CHAPTER 11

Binomial Theorem and Mathematical Induction

SYNOPSIS

The process of mathematical induction is an indirect method which helps us to prove complex mathematical formulae, that cannot be easily proved by direct methods.

The Principle of Mathematical Induction

If P(n) is a statement such that,

(i) P(n) is true for n =1
(ii) P(n) is true for n = k + 1, when it is true for n = k, where k is a natural number then the statement P(n) is true for all natural numbers.

❍ An algebraic expression containing only two terms is called a binomial expression.

$(x + y)^1 = x + y$

$(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^2$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

In the above examples, the coefficients of the variables in the expansions of the powers of the binomial expression are called binomial coefficients.

When the binomial coefficients are listed, for different values of n ∈ N, we see a definite pattern being followed. This pattern is given by the Pascal Triangle.

Pascal Triangle

The exponent of the binomial	The coefficients of the terms in the expansion
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1

This pattern, shown above, can be used to write the binomial expansion for higher powers.

❍ **Binomial theorem:** If n is a positive integer,

$(x + y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}y + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_r x^{n-r} y^r + \ldots\ldots + {}^nC_n y^n$

Important Inferences from the above Expansion

1. The number of terms in the expansion is n + 1.
2. From left to right, in every successive term of the expansion the exponent of x increases with simultaneous decrease in the exponent of y by 1.

3. In each term of the expansion, the sum of the exponents of x and y is equal to the exponent (n) of the binomial expression.
4. The coefficients of the terms that are equidistant from the beginning and the end have numerically equal values, i.e., ${}^nC_0 = {}^nC_n$; ${}^nC_1 = {}^nC_{n-1}$; ${}^nC_2 = {}^nC_{n-2}$ and so on.
5. The general term in the expansion of $(x + y)^n$ is given by $T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$.
6. On substituting '– y' in the place of 'y' in the expansion, we get
$(x - y)^n = {}^nC_0\, x^n - {}^nC_1\, x^{n-1}\, y + {}^nC_2\, x^{n-2}\, y^2 - {}^nC_3\, x^{n-3}\, y^3 + \ldots + (-1)^n\, {}^nC_n y^n$
The general term in the expansion $(x - y)^n$ is $T_{r+1} = (-1)^r\, {}^nC_r x^{n-r}\, y^r$.

Middle Terms in the Expansion of $(x + y)^n$

Depending on the nature of n, i.e., whether n is even or odd, there may exist one or two middle terms.

Case 1: When n is an even number, then there is only one middle term in the expansion $(x + y)^n$, which is $\left(\frac{n}{2}+1\right)^{th}$ term.

Case 2: When n is odd number, there will be two middle terms in the expansion of $(x + y)^n$, which are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms.

- **Term independent of x:** In an expansion of the form $\left(x^p+\frac{1}{x^q}\right)^n$, the term for which the exponent of x is 0 is said to be the term that is independent of x or a constant term.
For example, in the expansion $\left(x+\frac{1}{x}\right)^2$ is
$= x^2 + 2 + \frac{1}{x^2}$, the 2nd term is independent of 'x'.
- **The greatest coefficient in the expansion of $(1 + x)^n$ (where n is a positive integer):**
The coefficient of the (r + 1)th term in the expansion of $(1 + x)^n$ is ${}^nC_r$.
${}^nC_r$ is maximum when r = n/2 (if n is even) and
$r = \frac{n+1}{2}$ or $\frac{n-1}{2}$ (if n is odd)

Numerically Greatest Term

The numerically greatest term in the expansion of $(1 + x)^n$ is found out using the following process. We calculate the value of $\frac{(n+1)|x|}{|x|+1}$:

If $\frac{(n+1)|x|}{|x|+1}$ = an integer, say 'k', then k^{th} and $(k + 1)^{th}$ terms are numerically greatest terms.

If $\frac{(n+1)|x|}{|x|+1}$ is not an integer, say k + a; where 0 < a < 1, $(k + 1)^{th}$ term is numerically greatest term.

