CHAPTER 5

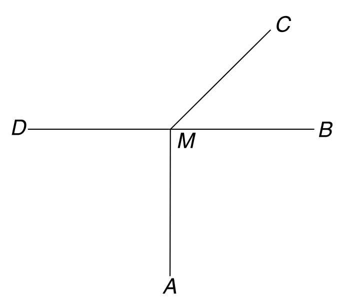
Heat Transfer

LEVEL 1

Q. 1: Four metal rods each of length L and cross sectional area A are joined at point M. Thermal conductivities of MA, MB and MD are equal and that of MC is thrice that of MA. The end points A, B, C and D are kept in large reservoirs. Heat flows into the junction from B at a rate of $P(\text{Js}^{-1})$ and from C at a rate of $3P$. Heat flows out of D at a rate of $5P$.

(a) Find relation between temperatures of points A, B and C.

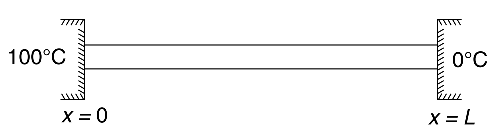
(b) Find temperature of D if temperature of A and M and T_A and T_M respectively.



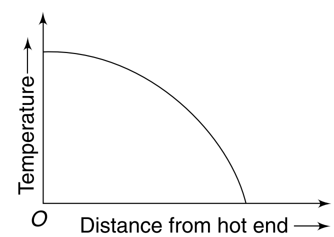
Q. 2: The two ends of a uniform metallic rod are maintained at 100°C and 0°C as shown in the figure. Assume that end of the rod at 100°C is at $x = 0$ and the other end at 0°C is at $x = L$. Plot the variation of temperature as x changes from 0 to L in steady state. Consider two cases.

(a) The rod is perfectly lagged.

(b) The rod is not lagged and surrounding is at 0°C.



Q.3: The ends of a metallic bar are maintained at different temperature and there is no loss/gain of heat from the sides of the bar due to conduction or radiation. In the steady state the temperature variation along the length of the bar is a shown in the figure what do you think about the cross sectional area of the bar?



Q.4: A thick spherical shell of inner and outer radii r and R respectively has thermal conductivity $k = \frac{\rho}{x^n}$, where ρ is a constant and x is distance from the centre of the shell. The inner and outer walls are maintained at temperature T_1 and T_2 ($< T_1$)

(a) Find the value of number n (call it n_0) for which the temperature gradient remains constant throughout the thickness of the shell.

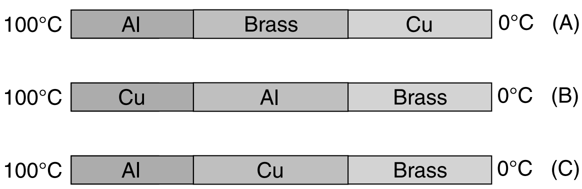
(b) For $n = n_0$, find the value of x at which the temperature is $\frac{T_1 + T_2}{2}$

(c) For $n = n_0$, calculate the rate of flow of heat through the shell.

Q. 5: Three bars of aluminium, brass and copper are of equal length and cross section. The three pieces are joined together as shown in A, B and C and the ends are maintained at 100°C and 0°C. The thermal conductivities of aluminium, brass and copper are in ratio 2 : 1 : 4. Assume no heat loss through curved surface of the bar and that the system is in steady state.

(a) In which of the three cases (A, B or C) the temperature difference across aluminium bar will be maximum?

(b) Draw a graph showing variation of temperature from one end of the bar to another in case B.



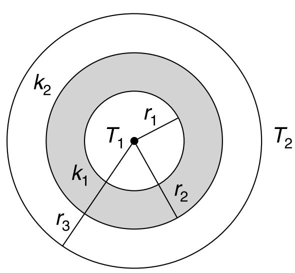
Q. 6: A lake is covered with ice 5 cm thick and the atmospheric temperature above the ice is –10°C. At what rate (in cm/hour) will the ice layer thicken?

Thermal conductivity of ice = 0.005 cgs unit, density of ice = 0.9 g/cc and latent heat of fusion of ice = 80 cal/g.

Q. 7. A liquid having mass m = 250 g is kept warm in a vessel by use of an electric heater. The liquid is maintained at 50°C when the power supplied by heater is 30 watt and surrounding temperature is 20°C. As the heater is switched off, the liquid starts cooling and it was observed that it took 10 second for temperature to fall down from 40°C to 39.9°C. Calculate the specific heat capacity of the liquid. Assume Newton's law of cooling to be applicable.

Q.8: (i) A cylindrical pipe of length L has inner and outer radii as a and b respectively. The inner surface of the pipe is at a temperature T_1 and the outer surface is at a lower temperature of T_2. Calculate the radial heat current if conductivity of the material is K.

(ii) A cylindrical pipe of length L has two layers of material of conductivity K_1 and K_2. (see figure). If the inner wall of the cylinder is maintained at T_1 and outer surface is at T_2 (< T_1), calculate the radial rate of heat flow.



Q. 9: A 3 mm diameter and 5 m long copper wire is insulated using a 2 mm thick plastic cover whose thermal conductivity is $K = 0.15\ Wm^{-1}K^{-1}$. The wire has a potential difference of 10 V between its ends and the current through it is 8A. The outer surface of the wire is at 30°C. Neglect convection.

(i) Calculate the temperature at the interface of the wire and the plastic cover.

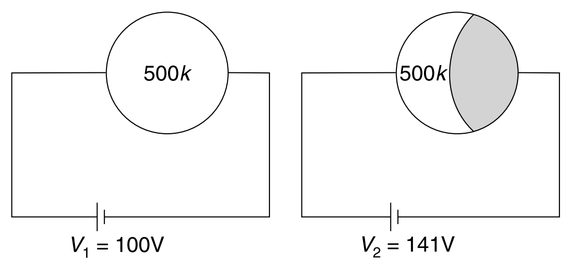
(ii) Determine whether doubling the thickness of the plastic cover will increase or decrease the interface temperature. [Given $\ell n(2.33) = 0.85$]

Q. 10: A potato at initial temperature T_0 is placed inside a hot convection oven maintained at a constant temperature $T_1(> T_0)$. Assume that the potato receives heat only because of convection phenomenon and the rate at which it receives heat is given as $hA(T_1 - T)$ where h is a constant, A is surface area of the potato and T is instantaneous temperature of the potato. Mass and specific heat capacity of the potato are m and s respectively. In how much time the potato will be at a temperature $T_2 = \dfrac{T_0 + T_1}{2}$? Assume no change in volume of the potato.

Q. 11: What is emissivity of a perfectly reflecting surface?

Q. 12: A body is in thermal equilibrium with surrounding. Absorptive power of the surface of the body is $a = 0.5$. E is the radiant energy incident in unit time on the surface of the body. How much energy propagates from its surface in unit time?

Q. 13: A copper sphere is maintained at 500 K temperature by connecting it to a battery of emf V_1 = 100 V (see figure). The surrounding temperature is 300 K. When half the surface of the copper sphere is completely blackened (so that the surface behaves almost like a black body), a cell of emf V_2 = 141 V is needed to maintain its temperature at 500 K. Calculate the emissivity of the copper surface.

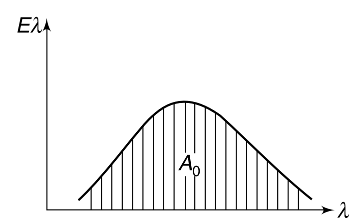


Q. 14: Stefan's contain (σ) derives from other known constant of nature, viz. Boltzmann constant, (k) planck's constant (h) and speed of light in vacuum (c). Value of the constant is

$$\sigma = 5.67 \times 10^{-8}\ Js^{-1}m^{-2}K^{-4}$$

If speed of light were 2% more than its present value, how much different (in percentage) the value of σ would have been?

Q. 15: An iron ball is heated to 727°C and it appears bright red. The plot of energy density distribution versus wavelength is as shown. The graph encloses an area A_0 under it. Now the ball is heated further and it appears bright yellow. Find the area (A) of the energy density graph now.



If the given that wavelengths for red and yellow light are 8000 Å and 6000 Å respectively.

Q. 16: A solid cylinder and a sphere of same material are suspended in a room turn by turn, after heating them to the same temperature. The cylinder and the sphere have same radius and same surface area.

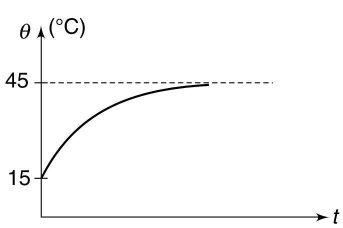
(a) Find the ratio of initial rate of cooling of the sphere to that of the cylinder.

(b) Will the ratio change if both the sphere and the cylinder are painted with a thin layer of lamp black?

Q. 17: A block body at temperature T radiates same amount of energy in the wavelength range λ_1 to $\lambda_1 + \Delta\lambda$ and λ_2 to $\lambda_2 + \Delta\lambda$. It is given that $\Delta\lambda << \lambda_1$ or λ_2 and $\lambda_2 > \lambda_1$. Prove that $\frac{b}{\lambda_1} > T > \frac{b}{\lambda_2}$ where b is Wien's constant.

