is



(A) $\dfrac{100(s+2)^2}{s(s+1)}$ (B) $\dfrac{10(.01s+1)}{s(.1s+1)}$

(C) $\dfrac{100(.01S+1)^2}{S(.1S+1)}$ (D) $\dfrac{(.01S+1)^2}{S(.1S+1)}$

19. Loop transfer function $G(s)H(s)$ of the magnitude plot shown in the figure is



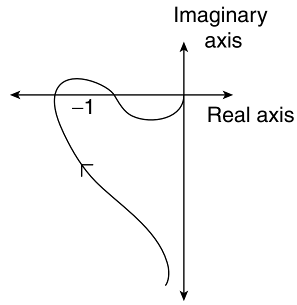
(A) $\dfrac{100(0.1s+1)^2}{s^2(0.01s+1)}$ (B) $\dfrac{100(s+10)^2}{(s+100)(s+1)^2}$

(C) Both (A) and (B) (D) None of these

20. Closed-loop control system with transfer function $\dfrac{G(s)}{1+G(s)H(s)}$ is stable when

(A) poles of the transfer function are on the left hand side of the s-plane.
(B) zeros of the characteristic equation are on the left half of the s-plane.
(C) poles of the characteristic equation are on the left half of the s-plane.
(D) Both A and B

21. Polar plot of an open-loop stable system is shown in the figure. The system is



(A) stable.
(B) unstable with one pole on the right-hand side of s-plane.
(C) unstable with two poles on the right-hand side of s-plane.
(D) marginally stable.

22. Which of the flowing are the effects of PD controller on system?

1. Reduces peak overshoot.
2. Reduces raise time.
3. Improves damping.
4. Reduces steady state error.

(A) 1, 2, 3 (B) ,2,3,4 (C) 2,3, 4 (D) 1,3,4

Practice Problems 2

Direction for questions 1 to 15: Select the correct alternative from the given Solutions.

1. Which of the following statements are true?
(i) Root locus is a frequency response plot.
(ii) The roots of characteristic equation are not a function of open-loop gain *K*.
(iii) Root locus technique is a tool for adjusting the location of closed-loop poles to achieve the desired system performance.
(iv) The exact root– locus is sketched by trail and error procedure.

(A) i and ii (B) ii and iii
(C) iii and iv (D) ii, iii, and iv

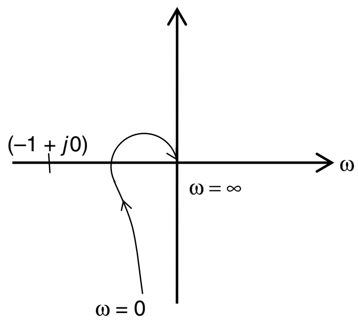
2. The following statements refer to the equation $P(s) + KQ(s) = 0$ where $P(s)$ and $Q(s)$ are polynomials of s with constant coefficients. Identify the statements which are true?
(i) The intersect of the asymptotes must always be on the real axis.
(ii) The breakaway points of the root loci must always be on the real axis.
(iii) Given the equation $1 + KG_1(s)H_1(s) = 0$ where $G_1(s)H_1(s)$ is a rational function of s and does not contain *K*, the roots of $\dfrac{dG_1(s)H_1(s)}{ds}$ are all break away points on the root loci $(-\infty < K < \infty)$.
(iv) At the breakaway points on the root loci, the root sensitivity is infinite.

(A) i and iv (B) i, ii, and iv
(C) ii and iii (D) ii, iii, and iv

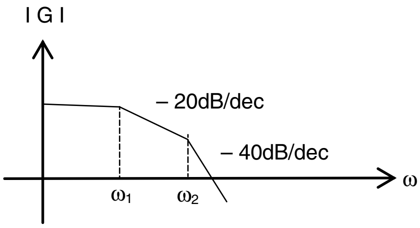
3. Which of the following statements are true?
(i) Adding a zero to the function $G(s)H(s)$ tends to push the root loci to the left.
(ii) Adding a zero to the forward path transfer function will generally improve the system damping, and thus always reduce the maximum over shoot of the system.
(iii) Adding a pole to $G(s)H(s)$ has the effect of pushing the root loci to the right.
(iv) Complementary root locus (CRL) refers to root loci with positive *k*.

