HERON'S FORMULA

CONTENTS

➢ SOME BASIC TERMS

❖ **Area :**

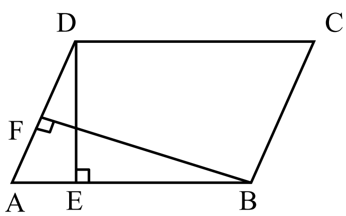
Place, which is covered by base of a body is called area of that body. Area is same from everywhere of the base.

❖ **Height :**

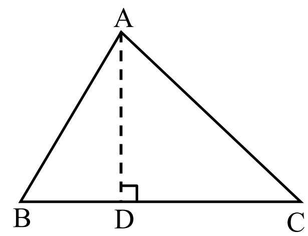
The perpendicular distance is called height and the side having foot of perpendicular, is called base.

❖ EXAMPLES ❖

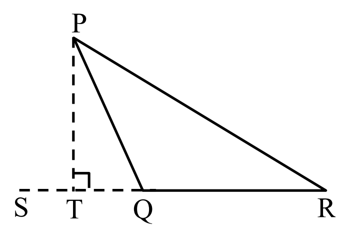
Ex.1 ABCD is a parallelogram. If DE & BF are perpendiculars from D and B on sides AB & DA respectively then their bases are AB and AD respectively.



Ex.2 ABC is an acute angle triangle. AD is height and BC is base



Ex.3 PQR is an obtuse angle triangle at Q. Then height of P from BC is PT but base is QR (not SR).



❖ **Area of triangle =** $\frac{1}{2}$ (base × height) square unit

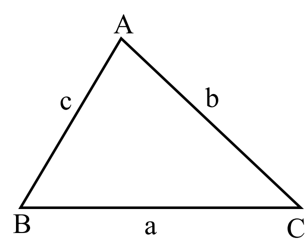
we can use this formula when we can find or given height & base.

❖ **Heron's formula :** If we have all sides of triangle and their is no way to find height then we use this formula for area of triangle.

$$\boxed{\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}}$$

Where s is semi perimeter of Δ.

$$s = \frac{1}{2} \text{ (sum of all sides)} = \frac{1}{2}(a + b + c)$$

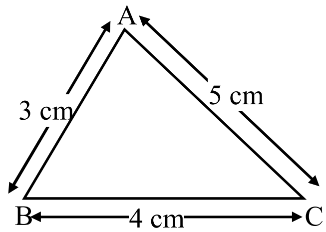


and a, b, c are sides of Δ.

Note : Use $\frac{1}{2}$ (base) (height) for area of right angle triangle, if any two sides are given.

❖ EXAMPLES ❖

Ex.4 For given figure find the s (s – a).



Sol. Perimeter = 2s = 3 + 4 + 5 = 12 cm

$\therefore$ semi perimeter = $\frac{12}{2}$ = 6 cm

$\therefore$ s (s – a) = 6 (6 – 4)

= 6 × 2

= 12 cm.

Ex.5 If semiperimeter of a triangle is 60 cm & its two sides are 45 cm, 40 cm then find third side.

Sol. Θ Semiperimeter = 60

$\therefore$ Perimeter = 2 × 60

$\Rightarrow$ Sum of all three sides = 120

(Let third side = x cm)

$\Rightarrow$ x + 45 + 40 = 120

$\Rightarrow$ x + 85 = 120

$\Rightarrow$ x = 120 – 85

$\Rightarrow$ x = 35 cm.

Ex.6 If perimeter of an equilateral triangle is 96 cm, then find its each side.

Sol. Θ Length of all sides are equal in equilateral Δ

Let length is x cm

$\therefore$ x + x + x = 96

$\Rightarrow$ 3x = 96

$\Rightarrow$ x = 32 cm.

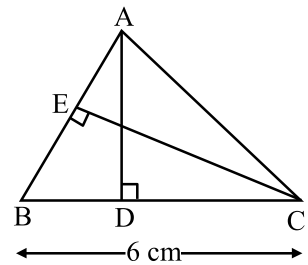
Ex.7 If one side from two equal sides of a Δ is 14 cm and semiperimeter is 22.5 cm then find the third side.

Sol. Let the third side is x cm

$\therefore$ x + 14 + 14 = 2 × 22.5

$\Rightarrow$ x = 45 – 28 = 17 cm.

Ex.8 Find the length of AD in given figure, if EC = 4 cm and AB = 5 cm.



