# CHAPTER 19 Differential Equations

## Section-A JEE Advanced/ IIT-JEE

### C MCQs with One Correct Answer

1. A solution of the differential equation *(1999 – 2 Marks)*
$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is
(a) $y = 2$ (b) $y = 2x$
(c) $y = 2x - 4$ (d) $y = 2x^2 - 4$

2. If $x^2 + y^2 = 1$, then *(2000S)*
(a) $yy'' - 2(y')^2 + 1 = 0$ (b) $yy'' + (y')^2 + 1 = 0$
(c) $yy'' + (y')^2 - 1 = 0$ (d) $yy'' + 2(y')^2 + 1 = 0$

3. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1)$ is equal to *(2003S)*
(a) $-1/2$ (b) $e + 1/2$
(c) $e - 1/2$ (d) $1/2$

4. If $y = y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x, y(0) = 1,$
then $y\left(\frac{\pi}{2}\right)$ equals *(2004S)*
(a) 1/3 (b) 2/3 (c) –1/3 (d) 1

5. If $y = y(x)$ and it follows the relation $x \cos y + y \cos x = \pi$ then $y''(0) =$ *(2005S)*
(a) 1 (b) –1 (c) $\pi - 1$ (d) $-\pi$

6. The solution of primitive integral equation $(x^2 + y^2)\, dy = xy\, dx$ is $y = y(x)$. If $y(1) = 1$ and $(x_0) = e$, then $x_0$ is equal to *(2005S)*
(a) $\sqrt{2(e^2-1)}$ (b) $\sqrt{2(e^2+1)}$
(c) $\sqrt{3}\, e$ (d) $\sqrt{\frac{e^2+1}{2}}$

7. For the primitive integral equation $ydx + y^2dy = x\, dy$; $x \in R, y > 0, y = y(x), y(1) = 1$, then $y(-3)$ is *(2005S)*
(a) 3 (b) 2 (c) 1 (d) 5

8. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with *(2005S)*
(a) variable radii and a fixed centre at (0, 1)
(b) variable radii and a fixed centre at (0, –1)
(c) fixed radius 1 and variable centres along the x–axis.
(d) fixed radius 1 and variable centres along the y–axis.

9. The function $y = f(x)$ is the solution of the differential equation
$\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$ in $(-1, 1)$ satisfying $f(0) = 0$. Then
$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)\, d(x)$ is *(JEE Adv. 2014)*
(a) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (b) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$
(c) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

### D MCQs with One or More than One Correct

1. The order of the differential equation whose general solution is given by
$y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x+C_5}$, where $C_1, C_2, C_3, C_4, C_5$, are arbitrary constants, is *(1998 – 2 Marks)*
(a) 5 (b) 4 (c) 3 (d) 2

2. The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where $c$ is a positive parameter, is of *(1999 – 3 Marks)*
(a) order 1 (b) order 2 (c) degree 3 (d) degree 4

3. A curve $y = f(x)$ passes through (1, 1) and at $P(x, y)$, tangent cuts the x–axis and y–axis at $A$ and $B$ respectively such that BP : AP = 3 : 1, then *(2006 – 5M, –1)*
(a) equation of curve is $xy' - 3y = 0$
(b) normal at (1, 1) is $x + 3y = 4$
(c) curve passes through (2, 1/8)
(d) equation of curve is $xy' + 3y = 0$

4. If $y(x)$ satisfies the differential equation $y' - y\tan x = 2x\sec x$ and $y(0) = 0$, then *(2012)*

(a) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ (b) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(c) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$ (d) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

5. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0.$

Then the equation of the curve is *(JEE Adv. 2013)*

(a) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (b) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(c) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ (d) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

6. Let $y(x)$ be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statement is (are) true? *(JEE Adv. 2015)*

(a) $y(-4) = 0$ (b) $y(-2) = 0$

(c) $y(x)$ has a critical point in the interval $(-1, 0)$

(d) $y(x)$ has no critical point in the interval $(-1, 0)$

7. Consider the family of all circles whose centers lie on the straight line $y = x$. If this family of circle is represented by the differential equation $Py'' + Qy' + 1 = 0$, where $P, Q$ are functions of $x$, $y$ and $y'$ $\left(\text{here } y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}\right)$, then which of the following statements is (are) true? *(JEE Adv. 2015)*

(a) $P = y + x$ (b) $P = y - x$

(c) $P + Q = 1 - x + y + y' + (y')^2$ (d) $P - Q = x + y - y' - (y')^2$

8. Let $f : (0, \infty) \to \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then *(JEE Adv. 2016)*

(a) $\lim_{x \to 0+} f'\left(\frac{1}{x}\right) = 1$ (b) $\lim_{x \to 0+} xf\left(\frac{1}{x}\right) = 2$

(c) $\lim_{x \to 0+} x^2 f'(x) = 0$ (d) $|f(x)| \le 2$ for all $x \in (0, 2)$

9. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0, x > 0$, passes through the point (1, 3). Then the solution curve *(JEE Adv. 2016)*

(a) intersects $y = x + 2$ exactly at one point

(b) intersects $y = x + 2$ exactly at two points

(c) intersects $y = (x + 2)^2$

(d) does NOT intersect $y = (x + 3)^2$

## E Subjective Problems

1. If $(a + bx)\, e^{y/x} = x$, then prove that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$ *(1983 – 3 Marks)*

2. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x–axis at $Q$. If $PQ$ is of constant length $k$, then show that the differential equation describing such curves is

$$y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$$ *(1994 – 5 Marks)*

Find the equation of such a curve passing through $(0, k)$.

3. Let $y = f(x)$ be a curve passing through (1, 1) such that the triangle formed by the coordinate axes and the tangent at any point of the curve lies in the first quadrant and has area 2. Form the differential equation and determine all such possible curves. *(1995 – 5 Marks)*

4. Determine the equation of the curve passing through the origin, in the form $y = f(x)$, which satisfies the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$. *(1996 – 5 Marks)*

5. Let $u(x)$ and $v(x)$ satisfy the differential equation $\frac{du}{dx} + p(x)u = f(x)$ and $\frac{dv}{dx} + p(x)v = g(x)$, where $p(x)$ $f(x)$ and $g(x)$ are continuous functions. If $u(x_1) > v(x_1)$ for some $x_1$ and $f(x) > g(x)$ for all $x > x_1$, prove that any point $(x, y)$ where $x > x_1$, does not satisfy the equatons $y = u(x)$ and $y = v(x)$. *(1997 – 5 Marks)*

6. A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point $P$ of the curve is equal to the distance of $P$ from the x–axis. Determine the equation of the curve. *(1999 – 10 Marks)*

7. A country has a food deficit of 10%. Its population grows continously at a rate of 3% per year. Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self–sufficient in food after n years, where n is the smallest integer bigger than or equal to $\frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$. *(2000 – 10 Marks)*

**8.** A hemispherical tank of radius 2 metres is initially full of water and has an outlet of 12 $cm^2$ cross–sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $v(t) = 0.6\sqrt{2gh(t)}$, where $v(t)$ and $h(t)$ are respectively the velocity of the flow through the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank. (Hint : Form a differential equation by relating the decrease of water level to the outflow). ***(2001 – 10 Marks)***

**9.** A right circular cone with radius $R$ and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = $k > 0$). Find the time after which the cone is empty. ***(2003 – 4 Marks)***

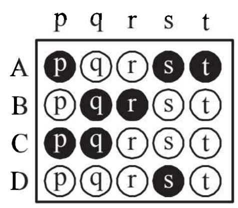
**10.** A curve '$C$' passes through (2, 0) and the slope at $(x, y)$ as $\frac{(x+1)^2 + (y-3)}{x+1}$. Find the equation of the curve. Find the area bounded by curve and x–axis in fourth quadrant. ***(2004 – 4 Marks)***

**11.** If length of tangent at any point on the curve $y = f(x)$ intecepted between the point and the x–axis is of length 1. Find the equation of the curve. ***(2005 – 4 Marks)***

## F Match the Following

*Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

*If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.*



**1.** Match the statements/expressions in **Column I** with the open intervals in **Column II**. ***(2009)***

| **Column I** | **Column II** |
|---|---|
| (A) Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2 + y' + y = 0$ | (p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |
| (B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$ | (q) $\left(0, \frac{\pi}{2}\right)$ |
| (C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies | (r) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$ |
| (D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing | (s) $\left(0, \frac{\pi}{8}\right)$ |
| | (t) $(-\pi, \pi)$ |

## H Assertion & Reason Type Questions

**1.** Let a solution $y = y(x)$ of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$.

**STATEMENT**-1 : $y(x) = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$ and

**STATEMENT**-2 : $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$

***(2008)***

(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is** a correct explanation for STATEMENT - 1

(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1

(c) STATEMENT - 1 is True, STATEMENT - 2 is False

(d) STATEMENT - 1 is False, STATEMENT - 2 is True

## I Integer Value Correct Type

**1.** Let $y'(x) + y(x)\,g'(x) = g(x),\ g'(x),\ y(0) = 0, x \in \boldsymbol{R}$, where $f'(x)$ denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on $R$ with $g(0) = g(2) = 0$. Then the value of $y(2)$ is ***(2011)***

## Section-B JEE Main / AIEEE

**1.** The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3}$ are **[2002]**

(a) $(1, \frac{2}{3})$ (b) $(3, 1)$

(c) $(3, 3)$ (d) $(1, 2)$

**2.** The solution of the equation $\frac{d^2y}{dx^2}=e^{-2x}$ **[2002]**

(a) $\frac{e^{-2x}}{4}$ (b) $\frac{e^{-2x}}{4}+cx+d$

(c) $\frac{1}{4}e^{-2x}+cx^2+d$ (d) $\frac{1}{4}e^{-4x}+cx+d$

**3.** The degree and order of the differential equation of the family of all parabolas whose axis is $x$ - axis, are respectively. **[2003]**

(a) 2, 3 (b) 2, 1 (c) 1, 2 (d) 3, 2.