(A) i, ii, and iii (B) i, ii, and iv
(C) ii, iii, and iv (D) ii and iv

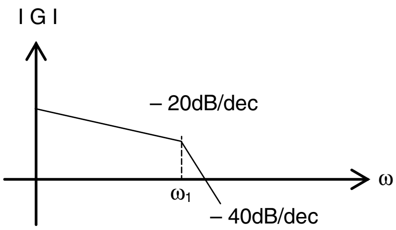
4. The Nyquist plot for a control system is shown in figure. The Bode plot for the same system will be



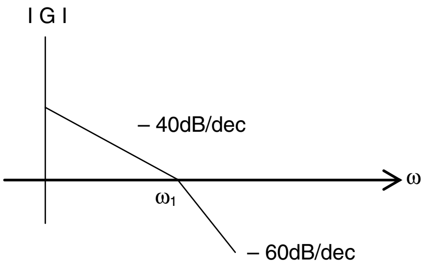
(A)



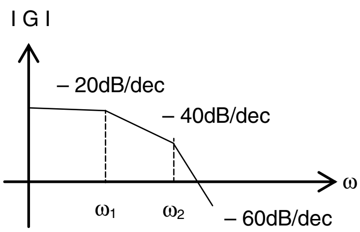
(B)



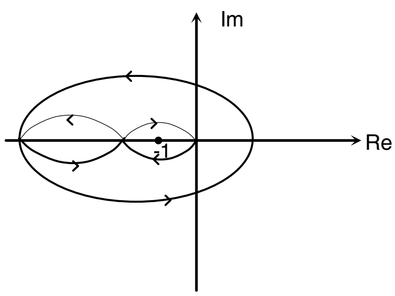
(C)



(D)



5. The Nyquist plot for the open–loop transfer function G(s) of a unity negative feedback system is shown in the figure. If G(s) has no pole in the right half of s-plane, the number of roots of the system characteristic equation in the right-half of s-plane is



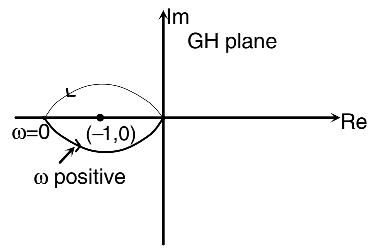
(A) 0 (B) 1 (C) 2 (D) 3

6. Which of the following points is not on the root locus of a system with the open-loop transfer function?

$$G(s)H(s) = \frac{K}{s(s+1)(s+3)}$$

(A) $s = -j\sqrt{3}$ (B) $s = -1.5$
(C) $s = -3$ (D) $s = -\infty$

7. The figure shows the Nyquist plot of the open-loop transfer function $G(s)H(s)$ of a system; If $G(s)H(s)$ has one right-hand pole, the closed-loop system is

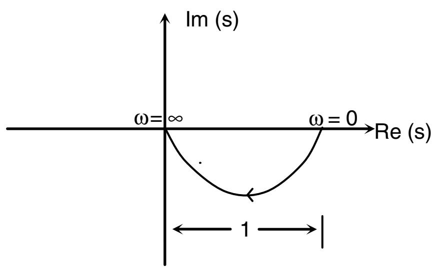


(A) always stable
(B) unstable with one closed-loop right-handpole
(C) unstable with two closed-loop right-hand poles
(D) unstable with three closed-loop right-hand poles

8. Given $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)}$, the point of intersection of the asymptotes of the root loci with the real axis is

(A) −4 (B) 1.33 (C) −1.33 (D) 4

9. The polar plot shown in the figure represents the transfer function



(A) $G(s) = \dfrac{1}{s}$

(B) $G(s) = \dfrac{1}{s(1+sT)}$

(C) $G(s) = \dfrac{1}{1+sT}$

(D) $G(s) = \dfrac{1}{(1+sT_1)(1+sT_2)}$

10. The open-loop transfer function of a unity gain feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+3)}$$

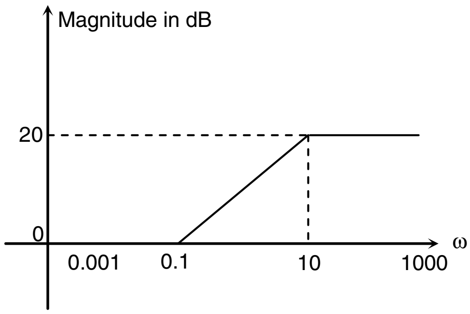
The gain margin of the system is dB is given by

(A) ∞ (B) 1 (C) 20 (D) 0

11. If the closed-loop transfer function of a control system is given by $T(s) = \dfrac{s-5}{(s+2)(s+3)}$, then it is

(A) an unstable system
(B) an uncontrollable system
(C) a minimum-phase system
(D) a non-minimum phase system