Solved Examples

1. Prove that $1 + 2 + 3 + \ldots. + n = \frac{n(n+1)}{2}$.

Solution: Let P(n): $1 + 2 + \ldots.+ n = \frac{n(n+1)}{2}$ be the given statement.

Step 1: Put n = 1

Then, L.H.S. = 1 and R.H.S. = $\frac{1(1+1)}{2} = 1$.

$\therefore$ L.H.S. = R.H.S.

$\Rightarrow$ P(n) is true for n = 1.

Step 2: Assume that P(n) is true for n = k.

$\therefore\ 1 + 2 + 3 + \ldots. + k = \frac{k(k+1)}{2}$

Adding (k+1) on both sides, we get

$$1 + 2 + 3 + \ldots. + k + (k + 1) = \frac{k(k+1)}{2} + (k + 1)$$

$$= (k + 1)\left(\frac{k}{2}+1\right)$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= \frac{(k+1)(\overline{k+1}+1)}{2}$$

$\Rightarrow$ P(n) is true for n = k + 1

$\therefore$ By the principle of mathematical induction P(n) is true for all natural numbers n.

Hence, $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$ for all $n \in N$

2. Prove that $1 + 3 + 5 + ... + (2n - 1) = n^2$

Solution: Let P(n): $1 + 3 + 5 + ... + (2n - 1) = n^2$ be the given statement

Step 1: Put n =1

Then, L.H.S. = 1

R.H.S. = $(1)^2 = 1$

$\therefore$ L.H.S. = R.H.S.

$\Rightarrow$ P(n) is true for n = 1.

Step 2: Assume that P(n) is true for n = k.

$\therefore$ $1 + 3 + 5 + ... + (2k - 1) = k^2$

Adding 2k+1 on both sides, we get

$1 + 3 + 5 + ... + (2k - 1) + (2k + 1) = k^2 + (2k + 1) = (k + 1)^2$

$\therefore$ $1 + 3 + 5 + ... + (2k - 1) + (2(k + 1) - 1) = (k + 1)^2$

$\Rightarrow$ P(n) is true for n = k + 1.

$\therefore$ By the principle of mathematical induction P(n) is true for all natural numbers 'n'.

Hence, $1 + 3 + 5 + ... + (2n - 1) = n^2$, for all $n \in N$

3. Prove that $3^{n+1} > 3(n + 1)$.

Solution: Let P(n): $3^{n+1} > 3(n + 1)$

Step 1: Put n = 1

Then, $3^2 > 3(2)$

$\Rightarrow$ p(n) is true for n = 1

Step 2: Assume that P(n) is true for n = k

Then, $3^{k+1} > 3(k + 1)$

Multiplying throughout with '3'.

$3^{k+1} \times 3 > 3(k + 1) \times 3 = 9k + 9 = 3(k + 2) + (6k + 3) > 3(k + 2)$

$\Rightarrow$ $3^{\overline{k+1}+1} > 3(\overline{k+1}+1)$

P(n) is true for n = k + 1

$\therefore$ By the principle of mathematical induction, P(n) is true for all $n \in N$.

Hence, $3^{n+1} > 3(n + 1)$, $\forall$ $n \in N$

4. Prove that 7 is a factor of $2^{3n} - 1$ for all natural numbers n.

Solution: Let P(n): 7 is a factor of $2^{3n} - 1$ be the given statement

Step 1: When n = 1,

$2^{3(1)} - 1 = 7$ and 7 is a factor of itself.

$\therefore$ P(n) is true for n = 1

Step 2: Let P(n) be true for n = k.

$\Rightarrow$ 7 is a factor of $2^{3k} - 1$.

$\Rightarrow$ $2^{3k} - 1 = 7M$, where $M \in N$.