Q. 18: Majority of radiation from the Sun is in visible and near infra-red (0.7 to 4 μm) region. What can you say about the composition of the radiation from the Earth?

Q. 19: A metal ball of mass 1.0 kg is kept in a room at 15°C. It is heated using a heater. The heater supplies heat to the ball at a constant rate of 24 W. The temperature of the ball rises as shown in the graph. Assume that the rate of heat loss from the surface of the ball to the surrounding is proportional to the temperature difference between the ball and the surrounding. Calculate the rate of heat loss from the ball when it was at temperature of 20°C.



Q. 20: A hot body is suspended inside a room that is maintained at a constant temperature. The temperature difference between the body and the surrounding becomes half in a time interval t_0. In how much time the temperature difference between the body and the surrounding will becomes $\frac{1}{4}$ the original value?

Q. 21: Newton's law of cooling says that the rate of cooling of a body is proportional to the temperature difference between the body and its surrounding when the difference in temperature is small.

(a) Will it be reasonable to assume that the rate of heating of a body is proportional to temperature difference between the surrounding and the body (for small difference in temperature) when the body is placed in a surrounding having higher temperature than the body?

(b) Assuming that our assumption made in (a) is correct estimate the time required for a cup of cold coffee to gain temperature from 10°C to 15°C when it is kept in a room having temperature 25°C. It was observed that the temperature of the cup increases from 5°C to 10°C in 4 min.

Q. 22: "Blue hot is hotter than red hot". Explain.

Q. 23: A planet of radius r_0 is at a distance r from the sun ($r >> r_0$). The sun has radius R. Temperature of the planet is T_0, and that of the surface of the sun is T_s. Calculate the temperature of another planet whose radius is $2r_0$ and which is at a distance $2r$ from the sun. Assume that the sun and the planets are black bodies.

Q. 24: A star having radius R has a small planet revolving around it at a distance d ($>> R$). The star and the planet both behave like black bodies and radiate maximum amount of energy at wavelength λ_s and λ_p respectively.

(i) Find d in terms of other given parameters.

(ii) Show that $\lambda_p >> \lambda_s$

Q. 25: A 20 mm diameter copper pipe is used to carry heated water. The external surface of the pipe is at T = 80°C and its surrounding is at T_0 = 20°C. The outer surface of the pipe radiates like a black body and also loses heat due to convection. The convective heat loss per unit area per unit time is given by $h(T - T_0)$ where $h = 6 \text{W (m}^2\text{K)}^{-1}$. Calculate the total heat lost by the pipe in unit time for one meter of its length.

Q. 26: Solar constant, I_s is defined as intensity of solar radiation incident on the Earth. Its value is close to 1.4 kW/m^2. Nearly 68% of this energy is absorbed by the Earth. The average temperature of Earth is about 290 K. Radius of the Earth is R_e = 6000 km and that of the Sun is R_s = 700,000 km. Earth - Sun distance is $r = 1.5 \times 10^8$ km. Assume Sun to be a black body.

(a) Estimate the effective emissivity of earth.

(b) Find the power of the sun.

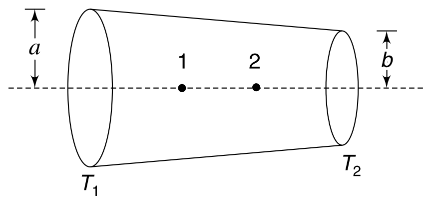
(c) Estimate the surface temperature of the Sun.

LEVEL 2

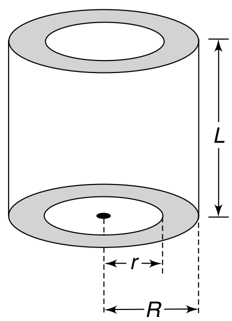
Q. 27: A tapering rod of length L has cross sectional radii of a and $b(< a)$ at its two ends. Its thermal conductivity is k. The end with radius a is maintained at a higher temperature T_1 and the other end is maintained at a lower temperature T_2. The curved surface is insulated.

(i) At which of the two points – 1 and 2 – shown in the figure will the temperature gradient be higher?

(ii) Calculate the thermal resistance of the rod.



Q. 28: A thick cylindrical shell made of material of thermal conductivity k has inner and outer radii r and R respectively and its length is L. When the curved surface of the cylinder are lagged (i.e., given insulation cover) and one end is maintained at temperature T_1 and the other end is maintained at $T_2(< T_1)$; the heat current along the length of the cylinder is H. In another experiment the two ends are lagged and the inner wall and outer wall are maintained at T_1 and T_2 respectively. Find the radial heat flow in this case.



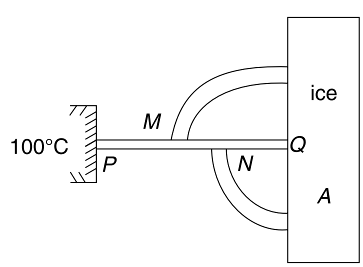
Q. 29: A double pan window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by 0.05 m thick stagnant air space. In the steady state, the room-glass interface and the glass-outdoor interface are at constant temperatures of 27°C and 0°C respectively. The thermal conductivity of glass is 0.8 $Wm^{-1}K^{-1}$ and of air 0.08 $Wm^{-1}K^{-1}$. Answer the following questions.

(a) Calculate the temperature of the inner glass-air interface.

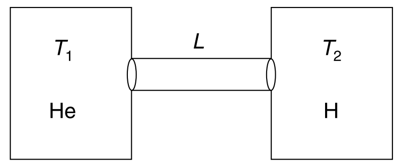
(b) Calculate the temperature of the outer glass-air interface.

(c) Calculate the rate of flow of heat through the window pane.

Q. 30: The container A contains ice at O°C. A conducting uniform rod PQ of length $4R$ is used to transfer heat to the ice in the container. The end P of the rod is maintained at 100°C and the other end Q is kept inside container A. The complete ice melts in 23 minutes. In another experiment, two conductors in shape of quarter circle of radii $2R$ and R are welded to the conductor PQ at M and N respectively and their other ends are inserted inside the container A. All conductors are made of same material and have same cross sectional area. Once again the end P is maintained at 100°C and this time the complete ice melts in t minute. Find t. Assume no heat loss from the curved surface of the rods. [Take $\pi \simeq 3.0$]



Q. 31: Two identical adiabatic containers of negligible heat capacity are connected by conducting rod of length L and cross sectional area A. Thermal conductivity of the rod is k and its curved cylindrical surface is well insulated from the surrounding. Heat capacity of the rod is also negligible. One container is filled with n moles of helium at temperature T_1 and the other one is filled with equal number of moles of hydrogen at temperature $T_2(<T_1)$. Calculate the time after which the temperature difference between two gases will becomes half the initial difference.



Q. 32: A copper slab is 2 mm thick. It is protected by a 2 mm layer of stainless steel on both sides. The temperature on one side of this composite slab is 400°C and 200°C on the other side. Value of thermal conductivities are-

$$k_{cu} = 400\ Wm^{-1}K^{-1} \text{ and } k_s = 16\ Wm^{-1}K^{-1}$$

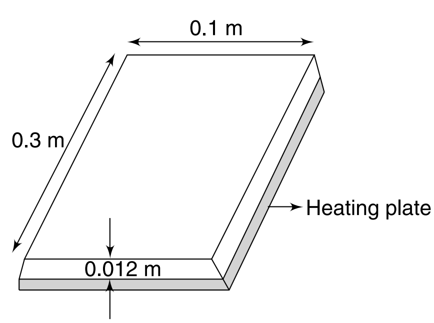
(a) Just by knowing that thermal conductivity of steel is much less than copper, find (approximately) the temperature of the copper slab.

(b) Plot the variation of temperature across the thickness of the composite wall.

Q. 33: A steel plate is 0.3 m long, 0.1 m wide and 0.012 m thick. The plate is placed on a heating plate of identical size maintained at 100°C. The heating plate is receiving energy through a 50 W heater. The heating plate losses heat only to the steel plate which is well insulated from all sides except at the top. The top surface of the steel plate is exposed to an airstream of temperature 20°C. The top surface of the steel plate radiates like a black body. Calculate the rate at which the top surface looses heat due to convection. The surface of steel plate in contact with heating plate is at 100°C.

Thermal conductivity of steel $k = 16\ Wm^{-1}K^{-1}$

Stefan's constant = $5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$.



Q. 34: A straight cylindrical wire is connected to a battery. The wire is maintained at constant temperature of T_1 when the room temperature is $T_0(< T_1)$. The wire is disconnected from the battery and half of it is cut-off. The remaining half of the wire is connected to the same battery in the same room. Find the constant temperature (T_2) attained by the wire. Assume that wire loses heat to the atmosphere through radiation from curved surface only.

Q. 35: A spherical shell is kept in an atmosphere at temperature T_0. The wavelength corresponding to maximum intensity of radiation for the shell is λ_0. A point source of constant power is switched on inside the shell. The power radiated by the source is $P = 0.4\sigma SeT_0^4$ where S, e and σ are outer surface area of the shell, emissivity of the outer surface of the shell and Stefan's constant respectively. Calculate the new wavelength (λ) corresponding to the maximum intensity of radiation from the shell. Assume that change in temperature (ΔT) of the shell is small compared to the ambient temperature T_0.