12. For the asymptotic Bode magnitude plot shown in figure, the system transfer function can be



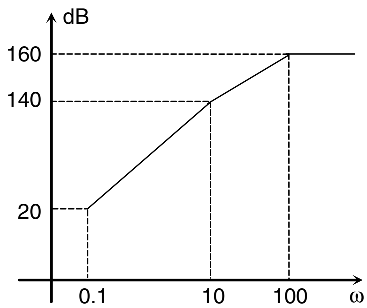
(A) $\dfrac{10s+1}{0.1s+1}$ (B) $\dfrac{100s+1}{0.1s+1}$

(C) $\dfrac{100s}{10s+1}$ (D) $\dfrac{0.1s+1}{10s+1}$

13. The root locus of the system $G(s)H(s) = \dfrac{K}{s(s+2)(s+3)}$ has the break-away point located at

(A) (−0.5, 0) (B) (−2.548, 0)
(C) (−4, 0) (D) (−0.784, 0)

14.



The approximate Bode magnitude plot of a minimum-phase system is shown inn the figure. The transfer function of the system is

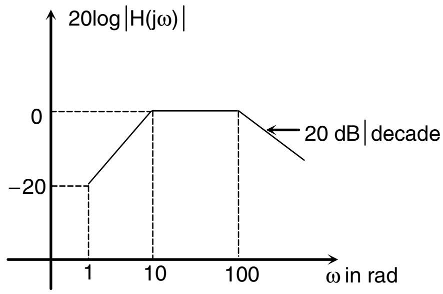
(A) $10^8 \dfrac{(s+0.1)^3}{(s+10)^2(s+100)}$

(B) $10^7 \dfrac{(s+0.1)^3}{(s+10)(s+100)}$

(C) $10^8 \dfrac{(s+0.1)^2}{(s+10)^2(s+100)}$

(D) $10^9 \dfrac{(s+0.1)^3}{(s+10)(s+100)^2}$

15. Consider the Bode magnitude plot shown in the figure. The transfer function $H(s)$ is



(A) $\dfrac{(s+10)}{(s+1)(s+100)}$

(B) $\dfrac{10(s+1)}{(s+10)(s+100)}$

(C) $\dfrac{10^2(s+1)}{(s+10)(s+100)}$

(D) $\dfrac{10^3(s+100)}{(s+1)(s+10)}$

Previous Years' Questions

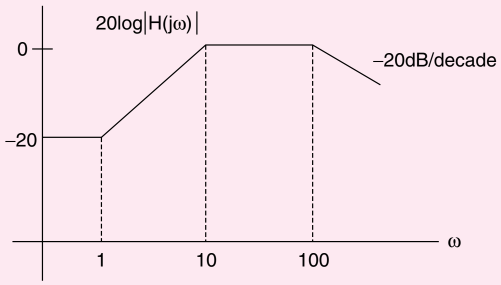
1. Given $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)}$, the point of intersection of the asymptotes of the root loci with the real axis is **[2004]**

(A) −4 (B) 1.33
(C) −1.33 (D) 4

2. The gain margin for the system with open-loop transfer function $G(s)H(s) = \dfrac{2(1+s)}{s^2}$, is **[2004]**

(A) ∞ (B) 0
(C) 1 (D) −∞

3. Consider the Bode magnitude plot shown in the figure. The transfer function H(s) is **[2004]**



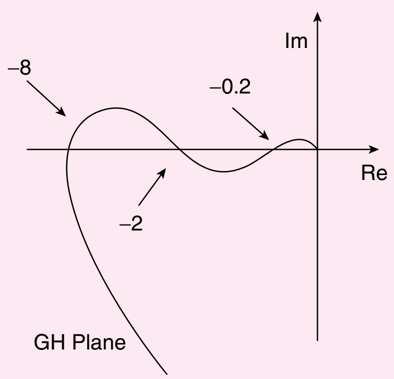
(A) $\dfrac{(s+10)}{(s+1)(s+100)}$ (B) $\dfrac{10(s+10)}{(s+1)(s+100)}$

(C) $\dfrac{10^2(s+1)}{(s+10)(s+100)}$ (D) $\dfrac{10^3(s+100)}{(s+1)(s+10)}$

4. For the polynomial $P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$, the number of roots which lie in the right half of the s-plane is **[2004]**