Sol. Θ area of ΔABC = $\frac{1}{2}$(AB × EC)

= $\frac{1}{2}$ (5 × 4) = 10 square cm.

also area of ΔABC = $\frac{1}{2}$(BC × AD)

= $\frac{1}{2}$(6 × AD) = 3AD square cm

$\therefore$ 3AD = 10

$\Rightarrow$ AD = $\frac{10}{3} = 3.33$ cm.

Note : $\sqrt{2}$ = 1.41, $\sqrt{3}$ = 1.73, $\sqrt{5}$ = 2.23, $\sqrt{6}$ = 2.45, $\sqrt{7}$ = 2.64, $\sqrt{8}$ = 2.82, $\sqrt{11}$ = 3.31, $\sqrt{15}$ = 3.87

Ex.9 Find the area of a triangle whose sides are of lengths 52 cm, 56 cm and 60 cm respectively.

Sol. Let a = 52 cm, b = 56 cm and c = 60 cm.

Perimeter of the triangle = (a + b + c) units

= (52 + 56 + 60) cm = 168 cm

$\therefore$ $s = \frac{1}{2}(a + b + c) = \left(\frac{1}{2} \times 168\right)$cm = 84 cm

(s – a) = (84 – 52) cm = 32 cm,

(s – b) = (84 – 56) cm = 28 cm

and (s – c) = (84 – 60) cm = 24 cm

By Heron's formula, the area of the given triangle is

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{84 \times 32 \times 28 \times 24}\text{cm}^2$

$= \sqrt{14 \times 6 \times 16 \times 2 \times 14 \times 2 \times 6 \times 4}\ \text{cm}^2$

= (14 × 6 × 4 × 2 × 2) cm^2 = 1344 cm^2.

Ex.10 Using Heron's formula, find the area of an equilateral triangle of side a units.

Sol. We have : $s = \frac{1}{2}(a + a + a) = \frac{3a}{2}$

$\therefore\ (s - a) = \left(\frac{3a}{2} - a\right) = \frac{a}{2}$,

$(s - b) = \left(\frac{3a}{2} - a\right) = \frac{a}{2}$

and $(s - c) = \left(\frac{3a}{2} - a\right) = \frac{a}{2}$

So, by Heron's formula, we have :

area = $\sqrt{s(s-a)(s-b)(s-c)}$ sq units

$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$ sq units $= \left(\frac{\sqrt{3}a^2}{4}\right)$ sq units

Hence, area of equilateral triangle of side a is $\left(\frac{\sqrt{3}a^2}{4}\right)$ sq units.

Note : $\Delta = \left(\frac{\sqrt{3}}{4} \times a^2\right) = \left(\frac{1}{2} \times a \times \frac{\sqrt{3}a}{2}\right)$

$= \left(\frac{1}{2} \times \text{base} \times \text{height}\right)$

$\therefore$ height $= \frac{\sqrt{3}a}{2}$ units

Ex.11 Find the area of an isosceles triangle each of whose equal sides is 13 cm and whose base is 24 cm.

Sol. Here, a = 13 cm, b = 13 cm and c = 24 cm.

$\therefore\ s = \frac{1}{2}(a + b + c) = \frac{1}{2}(13 + 13 + 24)$ cm = 25 cm.

$(s - a) = (25 - 13)$ cm = 12 cm,

$(s - b) = (25 - 13)$ cm = 12 cm

and $(s - c) = (25 - 24)$ cm = 1 cm.

So, by Heron's formula,

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$= \sqrt{25 \times 12 \times 12 \times 1}\ cm^2 = (5 \times 12) cm^2 = 60\ cm^2$.

Hence, the area of the given triangle is 60 cm^2.

Ex.12 The perimeter of a triangular field is 450 m and its sides are in the ratio 13 : 12 : 5. Find the area of the triangle.

Sol. a : b : c = 13 : 12 : 5 $\Rightarrow$ a = 13x, b = 12x & c = 5x

$\therefore$ Perimeter = 450 $\Rightarrow$ 13x + 12x + 5x = 450

$\Rightarrow$ 30x = 450 $\Rightarrow$ x = 15.

So, the sides of the triangle are

a = 13 × 15 = 195 m, b = 12 × 15 = 180 m

and c = 5 × 15 = 75 m

It is given that perimeter = 450

$\Rightarrow$ 2s = 450

$\Rightarrow$ s = 225

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{225(225-195)(225-180)(225-75)}$

$\Rightarrow$ Area $= \sqrt{225 \times 30 \times 45 \times 150}$

$= \sqrt{5^2 \times 3^2 \times 3 \times 5 \times 2 \times 3^2 \times 5 \times 5^2 \times 2 \times 3}$

$\Rightarrow$ Area $= \sqrt{5^6 \times 3^6 \times 2^2} = 5^3 \times 3^3 \times 2 = 6750\ m^2$.