**4.** The solution of the differential equation

$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$, is **[2003]**

(a) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$ (b) $(x-2)=ke^{2\tan^{-1}y}$

(c) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$ (d) $xe^{\tan^{-1}y}=\tan^{-1}y+k$

**5.** The differential equation for the family of circle $x^2+y^2-2ay=0$, where a is an arbitrary constant is **[2004]**

(a) $(x^2+y^2)y'=2xy$ (b) $2(x^2+y^2)y'=xy$

(c) $(x^2-y^2)y'=2xy$ (d) $2(x^2-y^2)y'=xy$

**6.** Solution of the differential equation $ydx+(x+x^2y)dy=0$ is **[2004]**

(a) $\log y=Cx$ (b) $-\frac{1}{xy}+\log y=C$

(c) $\frac{1}{xy}+\log y=C$ (d) $-\frac{1}{xy}=C$

**7.** The differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$, where $c>0$, is a parameter, is of order and degree as follows : **[2005]**

(a) order 1, degree 2 (b) order 1, degree 1

(c) order 1, degree 3 (d) order 2, degree 2

**8.** If $x\frac{dy}{dx}=y(\log y-\log x+1)$, then the solution of the equation is **[2005]**

(a) $y\log\left(\frac{x}{y}\right)=cx$ (b) $x\log\left(\frac{y}{x}\right)=cy$

(c) $\log\left(\frac{y}{x}\right)=cx$ (d) $\log\left(\frac{x}{y}\right)=cy$

**9.** The differential equation whose solution is $Ax^2+By^2=1$ where A and B are arbitrary constants is of **[2006]**

(a) second order and second degree

(b) first order and second degree

(c) first order and first degree

(d) second order and first degree

**10.** The differential equation of all circles passing through the origin and having their centres on the x-axis is **[2007]**

(a) $y^2=x^2+2xy\frac{dy}{dx}$ (b) $y^2=x^2-2xy\frac{dy}{dx}$

(c) $x^2=y^2+xy\frac{dy}{dx}$ (d) $x^2=y^2+3xy\frac{dy}{dx}$.

**11.** The soluton of the differential equation

$\frac{dy}{dy}=\frac{x+y}{x}$ satisfying the condition $y(1)=1$ is **[2008]**

(a) $y=\ln x+x$ (b) $y=x\ln x+x^2$

(c) $y=xe^{(x-1)}$ (d) $y=x\ln x+x$

**12.** The differential equation which represents the family of curves $y=c_1e^{c_2x}$, where $c_1$, and $c_2$ are arbitrary constants, is **[2009]**

(a) $y''=y'y$ (b) $yy''=y'$

(c) $yy''=(y')^2$ (d) $y'=y^2$

**13.** Solution of the differential equation

$\cos x\,dy=y(\sin x-y)\,dx,\ 0<x<\frac{\pi}{2}$ is **[2010]**

(a) $y\sec x=\tan x+c$ (b) $y\tan x=\sec x+c$

(c) $\tan x=(\sec x+c)y$ (d) $\sec x=(\tan x+c)y$

**14.** If $\frac{dy}{dx}=y+3>0$ and $y(0)=2$, then $y(\ln 2)$ is equal to : **[2011]**

(a) 5 (b) 13

(c) $-2$ (d) 7

**15.** Let $I$ be the purchase value of an equipment and $V(t)$ be the value after it has been used for $t$ years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where $k > 0$ is a constant and $T$ is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is **[2011]**

(a) $I - \frac{kT^2}{2}$ (b) $I - \frac{k(T-t)^2}{2}$

(c) $e^{-kT}$ (d) $T^2 - \frac{1}{k}$

**16.** The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\, p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is : **[2012]**

(a) $2 ln\, 18$ (b) $ln\, 9$ (c) $\frac{1}{2} ln\, 18$ (d) $ln\, 18$

**17.** At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers $x$ is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is **[JEE M 2013]**

(a) 2500 (b) 3000 (c) 3500 (d) 4500

**18.** Let the population of rabbits surviving at time $t$ be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals: **[JEE M 2014]**

(a) $600 - 500\, e^{t/2}$ (b) $400 - 300\, e^{-t/2}$

(c) $400 - 300\, e^{t/2}$ (d) $300 - 200\, e^{-t/2}$

**19.** Let y(x) be the solution of the differential equation

$$(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1).$$ Then y (e) is equal to: **[JEE M 2015]**

(a) 2 (b) 2e (c) e (d) 0

**20.** If a curve y = f(x) passes through the point (1, –1) and satisfies the differential equation, $y(1 + xy)\, dx = x\, dy$, then $f\left(-\frac{1}{2}\right)$ is equal to : **[JEE M 2016]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $-\frac{2}{5}$ (b) $-\frac{4}{5}$

# 19 Differential Equations

## Section-A : JEE Advanced/ IIT-JEE

**C** **1.** (c) **2.** (b) **3.** (a) **4.** (a) **5.** (c) **6.** (c) **7.** (a) **8.** (c) **9.** (b)

**D** **1.** (c) **2.** (a, c) **3.** (c, d) **4.** (a, d) **5.** (a) **6.** (a, c) **7.** (b, c) **8.** (a) **9.** (a, d)

**E** **3.** $x+y=2$ and $xy=1,\ x,\ y>0$ **4.** $y=\frac{1}{3}\left[\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right)-5x\right]$

**6.** $x^2+y^2-2x=0,\ x-1=0$ **8.** $\frac{14\pi}{27\sqrt{g}}(10)^5$ units **9.** H/k **10.** $\frac{4}{3}$ sq. units

**11.** $\log\left|\frac{1-\sqrt{1-y^2}}{y}\right|+\sqrt{1-y^2}=\pm x+c$

**F** **1.** (A) $\to$ p,q,r,s,t; (B) $\to$ p,t; (C) $\to$ p,q,r,t; (D) $\to$ s

**H** **1.** (c)

**I** **1.** 0

## Section-B : JEE Main/ AIEEE

**1.** (c) **2.** (b) **3.** (d) **4.** (c) **5.** (c) **6.** (b) **7.** (c) **8.** (c) **9.** (d) **10.** (a) **11.** (d) **12.** (c) **13.** (d) **14.** (d) **15.** (a) **16.** (a) **17.** (c) **18.** (c) **19.** (a) **20.** (b)

## Section-A JEE Advanced/ IIT-JEE

### C. MCQs with ONE Correct Answer

**1.** **(c)** $\left(\frac{dy}{dx}\right)^2-x.\frac{dy}{dx}+y=0.$

By actual verification we find that the choice (c), i.e. $y=2x-4$ satisfies the given differential equation.