(A) 4 (B) 2
(C) 3 (D) 1

5. The polar diagram of a conditionally stable system for open-loop gain $K = 1$ is shown in figure. The open-loop transfer function of the system is known to be stable. The closed-loop system is stable for **[2005]**



(A) $K < 5$ and $\frac{1}{2} < K < \frac{1}{8}$

(B) $K < \frac{1}{8}$ and $\frac{1}{2} < K < 5$

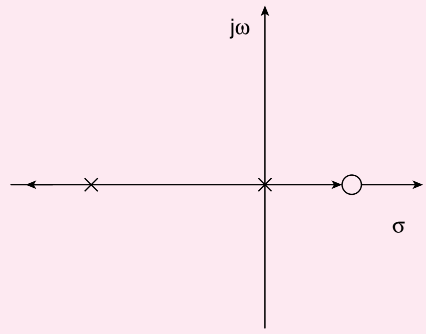
(C) $K < \frac{1}{8}$ and $5 < K$

(D) $K > \frac{1}{8}$ and $K < 5$

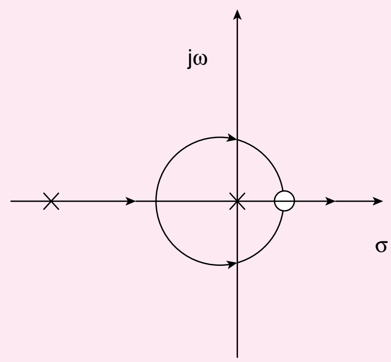
6. A unity feedback system is given as $G(s) = \dfrac{K(1-s)}{s(s+3)}$.

Indicate the correct root locus diagram **[2005]**

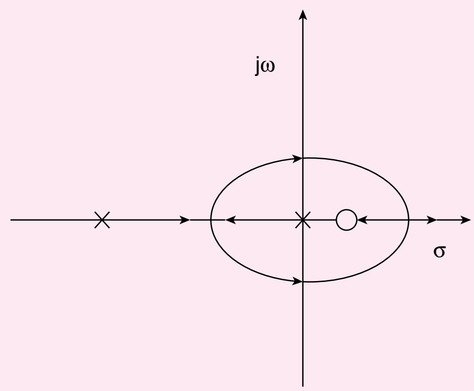
(A)



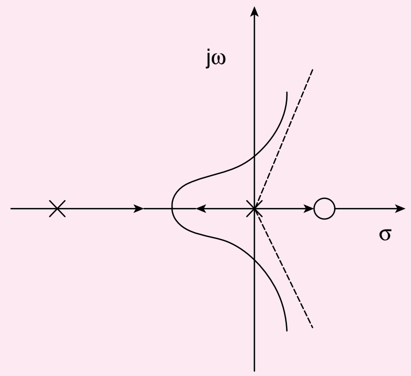
(B)



(C)



(D)



7. The open-loop transfer function of a unity-gain feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)}.$$

The gain margin of the system in dB is given by **[2006]**

(A) 0 (B) 1 (C) 20 (D) ∞

8. The Nyquist plot of $G(j\omega)H(j\omega)$ for a closed-loop control system passes through $(-1, j0)$ point in the GH plane. The gain margin of the system in dB is equal to **[2006]**

(A) infinite (B) greater than zero
(C) less than zero (D) zero

9. The transfer function of a phase-lead compensator is given by

$$G_C(S) = \frac{1+3T.S}{1+T.S} \text{ where } T > 0$$

The maximum phase–shift provided by such a compensator is **[2006]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Direction for questions 10 and 11:

Consider a unity -gain feedback control system whose open–loop transfer function is

$$G(S) = \frac{as+1}{s^2}$$

10. The value of a so that the system has a phase margin equal to $\frac{\pi}{4}$ is approximately equal to **[2006]**

(A) 2.40 (B) 1.40 (C) 0.84 (D) 0.74

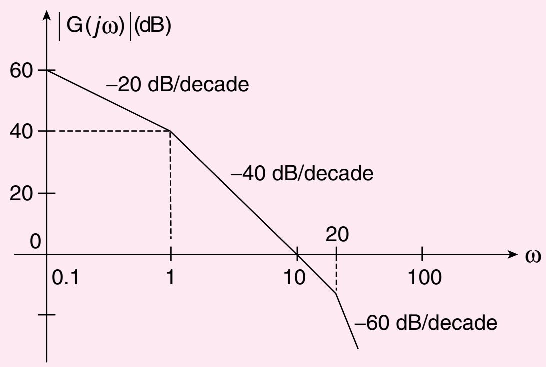
11. With the value of a set for a phase–margin of $\frac{\pi}{4}$, the value of unit-impulse response of the open -loop system at $t = 1$ s is equal to **[2006]**