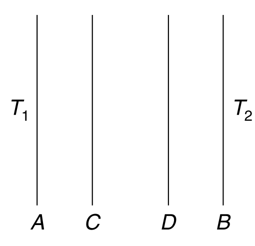
Q. 36: A body at a temperature of 50°C cools to 49°C in time Δt when it is placed in a room maintained at –3°C. The same body cools from 50°C to 49°C in time $\Delta t'$ when placed in a room that is maintained at 24°C. Find $\Delta t'$ in terms of Δt. Assume heat loss through radiation only and the specific heat capacity of the body remains constant with change in temperature.

Q. 37: A plane surface A is at a constant temperature $T_1 = 1000$ K. Another surface B parallel to A, is at a constant lower temperature $T_2 = 300$ K. There is no medium in the space between two surfaces. The rate of energy transfer from A to B is equal to r_1(J/s). In order to reduce rate of heat flow due to radiation, a heat shield consisting of two thin plates C and D, thermally insulated from each other, is placed between A and B in parallel. Now the rate of heat transfer (in steady state) reduces to r_2. Neglect any effect due to finite size of the surfaces, assume all surfaces to be black bodies and take Stefan's constant $\sigma = 6 \times 10^{-8}$ Wm^{-2}K^{-4}. Area of all surfaces $A = 1$m^2.

(i) Find r_1

(ii) Find the ratio $\frac{r_2}{r_1}$

(iii) Find the ratio $\frac{r_2}{r_1}$ if temperature of A and B were 2000 K and 600 K respectively.



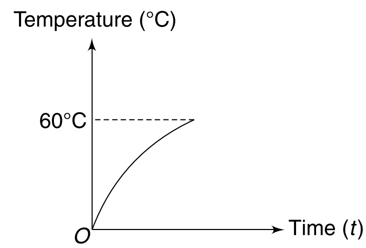
Q. 38: A block is kept in a room which is at 20°C. To raise the temperature of the block, heat is given to it at a constant rate of 600 watt (using an electric heater). The temperature of the block rises with time as shown in the graph. The slope of the graph at time $t = 0$ is 3°C s^{-1}. Once the temperature rises to 60°C, the heater is switched off and another heater is switched on to maintain the temperature of the block at 60°C. This new heater supplies heat at a constant rate of 100 watt. Assume that heat capacity of the block remains constant for the range of temperature involved.

(a) Explain why the slope of the given graph is decreasing with time.

(b) Calculate the heat capacity of the block.

(c) If the 100 W heater is also switched off, what will be initial rate of cooling of the block?

(d) Assuming that rate of heat loss by the block to the surrounding is proportional to difference in its temperature with surrounding, calculate the heat radiated per second by the block when it was at 30°C.



Q. 39: Mass m of a liquid A is kept in a cup and it is at a temperature of 90°C. When placed in a room having temperature of 20°C, it takes 5 min for the temperature of the liquid to drop to 30°C. Another liquid B has nearly same density as that of A and its sample of mass m kept in another identical cup at 50°C takes 5 min for its temperature to fall to 30°C when placed in room having temperature 20°C. If the two liquids at 90°C and 50°C are mixed in a calorimeter where no heat is allowed to leak, find the final temperature of the mixture. Assume that Newton's law of cooling is applicable for given temperature ranges.

Q. 40: Consider the Sun to be a black body at temperature $T_S = 5780$ K. Radius of the Sun is $r_S = 6.96 \times 10^8$ m. The Earth - Sun distance is $R = 1.49 \times 10^{11}$m. Assume that 30% of the solar radiation that hits the earth is scattered back into space without absorption.

(a) Calculate the steady state average temperature of the earth assuming it to be a black body. Take $(0.7)^{\frac{1}{4}} = 0.91$

(b) We know that average temperature of earth is ≃ 288 K. How does this value compare with that obtained in (a)?

The difference is due to greenhouse effect. Comment on the following statement - "Emissivity of earth is reduced more than absorptivity due to green house effect."

Q. 41: A and B are two sphere made of same material. Radius of A is double that of B and initially they are at same temperature (T). Both of them are kept far apart in a room at temperature $T_0(< T)$. Calculate the ratio of initial rate of cooling (i.e. rate of fall of temperature) of sphere A and B if

(a) the spheres are solid,

(b) the spheres are hollow made of thin sheets of same thickness

Q. 42: Heat received by the Earth due to solar radiations is 1.35 KWm^{-2}. It is also known that the temperature of the Earth's crust increases 1°C for every 30 m of depth. The average thermal conductivity of the Earth's crust is $K = 0.75\ J(msK)^{-1}$ and radius of the Earth is $R = 6400$ km.

(i) Calculate rate of heat loss by the Earth's core due to conduction.

(ii) Assuming that the Earth is a perfect block body estimate the temperature of its surface.

Q. 43: A truck of mass M has 4 wheels. The surface area of the metal disc in each wheel is A. When brakes are applied the brake shoe in each wheel rubs against the metal disc. This produces heat. We will assume that this heat is used to heat the disc in the wheel only. Each disc has mass m and is made of material of specific heat capacity s. One day the truck was going downhill on a road inclined at angle θ to the horizontal. To maintain a constant speed v the driver had to apply brakes. The only other dissipative force, apart from the brake force, on the truck is the air resistance force that is equal to F. Assume no heating of discs due to air friction.

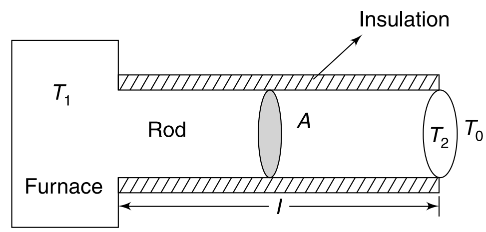
(a) Find the initial rate of rise of temperature of each metal disc after the brakes are applied.

(b) Find the final temperature (T) of the disc assuming that it is a black body and it loses heat only through radiation. Take the atmospheric temperature to be T_0

LEVEL 3

Q. 44: A solid body A of mass m and specific heat capacity 's' has temperature $T_1 = 400$ K. It is placed, at time $t = 0$, in atmosphere having temperature $T_0 = 300$ K. It cools, following Newton's law of cooling and its temperature was found to be $T_2 = 350$ K at time t_0. At time t_0, the body A is connected to a large water bath maintained at atmospheric temperature T_0, using a conducting rod of length L, cross section A and thermal conductivity k. The cross sectional area A of the connecting rod is small compared to the overall surface area of body A. Find the temperature of A at time $t = 2t_0$.

Q. 45: A cylindrical rod of length l, thermal conductivity k and area of cross section A has one end in a furnace maintained at constant temperature. The other end of the rod is exposed to surrounding. The curved surface of the rod is well insulated from the surrounding. The surrounding temperature is T_0 and the furnace is maintained at $T_1 = T_0 + \Delta T_1$. The exposed end of the rod is found to be slightly warmer then the surrounding with its temperature maintained $T_2 = T_0 + \Delta T_2$ [$\Delta T_2 << T_0$]. The exposed surface of the rod has emissivity e. Prove that ΔT_1 is proportional to ΔT_2 and find the proportionality constant.



ANSWERS

1. (a) $T_A = T_B = T_C$ (b) $T_D = 6T_M - 5T_A$

3. Cross sectional area of the bar decreases from hot end to the cold end

4. (a) $n_0 = 2$ (b) $\frac{r + R}{2}$

(c) $\frac{4\pi\rho(T_1 - T_2)}{(R - r)}$

6. 0.5 cm hr^{-1}

7. 8000 $\frac{J}{kg°C}$

8. (i) $\frac{(T_1 - T_2)2\pi KL}{\ell n\left(\frac{b}{a}\right)}$

(ii) $\frac{T_1 - T_2}{\frac{1}{2\pi K_1 L}\ell n\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi K_2 L}\ell n\left(\frac{r_3}{r_2}\right)}$

9. (i) 44.4°C (ii) Increase

10. $t = \frac{ms}{hA}\ell n2$

11. Zero

12. E

13. $\frac{1}{3}$

14. 4% less than its present value

15. $A = \left(\frac{4}{3}\right)^4 A_0$

16. (a) $\frac{3}{4}$ (b) No

18. Earth radiates almost 100% in far infra-red region

19. 4 Watt

20. $t = 2t_0$

21. (a) Yes (b) 5.6 min.

23. $\frac{T_0}{\sqrt{2}}$

24. (i) $d = \frac{R}{2}\left(\frac{\lambda_p}{\lambda_s}\right)^2$

25. 51.7 Wm^{-1}

26. (a) 0.6 (b) 3.96×10^{26}
(c) 5800 K

27. (i) At point 2 (ii) $\frac{L}{k\pi ab}$

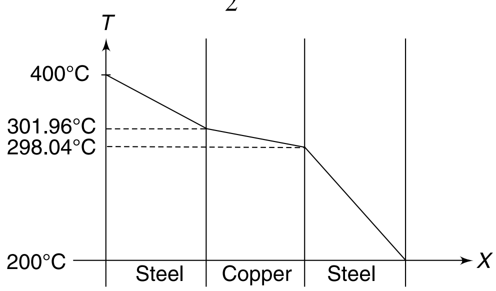
28. $\frac{2L^2H}{(R^2 - r^2)\ell n\left(\frac{R}{r}\right)}$

29. (a) 26.5°C (b) 0.5°C
(c) 40.5 J/s

30. 17.5 min

31. $\frac{15nRL}{16kA}\ln 2$

32. (a) Temperature drop will take palace almost entirely in steel. The copper plate will have almost uniform temperature of $\frac{400 + 200}{2} = 300°C$

(b) 

33. 30.05 watt.