$\Rightarrow$ $2^{3K} = 7M + 1 \rightarrow (1)$

Now consider $2^{3(k+1)} - 1 = 2^{3k+3} - 1 = 2^{3k}.2^3 - 1$

$= 8(7M+1) - 1$ (using (1)) $= 56M + 7$ (As $2^{3k} = 7M + 1$)

$\therefore$ $2^{3(k+1)} - 1 = 7(8M + 1)$

$\Rightarrow$ 7 is a factor of $2^{3(k+1)} - 1$

$\Rightarrow$ P(n) is true for n = k + 1

$\therefore$ By the principle of mathematical induction, P(n) is true for all natural numbers n.

Hence, 7 is a factor $2^{3n} - 1$ for all $n \in N$.

5. Find the middle term in the expansion of $(2x + 3y)^8$.

Solution: Since n is even number, $\left(\frac{8}{2}+1\right)^{th}$ term i.e., 5th term is the middle term in $(2x + 3y)^8$.

$T_5 = T_{4+1} = {}^8C_4 (2x)^{8-4} (3y)^4 = {}^8C_4 (2x)^4 (3y)^4$

6. Find the middle terms in the expansion of $(5x - 7y)^7$.

Solution: Since n is an odd number, the expansion contains two middle terms.

$\left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+3}{2}\right)^{th}$ terms are the two middle terms in the expansion of $(5x - 7y)^7$.

$T_4 = T_{3+1} = (-1)^3 \, {}^7C_3 (5x)^{7-3} (7y)^3$

$= -{}^7C_3 (5x)^4 (7y)^3$

$T_5 = T_{4+1} = (-1)^4 \times {}^7C_4 (5x)^{7-4} \times (7y)^4$

$= -{}^7C_4 (5x)^3 (7y)^4$

7. Find the term independent of x in $\left(x+\frac{1}{x}\right)^4$.

Solution: Let T_{r+1} be the term independent of x in the given expansion.

$\therefore$ $T_{r+1} = {}^4C_r x^{4-r} \left(\frac{1}{x}\right)^r = {}^4C_r \frac{x^{4-r}}{x^r} = {}^4C_r x^{4-2r}$

For the term independent of x the power of x should be zero.

$\therefore \quad 4 - 2r = 0$ or $r = 2$.

$\Rightarrow \quad T_{2+1} = T_3$, is the independent term of the expansion.

Note: If r is not a positive integer, then the expansion does not contain constant term.

8. Find the coefficient of x^2 in $\left(x^2 + \frac{1}{x^3}\right)^6$.

Solution: Let T_{r+1} be the term containing x^2.

$$T_{r+1} = {}^6C_r\,(x^2)^{6-r}\left(\frac{1}{x^3}\right)^r$$

$$= {}^6C_r\,x^{12-2r}\,\frac{1}{x^{3r}} = {}^6C_r\,x^{12-5r}$$

As the coefficient of x is 2

$12 - 5r = 2 \Rightarrow r = 2$.

$\therefore$ Coefficient of $x^2 = {}^6C_2 = 15$.

9. If the expansion $\left(x^2 + \frac{1}{x^3}\right)^n$ is to contain an independent term, then what should be the value of n?

Solution: General term, $T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot y^r$, for $(x + y)^n$

$\Rightarrow$ general term of $\left(x^2 + \frac{1}{x^3}\right)^n$ is

$${}^nC_r \cdot x^{2n-2r} \cdot \frac{1}{x^{3r}} = {}^nC_r \cdot x^{2n-5r}$$

For a term to be independent of x, $2n - 5r$ should be equal to zero, i.e., $2n - 5r = 0$.

$\Rightarrow \quad r = \frac{2}{5}n$, since r can take only integral values, n has to be a multiple of 5.

10. Find the sum of the co-efficient of the terms of the expansion $(1 + x + 2x^2)^6$.

Solution: Substituting $x = 1$, we have $(1 + 1 + 2)^6$, which gives us the sum of the co-efficients of the terms of the expansion.