(A) 3.40 (B) 2.40 (C) 1.84 (D) 1.74

12. A unity feedback control system has an open-loop transfer function $G(s) = \frac{K}{s(s^2+7s+12)}$. The gain K for which $s = -1 + j1$ will be lie on the root locus of this system is **[2007]**

(A) 4 (B) 5.5 (C) 6.5 (D) 10

13. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function $G(s)$ corresponding to this Bode plot is **[2007]**



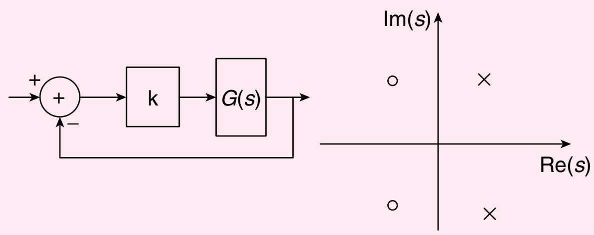
(A) $\frac{1}{(s+1)(s+20)}$ (B) $\frac{1}{s(s+1)(s+20)}$

(C) $\frac{100}{s(s+1)(s+20)}$ (D) $\frac{100}{s(s+1)(1+0.05s)}$

14. The number of open right half plane poles of $G(s) = \frac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$ is **[2008]**

(A) 0 (B) 1 (C) 2 (D) 3

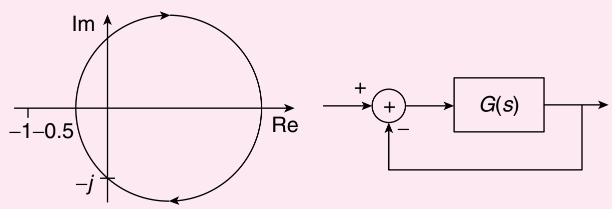
15. The feedback configuration and the pole-zero locations of $G(s) = \frac{s^2-2s+2}{s^2+2s+2}$ are shown below. The root locus for negative values of k, that is for $-\infty < k < 0$ has breakaway/break-in points and angle of departure at pole P (with respect to the positive real axis) equal to **[2009]**



(A) $\pm\sqrt{2}$ and 0° (B) $+\sqrt{2}$ and 45°
(C) $\pm\sqrt{3}$ and 0° (D) $\pm\sqrt{3}$ and 45°

Direction for questions 16 and 17:

The Nyquist plot of a stable transfer function $G(s)$ is shown in the figure. We are interested in the stability of the closed-loop system in the feedback configuration shown here.



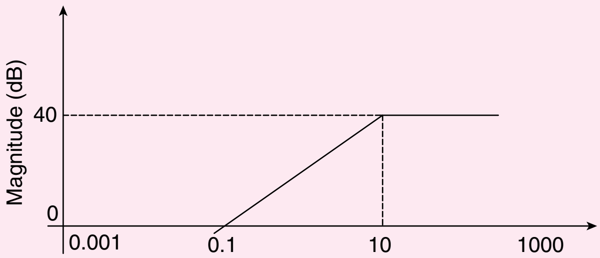
16. Which of the following statements is true? **[2009]**

(A) $G(s)$ is an all-pass filter
(B) $G(s)$ has a zero in the right-half plane
(C) $G(s)$ is the impedance of a passive network
(D) $G(s)$ is marginally stable

17. The gain and phase margins of $G(s)$ for closed-loop stability are **[2009]**

(A) 6dB and 180° (B) 3dB and 180°
(C) 6dB and 90° (D) 3dB and 90°

18. For the asymptotic Bode magnitude plot shown in the following figure, the system transfer function can be **[2010]**

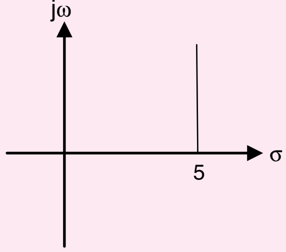


(A) $\dfrac{10s+1}{0.1s+1}$ (B) $\dfrac{100s+1}{0.1s+1}$

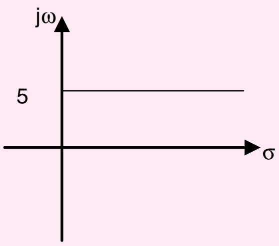
(C) $\dfrac{100s}{10s+1}$ (D) $\dfrac{0.1s+1}{10s+1}$