34. $T_2 = (4T_1^4 - 3T_0^4)^{1/4}$

35. $\frac{10}{11}\lambda_0$

36. $1.55\Delta t$

37. (i) $r_1 = 59514$ W (ii) $\frac{r_2}{r_1} = \frac{1}{3}$
(iii) $\frac{r_2}{r_1} = \frac{1}{3}$

38. (b) 200 $J°C^{-1}$ (c) 0.5°C s^{-1} (d) 25 W

39. 66°C

40. (a) 254 K

41. (a) $\frac{1}{2}$ (b) 1

42. (i) 1.29×10^{13} W (ii) 276 K

43. (a) $\frac{dT}{dt} = \left(\frac{Mg\sin\theta - F}{4m\cdot s}\right)v$

(b) $T = \left[\frac{(Mg\sin\theta - F)v}{\sigma A} + T_0^4\right]^{1/4}$

44. $T = 300 + 50e^{-\left[\frac{\ln 2}{t_0} + \frac{kA}{msL}\right]t_0}$

45. $\frac{4e\sigma LT_0^3}{k} + 1$

SOLUTIONS

1. (a) $K_A = K_B = K$ and $K_C = 3K$
Heat flow rate from A to M is P.
Since rate of heat flow through $CM = 3 \times$ heat flow through AM
This is possible if $T_C = T_A$

$$\therefore \quad T_A = T_B = T_C$$

(b)
$$\frac{5\cdot KA(T_A - T_M)}{L} = \frac{KA(T_M - T_D)}{L}$$

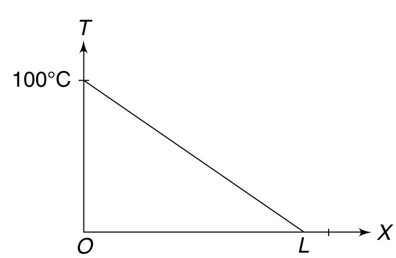
$$\Rightarrow \quad 5T_A - 5T_M = T_M - T_D$$

$$\Rightarrow \quad T_D = 6T_M - 5T_A$$

2. (a) For lagged bar, the heat current is constant at all sections.

$$\frac{dQ}{dt} = KA\frac{dT}{dx} = \text{a constant}$$

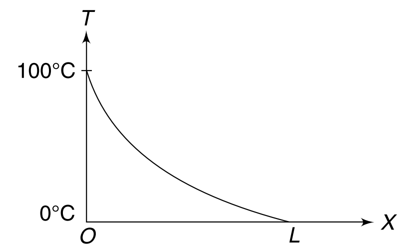
$$\therefore \quad \frac{dT}{dx} = \text{a constant}$$



(b) For not - lagged bar, as one moves from $x = 0$ to $x = L$, more and more heat is lost to the surrounding [as surface area goes on increasing]. It means the heat current goes on decreasing.

$\therefore \quad KA\left(\frac{dT}{dx}\right) = \frac{dQ}{dt}$ goes on decreasing from $x = 0$ to $x = L$

$\therefore \quad \frac{dT}{dx}$ goes on decreasing, i.e., slope of the graph goes on decreasing



4. (a) Thermal resistance of a shell of radius x and thickness dx will be

$$dR_{th} = \frac{dx}{k4\pi x^2} = \frac{1}{\rho \cdot 4\pi}\frac{x^n}{x^2}dx = \frac{1}{\rho \cdot 4\pi}x^{n-2}dx \quad ...(i)$$

Radial heat current (which is constant) will be

$$\frac{dT}{dR_{th}} = H \text{ (constant)}$$

$$\frac{4\pi\rho dT}{x^{n-2}dx} = H$$

$$\frac{dT}{dx} = \text{a constant if } n = 2$$

$$\therefore \quad n_0 = 2$$

(b) Since temperature gradient is uniform, hence temperature will be $\frac{T_1 + T_2}{2}$ at $x = \frac{r+R}{2}$

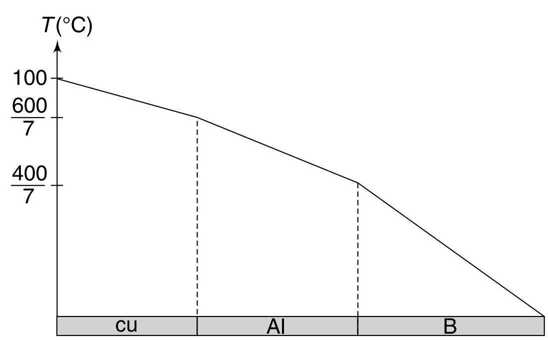
(c) From (i) $dR_{th} = \frac{1}{4\pi\rho}dx$

$$\therefore \quad R_{th} = \frac{1}{4\pi\rho}\int_r^R dx = \frac{R-r}{4\pi\rho}$$

$$\therefore \quad H = \frac{T_1 - T_2}{R_{th}} = \frac{4\pi\rho(T_1 - T_2)}{(R - r)}$$

5. (a) In all three cases it will be same.

(b)



6. Consider layer of ice that is x cm thick. Just below the ice temperature is 0°C and above it the temperature is $-\theta$°C. Let A be the area of the ice layer.

Heat conducted through ice in next dt time will be

$$dQ = \frac{kA\theta}{x}dt$$

If ice layer thickens by dx, the mass of ice frozen in time dt will be $= \rho A dx$

$$\therefore \quad dQ = (\rho A dx)L$$

$$\Rightarrow \quad \frac{kA\theta}{x}dt = \rho AL\, dx$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{k\theta}{x\rho L} = \frac{(0.005 \text{ cal cm}^{-1}\text{s}^{-1}\ {}^\circ\text{C}^{-1}) \times (10^\circ\text{C})}{(5 \text{ cm})(0.9\text{g cm}^{-3})(80 \text{ cal g}^{-1})} = 0.00014 \text{ cm s}^{-1}$$

$$= 0.00014 \times 3600 \text{ cm hr}^{-1} = 0.5 \text{ cm hr}^{-1}$$

7. $$-ms\frac{d\theta}{dt} = k(\theta - \theta_0)$$

Hence, $$30 \text{ watt} = k(50 - 20) \quad \text{... (i)}$$

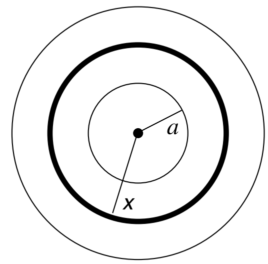
And $$m.s.\frac{0.1}{10} = k(40 - 20) \quad \text{....(ii)}$$

(ii) ÷ (i)

$$\frac{\frac{1}{4}s \times 0.01}{30} = \frac{20}{30}$$

$$s = 8000 \text{ J/Kg°C}$$

8. (i) Consider a layer (shown in dark) of radius x and thickness dx.



Resistance of this layer is $dR = \dfrac{dx}{K\cdot 2\pi Lx}$

∴ Resistance of entire pipe for radial heat flow is

$$R = \frac{1}{2\pi KL}\int_a^b \frac{dx}{x} = \frac{1}{2\pi KL}\,\ell n\left(\frac{b}{a}\right)$$

∴ $$\text{Heat current} = \frac{\Delta T}{R} = \frac{(T_1 - T_2)2\pi KL}{\ell n(b/a)}$$

(ii) In this case two resistances are in series

$$R = R_1 + R_2 = \frac{1}{2\pi K_1 L}\,\ell n\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi K_2 L}\,\ell n\left(\frac{r_3}{r_2}\right)$$

∴ $$\text{Heat current} = \frac{T_1 - T_2}{\dfrac{1}{2\pi K_1 L}\,\ell n\left(\dfrac{r_2}{r_1}\right) + \dfrac{1}{2\pi K_2 L}\,\ell n\left(\dfrac{r_3}{r_2}\right)}$$

9. (i) Rate of heat generation

$$H = VI = 10 \times 8 = 80 \text{ W}$$

Thermal resistance of the plastic cover is

$$R = \frac{\ell n(r_2/r_1)}{2\pi KL} = \frac{\ell n(3.5/1.5)}{2\pi \times 0.15 \times 5} = 0.18\text{°C/W}$$

In steady state, the heat produced in the copper wire is conducted by the plastic cover to the atmosphere.

∴ $$H = \frac{\Delta T}{R}$$

⇒ $$\Delta T = H.R = 80 \times 0.18 = 14.4\text{°C}$$

∴ Temperature at interface $= 30 + 14.4 = 44.4$°C

(ii) If thickness is increased, the resistance R increases.