$\therefore$ Sum $= 4^6$

PRACTICE EXERCISE 11 (A)

Directions for questions 1 to 40: Select the correct alternative from the given choices.

1. The sum of the first 'n' even natural numbers is

(1) $2n^2$ (2) $n(n + 1)$
(3) $n(n + 1)^2$ (4) $(n + 1)^3$

2. $n^2 + n + 1$ is a/an ______ number for all $n \in N$

(1) even (2) odd
(3) prime (4) None of these

3. $1 + 5 + 9 + \ldots\ldots + (4n - 3)$ is equal to

(1) $n(4n - 3)$ (2) $(2n - 1)$
(3) $n(2n - 1)$ (4) $(4n - 3)^2$

4. For all $n \in N$, which of the following is a factor of $2^{3n} - 1$?

(1) 3 (2) 5
(3) 7 (4) None of these

5. The smallest positive integer n for which $n! < \frac{(n-1)^n}{2}$ holds is

(1) 4 (2) 3
(3) 2 (4) 1

6. In the 8th term of $(x + y)^n$, the exponent of x is 3, then the exponent of x in 5th term is

(1) 5 (2) 4
(3) 2 (4) 6

7. The third term from the end in the expansion of $\left(\frac{4x}{3y}-\frac{3y}{2x}\right)^9$ is

(1) ${}^9C_7 \frac{3^5}{2^3}\frac{y^5}{x^5}$ (2) ${}^{-9}C_7 \frac{3^5}{2^3}\frac{y^5}{x^5}$
(3) ${}^9C_7 \frac{3^3}{2^5}\frac{y^5}{x^3}$ (4) None of these

8. For what values of n is $14^n + 11^n$ divisible by 5?

(1) when n is an even positive integer
(2) For all values of n
(3) When n is a prime number
(4) When n is a odd positive integer

9. If the expansion of $\left(x^3+\frac{1}{x^2}\right)^n$ contains a term independent of x, then the value of n can be

(1) 18 (2) 20
(3) 24 (4) 22

10. If the third term in the expansion of $\left(x+x^{\log_2 x}\right)^6$ is 960, then the value of x is

(1) 2 (2) 3
(3) 4 (4) 8

11. If ${}^nC_3 = {}^nC_{15}$, then ${}^{20}C_n$ is

(1) 18 (2) 380
(3) 190 (4) 300

12. In $(x + y)^n - (x - y)^n$ if the number of terms is 5, then find n.

(1) 6 (2) 5
(3) 10 (4) 9

13. If the sum of the coefficients in the expansion $(4ax - 1 - 3a^2x^2)^{10}$ is 0 then the value of a can be

(1) 2 (2) 4
(3) 1 (4) 7

14. Find the coefficient of x^4 in the expansion of $\left(2x^2+\frac{3}{x^3}\right)^7$.

(1) ${}^7C_2\, 2^5\, 3^3$ (2) ${}^7C_2\, 2^5\, 3^2$
(3) ${}^7C_2\, 3^5\, 2^2$ (4) ${}^7C_3\, 2^5\, 3^2$

15. Find the sum of coefficients of all the terms of the expansion $(ax + y)^n$.

(1) ${}^nC_0a^n + {}^nC_1a^{n-1}x^{n-1}y + {}^nC_2a^{n-2}x^{n-2}y^2 + \ldots\ldots + {}^nC_ny^n$
(2) ${}^nC_0a^n + {}^nC_1a^{n-1} + {}^nC_2a^{n-2} + \ldots\ldots + {}^nC_n$
(3) 2^n
(4) None of these

16. Find the coefficient of the term independent of x in the expansion of $\left(6x^3-\frac{5}{x^6}\right)^{12}$.

(1) ${}^{12}C_4\, 5^8\, 6^4$ (2) ${}^{12}C_4\, 5^8\, 6^3$
(3) ${}^{12}C_4\, 6^8\, 5^4$ (4) ${}^{12}C_4\, 6^8\, 5^3$

17. In the expansion of $(a + b)^n$, the coefficients of 15th and 11th terms are equal. Find the number of terms in the expansion.