**2.** **(b)** Given $x^2+y^2=1$. Differentiating w.r.t. $x$, we get $2x+2yy'=0$ or $x+yy'=0$. Again differentiating w.r.t. $x$, we get $1+y'y'+yy''=0$ or $1+(y')^2+yy''=0$

**3.** **(a)** The given differential equation is

$$\frac{dy}{dt}-\frac{t}{1+t}y=\frac{1}{1+t}$$

I.F. $=e^{-\int\frac{t}{1+t}dt}=e^{-\int\left(1-\frac{1}{1+t}\right)dt}=e^{-(t-\log(1+t))}$

$=e^{-t}.e^{\log(1+t)}=(1+t)e^{-t}$ $\therefore$ Solution is

$$y.e^{-t}(1+t)=\int\frac{1}{(1+t)}e^{-t}(1+t)\,dt+C$$

$\Rightarrow\ y.e^{-t}(1+t)=-e^{-t}+C\ \Rightarrow\ y=-\frac{1}{1+t}+\frac{Ce^t}{1+t}$

Given that $y(0)=-1\ \Rightarrow\ -1=-1+C\ \Rightarrow C=0$

$\therefore\ y=-\frac{1}{1+t}\ \therefore\ y(1)=-\frac{1}{1+1}=-\frac{1}{2}$

**4.** **(a)** $\frac{dy}{dx}\left(\frac{2+\sin x}{1+y}\right)=-\cos x, y(0)=1$

$\Rightarrow\ \frac{dy}{(1+y)}=\frac{-\cos x}{2+\sin x}dx$

Integrating both sides

$\Rightarrow\ \ln(1+y)=-\ln(2+\sin x)+C$

Put $x=0$ and $y=1\ \Rightarrow\ \ln(2)=-\ln 2+C\ \Rightarrow\ C=\ln 4$

Put $x=\frac{\pi}{2}$ $\ln(1+y)=-\ln 3+\ln 4=\ln\frac{4}{3}\ \Rightarrow\ y=\frac{1}{3}$

**5.** **(c)** Given that $y=y(x)$

and $x\cos y+y\cos x=\pi$ ...(1)

For $x=0$ in (1) we get $y=\pi$

Differentiating (1) with respect to $x$, we get

$-x\sin y.y'+\cos y+y'\cos x-y\sin x=0$

$\Rightarrow\ y'=\frac{y\sin x-\cos y}{\cos x-x\sin y}$ ...(2)

$\Rightarrow \quad y'(0) = 1 \qquad$ (Using $y(0) = \pi$)

Differentiating (2) with respect to $x$, we get

$$(y'\sin x + y\cos x + \sin y . y')(\cos x - x\sin y)$$

$$y'' = \frac{-(-\sin x - \sin y - x\cos y y')(y\sin x - \cos y)}{(\cos x - x\sin y)^2}$$

$$\Rightarrow \quad y''(0) = \frac{\pi(1) - 1}{1} = \pi - 1$$

**6. (c)** The given D.E. is $(x^2 + y^2)dy = xy\,dx$ s.t. $y(1) = 1$ and $y(x_0) = e$

The given eq$^n$ can be written as

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$$

Put $y = vx$, $\therefore\ v + x\frac{dv}{dx} = \frac{v}{1+v^2}$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v^3}{1+v^2} \Rightarrow \int \frac{1+v^2}{v^3}dv + \int \frac{dx}{x} = 0$$

$$\Rightarrow \quad -\frac{1}{2v^2} + \log|v| + \log|x| = C$$

$$\Rightarrow \quad \log y = C + \frac{x^2}{2y^2} \quad (\text{using } v = y/x)$$

Also, $y(1) = 1 \Rightarrow \log 1 = C + \frac{1}{2} \Rightarrow C = -\frac{1}{2}$

$\therefore \quad \log y = \frac{x^2 - y^2}{2y^2}$, But given $y(x_0) = e$

$$\Rightarrow \quad \log e = \frac{x_0^2 - e^2}{2e^2} \Rightarrow x_0^2 = 3e^2 \Rightarrow x_0 = \sqrt{3}e$$

**7. (a)** The given eq$^n$ is

$$ydx + y^2dy = x\,dy\ ; x \in R, y > 0, y(1) = 1$$

$$\Rightarrow \quad \frac{ydx - xdy}{y^2} + dy = 0 \Rightarrow \frac{d}{dx}\left(\frac{x}{y}\right) + dy = 0$$

On integration, we get $\frac{x}{y} + y = C$

$y(1) = 1 \Rightarrow 1 + 1 = C \Rightarrow C = 2$

$\therefore \quad \frac{x}{y} + y = 2$

Now to find $y(-3)$, putting $x = -3$ in above eq$^n$, we get

$$-\frac{3}{y} + y = 2 \Rightarrow y^2 - 2y - 3 = 0 \Rightarrow y = 3, -1$$

But given that $y > 0$, $\therefore\ y = 3$

**8. (c)** $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y} \Rightarrow \frac{-2y}{\sqrt{1-y^2}}dy + 2dx = 0$

$$\Rightarrow \quad 2\sqrt{1-y^2} + 2x = 2c \Rightarrow \sqrt{1-y^2} + x = c$$

$$\Rightarrow \quad (x-c)^2 + y^2 = 1$$

which is a circle of fixed radius 1 and variable centre $(c, 0)$ lying on $x$-axis.

**9. (b)** Given D.E. can be written as

$$\frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{x^4 + 2x}{\sqrt{1-x^2}}$$

If $e^{\int \frac{-x}{1-x^2}dx} = e^{\frac{+1}{2}\log(1-x^2)} = \sqrt{1-x^2}$

$\therefore$ Solution is given by

$$y\sqrt{1-x^2} = \int \sqrt{1-x^2}.\frac{x^4 + 2x}{\sqrt{1-x^2}}dx$$

$$y\sqrt{1-x^2} = \frac{x^5}{5} + x^2 + c$$

$f(0) = 0 \Rightarrow$ At $x = 0, y = 0$

$\therefore\ c = 0$

$$\therefore \quad y\sqrt{1-x^2} = \frac{x^5}{5} + x^2$$

or $\quad y = f(x) = \dfrac{\frac{x^5}{5} + x^2}{\sqrt{1-x^2}}$

$$\therefore \quad I = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{\frac{x^5}{5} + x^2}{\sqrt{1-x^2}}dx$$

$$= 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx \quad \left(\because \frac{x^5}{\sqrt{1-x^2}} \text{ is odd}\right)$$

put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$$\therefore I = 2\int_0^{\frac{\pi}{3}} \sin^2\theta d\theta = \int_0^{\frac{\pi}{3}} (1 - \cos 2\theta)d\theta$$

$$= \left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

## D. MCQs with ONE or MORE THAN ONE Correct

**1. (c)** The given solution of D.E. is

$$y = (c_1 + c_2)\cos(x + c_3) = c_4 e^{x + c_5}$$

$$= (c_1 + c_2)\cos(x + c_3) - c_4.e^{c_5}.e^x$$

$$= A\cos(x + c_3) - Be^x$$

[Taking $c_1 + c_2 = A, c_4 e^{c_5} = B$]

Thus, there are actually three arbitrary constants and hence this differential equation should be of order 3.

**2.** **(a, c)** $2yy_1 = 2c \Rightarrow c = yy_1$

Eliminating, $c$, we get,

$$y^2 = 2yy_1(x+\sqrt{yy_1}) \text{ or } (y-2xy_1)^2 = 4yy_1^3$$

It involves only Ist order derivative, its order is 1 but its degree is 3 as $y_1^3$ is there.

**3.** **(c, d)** Tangent to the curve $y = f(x)$ at $(x, y)$ is

$$Y - y = \frac{dy}{dx}(X - x)$$

$$\therefore \quad A\left(\frac{x\frac{dy}{dx}-y}{\frac{dy}{dx}}, 0\right);\ B\left(0, -x\frac{dy}{dx}+y\right)$$

$$\because \quad BP : PA = 3:1 \Rightarrow x = \frac{\dfrac{3\left(x\frac{dy}{dx}-y\right)}{\frac{dy}{dx}} + 1\times 0}{4}$$

$$\Rightarrow x\frac{dy}{dx} + 3y = 0 \Rightarrow \int\frac{dy}{y} = \int -3\frac{dx}{x}$$

$$\Rightarrow \log y = -3\log x + \log c \Rightarrow y = \frac{c}{x^3}$$

As curve passes through (1, 1), $c = 1$

$\therefore$ curve is $x^3 y = 1$, which also passes through $\left(2, \frac{1}{8}\right)$.