∴ $\Delta T = HR$ will increase

⇒ Temperature at the interface will increase

10. $$\frac{dQ}{dt} = hA(T_1 - T)$$

$$ms\frac{dT}{dt} = hA(T_1 - T)$$

⇒ $$\int_{T_0}^{T_2} \frac{dT}{T_1 - T} = \frac{hA}{ms}\int_0^t dt$$

$$[\ell n(T_1 - T)]_{T_0}^{T_2} = -\frac{hAt}{ms}$$

$$\Rightarrow \quad \ell n\left(\frac{T_1 - T_2}{T_1 - T_0}\right) = -\frac{hAt}{ms}$$

Put $$T_2 = \frac{T_1 + T_0}{2}$$

$$\ell n\left(\frac{1}{2}\right) = -\frac{hAt}{ms} \Rightarrow \ell n 2 = \frac{hAt}{ms}$$

$$\Rightarrow \quad t = \frac{ms}{hA}\ell n 2$$

12. 0.5 E = energy reflected back
0.5 E = energy radiated
Total energy propagating from the surface of the body = E

13. Power supplied by battery in first case

$$P_1 = \frac{100^2}{R} \qquad [R = \text{ resistance of sphere}]$$

Power supplied after blackening of the half sphere $P_2 = \dfrac{141^2}{R} = \dfrac{(\sqrt{2} \times 100)^2}{R} = 2P_1$

In first case $\quad \sigma eA\,[500^4 - 300^4] = P_1 \qquad$ ...(i)

In second case $\quad \dfrac{\sigma eA}{2}[500^4 - 300^4] + \dfrac{\sigma A}{2}[500^4 - 300^4] = 2P_1 \qquad$ ...(ii)

Dividing (ii) by (i)

$$\frac{\frac{e}{2} + \frac{1}{2}}{e} = 2 \Rightarrow e = \frac{1}{3}$$

14. From dimensional analysis it can be shown that

$$\sigma = a\frac{k^4}{c^2h^3} \qquad [a = \text{constant}]$$

$$\therefore \quad \frac{\Delta\sigma}{\sigma} \simeq -2\frac{\Delta c}{c}$$

$$\therefore \quad \frac{\Delta\sigma}{\sigma} \times 100 = -2\frac{\Delta c}{c} \times 100 = -2 \times 2 = -4\%$$

15. $\lambda_m T = b$ = a constant

$$\therefore \quad \lambda_R\,(727 + 273) = \lambda_y T_y$$

[T_y = temperature when ball is yellow]

$$\therefore \quad T_y = \frac{8000}{6000} \times 1000 = \frac{4000}{3}\,K$$

$$\frac{A}{A_0} = \left(\frac{4000/3}{1000}\right)^4 \Rightarrow A = \left(\frac{4}{3}\right)^4 A_0$$

16. Let length of the cylinder be ℓ

$$2\pi r\ell + 2\cdot\pi r^2 = 4\pi r^2$$

$$\Rightarrow \quad \ell = r$$

$$\frac{m_{sp}}{m_{cy}} = \frac{V_{sp}}{V_{cy}} = \frac{\frac{4}{3}\pi r^3}{\pi r^2\cdot\ell} = \frac{4}{3}$$

T = temperature of sphere/cylinder

$$T_0 = \text{temperature of room}$$

$$-m_{sp} \cdot s \cdot \left(\frac{dT}{dt}\right)_{sp} = e\sigma A(T^4 - T_0^4)$$

$$-m_{cy} \cdot s \cdot \left(\frac{dT}{dt}\right)_{cy} = e\sigma A(T^4 - T_0^4)$$

$\therefore$
$$\frac{-(dT/dt)_{sp}}{-(dT/dt)_{cy}} = \frac{m_{cy}}{m_{sp}} = \frac{3}{4}$$

17. The maximum radiation is given out at wavelength λ_0 such that

$$\lambda_1 < \lambda_0 < \lambda_2$$

But
$$\lambda_0 = \frac{b}{T}$$

$\therefore$
$$\lambda_1 < \frac{b}{T} < \lambda_2$$

$\Rightarrow$
$$\frac{1}{\lambda_1} > \frac{T}{b} > \frac{1}{\lambda_2}$$

$\Rightarrow$
$$\frac{b}{\lambda_1} > T > \frac{b}{\lambda_2}$$

18. Temperature of the Earth << Temperature of the Sun.

$\therefore$ Earth will radiate predominantly in longer wavelength region

Almost all radiation from the earth has $\lambda > 4\,\mu m$.

19. Final constant temperature = 45°C

At 45°C, rate of heat loss = 24W

$$24\text{W} = k(45 - 15) \quad ...(i)$$

It rate of heat loss at 20°C is P then

$$P = k(20 - 15) \quad ...(ii)$$

(ii) ÷ (i)

$$\frac{P}{24} = \frac{5}{30} \Rightarrow P = 4 \text{ Watt.}$$

20. Let θ_0 = room temperature

θ_1 = initial temperature of the body

If θ = temperature of the body at time t then

$$-\frac{d\theta}{dt} = k(\theta - \theta_0)$$

$$\int_{\theta_1}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k\int_0^t dt$$

$$\ell n(\theta - \theta_0) - \ell n(\theta_1 - \theta_0) = -kt$$

$\therefore$
$$\frac{\theta - \theta_0}{\theta_1 - \theta_0} = e^{-kt}$$

$$\theta - \theta_0 = (\theta_1 - \theta_0)e^{-kt}$$

$\Rightarrow$
$$\Delta\theta = \Delta\theta_0 e^{-kt}$$

Where $\Delta\theta$ = temperature difference between the body and the surrounding

$$\Delta\theta_0 = \text{initial value of } \Delta\theta$$

As per question

$$\frac{1}{2}\Delta\theta_0 = \Delta\theta_0 e^{-kt_0} \quad \Rightarrow \quad e^{-kt_0} = \frac{1}{2}$$

When $\Delta\theta = \frac{1}{4}\Delta\theta_0 \Rightarrow \frac{1}{4} = e^{-kt}$

Obviously, $t = 2t_0$

21. (b) The average temperature of the cup between 5°C to 10°C is 7.5°C

The temperature difference with surrounding can be taken as 25°C – 7.5°C = 17.5°C

$$\frac{5°C}{4\text{min}} = k(17.5) \qquad \text{...(i) } k = a \text{ constant}$$

The average temperature of coffee cup between 10°C and 15°C is 12.5°C

$$\therefore \quad \frac{5°C}{t} = k(12.5) \qquad \text{...(ii)}$$

(i) ÷ (ii)

$$\frac{t}{4} = \frac{17.5}{12.5} \Rightarrow t = 5.6 \text{ min}$$

23. Power radiated by the sun $= \sigma 4\pi R^2 T_s^4$

Intensity of the sun light at a distance r from the sun is

$$I = \frac{\sigma 4\pi R^2 T_s^4}{4\pi r^2} = \sigma T_s^4 \frac{R^2}{r^2}$$

Power received by the planet $= \pi r_0^2 \cdot \sigma T_s^4 \frac{R^2}{r^2}$

If T_0 = temperature of the planet then power radiated by the planet at T_0 must be equal to power received.

$$\therefore \quad \sigma 4\pi r_0^2 T_0^4 = \pi r_0^2 \sigma T_s^4 \frac{R^2}{r^2}$$

$$\therefore \quad T_0 = \left[\frac{R^2}{4r^2} T_s^4\right]^{\frac{1}{4}}$$

∴ Temperature of the planet does not depend on its radius

If distance r is doubled, the temperature will become $\frac{1}{\sqrt{2}}$ times.

24. $$P_{\text{star}} = \sigma A T_s^4 = \sigma 4\pi R^2 T_s^4$$

Intensity of radiation at the surface of the planet is

$$I = \frac{P_{\text{star}}}{4\pi d^2} = \frac{\sigma R^2 T_s^4}{d^2}$$

Energy incident (= energy absorbed) on the planet per unit time

$$P_{\text{planet}} = I\pi r^2 \qquad [r = \text{radius of planet}]$$

$$= \frac{\pi\sigma R^2 T_s^4}{d^2} r^2$$

In thermal equilibrium the planet will radiate same amount of energy per unit time

$$\therefore \quad \sigma 4\pi r^2 T_p^4 = \frac{\pi\sigma R^2 T_s^4 r^2}{d^2}$$

$$\therefore \quad \left(\frac{T_p}{T_s}\right)^4 = \frac{R^2}{4d^2} \qquad \therefore \quad d = \frac{R}{2}\left(\frac{T_s}{T_p}\right)^2$$

$$d = \frac{R}{2}\left(\frac{\lambda_p}{\lambda_s}\right)^2 \qquad [\because \; T\lambda = b = \text{a constant}]$$

(ii) Since $d >> R$

$\therefore \qquad \lambda_p >> \lambda_s$

25. $q_{\text{conv}} = h(T - T_0) = 6(80 - 20) = 360 \text{ Wm}^{-2}$

For 1 m length of the pipe

$$Q_{\text{conv}} = q_{\text{conv}} A = q_{\text{conv}} \times 2\pi r$$
$$= 360 \times 2 \times 3.14 \times 0.01 = 22.6 \text{ Wm}^{-1}$$
$$q_{\text{rad}} = \sigma(T^4 - T_0^4) = 5.67 \times 10^{-8}(353^4 - 293^4) = 462 \text{ Wm}^{-2}$$