19. For the transfer function $G(j\omega) = 5 + j\omega$, the corresponding Nyquist plot for positive frequency has the form **[2011]**

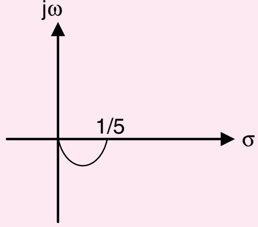
(A)



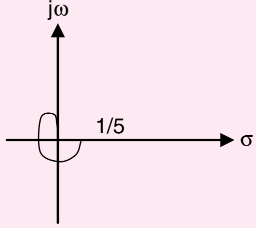
(B)



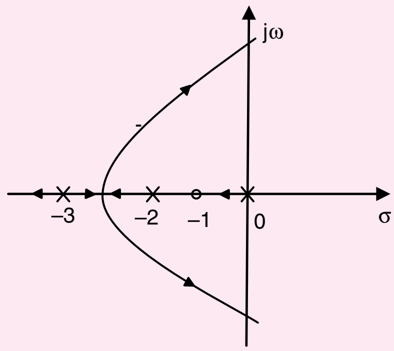
(C)



(D)



20. The root locus plot for a system is shown as follows. The open-loop transfer function corresponding to this plot is given by **[2011]**



(A) $G(s)H(s) = k\dfrac{s(s+1)}{(s+2)(s+3)}$

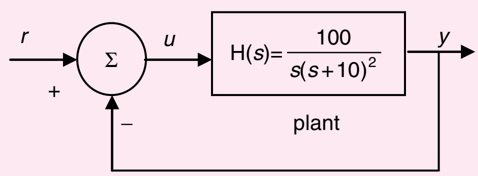
(B) $G(s)H(s) = k\dfrac{(s+1)}{s(s+2)(s+3)^2}$

(C) $G(s)H(s) = k\dfrac{1}{s(s-1)(s+2)(s+3)}$

(D) $G(s)H(s) = k\dfrac{(s+1)}{s(s+2)(s+3)}$

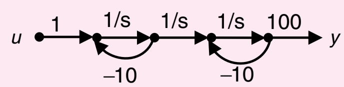
Direction for questions 21 and 22:

The input–output transfer function of a plant $H(s) = \dfrac{100}{s(s+10)^2}$. The plant is placed in a unity negative feedback configuration as shown in the figure below.

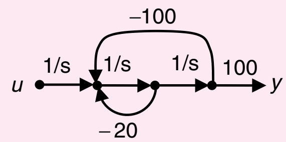


21. The signal flow graph that does not model the plant transfer function $H(s)$ is **[2011]**

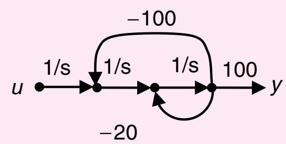
(A)



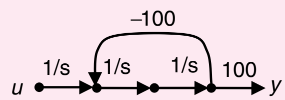
(B)



(C)

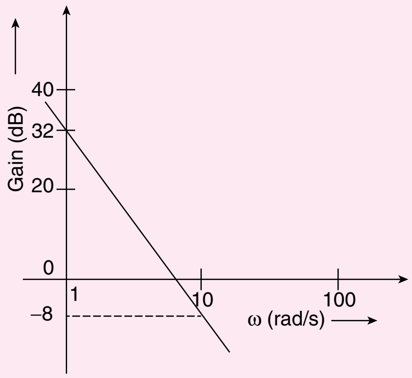


(D)



22. The gain margin of the system under closed-loop unity negative feedback is **[2011]**
(A) 0 dB (B) 20 dB
(C) 26 dB (D) 46 dB

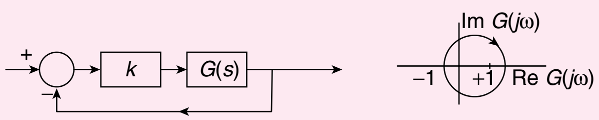
23. The Bode plot of transfer function $G(s)$ is shown in the figure below. **[2013]**



The gain ($20 \log|G(s)|$) is 32 dB and -8 dB at 1 rad/s and 10 rad/s, respectively. The phase is negative for all ω. Then, $G(s)$ is

(A) $\frac{39.8}{s}$ (B) $\frac{39.8}{s^2}$ (C) $\frac{32}{s}$ (D) $\frac{32}{s^2}$

24. Consider the feedback system shown in the figure. The Nyquist plot of $G(s)$ is also shown. Which one of the following conclusions is correct? **[2014]**

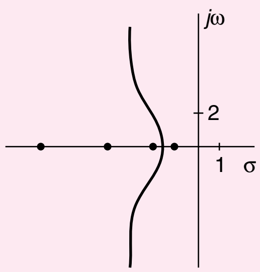


(A) $G(s)$ is an all-pass filter
(B) $G(s)$ is a strictly proper transfer function
(C) $G(s)$ is a stable and minimum-phase transfer function
(D) The closed-loop system is unstable for sufficiently large and positive k.