Ex.13 Find the percentage increase in the area of a triangle if its each side is doubled.

Sol. Let a, b, c be the sides of the old triangle and s be its semi-perimeter. Then,

$s = \frac{1}{2}(a + b + c)$

The sides of the new triangle are 2a, 2b and 2c. Let s' be its semi-perimeter. Then,

$s' = \frac{1}{2} \times (2a + 2b + 2c)$

$= a + b + c = 2s$

Let Δ and Δ' be the areas of the old and new triangles respectively. Then,

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ and

$\Delta' = \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$

$\Rightarrow \Delta' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

[$\because$ s' = 2s]

$\Rightarrow \Delta' = 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$

$\therefore$ Increase in the area of the triangle

$= \Delta' - \Delta = 4\Delta - \Delta = 3\Delta$

Hence, percentage increase in area

$= \left(\frac{3\Delta}{\Delta} \times 100\right) = 300\%$

Ex.14 The lengths of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find (i) the area of the triangle and (ii) the height corresponding to the longest side.

Sol. Perimeter = 144 cm and ratio of sides = 3 : 4 : 5

Sum of ratio terms = (3 + 4 + 5) = 12.

Let the lengths of the sides be a, b and c respectively.

Then, $a = \left(144 \times \frac{3}{12}\right)$ cm = 36 cm,

$b = \left(144 \times \frac{4}{12}\right)$ cm = 48 cm

and $c = \left(144 \times \frac{5}{12}\right)$ cm = 60 cm.

$\therefore\ s = \frac{1}{2}(a + b + c) = \frac{1}{2}(36 + 48 + 60)$ cm

= 72 cm.

(s – a) = (72 – 36) cm = 36 cm,

(s – b) = (72 – 48) cm = 24 cm

and (s – c) = (72 – 60) cm = 12 cm.

(i) By Heron's formula, the area of the triangle is given by

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{72 \times 36 \times 24 \times 12}\ cm^2$

$= \sqrt{36 \times 36 \times 24 \times 24}\ cm^2$

$= (36 \times 24)\ cm^2 = 864\ cm^2$.

Hence, the area of the given triangle is $864\ cm^2$.

(ii) Let base = longest side = 60 cm and the corresponding height = h cm.

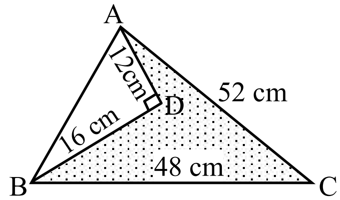
Then, area $= \left(\frac{1}{2} \times \text{base} \times \text{height}\right)$ sq units

$= \left(\frac{1}{2} \times 60 \times h\right) cm^2 = (30h)\ cm^2$.

$\therefore\ 30h = 864 \Rightarrow h = \left(\frac{864}{30}\right) = 28.8$.

Hence, the height corresponding to the longest side is 28.8 cm.

Ex.15 Find the area of the shaded region in figure :



Sol. By Pythagoras theorem, in ΔADB

$AB = \sqrt{AD^2 + BD^2}$

$= \sqrt{12^2 + 16^2}$

$= \sqrt{144 + 256}$

$= \sqrt{400}$

AB = 20 cm.

$\therefore$ area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$\left\{s = \frac{20 + 48 + 52}{2} = 60\,cm\right\}$

$= \sqrt{60(60-20)(60-48)(60-52)}$

$= \sqrt{60 \times 40 \times 12 \times 8}$

$= \sqrt{(12 \times 5) \times (5 \times 8) \times 12 \times 8}$

$= \sqrt{5^2 \times 8^2 \times 12^2} = 5 \times 8 \times 12 = 480\ cm^2$.

also area of $\Delta ADB = \frac{1}{2}$ (AD) (BD)

$= \frac{1}{2} \times 12 \times 16 = 96\ cm^2$

$\therefore$ Shaded area = ar(ΔABC) – ar (ΔADB)

$= 480 - 96 = 384\ cm^2$.

Ex.16 Find the area of an isosceles triangle of its sides are a cm, a cm and b cm.

Sol. Semi perimeter $= \frac{a + a + b}{2} = \frac{2a + b}{2}$ cm.