For 1m length of the pipe

$$Q_{\text{rad}} = q_{\text{rad}} A = 462 \times 2 \times 3.14 \times 0.01 = 29.1 \text{ Wm}^{-1}$$

$\therefore \qquad Q_{\text{conv}} + Q_{\text{rad}} = 22.6 + 29.1 = 51.7 \text{ Wm}^{-1}$

26. (a) Amount of heat radiated by the Earth = solar flux from the sun

$$e\sigma 4\pi R_e^2 T_e^4 = 0.68 I_s \cdot \pi R_e^2$$

$$e = \frac{0.68 I_s}{4\sigma T_e^4} = \frac{0.68 \times 1.4 \times 10^3 \text{ W/m}^2}{4\left(5.67 \times 10^{-8} \dfrac{W}{m^2K^4}\right)(290K)^4} \simeq 0.6$$

(b) $P_{\text{sun}} = 4\pi r^2 I_s$

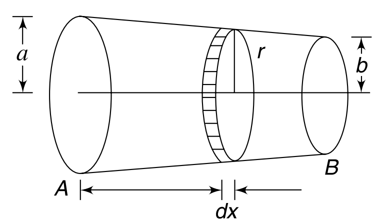
$$= 4\pi(1.5 \times 10^{11}\text{m})^2 \cdot (1.4 \times 10^3 \text{ W/m}^2) = 3.96 \times 10^{26} \text{ W}$$

(c) $P_{\text{sun}} = \sigma 4\pi R_s^2 T_s^4$

$$\Rightarrow \qquad T_s^4 = \frac{3.96 \times 10^{26} \text{ W}}{\left(5.67 \times 10^{-8} \dfrac{W}{m^2K^4}\right)4\pi(7 \times 10^8\text{m})^2} = 1.13 \times 10^{15}$$

$$T_s \simeq 0.58 \times 10^4 = 5800 \text{ K}$$

27. (i) Heat current $\quad H = kA\left(\dfrac{dT}{dx}\right)$



Heat current is same at every cross section.

Since $A_1 > A_2$

Hence $\quad \left(\dfrac{dT}{dx}\right)_1 < \left(\dfrac{dT}{dx}\right)_2$

(ii) Radius at a distance x from the end A is

$$r = a - \left(\frac{a-b}{L}\right)x$$

$\therefore$ Area of cross section $\quad A = \pi r^2 = \pi\left[a - \left(\dfrac{a-b}{L}\right)x\right]^2$

Resistance of a disc shaped element will be

$$dR = \frac{dx}{kA} = \frac{dx}{k\pi\left[a - \left(\dfrac{a-b}{L}\right)x\right]^2}$$

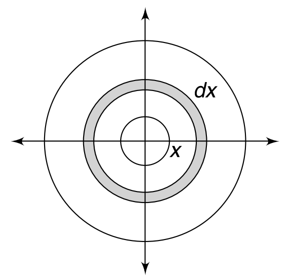
$\therefore$ Resistance of the rod is

$$R = \int dR = \frac{1}{k\pi}\int_0^L \frac{dx}{\left[a - \left(\frac{a-b}{L}\right)x\right]^2}$$

$$= \frac{L}{k\pi(a-b)}\left[\frac{1}{a - \left(\frac{a-b}{L}\right)x}\right]_0^L = \frac{L}{k\pi ab}$$

28. In first case $H = \frac{k\pi(R^2 - r^2)\Delta T}{L}$ …(i)

In case of radial heat how the thermal resistance can be calculate as



$$dR_{th} = \frac{dx}{k \cdot 2\pi x \cdot L}$$

$$R_{th} = \frac{1}{2\pi Lk}\int_r^R \frac{dx}{x} = \frac{1}{2\pi Lk}\,\ell n\left(\frac{R}{r}\right)$$

$\therefore$ $$H' = \frac{\Delta T}{R_{th}} = \frac{2\pi Lk\Delta T}{\ell n\left(\frac{R}{r}\right)} \quad \text{...(i)}$$

$$\frac{H'}{H} = \frac{2\pi Lk\Delta T}{\ell n\,\frac{R}{r}} \cdot \frac{L}{k \cdot \pi(R^2 - r^2)\Delta T}$$

$$H' = \frac{2L^2 H}{(R^2 - r^2)\ell n\left(\frac{R}{r}\right)}$$

29. Let T_2 and T_3 be the temperatures of the glass 1-air interface and air-glass 2 interface respectively

The equations of heat flow are $\frac{dQ_1}{dt} = \frac{dQ_2}{dt} = \frac{dQ_3}{dt}$

$$\Rightarrow \quad \frac{dQ}{dt} = \frac{K_g A(T_1 - T_2)}{d_g} = \frac{K_a A(T_2 - T_3)}{d_a} = \frac{K_g A(T_3 - T_4)}{d_g} \quad \text{...(A)}$$

$$\Rightarrow \quad \frac{dQ}{dt} = \frac{K_g A(300 - T_2)}{d_g} = \frac{K_a A(T_2 - T_3)}{d_a} = \frac{K_g A(T_3 - 273)}{d_g}$$

$$\Rightarrow \quad 50(300 - T_2) = T_2 - T_3 \quad \text{...(i)}$$

And $$300 - T_2 = T_3 - 273 \quad \text{...(ii)}$$

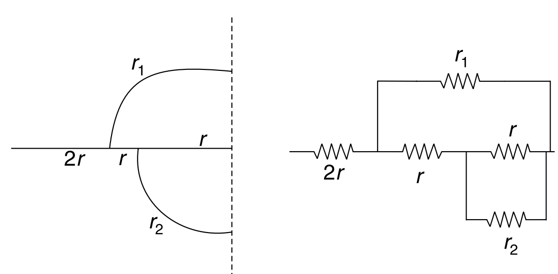
Solve (i) and (ii) to get T_2 and T_3. Put this in (A) to get $\frac{dQ}{dt}$

30. Thermal resistance of length R of the rod is $r = \frac{R}{kA}$ [k = thermal conductivity]

With only rod PQ, the thermal resistance is $4r$. Heat current $H_1 = \frac{\Delta T}{4r}$

$$t_1 \propto \frac{1}{H_1} \quad \Rightarrow \quad t_1 \propto (4r) \quad \text{…(i)}$$

With circular parts added the equivalent thermal resistance will be obtained by series–parallel combination shown in figure.



Where $$r_1 = \frac{\pi R}{kA} = \pi r \simeq 3r$$

$$r_2 = \frac{\pi R/2}{kA} = \frac{\pi}{2}\,r \simeq \frac{3}{2}\,r$$

r in parallel to r_2 gives $$\frac{rr_2}{r+r_2} = \frac{\frac{3}{2}r}{1+\frac{3}{2}} = \frac{3r}{5}$$

$\frac{3r}{5}$ and r are in series which gives $\frac{8r}{5}$

$\frac{8r}{5}$ is in parallel to r_1, which gives $\dfrac{\frac{8r}{5}\times 3r}{\frac{8r}{5}+3r} = \frac{24r}{23}$

$\frac{24r}{23}$ and $2r$ are in series; which gives $\frac{70r}{23}$

$$\therefore \quad H_2 = \frac{\Delta T}{\frac{70r}{23}} \Rightarrow H_2 \propto \frac{23}{70r}$$

$$\therefore \quad t_2 \propto \frac{70r}{23} \qquad \text{...(ii)}$$

From (i) and (ii)

$$\frac{t_2}{t_1} = \frac{70}{23\times 4} \Rightarrow t_2 = \frac{70}{4}\text{ min}$$

31. Heat is transferred from He to H.

Let temperature of the two containers be θ_1 and θ_2 at time t.