**4.** **(a, d)**

The given differential equation is

$$\frac{dy}{dx} - y\tan x = 2x\sec x,$$

$$\text{I.F.} = e^{-\int \tan x dx} = \cos x$$

$$\therefore \quad y.\cos x = \int 2x\,dx = x^2 + c \quad y(0) = 0 \Rightarrow c = 0$$

$$\therefore y = x^2 \sec x$$

Now at $x = \frac{\pi}{4}$, $y = \frac{\pi^2}{16}\times\sqrt{2} = \frac{\pi^2}{8\sqrt{2}}$

At $x = \frac{\pi}{3}$, $y = \frac{\pi^2}{9}\times 2 = \frac{2\pi^2}{9}$

At $x = \frac{\pi}{4}$, $y' = \frac{2\pi}{4}\times\sqrt{2} + \frac{\pi^2}{8\sqrt{2}}\times 1 = \frac{\pi^2}{8\sqrt{2}} + \frac{\pi}{\sqrt{2}}$

At $x = \frac{\pi}{3}$, $y' = \frac{2\pi}{3}\times 2 + \frac{2\pi^2}{9}\times\sqrt{3} = \frac{2\pi^2}{3\sqrt{3}} + \frac{4\pi}{3}$

**5.** **(a)** $\frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get $x\frac{dv}{dx} = \sec v$

or $\int \cos v\, dv = \int\frac{dx}{x}$

$\Rightarrow \sin v = \log x + c \qquad (\because x > 0)$

$\Rightarrow \sin\frac{y}{x} = \log x + c$

It passes through $\left(1, \frac{\pi}{6}\right) \Rightarrow C = \frac{1}{2}$

$\therefore \quad \sin\frac{y}{x} = \log x + \frac{1}{2}$

**6.** **(a, c)** $\frac{dy}{dx} + \frac{e^x}{1+e^x}y = \frac{1}{1+e^x}$

I.F. $= 1 + e^x$

$\therefore$ Soln: $y(1+e^x) = x + c$

$y(0) = 2 \Rightarrow c = 4$

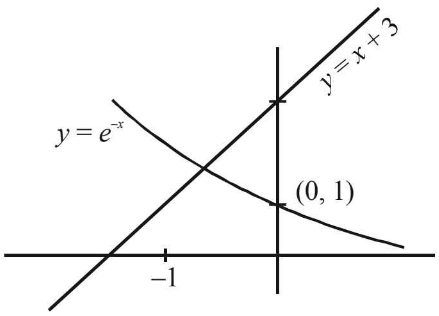
$\therefore \quad y = \frac{x+4}{e^x+1} \quad \therefore y(-4) = 0$

Also $\frac{dy}{dx} = \frac{(e^x+1) - e^x(x+4)}{(e^x+1)^2}$

For critical point $\frac{dy}{dx} = 0$

$\Rightarrow e^x(x+3) = 1 \quad \Rightarrow x + 3 = e^{-x}$

Its solution will be intersection point of $y = x + 3$ and $y = e^{-x}$




Clearly there is a critical point in (–1, 0).

**7.** **(b, c)** Let the equation of circle be

$x^2 + y^2 + 2gx + 2gy + c = 0$

$\Rightarrow 2x + 2yy' + 2g + 2gy' = 0$

$\Rightarrow x + yy' + g + gy' = 0 \qquad$ ...(i)

Differentiating again,

$$1 + yy'' + (y')^2 + gy'' = 0 \Rightarrow g = -\left[\frac{1+(y')^2+yy''}{y''}\right]$$

Substituting value of g in eqn. (i)

$$x + yy' - \frac{1+(y')^2+yy''}{y''} - \left(\frac{1+(y')^2+yy''}{y''}\right)y' = 0$$

$\Rightarrow xy'' + yy'y'' - 1 - (y')^2 - yy'' - y' - (y')^3 - yy'y'' = 0$

$\Rightarrow (x-y)y'' - y'(1+y'+(y')^2) = 1$

or $(y-x)y'' + [1+y'+(y')^2]y' + 1 = 0$

$Py'' + Qy' + 1 = 0$

$\therefore \quad P = y - x,\ Q = 1 + y' + (y')^2$

Also $P + Q = 1 - x + y + y' + (y')^2$

**8.** **(a)** $f'(x) = 2 - \frac{f(x)}{x}$

$\Rightarrow f'(x) + \frac{1}{x} f(x) = 2$

If $= e^{\log x} = x$

$\therefore f(x).x = \int 2x\,dx = x^2 + C$

or $f(x) = x + \frac{C}{x}, C \neq 0$ as $f(x) \neq 1$

(a) $\lim_{x\to 0^+} f'\left(\frac{1}{x}\right) = \lim_{x\to 0^+} (1 - Cx^2) = 1$

(b) $\lim_{x\to 0^+} x f\left(\frac{1}{x}\right) = \lim_{x\to 0^+} x\left(\frac{1}{x} + Cx\right) \lim_{x\to 0^+} 1 + Cx^2 = 1$

(c) $\lim_{x\to 0^+} x^2 f'x$

$= \lim_{x\to 0^+} x^2\left(1 - \frac{C}{x^2}\right) = \lim_{x\to 0^+} x^2 - C = -C$

(d) for $C \neq 0$, $f(x)$ is unbounded as $0 < x < 2$

$\Rightarrow \frac{C}{2} < \frac{C}{x} < \infty \Rightarrow \frac{C}{2} < x + \frac{C}{x} < \infty$

**9.** **(a, d)** $[(x+2)^2 + y(x+2)]\frac{dy}{dx} = y^2$

$\Rightarrow y^2 \frac{dx}{dy} - (x+2)y = (x+2)^2$

$\Rightarrow \frac{1}{(x+2)^2}\frac{dx}{dy} - \frac{1}{(x+2)y} = \frac{1}{y^2}$

Let $\frac{-1}{x+2} = u \Rightarrow \frac{1}{(x+2)^2}\frac{dx}{dy} = \frac{du}{dy}$

$\therefore$ Eqn becomes

$\frac{du}{dy} + \frac{1}{y}u = \frac{1}{y^2}$

I.F. $= e^{\log y} = y$

$\therefore$ Solution is : $u \times y = \int y \times \frac{1}{y^2} dy = \log y + C$

$\Rightarrow \frac{-y}{x+2} = \log y + C$

As it passes through $(1, 3) \Rightarrow C = -1 - \log 3$

$\therefore \frac{-y}{x+2} = \log y - 1 - \log 3$

$\Rightarrow \log \frac{y}{3} = 1 - \frac{y}{x+2}$ ...(1)

Intersection of (1) and $y = x + 2$

$\log \frac{y}{3} = 0 \Rightarrow y = 3 \Rightarrow x = 1$

$\therefore$ (1, 3) is the only intersection point.

Intersection of (1) and $y = (x+2)^2$

$\log \frac{(x+2)^2}{3} = 1 - (x+2)$ or $\log \frac{(x+2)^2}{3} + (x+2) = 1$

$\because \frac{(x+2)^2}{3} > \frac{4}{3} > 1, \forall x > 0$

$\therefore$ LHS $> 2$, $\forall$ $x > 0 \Rightarrow$ no solution.

## E. Subjective Problems

**1.** **(a)** $(a+bx)e^{\frac{y}{x}} = x$

$\Rightarrow e^{\frac{y}{x}} = \frac{x}{a+bx}$ ...(1)

Diff. w.r. to $x$, we get

$$e^{\frac{y}{x}} \frac{\left[(x)\frac{dy}{dx} - y\right]}{x^2} = \frac{a+bx-bx}{(a+bx)^2}$$

or $\left(x\frac{dy}{dx} - y\right)e^{\frac{y}{x}} = \frac{ax^2}{(a+bx)^2}$ ...(2)

From (1) using $e^{\frac{y}{x}} = \frac{x}{a+bx}$, we get

$\left(x\frac{dy}{dx} - y\right) = \frac{ax}{a+bx}$ ...(3)

Differentiating (3) w. r. to $x$, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(a+bx)a - axb}{(a+bx)^2}$$

or $x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$

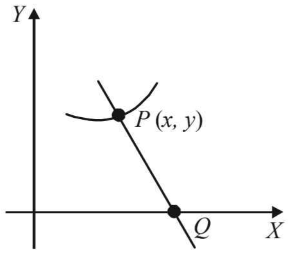
$\Rightarrow x^3\frac{d^2y}{dx^2} = \left(\frac{ax}{a+bx}\right)^2$ ...(4)

Comparing (3) and (4) we get

$$x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$$

**2.** The length of normla $PQ$ to any curve $y = f(x)$ is given by

$y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$




According to question length of $PQ = k$

$\Rightarrow \left(y\frac{dy}{dx}\right)^2 + y^2 = k^2$

$\Rightarrow y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

which is the required differential equation of given curve.
On solving this D.E. we get the eq$^n$ of curve as follows

$$\int \frac{y\,dy}{\sqrt{k^2-y^2}} = \int \pm dx \Rightarrow -\frac{1}{2}.2\sqrt{k^2-y^2} = \pm x + C$$

$$-\sqrt{k^2-y^2} = \pm x + C$$

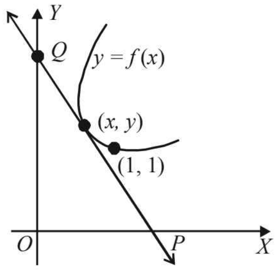
As it passes through $(0, k)$ we get $C = 0$

$\therefore$ Eq$^n$ of curve is

$$-\sqrt{k^2-y^2} = \pm x \text{ or } x^2+y^2 = k^2$$

**3.** Equation of the tangent to the curve $y = f(x)$ at point $(x, y)$ is $Y - y = f'(x)(X - x)$ ...(1)

The line (1) meets $X$-axis at $P\left(x - \frac{y}{f'(x)}, 0\right)$ and Y-axis in $Q(0, y - xf'(x))$