$\therefore\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{\left(\frac{2a+b}{2}\right)\left(\frac{2a+b}{2} - a\right)\left(\frac{2a+b}{2} - a\right)\left(\frac{2a+b}{2} - b\right)}$

$\Delta = \sqrt{\left(\frac{2a+b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{2a-b}{2}\right)}$

$\Delta = \frac{b}{2 \times 2}\sqrt{(2a+b)(2a-b)}$

$\Delta = \frac{b}{4}\sqrt{4a^2 - b^2}$ square cm.

Ex.17 A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board ?

[NCERT]

Sol. Let 2s be the perimeter of the signal board. Then,

$$2s = a + a + a \Rightarrow s = \frac{3a}{2}$$

Let Δ be the area of the given equilateral triangle. Then,

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \quad \Delta = \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

[Θ a = b = c]

$$\Rightarrow \quad \Delta = \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4}a^2$$

If, perimeter = 180 cm. Then,

$$2s = 180 \Rightarrow 3a = 180 \Rightarrow a = 60$$

$$\therefore \quad \Delta = \frac{\sqrt{3}}{4} \times (60)^2 = 900\sqrt{3}\ \text{cm}^2.$$

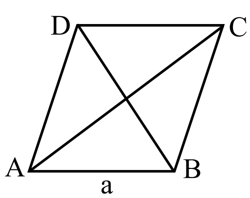
➤ AREA OF QUADRILATERAL

If all four sides and a diagonal are given then by the diagonal we get two triangles. By Heron's formula, we can find area of both triangles and by adding them, we get area of quadrilateral.

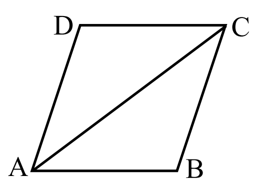
➤ AREA OF RHOMBUS

(1) If both diagonals are given (or we can find their length) then area = $\frac{1}{2}$ (Product of diagonals)

(2) If we use Heron's formula then we find area of one triangle made by two sides and a diagonal then twice of this area is area of rhombus.



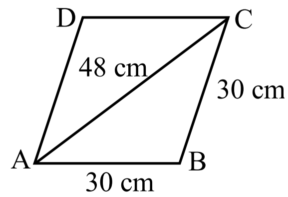
area = $\frac{1}{2}$(AC×DB) sq. units



area = 2(ΔABC) sq. units

Ex.18 A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting ? **[NCERT]**

Sol.



s = (30 + 30 + 48)/2 = (108)/2 = 54 m

area = 2(ΔABC)

$= 2\sqrt{s(s-a)(s-b)(s-c)}$

$= 2\sqrt{54(54-30)(54-30)(54-48)}$

$= 2\sqrt{9\times 6\times 24\times 24\times 6}$

area = 2 × 3 × 6 × 24

Θ area required for 18 cows

= (48 × 18) sq. units = 864 sq. units

∴ area required for 1 cow

$= \frac{48\times 18}{18}$ = 48 sq. units

OR

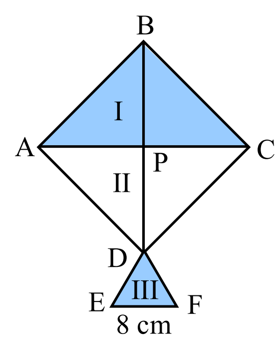
$$\text{area} = 2\left[\frac{b}{4}\sqrt{4a^2-b^2}\right]$$

$= 2 \times \frac{48}{4}\sqrt{4(30)^2-(48)^2}$

$= 24\sqrt{3600-2304} = 24\sqrt{1296}$

= 24 × 36 = 864 sq. units.

Ex.19 A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it ? **[NCERT]**



Sol. Θ ABCD is square

∴ Both diagonals are equal = 32 cm (each) also diagonals bisect each other at right angle

∴ AC = 32 cm & BP = PD = $\frac{32}{2}$ = 16 cm

$\therefore$ area of (ABC) = ar (ADC)

$$= \frac{1}{2}(32) \times 16 = 256 \text{ cm}^2$$

and area of ΔDEF

$= \sqrt{s(s-a)(s-b)(s-c)}$, $s = \frac{8+6+6}{2} = 10\text{cm}$

$= \sqrt{10(10-8)(10-6)^2}$

$= 4\sqrt{2\times5\times2}$

$= 4\times2\sqrt{5}$

$= 8\sqrt{5}\text{ cm}^2 = 8 \times 2.236 = 17.88 \text{ cm}^2$

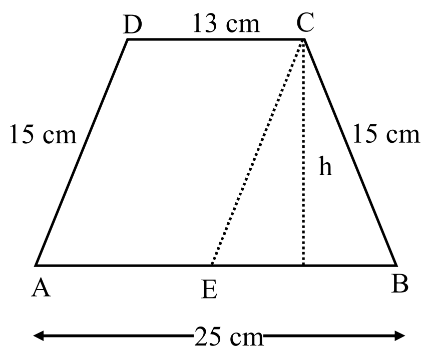
$\therefore$ required areas are 256 cm², 256 cm², 17.88 cm².