25. The phase margin in degrees of

$G(s) = \frac{10}{(s+0.1)(s+1)(s+10)}$ calculated using the asymptotic Bode plot is ______ **[2014]**

26. In the root locus plot shown in the figure, the pole/zero marks and the arrows have been removed. Which one of the following transfer functions has this root locus? **[2014]**



(A) $\frac{s+1}{(s+2)(s+4)(s+7)}$

(B) $\frac{s+4}{(s+1)(s+2)(s+7)}$

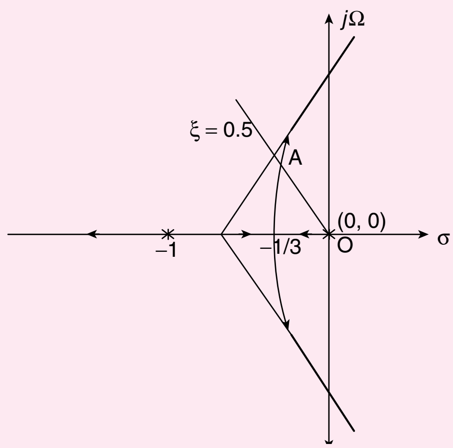
(C) $\frac{s+7}{(s+1)(s+2)(s+4)}$

(D) $\frac{(s+1)(s+2)}{(s+7)(s+4)}$

27. Consider a transfer function

$G_p(s) = \frac{ps^2+3ps-2}{s^2+(3+p)s+(2-p)}$ with p a positive real parameter. The maximum value of p until which G_p remains stable is ____. **[2014]**

28. The characteristic equation of a unity negative feedback system is $1 + KG(s) = 0$. The open-loop transfer function $G(s)$ has one pole at 0 and two poles at -1. The root locus of the system for varying K is shown in the figure. **[2014]**

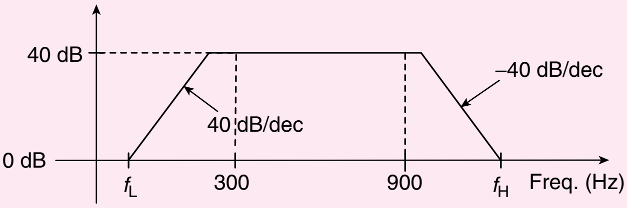


The constant damping ratio line, for $\xi = 0.5$, intersects the root locus at point A. The distance from the origin to point A is given as 0.5. The value of K at point A is ________.

29. A unity negative feedback system has the open-loop transfer function $G(s) = \frac{K}{s(s+1)(s+3)}$. The value of the gain $K(>0)$ at which the root locus crosses the imaginary axis is _______. **[2015]**

30. The polar plot of the transfer function $G(s) = \frac{10(s+1)}{s+10}$ for $0 \le \omega < \infty$ will be in the **[2015]**
(A) first quadrant (B) second quadrant
(C) third quadrant (D) fourth quadrant

31. Consider the Bode plot shown in the figure. Assume that all the poles and zeros are real-valued.



The value of $f_H - f_L$ (in Hz) is ______. **[2015]**

32. The phase margin (in degrees) of the system $G(s) = \dfrac{10}{s(s+10)}$ is ______. **[2015]**

33. Match the inferences X, Y and Z about a system, to the corresponding properties of the elements of first column in Routh's table of the system characteristic equation. **[2016]**

X: The system is stable	P: When all elements are positive
Y: The system is unstable	Q: When any one element is zero
Z: The test breaks down	R: When there is a change in sign of coefficients

(A) $x \to P, Y \to Q, Z \to R$
(B) $X \to Q, Y \to P, Z \to R$
(C) $X \to R, Y \to Q, Z \to P$
(D) $X \to P, Y \to R, Z \to Q$

34. A closed loop control system is stable if the Nyquist plot of the corresponding open loop transfer function **[2016]**

(A) Encircles the s-plane point $(-1 + j0)$ in the counter clockwise direction as many times as the number of right half s-plane poles.
(B) Encircles the s-plane point $(0 - j1)$ in the clockwise direction as many times as the number of right half s-plane poles.
(C) Encircles the s-plane point $(-1 + j0)$ in the counter clockwise direction as many times as the number of left half s-plane poles.
(D) Encircles the s-plane point $(-1 + j0)$ in the counter clockwise direction as many times as the number of right half s-plane zeros.