(1) 26 (2) 25
(3) 20 (4) 24

18. If ${}^{19}C_r$ and ${}^{19}C_{r-1}$ are in the ratio 2: 3, then find ${}^{14}C_r$.

(1) 91 (2) 81
(3) 71 (4) 61

19. If sum of the coefficients of the first two odd terms of the expansion $(x + y)^n$ is 16 then find n.

(1) 10 (2) 8
(3) 7 (4) 6

20. The number of terms in the expansion of $[(2x + 3y)^4 (4x - 6y)^4]^9$ is

(1) 36 (2) 37
(3) 10 (4) 40

21. If $p(n) = (n - 2)(n - 1)n(n + 1)(n + 2)$, then greatest number which divides p(n) for all $n \in N$ is

(1) 12 (2) 24
(3) 120 (4) None of these

22. $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots\ldots\ldots+\frac{1}{n(n+1)}$ is equal to

(1) $\frac{1}{n+1}$ (2) $\frac{n+2}{n(n+1)}$
(3) $\frac{n+3}{n(n+1)}$ (4) $\frac{n}{n+1}$

23. The inequality $(2n + 7) < (n + 3)^2$ is true for

(1) All negative numbers.
(2) All whole numbers.
(3) All natural numbers.
(4) None of these

24. For $n \in N$, $a^{2n-1} + b^{2n-1}$ is divisible by

(1) $a + b$ (2) $(a + b)^2$
(3) $a^3 + b^3$ (4) $a^2 + b^2$

25. The greatest number which divides $25^n - 24n - 1$ for all $n \in N$ is

(1) 24 (2) 578
(3) 27 (4) 576

26. Find the coefficient of the independent term in the expansion of $\left(x^{\frac{1}{2}}+7x^{-\frac{1}{3}}\right)^{10}$.

(1) ${}^{10}C_4\, 7^4$ (2) ${}^{10}C_6\, 7^6$
(3) ${}^{10}C_6\, 7^5$ (4) ${}^{10}C_4\, 7^7$

27. Find the term which has the exponent of x as 8 in the expansion of $\left(x^{\frac{5}{2}}-\frac{3}{x^3\sqrt{x}}\right)^{10}$.

(1) T_2 (2) T_3
(3) T_4 (4) Does not exist

28. In the expansion $(1 + x)^n$ if the coefficient of the 8th term is 4 times the coefficient of the 9th term, then find the total number of terms in the expansion.

(1) 9 (2) 10
(3) 8 (4) None of these

29. The remainder when $9^{49} + 7^{49}$ is divided by 64 is

(1) 24 (2) 8
(3) 16 (4) 38

30. If 'p' and 'q' are the coefficients of x^a and x^b respectively in $(1 + x)^{a+b}$, then

(1) $2p = q$ (2) $p + q = 0$
(3) $p = q$ (4) $p = 2q$

31. The number of rational terms in the expansion of $\left(x^{\frac{1}{5}}+y^{\frac{1}{10}}\right)^{45}$ is

(1) 5 (2) 6
(3) 4 (4) 7

32. If three consecutive coefficients in the expansion of $(1 + x)^n$, where n is a natural number are 36, 84 and 126 respectively, then n is

(1) 8 (2) 9
(3) 10 (4) Cannot be determined

33. $\sum_{r=2}^{16} {}^{16}C_r =$

(1) $2^{15} - 15$ (2) $2^{16} - 16$
(3) $2^{16} - 17$ (4) $2^{17} - 17$

34. Find the value(s) of k such that the term independent of x in $\left(3x^2+\frac{k}{2x}\right)^6$ is 135.

(1) ± 2 (2) ± 1
(3) ± 4 (4) ± 3

35. Find the value of $(51)^4$ by using binomial theorem.

(1) 6765021 (2) 6765201
(3) 6765211 (4) 6765101

36. Number of non-zero terms in the expansion of $(5\sqrt{5}x+\sqrt{7})^6+(5\sqrt{5}x-\sqrt{7})^6$ is

(1) 4 (2) 10
(3) 12 (4) 14

37. Find the two successive terms in the expansion of $(2 + 3x)^8$ whose coefficients are in the ratio 1: 3.

(1) 2nd and 3rd terms (2) 3rd and 4th terms
(3) 4th and 5th terms (4) 5th and 6th terms