Area of triangle $OPQ$ is

$$= \frac{1}{2}(OP)(OQ) = \frac{1}{2}\left(x - \frac{y}{f'(x)}\right)(y - xf'(x))$$

$$= -\frac{(y - xf'(x))^2}{2f'(x)}$$

We are given that area of $\Delta OPQ = 2$

$$\Rightarrow \frac{-(y - xf'(x))^2}{2f'(x)} = 2 \Rightarrow (y - xf'(x))^2 + 4f'(x) = 0$$

$$\Rightarrow (y - px)^2 + 4p = 0 \quad ...(2)$$

where $p = f'(x) = \frac{dy}{dx}$

Since $OQ > 0, y - xf'(x) > 0.$ Also note that

$$p = f'(x) < 0$$

We can write (2) as $y - px = 2\sqrt{-p}$

$$\Rightarrow y = px + 2\sqrt{-p} \quad ...(3)$$

Differentiating (3) with respect to $x$, we get

$$\frac{dy}{dx} = p + \frac{dp}{dx}x + 2\left(\frac{1}{2}\right)(-p)^{-1/2}(-1)\frac{dp}{dx}$$

$$\Rightarrow \frac{dp}{dx}x - (-p)^{-1/2}\frac{dp}{dx} = 0$$

$$\Rightarrow \frac{dp}{dx}[x - (-p)^{-1/2}] = 0 \Rightarrow \frac{dp}{dx} = 0 \text{ or } x = (-p)^{-1/2}$$

If $\frac{dp}{dx} = 0$, then $p = c$ where $c < 0$ $[\because p < 0]$

Putting this value in (3) we get

$$y = cx + 2\sqrt{-c} \quad ...(4)$$

This curve will pass through (1, 1) if

$$1 = c + 2\sqrt{-c} \Rightarrow -c - 2\sqrt{-c} + 1 = 0$$

$$\Rightarrow (\sqrt{-c} - 1)^2 = 0$$

or $\sqrt{-c} = 1 \Rightarrow -c = 1$ or $c = -1$

Putting the value of $c$ in (4), we get

$$y = -x + 2, \text{ or } x + y = 2$$

Next, putting $x = (-p)^{-1/2}$ or $-p = x^{-2}$ in (3) we get

$$y = \frac{-x}{x^2} + 2\left(\frac{1}{x}\right) = \frac{1}{x}$$

$$\Rightarrow xy = 1 (x > 0, y > 0)$$

Thus, the two required curves are $x + y = 2$ and $xy = 1, (x > 0, y > 0).$

**4.** Put $10x + 6y = v$

$$\therefore 10 + 6\frac{dy}{dx} = \frac{dv}{dx} \qquad \therefore \frac{dv}{dx} - 10 = 6\sin v$$

$$\Rightarrow \frac{dv}{6\sin v + 10} = dx \text{ or } \frac{dv}{12\sin\frac{v}{2}\cos\frac{v}{2} + 10} = dx$$

Divide numerator and denomenator by $\cos^2\left(\frac{v}{2}\right)$ and put $\tan\left(\frac{v}{2}\right) = t$

$$\therefore \frac{2dt}{12t + 10(1+t^2)} = dx \text{ or } \frac{dt}{5t^2 + 6t + 5} = dx$$

$$\frac{dt}{\left(t + \frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 5dx$$

$$\text{or } \frac{5}{4}\tan^{-1}\frac{5t+3}{4} = 5x + 5c \text{ or } \tan^{-1}\frac{5t+3}{4} = 4x + c$$

At origin $x = 0, y = 0$

$$\therefore v = 0, \therefore t = \tan\frac{v}{2} = 0$$

Hence, from above

$$\tan^{-1}\frac{3}{4} = c \Rightarrow \tan^{-1}\frac{5t+3}{4} - \tan^{-1}\frac{3}{4} = 4x$$

$$\text{or } \frac{\frac{5t+3}{4} - \frac{3}{4}}{1 + \frac{5t+3}{4}\frac{3}{4}} = \tan 4x \text{ or } \frac{20t}{25 + 15t} = \tan 4x$$

$$\text{or } 4t = (5 + 3t)\tan 4x \text{ or } t(4 - 3\tan 4x) = 5\tan 4x$$

or $\tan\frac{v}{2}=\frac{5\tan 4x}{4-3\tan 4x}$

or $\tan(5x+3y)=\frac{5\tan 4x}{4-3\tan 4x}$

or $5x+3y=\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right)$

or $y=\frac{1}{3}\left(\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right)-5x\right)$

**5.** (i) $y=u(x)$ and $y=v(x)$ are solutions of given differential equations.

(ii) $u(x_1)>v(x_1)$ for some $x_1$

(iii) $f(x)>g(x), \forall x>x_1$

$$\frac{du}{dx}+p(x)u=f(x)$$

$$\therefore \frac{d}{dx}\left[u.e^{\int p\,dx}\right]=f(x)e^{\int p\,dx}$$

Similarly, $\frac{d}{dx}\left[v.e^{\int p\,dx}\right]=g(x)e^{\int p\,dx}$

Subtracting, $\frac{d}{dx}\left[(u-v).e^{\int p\,dx}\right]=[f(x)-g(x)]e^{\int p\,dx}$

From above since $f(x)>g(x), \forall x>x_1$ and exponential function is always +ive, then R.H.S. is +ive.

$$\therefore \frac{d}{dx}\left[(u-v).e^{\int p\,dx}\right]>0 \text{ or } \frac{dF}{dx}>0$$

Hence the function $F=(u-v)e^{\int pdx}$ is an increasing function.

Again $u(x_1)>v(x_1)$ for some $x_1$

$\therefore$ $F=(u-v)e^{\int pdx}$ is +ive at $x=x_1$

$\Rightarrow$ $F=(u-v)e^{\int pdx}$ is +ive $\forall x>x_1$

($F$ being increasing function)

$\therefore$ $u(x)>v(x), \forall x>x_1$

$\therefore$ Hence there is no point $(x,y)$ such that $x>x_1$ which can satisfy the equations.

$y=u(x)$ and $y=v(x)$.

**6.** Equation of normal is, $\frac{dx}{dy}(X-x)+Y-y=0$

$$\therefore \frac{\left|x\frac{dx}{dy}+y\right|}{\sqrt{1+\left(\frac{dx}{dy}\right)^2}}=|y|$$

$$\Rightarrow x^2\left(\frac{dx}{dy}\right)^2+y^2+2xy\frac{dx}{dy}=y^2+y^2\left(\frac{dx}{dy}\right)^2$$

$$\Rightarrow \left(\frac{dx}{dy}\right)=0, \text{ or } \frac{dx}{dy}=\frac{2xy}{y^2-x^2}$$

If $\frac{dx}{dy}=0$, then $x=c$, when $x=1, y=1, c=1$.

$\therefore$ $x=1$ ...(1)

When $\frac{dx}{dy}=\frac{2xy}{y^2-x^2}$ (homogeneous)

Putting $x=vy \Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$

$$\therefore v+y\frac{dv}{dy}=\frac{2v}{1-v^2}$$

$$\Rightarrow y\frac{dv}{dy}=\frac{2v}{1-v^2}-v=\frac{2v-v+v^3}{1-v^2}=\frac{v+v^3}{1-v^2}$$

$$\Rightarrow \frac{(1-v^2)dv}{v(1+v^2)}=\frac{dy}{y} \Rightarrow \left(\frac{1}{v}-\frac{2v}{(1+v^2)}\right)dv=\frac{dy}{y}$$

$$\Rightarrow \log v-\log(1+v^2)=\log y+\log c$$

or $\frac{v}{1+v^2}=cy \Rightarrow \frac{xy}{x^2+y^2}=cy$

$$\Rightarrow \frac{x}{x^2+y^2}=c,$$

Putting $x=1, y=1$ gives $c=\frac{1}{2}$

$\therefore$ Solution is $x^2+y^2-2x=0$ ...(2)

Hence the solutions are,

$$x^2+y^2-2x=0,\ x-1=0.$$

**7.** Let $X_0$ be initial population of the country and $Y_0$ be its initial food production. Let the average consumption be a units. Therefore, food required initially $aX_0$. It is given

$Y_0=aX_0\left(\frac{90}{100}\right)=0.9aX_0$ ...(i)

Let $X$ be the population of the country in year $t$.

Then $\frac{dX}{dt}=$ rate of change of population

$$=\frac{3}{100}X=0.03X$$

$\frac{dX}{X}=0.03\,dt$; Integrating, $\int\frac{dX}{X}=\int 0.03\,dt$

$\Rightarrow$ $\log X=0.03t+c \Rightarrow X=A.e^{0.03t}$

At $t=0, X=X_0$, thus $X_0=A, X=X_0e^{0.03t}$

Let $Y$ be the food production in year $t$.