35. The open loop transfer function of a unity feedback control system is

$$G(s) = \frac{K}{s^2 + 5s + 5}.$$

The value of K at the breakaway point of the feedback control system's root locus plot is ________ .

36. The transfer function of a linear time invariant system is given by

$H(s) = 2s^4 - 5s^3 + 5s - 2$

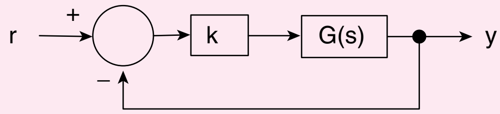
The number of zeros in the right half of the s-plane is ________ . **[2016]**

37. The number and direction of encirclements around the point $-1 + j0$ in the complex plane by the Nyquist plot of $G(s) = \dfrac{1-s}{4+2s}$ is: **[2016]**

(A) Zero
(B) One, anti clockwise
(C) One, clockwise
(D) Two, clockwise

38. In the feedback system below.

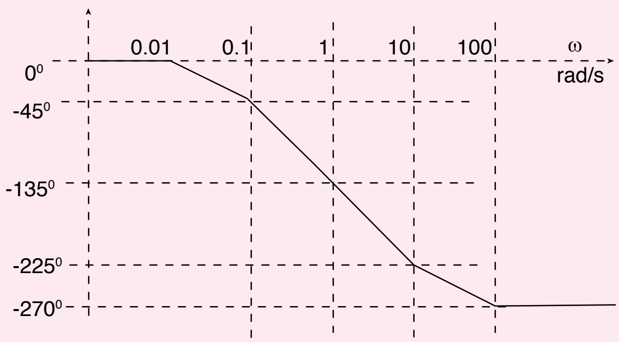
$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}.$$



The positive value of k for which the gain margin of the loop is exactly 0 dB and the phase margin of the loop is exactly zero degree is ________. **[2016]**

39. The asymptotic Bode phase plot of

$G(s) = \dfrac{k}{(s+0.1)(s+10)(s+p_1)}$, with k and p_1 both positive is shown in the following figure:



The value of p_1 is ________. **[2016]**

40. The first two rows in the Routh table for the characteristic equation of a certain closed loop control system are given as

S^3	1	(2K + 3)
S^2	2K	4

The range of K for which the system is stable is **[2016]**

(A) $-2.0 < K < 0.5$
(B) $0 < k < 0.5$
(C) $0 < k < \infty$
(D) $0.5 < K < \infty$

41. The forward path transfer function and the feedback path transfer function of a single loop negative feedback control system are given as

$$G(s) = \frac{K(s+2)}{s^2+2s+2} \text{ and } H(s) = 1,$$

respectively. If the variable parameter K is real positive, then the location of the breakaway point on the root locus diagram of the system is__________ . **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** B	**3.** A	**4.** C	**5.** C	**6.** C	**7.** D	**8.** C	**9.** B	**10.** C
11. D	**12.** B	**13.** D	**14.** C	**15.** C	**16.** B	**17.** B	**18.** C	**19.** A	**20.** C
21. A									

Practice Problems 2

1. C	**2.** A	**3.** A	**4.** D	**5.** A	**6.** B	**7.** A	**8.** C	**9.** C	**10.** A
11. D	**12.** A	**13.** D	**14.** A	**15.** C					

Previous Years' Questions

1. C	**2.** D	**3.** C	**4.** B	**5.** A	**6.** C	**7.** D	**8.** D	**9.** D	**10.** C
11. C	**12.** D	**13.** D	**14.** C	**15.** B	**16.** C	**17.** C	**18.** A	**19.** A	**20.** B
21. D	**22.** C	**23.** B	**24.** D	**25.** 42 to 48		**26.** B	**27.** 1.9 to 2.1		
28. 0.375	**29.** 12	**30.** A	**31.** 8,970	**32.** 84 to 84.5		**33.** D	**34.** D	**35.** 1.25	**36.** 3
37. A	**38.** 60	**39.** 1	**40.** D	**41.** −3.414					