38. The ratio of the coefficients of x^4 to that of the term independent of x in the expansion of $\left(x^2 + \frac{9}{x^2}\right)^{18}$ is

(1) 1: 6 (2) 3: 8
(3) 1: 10 (4) 1: 8

39. Find the value of r for which t_{r+1} is the independent term of x in the expansion of $\left(a_1 x^{k_1} + \frac{a_2}{x^{k_2}}\right)^m$.

(1) $r = \frac{mk_2}{k_1 + k_2}$ (2) $r = \frac{mk_1}{k_1 + k_2}$
(3) $r = \frac{mk_1}{k_1 - k_2}$ (4) $r = \frac{mk_2}{k_2 - k_1}$

40. Find the coefficient of a^3 in the expansion of $\left(\sqrt{a} + \frac{1}{a\sqrt{a}}\right)^{22}$.

(1) ${}^{22}C_4$
(2) ${}^{22}C_6$
(3) ${}^{22}C_8$
(4) ${}^{22}C_{10}$

PRACTICE EXERCISE 11 (B)

Directions for questions 1 to 40: Select the correct alternative from the given choices.

1. The sum of the first 'n' odd natural numbers is

(1) $2n - 1$ (2) $n(2n - 1)$
(3) n^2 (4) n^3

2. $n^2 - n + 1$ is an odd number for all

(1) $n > 1$
(2) $n > 2$
(3) $n \geq 1$
(3) $n \geq 5$

3. For $n \in N$, $2^{3^n} + 1$ is divisible by

(1) 3^{n+11} (2) 3^{n-11}
(3) 3^{n+1} (4) 3^{n+111}

4. $2^n - 1$ gives the set of all odd natural numbers for all $n \in N$. Comment on the given statement.

(1) True for all values of n
(2) False
(3) True for only odd values of n
(4) True for only prime values of n

5. The inequality $2^n > n$ is true for

(1) all whole numbers
(2) all positive integers
(3) all negative integers
(4) all integers

6. In the 5th term of $(x + y)^n$, the exponent of y is 4, then the exponent of y in the 8th term is

(1) 1 (2) 7
(3) 5 (4) 9

7. The third term from the end in the expansion of $(3x - 2y)^{15}$ is

(1) ${}^{-15}C_{13} 3^{13} 2^2 x^{13} y^2$ (2) ${}^{15}C_{13} 3^{13} 2^2 x^{13} y^2$
(3) ${}^{15}C_2 3^2 2^{13} x^2 y^{13}$ (4) ${}^{-15}C_2 3^2 2^{13} x^2 y^{13}$

8. $7^{n+1} + 3^{n+1}$ is divisible by

(1) 10 for all natural numbers n
(2) 10 for odd natural numbers n
(3) 10 for even natural numbers n
(4) None of these

9. The elements in the fifth row of Pascal triangle is

(1) 1, 5, 10, 10, 5, 1
(2) 1, 6, 15, 20, 15, 6, 1
(3) 1, 4, 6, 4, 1
(4) 1, 7, 21, 35, 35, 21, 7, 1

10. If the coefficients of 6th and 5th terms of expansion $(1 + x)^n$ are in the ratio 7: 5, then find the value of n.

(1) 11 (2) 12
(3) 10 (4) 9

11. Find the sixth term in the expansion of $\left(2x^2 - \frac{3}{7x^3}\right)^{11}$.

(1) ${}^{-11}C_5 \frac{2^6 3^5}{7^5} x^3$ (2) ${}^{11}C_5 \frac{2^6 3^5}{7^5} x^{-3}$
(3) ${}^{-11}C_5 \frac{2^6 3^5}{7^5} x^{-3}$ (4) None of these