Then $Y=Y_0\left(1+\frac{4}{100}\right)^t=0.9\,aX_0(1.04)^t$

(since $Y=0.9aX_0$ from (i))

Food consumption in the year $t$ is $a X_0 e^{0.03t}$

Again $Y - aX \ge 0$ (given)

$$\Rightarrow \quad 0.9a(1.04)^t \ge aX_0 e^{0.03t} \Rightarrow \frac{(1.04)^t}{e^{0.03t}} \ge \frac{1}{0.9} = \frac{10}{9}$$

Taking log on both sides,

$$t[\ln(1.04) - 0.03] \ge \ln 10 - \ln 9$$

$$\Rightarrow \quad t \ge \frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$$

Thus, the least integral value of the year $n$, when the country becomes self-sufficient, is the smallest integer greater than or equal to $\frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$.

**8.** Let the water level be at a height $h$ after time $t$, and water level falls by $dh$ in time $dt$ and the corresponding volume of water gone out be $dV$.

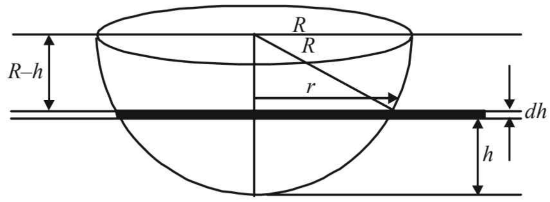
$\Rightarrow \quad |dV| = |\pi r^2 dh|$ $\quad$ ($\because$ $dh$ is very small)

$\Rightarrow \quad \frac{dV}{dt} = -\pi r^2 \frac{dh}{dt}$ $\quad$ ($\because$ as $t$ increases, $h$ decreases)

Now, velocity of water, $v = \frac{3}{5}\sqrt{2gh}$

Rate of flow of water $= Av \, (A = 12\, \text{cm}^2)$

$$\Rightarrow \quad \frac{dV}{dt} = \left(\frac{3}{5}\sqrt{2gh}A\right) = -\pi r^2 \frac{dh}{dt}$$




Also from figure,

$$R^2 = (R-h)^2 + r^2 \Rightarrow r^2 = 2hR - h^2$$

$$\text{So, } \frac{3}{5}\sqrt{2g}.\sqrt{h}A = -\pi(2hR - h^2).\frac{dh}{dt}$$

$$\Rightarrow \quad \frac{2hR - h^2}{\sqrt{h}} dh = -\frac{3}{5\pi}\sqrt{2g}.A.dt$$

Integrating,

$$\int_R^0 (2R\sqrt{h} - h^{3/2})dh = -\frac{3\sqrt{2g}}{5\pi}.A.\int_0^T dt$$

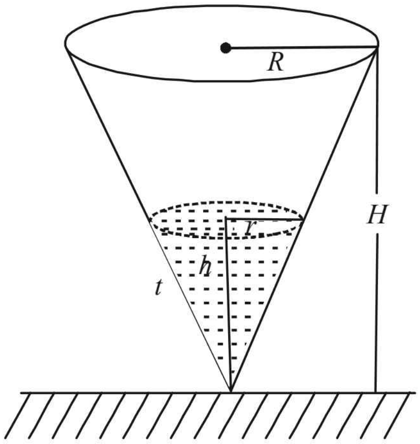
$$\Rightarrow \quad T = \frac{5\pi}{3A\sqrt{2g}}\left(2R.\frac{h^{\frac{3}{2}}}{\frac{3}{2}} - .\frac{h^{\frac{5}{2}}}{\frac{5}{2}}\right)_R^0$$

$$= \frac{5\pi}{3A\sqrt{2g}}\left(-\frac{2}{5}R^{5/2} + \frac{4R}{3}.R^{3/2}\right) = \frac{5\pi}{3A\sqrt{2g}}.\frac{14}{15}.R^{5/2}$$

$$= \frac{56\pi}{9A\sqrt{g}}(10)^5 = \frac{56\pi}{9 \times 12\sqrt{g}}(10)^5 = \frac{14\pi}{27\sqrt{g}}(10)^5 \text{ units.}$$

**9.** Let at time $t$, $r$ and $h$ be the radius and height of cone of water.

$\therefore$ At time $t$, surface area of liquid in contact with air $= \pi r^2$




$$ATQ \quad -\frac{dV}{dt} \propto \pi r^2$$

[$\because$ '–'ve sign shows that $V$ decreases with time.]

$$\Rightarrow \quad \frac{dV}{dt} = -k\pi r^2 \quad \Rightarrow \quad \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right] = -k\pi r^2$$

$$\Rightarrow \quad \frac{1}{3}\pi\frac{d}{dt}[r^2 h] = -k\pi r^2$$

But from figure $\frac{r}{h} = \frac{R}{H}$ $\quad$ [Using similarity of $\Delta$'s ]

$$\Rightarrow \quad h = \frac{rH}{R}$$

$$\therefore \quad \text{We get, } \frac{1}{3}\frac{d}{dt}\left[r^2.\frac{rH}{R}\right] = -kr^2$$

$$\Rightarrow \quad \frac{r^2 H}{R}\frac{dr}{dt} = -kr^2$$

$$\Rightarrow \quad \frac{dr}{dt} = -\frac{kR}{H} \Rightarrow r = \frac{-kR}{H}t + C$$

But at $t = 0, r = R \Rightarrow R = 0 + C \Rightarrow C = R$

$$\therefore \quad r = \frac{-kRt}{H} + R$$

Now let the time at which cone is empty be $T$ then at $T$, $r = 0$ (no liquid is left)

$$\therefore \quad 0 = \frac{-kRT}{H} + R \Rightarrow T = \frac{H}{k}$$

**10.** According to question

$$\text{slope of curve } C \text{ at } (x, y) = \frac{(x+1)^2 + (y-3)}{(x+1)}$$

$$\Rightarrow \quad \frac{dy}{dx} = (x+1) + \frac{y-3}{x+1}$$

$$\Rightarrow \quad \frac{dy}{dx} - \left(\frac{1}{x+1}\right)y = x + 1 - \frac{3}{x+1}$$

$$\text{I.F.} = e^{-\int \frac{1}{x+1}dx} = e^{-\log(x+1)} = \frac{1}{x+1}$$

$\therefore$ Sol$^n$ is $y\dfrac{1}{x+1} = \int\left[1-\dfrac{3}{(x+1)^2}\right]dx$

$$\frac{y}{x+1} = x + \frac{3}{x+1} + C$$

$$y = x(x+1) + 3 + C(x+1) \quad ...(1)$$

As the curve passes through (2, 0)

$\therefore$ $0 = 2.3 + 3 + C.3$

$\Rightarrow$ $C = -3$

$\therefore$ Eq$^n$. (1) becomes

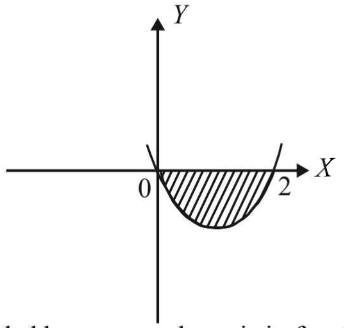
$$y = x(x+1) + 3 - 3x - 3$$

$$y = x^2 - 2x \quad ...(2)$$

which is the required eq$^n$ of curve.

This can be written as $(x-1)^2 = (y+1)$

[Upward parabola with vertex at $(1, -1)$, meeting $x$-axis at (0, 0) and (2, 0)]




Area bounded by curve and $x$-axis in fourth quadrant is as shaded region in fig. given by

$$A = \left|\int_0^2 y\,dx\right| = \left|\int_0^2 (x^2-2x)\,dx\right| = \left[\frac{x^3}{3} - x^2\right]_0^2$$

$$= \left|\frac{8}{3} - 4\right| = \frac{4}{3} \text{ sq. units.}$$

**11.** We know that length of tangent to curve $y = f(x)$ is given by

$$\left|\frac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\left(\frac{dy}{dx}\right)}\right|$$

As per question $\left|\dfrac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\left(\frac{dy}{dx}\right)}\right| = 1$

$\Rightarrow$ $y^2\left(1+\left(\dfrac{dy}{dx}\right)^2\right) = \left(\dfrac{dy}{dx}\right)^2$

$\Rightarrow$ $\left(\dfrac{dy}{dx}\right)^2 = \dfrac{y^2}{1-y^2}$ $\Rightarrow$ $\dfrac{dy}{dx} = \pm\dfrac{y}{\sqrt{1-y^2}}$

$\Rightarrow$ $\displaystyle\int\frac{\sqrt{1-y^2}}{y}dy = \int \pm dx$

$\Rightarrow$ Put $y = \sin\theta$ so that $dy = \cos\theta\, d\theta$

$\Rightarrow$ $\displaystyle\int\frac{\cos\theta}{\sin\theta}\cos\theta\, d\theta = \pm x + c$