Temperature difference is $\theta = \theta_1 - \theta_2$

In time dt change in temperature difference is

$$d\theta = d\theta_1 - d\theta_2 \qquad \text{...(i)}$$

Heat transferred in time dt is

$$dQ = \frac{kA(\theta_1-\theta_2)}{L}dt = \frac{kA\theta\cdot dt}{L}$$

For helium dQ is negative and for hydrogen it is positive

$$\therefore \quad n(C_v)_{\text{He}}\, d\theta_1 = -dQ$$

$$d\theta_1 = -\frac{2}{3Rn}dQ$$

Similarly for hydrogen $$d\theta_2 = \frac{2dQ}{5R\cdot n}$$

From (i) $$d\theta = \left(-\frac{2}{3Rn} - \frac{2}{5Rn}\right)dQ$$

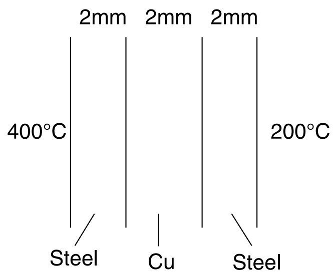
$$d\theta = -\frac{16}{15nR}\cdot\frac{kA\theta}{L}dt$$

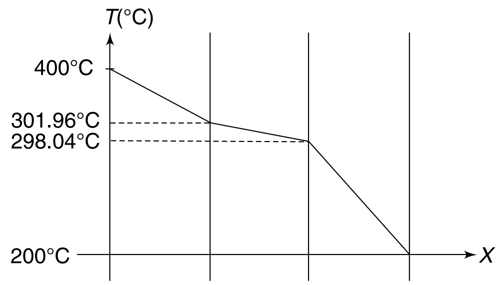
$$\therefore \quad \int_0^t dt = -\frac{15nRL}{16kA}\int_{T_1-T_2}^{\frac{T_1-T_2}{2}}\frac{d\theta}{\theta}$$

$$t = -\frac{15nRL}{16kA}\ln\frac{1}{2} = \frac{15nRL}{16kA}\ln 2$$

32. In steady state heat current is same at any cross section.

$$\therefore \quad \frac{dQ}{dt} = \frac{Ak_s\Delta T_s}{2\text{ mm}} = \frac{Ak_{\text{cu}}\Delta T_{\text{cu}}}{2\text{ mm}}$$





$$\therefore \quad \Delta T_s = \frac{k_{cu}}{k_s}\Delta T_{cu} \Rightarrow \Delta T_s = \frac{400}{16}\Delta T_{cu}$$

$$\Rightarrow \quad \Delta T_s = 25\Delta T_{cu}$$

Given, $2\Delta T_s + \Delta T_{cu} = 200°C \quad \therefore \quad 51\Delta T_{cu} = 200°C$

$$\Rightarrow \quad \Delta T_{cu} = 3.92°C$$

$$\Delta T_s = \frac{200 - 3.92}{2} = 98.04°C$$

33. Let the upper surface of the steel plate be at temperature t°C

$$\frac{dQ}{dt} = \frac{kA(100 - t)}{L}$$

$$50 = \frac{16 \times 0.03 \times (100 - t)}{0.012}$$

$$\therefore \quad t = 98.75°C$$

If heat loss due to convection is H J/s,

$$H + \sigma A(T^4 - T_0^4) = 50$$

$$H + 5.67 \times 10^{-8} \times 0.03[(371.75)^4 - (293)^4] = 50$$

$$H = 30.05 \text{ watt}$$

34. In equilibrium

Electrical power dissipated in the wire = Heat radiated to the surrounding

$$\therefore \quad \frac{V^2}{R} = e\sigma A(T^4 - T_0^4)$$

V = emf of the battery

R = resistance of the wire

A = Area of curved surface of the wire

For two situations the above equation can be written as

$$\frac{V^2}{R} = e\sigma A(T_1^4 - T_0^4)$$

And $$\frac{V^2}{R/2} = e\sigma \frac{A}{2}(T_2^4 - T_0^4)$$

Taking ratio $$2 = \frac{1}{2}\left(\frac{T_2^4 - T_0^4}{T_1^4 - T_0^4}\right)$$

$$\Rightarrow \quad T_2 = \left[4T_1^4 - 3T_0^4\right]^{\frac{1}{4}}$$

35. Let the temperature of the shell after the source is switched on be T.

$$T = T_0 + \Delta T$$

Power of the source = net power radiated by the outer surface of the shell

$$0.4\sigma Se\, T_0^4 = e\sigma S(T^4 - T_0^4)$$

$$\Rightarrow \quad 0.4T_0^4 = (T_0 + \Delta T)^4 - T_0^4$$

$$\Rightarrow \quad 0.4T_0^4 = T_0^4\left[\left(1 + \left(\frac{\Delta T}{T_0}\right)\right)^4 - 1\right]$$

$$\Rightarrow \quad 0.4 \simeq 1 + 4\frac{\Delta T}{T_0} - 1 \quad \Rightarrow \quad \frac{\Delta T}{T_0} = \frac{1}{10}$$

$$\Rightarrow \quad T - T_0 = \frac{T_0}{10} \quad \Rightarrow \quad T = \frac{11}{10}T_0$$

Wien's law gives- $\lambda T = \lambda_0 T_0$

$$\lambda = \lambda_0\frac{T_0}{T} = \lambda_0\left(\frac{10}{11}\right)$$

36 Rate of heat loss by a body maintained at temperature T when placed in a room at T_0 is

$$\frac{dQ}{dt} = e\sigma A(T^4 - T_0^4)$$

If temperature difference is small we can write $T = T_0 + \Delta T$

Where $\Delta T << T_0$

$$\therefore \quad T^4 = (T_0 + \Delta T)^4 = T_0^4\left(1 + \frac{\Delta T}{T_0}\right)^4 \simeq T_0^4\left(1 + \frac{4\Delta T}{T_0}\right)$$

$$\therefore \quad \frac{dQ}{dt} \simeq e\sigma A T_0^4\left[1 - \left(1 + \frac{4\Delta T}{T_0}\right)\right] = 4e\sigma A T_0^3\,\Delta T$$

$$\Rightarrow \quad -ms\frac{dT}{dt} = 4e\sigma A T_0^3\,\Delta T$$

$$\Rightarrow \quad -\frac{dT}{dt} = \left(\frac{4e\sigma A T_0^3}{ms}\right)\Delta T$$

$$\Rightarrow \quad -\frac{dT}{dt} = k\Delta T$$

This is Newton's law of cooling. We need to take care that the constant k depends on T_0^3. When room temperature is -3°C and 24°C let the value of constant be k_1 and k_2. Then $\frac{k_2}{k_1} = \left(\frac{297}{270}\right)^3 = (1.1)^3 = 1.33$

$\therefore$ When ball is in first room

$$\frac{1°\text{C}}{\Delta t} = k_1[49.5 - (-3)] \qquad \text{...(i)}$$

[49.5 = average temperature of the body during cooling]

In second room

$$\frac{1°\text{C}}{\Delta t'} = k_2[49.5 - 24] \qquad \text{...(ii)}$$

$\therefore$ (i) ÷ (ii)

$$\frac{\Delta t'}{\Delta t} = \frac{k_1}{k_2} \times \frac{52.5}{25.5}$$

$$\therefore \quad \Delta t' = \frac{1}{1.33} \times \frac{52.5}{25.5}\Delta t = 1.55\Delta t$$

37 (i) When place C and D are not present

$$r_1 = \sigma A(T_1^4 - T_2^4) = 6 \times 10^{-8} \times 1[10^{12} - 3^4 \times 10^8]$$
$$= 6 \times [10000 - 81] = 59514 \text{ Watt}$$

(ii) Let temperature of C and D be T_3 and T_4 respectively in the steady state.
Rate of heat gain by C = rate of heat loss by C

$$\therefore \quad \sigma A(T_1^4 - T_3^4) = \sigma A(T_3^4 - T_4^4)$$
$$\Rightarrow \quad T_1^4 + T_4^4 = 2T_3^4 \quad \text{...(i)}$$

Similarly for plate D

$$\sigma A(T_3^4 - T_4^4) = \sigma A(T_4^4 - T_2^4)$$
$$\therefore \quad T_3^4 + T_2^2 = 2T_2^4 \quad \text{....(ii)}$$

Solving (i) and (ii)

$$T_3^4 = \frac{1}{3}(2T_1^4 + T_2^4)$$
$$\therefore \quad r_2 = \sigma A(T_1^4 - T_3^4) = -\sigma A\left[T_1^4 - \frac{2}{3}T_1^4 - \frac{1}{3}T_2^4\right]$$
$$= \frac{1}{3}\sigma A\left[T_1^4 - T_2^4\right] = \frac{r_1}{3}$$

(iii) $\frac{r_2}{r_1}$ does not depend on values of T_1 and T_2

$\therefore$ Ratio will remain unchanged

38. (b) Initially, block is at room temperature

$$\therefore \quad P_{\text{loss to surrounding}} = 0$$
$$P_{\text{Heater}} = P_{\text{used to increase int. energy}} + P_{\text{loss to surrounding}}$$

At $t = 0$,

$$P_{\text{Heater}} = C\frac{dT}{dt} \qquad [C = \text{heat capacity}]$$
$$600 \text{ W} = C \times 3\frac{°\text{C}}{s}$$
$$\therefore \quad C = 200 \text{ J°C}^{-1}$$

(b) At 60°C

$$P_{\text{loss to surrounding}} = 100 \text{ W}$$

After heater is switched off

$$C\frac{dT}{dt} = -100 \text{ W}$$
$$\frac{dT}{dt} = -0.5\text{°Cs}^{-1}$$

(c) At 60°C, the block is losing energy at a rate of 100 W

$$\therefore \quad k(60 - 20) = 100 \quad \text{...(i)}$$
$$\text{At} \quad 30°\text{C}; \; k(30 - 20) = P_{30°\text{C}} \quad \text{...(ii)}$$

(ii) ÷ (i)

$$P_{30°\text{C}} = 25 \text{ Watt}$$

39. For cooling of liquid A:

$$\frac{\Delta\theta}{\Delta t} = \frac{k}{s_A}(\theta_{av} - \theta_0)$$

$$\frac{(90-30)^\circ\text{C}}{5\text{ min}} = \frac{k}{s_A}\left(\frac{90+30}{2} - 20\right)$$

$$12 = \frac{k}{s_A}(40)$$

$$\Rightarrow \qquad s_A = \frac{10k}{3} \qquad \text{...(i)}$$

For cooling of liquid B we have

$$\frac{50-30}{5} = \frac{k}{s_B}\left(\frac{50+30}{2} - 20\right)$$

$$4 = \frac{k}{s_B}(20)$$

$$\therefore \qquad s_B = 5k \qquad \text{...(ii)}$$

When the two liquids are mixed

Heat lost by A = Heat gained by B

$$m\cdot s_A(90-\theta) = m\cdot s_B(\theta - 50)$$

$$\frac{10k}{3}(90-\theta) = 5k(\theta - 50)$$

$$180 - 2\theta = 3\theta - 150$$

$$\Rightarrow \qquad 5\theta = 330 \quad \Rightarrow \quad \theta = 66^\circ\text{C}$$

Note: Same density of two liquids means that they will occupy same volume in the identical cups. Therefore, the surface area through which the heat leaks is same for both.