AREA OF TRAPEZIUM

Ex.20 Find the area of a trapezium whose parallel sides 25 cm, 13 cm and other sides are 15 cm and 15 cm.

Sol. Let ABCD be the given trapezium in which AB = 25 cm, CD = 13 cm, BC = 15 cm and AD = 15 cm.

Draw CE || AD.



Now, ADCE is a parallelogram in which AD || CE and AE || CD.

$\therefore$ AE = DC = 13 cm and BE = AB – AE = 25 – 13 = 12 cm

In Δ BCE, we have

$$s = \frac{15+15+12}{2} = 21$$

$\therefore$ Area of ΔBCE = $\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ Area of ΔBCE

$= \sqrt{21(21-15)(21-15)(21-12)}$

$\Rightarrow$ Area of ΔBCE

$= \sqrt{21\times6\times6\times9} = 18\sqrt{21}\text{ cm}^2$ ....(i)

Let h be the height of ΔBCE, then

Area of ΔBCE = $\frac{1}{2}$ (Base × Height)

$= \frac{1}{2} \times 12 \times h = 6h$ .....(ii)

From (i) and (ii), we have,

$6h = 8\sqrt{21} \Rightarrow h = 3\sqrt{21}$ cm

Clearly, the height of trapezium ABCD is same as that of ΔBCE.

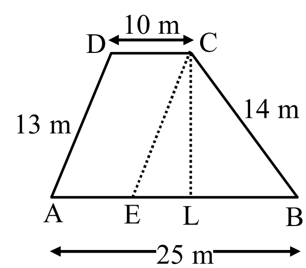
$\therefore$ Area of trapezium = $\frac{1}{2}$ (AB + CD) × h

$\Rightarrow$ Area of trapezium

$= \frac{1}{2}(25 + 13) \times 3\sqrt{21}\text{ cm}^2 = 57\sqrt{21}\text{ cm}^2$.

Ex.21 A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The nonparallel sides are 14 m and 13 m. Find the area of the field. **[NCERT]**

Sol. From C, draw CE || DA. Clearly, ADCE is a parallelogram having AD || CE and DC || AE such that AD = 13 m and DC = 10 m.



$\therefore$ AE = DC = 10 m and CE = AD = 13 m

$\Rightarrow$ BE = AB – AE = (25 – 10) m = 15 m

Thus in BCE, we have

BC = 14 m, CE = 13 m and BE = 15 m

Let s be the semi-perimeter of ΔBCE. Then,

2s = BC + CE + BE = 14 + 13 + 15 = 42

$\Rightarrow$ s = 21

$\therefore$ Area of ΔBCE

$= \sqrt{21\times(21-14)\times(21-13)\times(21-15)}$

$\Rightarrow$ Area of ΔBCE = $\sqrt{21\times7\times8\times6}$

$\Rightarrow$ Area of ΔBCE = $\sqrt{7^2\times3^2\times4^2} = 84\text{ m}^2$

Also, Area of ΔBCE = $\frac{1}{2}$(BE × CL)

$$\Rightarrow \quad 84 = \frac{1}{2}\times15\times\text{CL}$$

$\Rightarrow \qquad CL = \dfrac{168}{15} = \dfrac{56}{5}$

$\Rightarrow$ Height of parallelogram ADCE = CL = $\dfrac{56}{5}$ m

$\therefore$ Area of parallelogram ADCE

= Base × Height = AE × CL = $10 \times \dfrac{56}{5} = 112\,m^2$

Hence, Area of trapezium ABCD = Area of parallelogram ADCE + Area of ΔBCE

= (112 + 84) m^2 = 196 m^2.