$\Rightarrow$ $\displaystyle\int(\operatorname{cosec}\theta - \sin\theta)d\theta = \pm x + c$

$\Rightarrow$ $\log|\operatorname{cosec}\theta - \cot\theta| + \cos\theta = \pm x + c$

$\Rightarrow$ $\log\left|\dfrac{1-\sqrt{1-y^2}}{y}\right| + \sqrt{1-y^2} = \pm x + c$

## F. Match the Following

**1.** ***A-p, q, r, s, t; B-p, t; C-p, q, r, t; D-s***

(A) $(x-3)^2 y' + y = 0$

$\Rightarrow$ $(x-3)^2\dfrac{dy}{dx} = -y$

$\Rightarrow$ $\displaystyle\int\left(-\frac{1}{y}\right)dy = \int\frac{1}{(x-3)^2}dx$

or $\log|y| = \dfrac{1}{x-3} + \log c,\ x \neq 3$

$\Rightarrow$ $\log\left(\dfrac{y}{c}\right) = \dfrac{1}{x-3},\ x \neq 3$

$\Rightarrow$ $\dfrac{y}{c} = e^{\frac{1}{x-3}}$ or $y = ce^{\frac{1}{x-3}},\ x \neq 3$

$\therefore$ The solution set is $(-\infty, \infty) - \{3\}$

The interval $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right), \left(0, \dfrac{\pi}{2}\right), \left(\dfrac{\pi}{8}, \dfrac{5\pi}{4}\right), \left(0, \dfrac{\pi}{8}\right)$ and $(-\pi, \pi)$ contained in the domain

$\therefore$ ***(A)*** $\rightarrow$ ***p, q, r, s, t***

(B) $\displaystyle\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$

Let $(x-3) = t \Rightarrow dx = dt$

Also when $x \to 1, t \to -2$

and when $x \to 5, t \to 2$

$\therefore$ Integral becomes

$$\int_{-2}^{2}(t+2)(t+1)t(t-1)(t-2)dt$$

$$= \int_{-2}^{2} t(t^2-1)(t^2-4)dt = 0$$

as integrand is an odd function.

$O$ is contained by $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ and $(-\pi, \pi)$

$\therefore$ ***(B)*** $\rightarrow$ ***p, t.***

(C) Let $f(x)=\cos^2 x+\sin x$

$\Rightarrow\ f'(x)=-2\sin x\cos x+\cos x$

For critical point $f'(x)=0\ \Rightarrow \sin x=\frac{1}{2}$ or $\cos x=0$

$\Rightarrow\ x=\frac{\pi}{6},\frac{5\pi}{6},$ or $x=\frac{\pi}{2},-\frac{\pi}{2}$

Now $f''(x)=-2\cos 2x-\sin x$

$f''(x)|_{x=\pi/6}=-ve\quad f''(x)|_{x=5\pi/6}=-ve$

$f''(x)|_{x=\pi/2}=+ve$ and $f''(x)|_{x=-\pi/2}=+ve$

$\therefore\ \frac{\pi}{6}$ and $\frac{5\pi}{6}$ are the points of local maxima.

Clearly all the intervals given in column II except $\left(0,\frac{\pi}{8}\right)$ contain at least one point of local maxima.

$\therefore$ **(C) → p, q, r, t**

(D) Let $f(x)=\tan^{-1}(\sin x+\cos x)$

$=\tan^{-1}\left[\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\right]$

$f'(x)=\frac{1}{1+2\sin^2\left(x+\frac{\pi}{4}\right)}.\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$

For $f(x)$ to be an increasing function,

$f'(x)>0$

$\Rightarrow\ \cos\left(x+\frac{\pi}{4}\right)>0\ \Rightarrow -\frac{\pi}{2}<x+\frac{\pi}{4}<\frac{\pi}{2}$

$\Rightarrow\ -\frac{3\pi}{4}<x<\frac{\pi}{4}$

Clearly only $\left(0,\frac{\pi}{8}\right)\subset\left(-\frac{3\pi}{4},\frac{\pi}{4}\right)$

$\therefore$ **(D) → s.**

### H. Assertion & Reason Type Questions

1. **(c)** The given differential equation is

$x\sqrt{x^2-1}\,dy-y\sqrt{y^2-1}\,dx=0$

$\Rightarrow \int\frac{dy}{y\sqrt{y^2-1}}=\int\frac{dx}{x\sqrt{x^2-1}}\ \Rightarrow\ \sec^{-1}y=\sec^{-1}x+C$

$\Rightarrow$ y= $\sec\left[\sec^{-1}x+C\right]\qquad \because\ y(2)=\frac{2}{\sqrt{3}}$

$\Rightarrow\ \frac{2}{\sqrt{3}}=\sec\left(\sec^{-1}2+C\right)\ \Rightarrow\ \sec^{-1}\frac{2}{\sqrt{3}}-\sec^{-1}2=C$

$\Rightarrow C=\frac{\pi}{6}-\frac{\pi}{3}=-\frac{\pi}{6}\quad \therefore\ y=\sec\left[\sec^{-1}x-\frac{\pi}{6}\right]$

$\Rightarrow$ Statement 1 is true.

Also $\frac{1}{y}=\cos\left[\cos^{-1}\frac{1}{x}-\frac{\pi}{6}\right]$

$=\cos\left(\cos^{-1}\frac{1}{x}\right)\cos\frac{\pi}{6}+\sin\left(\cos^{-1}\frac{1}{x}\right)\sin\frac{\pi}{6}$

$\Rightarrow \frac{1}{y}=\frac{\sqrt{3}}{2x}+\frac{1}{2}\sqrt{1-\frac{1}{x^2}}$ $\therefore$ Statement 2 is false.

### I. Integer Value Correct Type

1. **(0)**

The given equation is $\frac{dy}{dx}+g'(x)y=g(x)\,g'(x)$

I.F. = $e^{\int g'(x)dx}=e^{g(x)}$

$\therefore$ Solution is $y.e^{g(x)}=\int e^{g(x)}g(x).g'(x)dx$

put g (x) = t so that g′(x) dx = dt

$=\int e^t t\,dt=e^t(t-1)+c$

$\therefore\ y.e^{g(x)}=e^{g(x)}[g(x)-1]+c$

As y (0) = 0 and g (0) = 0

$\therefore$ C = 1

$\therefore\ y.e^{g(x)}=e^{g(x)}[g(x)-1]+1$

As g (2) = 0, putting $x=2$ we get

$y(2).e^{g(2)}=e^{g(2)}[g(2)-1]+1\Rightarrow y(2)=0$

## Section-B JEE Main/ AIEEE

1. **(c)** $\left(1+3\frac{dy}{dx}\right)^2=\left(\frac{4d^3y}{dx^3}\right)^3\ \Rightarrow\ \left(1+3\frac{dy}{dx}\right)^2=16\left(\frac{d^3y}{dx^3}\right)^3$

2. **(b)** $\frac{d^2y}{dx^2}=e^{-2x};\ \frac{dy}{dx}=\frac{e^{-2x}}{-2}+c;\ y=\frac{e^{-2x}}{4}+cx+d$

3. **(c)** $y^2=4a(x-h),\ 2yy_1=4a\Rightarrow yy_1=2a$

Differentiating, $\Rightarrow y_1^2+yy_2=0$

Degree = 1, order = 2.

4. **(c)** $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

$\Rightarrow\frac{dx}{dy}+\frac{x}{(1+y^2)}=\frac{e^{\tan^{-1}y}}{(1+y^2)}$

$I.F=e^{\int\frac{1}{(1+y^2)}dy}=e^{\tan^{-1}y}$

$$x(e^{\tan^{-1}y}) = \int \frac{e^{\tan^{-1}y}}{1+y^2} e^{\tan^{-1}y} dy$$

$$x(e^{\tan^{-1}y}) = \frac{e^{2\tan^{-1}y}}{2} + C$$

$$\therefore\ 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$$

**5.** **(c)** $x^2 + y^2 - 2ay = 0$ ...........(1)

Differentiate,

$$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0 \Rightarrow a = \frac{x+yy'}{y'}$$

Put in (1), $x^2 + y^2 - 2\left(\frac{x+yy'}{y'}\right)y = 0$

$$\Rightarrow (x^2+y^2)y' - 2xy - 2y^2y' = 0 \Rightarrow (x^2 - y^2)y' = 2xy$$

**6.** **(b)** $ydx + (x + x^2y)dy = 0$

$$\Rightarrow \frac{dx}{dy} = -\frac{x}{y} - x^2 \Rightarrow \frac{dx}{dy} + \frac{x}{y} = -x^2,$$

It is Bernoullis form. Divide by $x^2$

$$x^{-2}\frac{dx}{dy} + x^{-1}\left(\frac{1}{y}\right) = -1.$$

put $x^{-1} = t, -x^{-2}\frac{dx}{dy} = \frac{dt}{dy}$

We get, $-\frac{dt}{dy} + t\left(\frac{1}{y}\right) = -1 \Rightarrow \frac{dt}{dy} - \left(\frac{1}{y}\right)t = 1$

It is linear in $t$.