12. Which term is the constant term in the expansion of $\left(2x-\frac{1}{3x}\right)^6$?

(1) 2nd term (2) 3rd term
(3) 4th term (4) 5th term

13. The number of terms which are not radicals in the expansion $\left(\sqrt{7}+4\right)^6+\left(\sqrt{7}-4\right)^6$, after simplification is

(1) 6 (2) 5
(3) 4 (4) 3

14. The sum of the coefficients in the expansion of $(x+y)^7$ is

(1) 119 (2) 64
(3) 256 (4) 128

15. The coefficient of x^4 in the expansion of $\left(4x^2+\frac{3}{x}\right)^8$ is

(1) ${}^8C_5 12^5$ (2) ${}^8C_4 12^4$
(3) ${}^8C_3 12^3$ (4) ${}^8C_6 12^6$

16. Find the sum of the coefficients of all the terms in the expansion of $(3x^2+y)^6$.

(1) 4096 (2) 4005
(3) 4003 (4) 4004

17. The term independent of x in the expansion of $\left(x^3-\frac{1}{x^2}\right)^{10}$ is

(1) ${}^{10}C_6$ (2) ${}^{10}C_7$
(3) ${}^{10}C_9$ (4) ${}^{-10}C_6$

18. If the 20th and 21st terms in the expansion of $(1+x)^{40}$ are equal, then the value of x is

(1) $\frac{20}{21}$ (2) $\frac{21}{20}$
(3) 25 (4) $\frac{1}{25}$

19. If $11\left[{}^{n-1}C_3\right]=24\left[{}^{n}C_2\right]$, then the value of n is

(1) 12 (2) 11
(3) 10 (4) 13

20. The sum of the elements in the sixth row of pascal triangle is

(1) 32 (2) 63
(3) 128 (4) 64

21. The number of factors of the greatest number that divides any number of the form p(n) where $p(n)=n(n+1)(n+2)(n+3)(n+4)(n+5)$ is

(1) 10 (2) 20
(3) 30 (4) 40

22. $\frac{2}{3}+\frac{2}{15}+\frac{2}{35}+\text{..........}+\frac{2}{(2n-1)(2n+1)}$ is equal to

(1) $\frac{2n}{2n-1}$ (2) $\frac{2n}{2n+1}$
(3) $\frac{2n}{2n-3}$ (4) $\frac{2n}{2n+3}$

23. $2.3^{n+1}+3.2^{n-1}$ (where $n \in N$) is divisible by

(1) 2 (2) 3
(3) 6 (4) 7

24. For all $n \in N$, x^n+1 is divisible by

(1) $x+1$
(2) $x-1$
(3) Both (1) and (2)
(4) None of these

25. For all $n \in N$, $41^n-40n-1$ is divisible by

(1) 41 (2) 40
(3) 300 (4) 500

26. Find the number of terms of the expansion $(1-x^2+y^3)^5$.

(1) 15 (2) 6
(3) 21 (4) 81

27. Find the coefficient of x^9 in the expansion of $\left(11x^2+\frac{3}{x^5}\right)^{15}$.

(1) ${}^{15}C_{12}\,11^3 3^{12}$ (2) ${}^{15}C_3\,11^{10} 3^5$
(3) ${}^{15}C_3\,3^2 11^3$ (4) ${}^{15}C_{12}\,11^{12} 3^3$

28. Find the independent term in the expansion of $\left(2x^2+\frac{7}{x^5}\right)^{10}$.

(1) ${}^{10}C_2\,2^8 7^2$ (2) ${}^{10}C_3\,2^7 7^3$
(3) ${}^{10}C_8\,2^8 7^3$ (4) Does not exist.