Ex.22 Students of a school staged a rally for cleanliness compaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA, while other through AC, CD and DA (see fig.). Then they cleaned the area enclosed within their lanes. If AB = 9 m, BC = 40 m, CD = 15 m, DA = 28 m, and ∠B = 90°. Which group cleaned more area and by how much? Find the total area cleaned by the students. **[NCERT]**

Sol. In ΔABC, we have

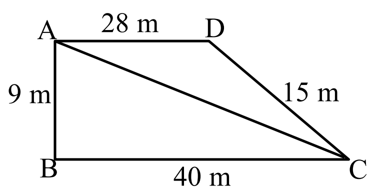
∠B = 90°

$\therefore \; AC^2 = AB^2 + BC^2$

[By Pythagoras Theorem]

$\Rightarrow AC^2 = 9^2 + 40^2 = 1681$

$\Rightarrow$ AC = 41



Computation of area of ΔABC :

Δ_1 = Area of ΔABC = $\dfrac{1}{2}(BC \times AB)$

[Θ ∠B = 90°]

$\Rightarrow \Delta_1 = \dfrac{1}{2}(40 \times 9)\; m^2 = 180\; m^2$

Computation of area of triangle ACD :

Let 2s be the perimeter of ΔACD. Then,

2s = AC + CD + DA = 41 + 15 + 28 = 84

$\Rightarrow$ s = 42 m

$\therefore \; \Delta_2$ = Area of ΔACD

$\Rightarrow \; \Delta_2 = \sqrt{s(s-AC)(s-CD)(s-DA)}$

$\Rightarrow \; \Delta_2 = \sqrt{42 \times (42-41) \times (42-15) \times (42-28)}$

$= \sqrt{42 \times 1 \times 27 \times 14} = 126\,m^2$

So, first group cleaned 180 m^2 and second group cleaned 126 m^2

Also, $\Delta_1 + \Delta_2 = (180 + 126)\; m^2 = 306\; m^2$

and, $\Delta_1 - \Delta_2 = (180 - 126)\; m^2 = 54\; m^2$

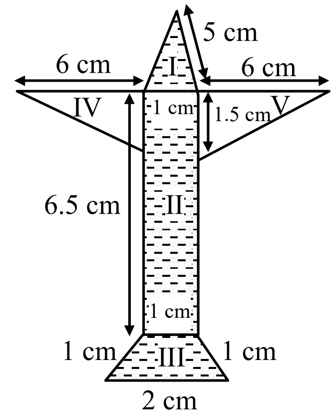
Thus, first group cleaned 54 m^2 more area than the second group and total area cleaned by all the students is 306 m^2.

Ex.23 Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used. **[NCERT]**

Sol. Area of region I;

Region I is enclosed by a triangle of sides

a = 5 cm, b = 5 cm and c = 1 cm



Let 2s be the perimeter of the triangle. Then,

$2s = 5 + 5 + 1 \Rightarrow s = \dfrac{11}{2}$ cm

$\therefore$ Area of region I

$= \sqrt{s(s-a)(s-b)(s-c)}\; cm^2$

$\Rightarrow$ Area of region I

$= \sqrt{\dfrac{11}{2} \times \left(\dfrac{11}{2} - 5\right) \times \left(\dfrac{11}{2} - 5\right) \times \left(\dfrac{11}{2} - 1\right)}\; cm^2$

$\Rightarrow$ Area of region I = $\sqrt{\dfrac{11}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{9}{2}}\; cm^2$

$= \dfrac{3}{4}\sqrt{11}\; cm^2 = \dfrac{3}{4} \times 3.32\; cm^2 = 2.49\; cm^2$

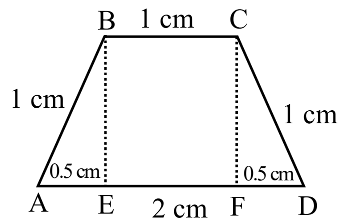
Area of region II :

Region II is a rectangle of length 6.5 cm and breadth 1 cm.

$\therefore$ Area of region II = 6.5 × 1 cm^2 = 6.5 cm^2

Area of region III :

Region III is an isosceles trapezium as shown in figure.



In ΔABE, we have

$$AB^2 = AE^2 + BE^2$$

$\Rightarrow\ 1 = 0.25 + BE^2$

$\Rightarrow\ BE = \sqrt{0.75} = \sqrt{\frac{3}{4}}$

$\therefore$ Area of region III $= \frac{1}{2}(AD + BC) \times BE$

$= \frac{1}{2}(2 + 1) \times \sqrt{\frac{3}{4}}\ cm^2 = \frac{3\sqrt{3}}{4}\ cm^2 = 1.3\ cm^2$

Area of region IV :

Region IV forms a right triangle whose two sides are of lengths 6 cm and 1.5 cm.