40. (a) Power radiated by the Sun is

$$P_S = 4\pi r_S^2 \cdot T_S^4$$

At the Earth, this energy is distributed over sphere of radius R. Energy passing through each square meter of the sphere is given by

$$\frac{P_S}{4\pi R^2} = \frac{r_S^2}{R^2}T_S^4$$

Cross section of the earth is πr_e^2. Amount of solar power absorbed by the earth is

$$P_{\text{ab}} = \pi r_E^2 \cdot \frac{r_S^2}{R^2}\cdot T_S^4 \times 0.7$$

Amount of energy radiated by earth per unit time is $= 4\pi r_E^2 \cdot \sigma T_E^4$

For steady state

$$4\pi r_E^2 \sigma T_E^4 = \pi r_E^2 \frac{r_S^2}{R^2}\cdot T_S^4 \times 0.7$$

$$T_E^4 = 0.7\frac{r_S^2 T_S^4}{R^2}$$

$$T_E = (0.7)^{\frac{1}{4}} \times (5780\text{ K})\sqrt{\frac{(6.96\times 10^8)^2}{2\times 1.49\times 10^{11}}} \simeq 254\text{ K}$$

(b) The statement is true.

41. $$ms\frac{dT}{dt} = -e\sigma A(T^4 - T_0^4)$$

(a) $$m = \rho\cdot\frac{4}{3}\pi r^3 \text{ and } A = 4\pi r^2$$

$$\therefore \quad \frac{dT}{dt} = -\frac{3e\sigma}{\rho s r}(T^4 - T_0^4)$$

$$\therefore \quad \frac{\left(\frac{dT}{dt}\right)_A}{\left(\frac{dT}{dt}\right)_B} = \frac{r_B}{r_A} = \frac{1}{2}$$

(b) $m = \rho 4\pi r^2 \Delta t$ and $A = 4\pi r^2$

[Δt = thickness of the wall]

$$\frac{dT}{dt} = \frac{-e\sigma(T^4 - T_0^4)}{\rho s \Delta t}$$

This is independent of r.

$\therefore$ Rate of cooling is same for both the sphere.

42. (i) Heat loss due to conduction is

$$\frac{dQ}{dt} = k\cdot(4\pi R^2)\frac{dT}{dR}$$

$$= 0.75 \times 4 \times 3.14 \times (6.4 \times 10^6)^2 \times \frac{1}{30} \qquad \left[\because \frac{dT}{dR} = 1°\text{C}/30\text{ m}\right]$$

$$= 1.29 \times 10^{13} \text{ Watt}$$

(ii) Power received from the Sun is

$$P_{\text{sun}} = (1.35 \times 10^3) \times \pi R^2$$

$$= 1.35 \times 10^3 \times 3.14 \times (6.4 \times 10^6)^2 \simeq 1.7 \times 10^{17} \text{ W}$$

Heat received from the sun is much larger than heat lost due to conduction.

If equilibrium temperature of the earth is T_0, then it must radiate at a rate = P_{sun} at this temperature.

$$\therefore \quad \sigma A T^4 = 1.7 \times 10^{17}$$

$$5.67 \times 10^{-8} \times 4 \times 3.14 \times (6.4 \times 10^6)^2\, T^4 = 1.7 \times 10^{17}$$

$$\therefore \quad T^4 = 58 \times 10^8$$

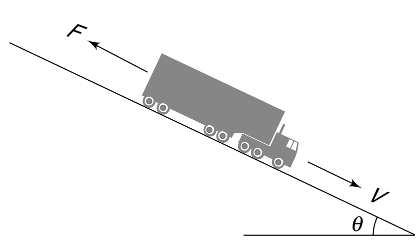
$$\therefore \quad T = (58)^{1/4} \times 10^2 \simeq (7.6)^{1/2} \times 100 \simeq 2.76 \times 100 \simeq 276 \text{ K}$$

43. (a) As per the question the ground friction force acting on wheels are non dissipative. You may assume that wheels are rolling without sliding.

Since, truck is moving with constant speed, power produced by gravitational force is equal to rate of energy dissipation due to air resistance and braking.

$(Mg\sin\theta)v = F\cdot v$ + Rate of heat dissipation in brakes

$\therefore$ Rate of heat dissipation in brakes = $[(Mg\sin\theta) - F]v$



$$\therefore \quad 4m\cdot s\frac{dT}{dt} = (Mg\sin\theta - F)v$$

$$\therefore \quad \frac{dT}{dt} = \frac{(Mg\sin\theta - F)v}{4\, m\cdot s}$$

(b) Let final temperature be T

Rate of heat loss from each disc due to radiation = $\sigma A(T^4 - T_0^4)$
This will be equal to rate of heat production in each brake

$$\therefore \quad \sigma A(T^4 - T_0^4) = [Mg\sin\theta - F]v$$

$$\Rightarrow \quad T^4 - T_0^4 = \frac{(Mg\sin\theta - F)v}{\sigma A}$$

$$\Rightarrow \quad T = \left[\frac{(Mg\sin\theta - F)v}{\sigma A} + T_0^4\right]^{1/4}$$

44. For interval 0 to t_0, rate of heat loss by body is

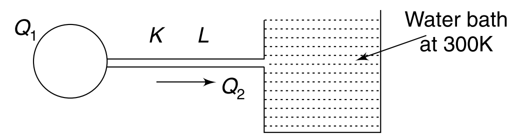
$$-ms\frac{dT}{dt} = b(T - T_0) \qquad \{b = \text{a constant}\}$$

$$\Rightarrow \quad \int_{T_1=400\text{k}}^{T_2=350\text{k}} \frac{dT}{T - T_0} = -\frac{b}{ms}\int_{t=0}^{t_0} dt$$

$$\Rightarrow \quad \ln = (350 - 300) - \ln(400 - 300) = -\frac{bt_0}{ms}$$

$$\Rightarrow \quad \ln 2 = \frac{bt_0}{ms} \qquad \text{...(i)}$$

During interval t_0 to $2t_0$, body loses heat by radiation as well as conduction



$$-ms\frac{dT}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt}$$

$$-ms\frac{dT}{dt} = b(T - 300) + \frac{kA(T - 300)}{L}$$

$$\int \frac{dT}{T - 300} = -\int\frac{bdt}{ms} - \int\frac{kAdt}{msL}$$

From (i) $\frac{b}{ms} = \frac{\ln 2}{t_0}$

$$\int_{350}^{T} \frac{dT}{T - 300} = -\left(\frac{\ln 2}{t_0} + \frac{kA}{msL}\right)\int_{t_0}^{2t_0} dt$$

$$\ln[T - 300] - \ln 50 = -\left(\frac{\ln 2}{t_0} + \frac{kA}{msL}\right)t_0$$

$$\frac{T - 300}{50} = e^{-\left[\frac{\ln 2}{t_0} + \frac{kA}{msL}\right]t_0}$$

$$\Rightarrow \quad T = 300 + 50\, e^{-\left[\frac{\ln 2}{t_0} + \frac{kA}{msL}\right]t_0}$$

45. In steady state heat conducted from hot end to the exposed end of the rod is being lost as radiation to the surrounding

$$\frac{\Delta Q}{\Delta t} = \frac{kA(T_1 - T_2)}{L} = eA\sigma\left[T_2^4 - T_0^4\right]$$

$$\Rightarrow \quad T_1 - T_2 = \frac{e\sigma L}{k}\left[(T_0 + \Delta T_2)^4 - T_0^4\right]$$

$$\Rightarrow \quad (T_0 + \Delta T_1) - (T_0 + \Delta T_2) = \frac{e\sigma L}{k} T_0^4\left[\left(1 + \frac{\Delta T_2}{T_0}\right)^4 - 1\right]$$

$$\Rightarrow \qquad \Delta T_1 - \Delta T_2 \simeq \frac{e\sigma L}{k} T_0^4 \left[1 + \frac{4\Delta T_2}{T_0} - 1\right]$$

$\because \quad \left(1 + \frac{\Delta T_2}{T_0}\right)^4 \simeq 1 + 4\frac{\Delta T_2}{T_0}$ as all other terms will be very small and can be neglected.

$$\Rightarrow \qquad \Delta T_1 = \left[\frac{4e\sigma L}{k} T_0^3 + 1\right]\Delta T_2$$