29. If sum of the first 3 coefficients is 16 in the expansion $\left(x+\frac{1}{x^3}\right)^n$, then find n.

(1) 10 (2) 8
(3) 5 (4) 4

30. The value of middle term in the expansion of $(100 - 2)^4$ by using the binomial theorem is

(1) −240000
(2) 240000
(3) −3200
(4) 3200

31. Find the coefficient of the term which contains the 10th power of x in the expansion of $\left(x^{\frac{1}{3}} + \frac{1}{x\sqrt[3]{x}}\right)^{40}$.

(1) 870 (2) 680
(3) 780 (4) None of these

32. If m and n are the coefficients of x^{a^2} and x^{b^2} respectively in $(1+x)^{a^2+b^2}$, then

(1) n = 2m (2) m + n = 0
(3) 2n = m (4) m = n

33. The number of rational terms in the expansion of $\left(x^{\frac{1}{2}} + y^{\frac{1}{3}}\right)^{18}$ is

(1) 5 (2) 2
(3) 3 (4) 4

34. If three consecutive coefficients in the expansion of $(1 + x)^n$ are 495, 220 and 66 respectively, then n =

(1) 15 (2) 12
(3) 13 (4) 14

35. Find the coefficient of x^7 in the expansion of $\left(7x + \frac{2}{x^2}\right)^{13}$.

(1) $78 \times 8^8 \times 4$ (2) $78 \times 7^6 \times 4^2$
(3) $78 \times 7^{11} \times 4$ (4) $78 \times 7^{11} \times 4^2$

36. Find the two successive terms in the expansion of $(3 + 4x)^7$ whose coefficients of x are in the ratio 1: 1.

(1) T_5, T_6 (2) T_3, T_4
(3) T_4, T_5 (4) Does not exist

37. Find the value of k for which the term independent of x in $\left(x^2 + \frac{k}{x}\right)^{12}$ is 7920.

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$
(3) $\sqrt{2}$ (4) 2

38. If the coefficients of x in the 4th and 7th terms of the expansion $(1 + x)^n$ are in the ratio 4: 7, then find n

(1) 8 (2) 9
(3) 12 (4) 10

39. The total number of terms in the expansion of $(x + y)^{50} + (x - y)^{50}$ is

(1) 51
(2) 26
(3) 102
(4) 25

40. The value of $\left(\sqrt{5}+2\right)^6 + \left(\sqrt{5}-2\right)^6$ is

(1) a positive integer
(2) a negative integer
(3) an irrational number
(4) a rational number but not an integer

ANSWER KEYS

PRACTICE EXERCISE 11 (A)

1. 2	**2.** 2	**3.** 3	**4.** 3	**5.** 1	**6.** 4	**7.** 2	**8.** 4	**9.** 2	**10.** 1
11. 3	**12.** 4	**13.** 3	**14.** 2	**15.** 2	**16.** 3	**17.** 2	**18.** 1	**19.** 4	**20.** 2
21. 3	**22.** 4	**23.** 2	**24.** 1	**25.** 4	**26.** 2	**27.** 4	**28.** 2	**29.** 3	**30.** 3
31. 1	**32.** 2	**33.** 3	**34.** 1	**35.** 2	**36.** 1	**37.** 2	**38.** 3	**39.** 2	**40.** 1

PRACTICE EXERCISE 11 (B)

1. 3	**2.** 3	**3.** 3	**4.** 2	**5.** 4	**6.** 2	**7.** 4	**8.** 3	**9.** 1	**10.** 1
11. 3	**12.** 3	**13.** 3	**14.** 4	**15.** 2	**16.** 1	**17.** 1	**18.** 1	**19.** 2	**20.** 4
21. 3	**22.** 2	**23.** 2	**24.** 4	**25.** 2	**26.** 3	**27.** 4	**28.** 4	**29.** 3	**30.** 2
31. 3	**32.** 4	**33.** 4	**34.** 2	**35.** 3	**36.** 4	**37.** 3	**38.** 4	**39.** 2	**40.** 1