$\therefore$ Area of region IV $= \frac{1}{2} \times 6 \times 1.5\ cm^2 = 4.5\ cm^2$

Area of region V :

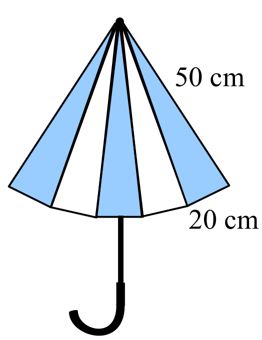
Region IV & V are congruent

$\therefore$ Area of region V $= 4.5\ cm^2$

Hence, total area of the paper used

$= (2.49 + 6.5 + 1.3 + 4.5 + 4.5)\ cm^2 = 19.29\ cm^2$

Ex.24 An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella? **[NCERT]**



Sol. Sides of one triangular piece of cloth are of lengths a = 20 cm, b = 50 cm and c = 50 cm

Let s be the semi-perimeter of the triangular piece. Then,

$$2s = a + b + c \Rightarrow 2s = 20 + 50 + 50 \Rightarrow s = 60$$

$\therefore\ \Delta$ = Area of one triangular piece

$= \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \Delta = \sqrt{60 \times (60-20) \times (60-50) \times (60-50)}\ cm^2$

$\Rightarrow \Delta = \sqrt{60 \times 40 \times 10 \times 10}\ cm^2$

$= \sqrt{6 \times 4 \times 10 \times 10 \times 10 \times 10}\ cm^2 = 200\sqrt{6}\ cm^2$

Area of cloth of each colour

$= 5 \times 200\sqrt{6}\ cm^2 = 1000\sqrt{6}\ cm^2$

Ex.25 A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 paisa per cm^2. **[NCERT]**

Sol. Lengths of the sides of the triangular tile are 28 cm, 9 cm and 35 cm.

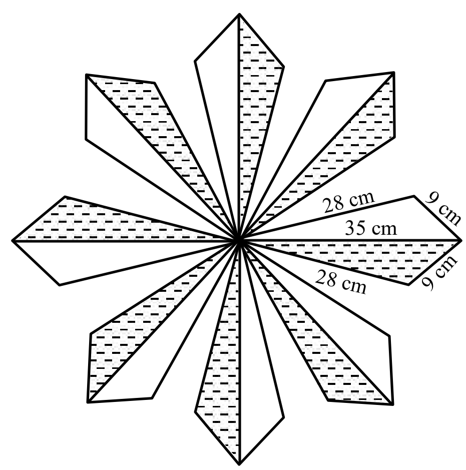
Let s be the semi-perimeter of a tile. Then,

$$s = \frac{28 + 9 + 35}{2}\ cm = 36\ cm$$

$\therefore$ Area of one tile

$= \sqrt{36 \times (36-28) \times (36-9) \times (36-35)}$

$= 36\sqrt{6}\ cm^2$



So, area of 16 tiles

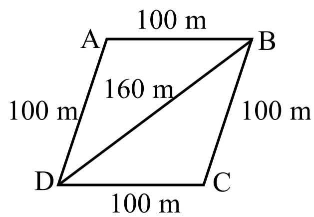
$= 16 \times 36\sqrt{6}\ cm^2 = 576\sqrt{6}\ cm^2$

Hence, cost of polishing the tiles at the rate of 50 paisa i.e. ₹ $\frac{1}{2}$ per cm^2

$=$ ₹ $576\sqrt{6} \times \frac{1}{2}$ = ₹ 705.45.

Ex.26 Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops to suffice the needs of their family. She divided the land in two equals parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get ? **[NCERT]**

Sol. Let ABCD be the field which is divided by the diagonal BD = 160 m into two equal parts.



Since ABCD is a rhombus of perimeter 400 m. Therefore,

$$AB = BC = CD = DA = \frac{400}{4} m = 100 \text{ m}$$

Let s be the semi-perimeter of ΔBCD

Then, $s = \frac{BC + CD + BD}{2} = \frac{100 + 100 + 160}{2}$ m

= 180 m

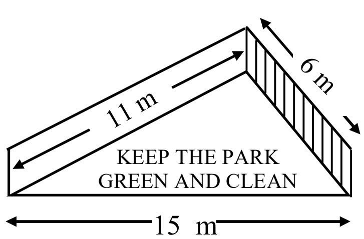
$\therefore$ Area of ΔBCD

$= \sqrt{180 \times (180-100) \times (180-100) \times (180-160)}\ m^2$

$= \sqrt{180 \times 80 \times 80 \times 20}\ m^2 = 4800\ m^2$

Hence, each of the two children will get an area of 4800 m^2.

Ex.27 There is a slide in a park. One of its side walls has been painted in some colour with a message ''KEEP THE PARK GREEN AND CLEAN'' (see figure). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour. **[NCERT]**



Sol. Clearly, the side wall is in the triangular form with sides a = 15 m, b = 6 m and c = 11 m.