Integrating factor $= e^{\int -\frac{1}{y}dy} = e^{-\log y} = y^{-1}$

$\therefore$ Solution is $t(y^{-1}) = \int (y^{-1})dy + c$

$$\Rightarrow \frac{1}{x}.\frac{1}{y} = \log y + c \Rightarrow \log y - \frac{1}{xy} = c$$

**7.** **(c)** $y^2 = 2c(x + \sqrt{c})$ ........ (i)

$2yy' = 2c.1$ or $yy' = c$ ........ (ii)

$\Rightarrow\ y^2 = 2yy'(x + \sqrt{yy'})$

[On putting value of c from (ii) in (i)]

On simplifying, we get

$(y - 2xy')^2 = 4yy'^3$ ........ (iii)

Hence equation (iii) is of order 1 and degree 3.

**8.** **(c)** $\frac{xdy}{dx} = y(\log y - \log x + 1)$

$$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$$

Put $y = vx$

$$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow v + \frac{xdv}{dx} = v(\log v + 1)$$

$$\frac{xdv}{dx} = v\log v \Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$$

Put $\log v = z \Rightarrow \frac{1}{v}dv = dz \Rightarrow \frac{dz}{z} = \frac{dx}{x}$

$\Rightarrow \ln z = \ln x + \ln c$

$x = cx$ or $\log v = cx$ or $\log\left(\frac{y}{x}\right) = cx.$

**9.** **(d)** $Ax^2 + By^2 = 1$ .........(1)

$Ax + By\frac{dy}{dx} = 0$ ..........(2)

$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0$ ........(3)

From (2) and (3)

$$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$$

Dividing both sides by –B, we get

$$\Rightarrow\ xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

Which is a DE of order 2 and degree 1.

**10.** **(a)** General equation of circles passing through origin and having their centres on the x-axis is

$x^2 + y^2 + 2gx = 0$ ...(i)

On differentiating w.r.t $x$, we get

$$2x + 2y.\frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y\frac{dy}{dx}\right)$$

$\therefore$ equation (i) be

$$x^2 + y^2 + 2\left\{-\left(x + y\frac{dy}{dx}\right)\right\}.x = 0$$

$$\Rightarrow x^2 + y^2 - 2x^2 - 2x\frac{dy}{dx}.y = 0 \Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$$

**11.** **(d)** $\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x}$

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

we get

$$v + x\frac{dv}{dx} = 1 + v \Rightarrow \int\frac{dx}{x} = \int dv$$

$\Rightarrow v = \ln x + c \Rightarrow y = x\ln x + cx$

As $y(1) = 1$

$\therefore\ c = 1$ So solution is $y = x\ln x + x$

**12.** **(c)** We have $y = c_1e^{c_2x}$

$\Rightarrow\ y' = c_1c_2e^{c_2x} = c_2y$

$$\Rightarrow\ \frac{y'}{y} = c_2 \Rightarrow \frac{y''y - (y')^2}{y^2} = 0 \Rightarrow y''y = (y')^2$$

**13.** **(d)** $\cos x\,dy = y(\sin x - y)dx$

$$\frac{dy}{dx} = y\tan x - y^2\sec x$$

$$\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x \quad ...(i)$$

Let $\frac{1}{y}=t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

From equation (i)

$$-\frac{dt}{dx}-t\tan x=-\sec x \Rightarrow \frac{dt}{dx}+(\tan x)t=\sec x$$

$$\text{I.F.}=e^{\int \tan x\,dx}=(e)^{\log|\sec x|}\sec x$$

Solution : $t(\text{I.F})=\int(\text{I.F})\sec x\,dx \Rightarrow \frac{1}{y}\sec x=\tan x+c$

**14. (d)** $\frac{dy}{dx}=y+3 \Rightarrow \int\frac{dy}{y+3}=\int dx \Rightarrow \ell n|y+3|=x+c$

Since $y(0)=2$, $\therefore \ell n5=c$

$\Rightarrow \ell n|y+3|=x+\ell n5$

When $x=\ell n 2$, then $\ell n|y+3|=\ell n 2+\ell n5$

$\Rightarrow \ell n|y+3|=\ell n 10$

$\therefore y+3=\pm 10 \Rightarrow y=7,-13$

**15. (a)** $\frac{dV(t)}{dt}=-k(T-t) \Rightarrow \int dVt=-k\int(T-t)dt$

$$V(t)=\frac{k(T-t)^2}{2}+c$$

$$V(0)=I \Rightarrow I=\frac{KT^2}{2}+C \Rightarrow C=I-\frac{KT^2}{2}$$

$$\therefore V(T)=0+C=I-\frac{KT^2}{2}$$

**16. (a)** Given differential equation is

$$\frac{dp(t)}{dt}=0.5p(t)-450$$

$$\Rightarrow \frac{dp(t)}{dt}=\frac{1}{2}p(t)-450 \Rightarrow \frac{dp(t)}{dt}=\frac{p(t)-900}{2}$$

$$\Rightarrow 2\frac{dp(t)}{dt}=-[900-p(t)] \Rightarrow 2\frac{dp(t)}{900-p(t)}=-dt$$

Integrate both sides, we get

$$-2\int\frac{dp(t)}{900-p(t)}=\int dt$$

Let $900-p(t)=u$

$\Rightarrow -dp(t)=du$

$\therefore$ We have, $2\int\frac{du}{u}=\int dt \Rightarrow 2\,ln\,u=t+c$

$\Rightarrow 2ln[900-p(t)]=t+c$

when $t=0, p(0)=850$

$2\,ln(50)=c$

$$\Rightarrow 2\left[ln\left(\frac{900-p(t)}{50}\right)\right]=t \Rightarrow 900-p(t)=50e^{\frac{t}{2}}$$

$$\Rightarrow p(t)=900-50e^{\frac{t}{2}}$$

let $p(t_1)=0$

$$0=900-50e^{\frac{t_1}{2}} \qquad \therefore t_1=2ln18$$

**17. (c)** Given, Rate of change is $\frac{dP}{dx}=100-12\sqrt{x}$

$\Rightarrow dP=(100-12\sqrt{x})\,dx$

By integrating

$$\int dP=\int(100-12\sqrt{x})\,dx$$

$P=100x-8x^{3/2}+C$

Given, when $x=0$ then $P=2000$

$\Rightarrow C=2000$

Now when $x=25$ then

$P=100\times 25-8\times(25)^{3/2}+2000$

$=4500-1000$

$\Rightarrow P=3500$

**18. (c)** Given differential equation is

$$\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$$

By separating the variable, we get

$$dp(t)=\left[\frac{1}{2}p(t)-200\right]dt$$

$$\Rightarrow \frac{dp(t)}{\frac{1}{2}p(t)-200}=dt$$

Integrating on both the sides,

$$\int\frac{d(p(t))}{\frac{1}{2}p(t)-200}=\int dt$$

Let $\frac{1}{2}p(t)-200=s \Rightarrow \frac{dp(t)}{2}=ds$

So, $\int\frac{d\,p(t)}{\left(\frac{1}{2}p(t)-200\right)}=\int dt$

$$\Rightarrow \int\frac{2ds}{s}=\int dt \Rightarrow 2\log s=t+c$$

$$\Rightarrow 2\log\left(\frac{p(t)}{2}-200\right)=t+c \Rightarrow \frac{p(t)}{2}-200=e^{\frac{1}{2}}k$$

Using given condition $p(t)=400-300\,e^{t/2}$

**19. (a)** Given, $\frac{dy}{dx}+\left(\frac{1}{x\log x}\right)y=2$

$$\text{I.F.}=e^{\int\frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$$

$y.\log x=\int 2\log x dx+c$

$y\log x=2[x\log x-x]+c$

Put $x=1$, $y.0=-2+c \Rightarrow c=2$

Put $x=e$

$y\log e=2e(\log e-1)+c \Rightarrow y(e)=c=2$

**20. (b)** $y(1+xy)dx=xdy$

$$\frac{xdy-ydx}{y^2}=xdx \Rightarrow \int -d\left(\frac{x}{y}\right)=\int xdx$$

$$-\frac{x}{y}=\frac{x^2}{2}+C \text{ as } y(1)=-1 \Rightarrow C=\frac{1}{2}$$

Hence, $y=\frac{-2x}{x^2+1} \Rightarrow f\left(\frac{-1}{2}\right)=\frac{4}{5}$