Let 2s be the perimeter of the side wall. Then,

$2s = a + b + c \Rightarrow 2s = 15 + 6 + 11 \Rightarrow s = 16$

$\therefore$ s – a = 16 – 15 = 1, s – b = 16 – 6 = 10

and s – c = 16 – 11 = 5

Hence, Area to be painted in colour

= Area of the side wall

$= \sqrt{s(s-a)(s-b)(s-c)}$

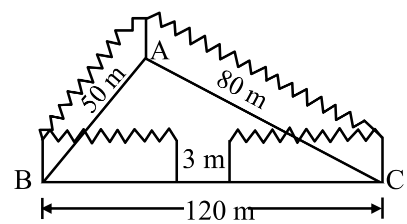
$= \sqrt{16 \times 1 \times 10 \times 5} = 20\sqrt{2}\ m^2$

Ex.28 A triangular park ABC has sides 120 m, 80 m and 50 m (see fig.). A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of ₹ 20 per metre leaving a space 3m wide for a gate on one side. **[NCERT]**

Sol. Computation of area : Clearly, the park is triangular with sides

a = BC = 120 m, b = CA = 80 m and

c = AB = 50 m



It s denotes the semi-perimeter of the park, then

$2s = a + b + c \Rightarrow 2s = 120 + 80 + 50$

$\Rightarrow s = 125$

$\therefore$ s – a = 125 – 120 = 5, s – b = 125 – 80 = 45 and s – c = 125 – 50 = 75.

Hence, Area of the park

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}\ m^2 = 375\sqrt{15}\ m^2$

Length of the wire needed for fencing

= Perimeter of the park – width of the gate

= 250 m – 3 m = 247 m

$\therefore$ Cost of fencing = ₹ (20 × 247)

= ₹ 4940.

Ex.29 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see fig.). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired both walls for 3 months. How much rent did it pay ? **[NCERT]**

Sol. The lengths of the sides of the walls are 122 m, 22 m and 120 m.

We have,

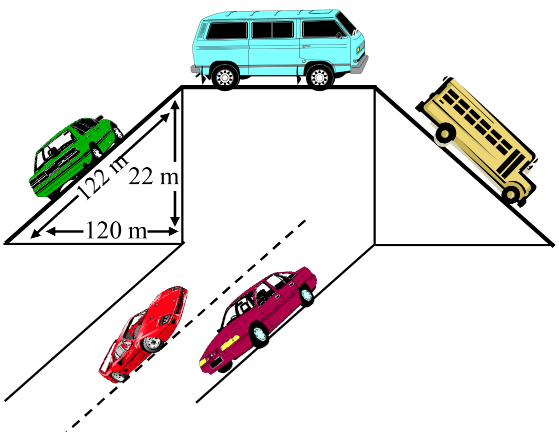
$$122^2 = 120^2 + 20^2$$

So, walls are in the form of right triangles.

$\therefore$ Area of two walls $= 2 \times \left(\frac{1}{2} \times \text{Base} \times \text{Height}\right)$

$\Rightarrow$ Area of two walls

$= 2 \times \left(\frac{1}{2} \times 120 \times 22\right) = 2640 \text{ m}^2$



We have,

Yearly rent = ₹ 5000 per m^2

$\therefore$ Monthly rent = ₹ $\left(\frac{5000}{12}\right)$ per m^2

Hence, rent paid by the company for 3 months

$= ₹ \left(\frac{5000}{12} \times 3 \times 2640\right) = ₹ 3300000.$

Ex.30 Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area. **[NCERT]**

Sol. Lets sides of Δ are 12x cm, 17x cm, 25x cm.

$\therefore$ 12x + 17x + 25x = 540 cm

$\Rightarrow$ 54x = 540

$\Rightarrow$ x = 10 cm

$\therefore$ sides are 120 cm, 170 cm, 250 cm

& $s = \frac{540}{2} = 270$ cm

$\therefore$ area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-120)(270-170)(270-250)}$

$= \sqrt{27 \times 10(150)(100)(20)}$

$= 10\sqrt{9 \times 3 \times 10 \times (5 \times 3 \times 10)(4 \times 5)}$

$= 10 \times 3 \times 2\sqrt{(3 \times 3)(5 \times 5)(10 \times 10)}$

$= 60 \times 3 \times 5 \times 10 = 9000 \text{ cm}^2$.

Ex.31 Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm. **[NCERT]**

Sol. Two sides of Δ are 18 cm, 10 cm & Perimeter = 42 cm.

$\therefore$ Third side = 42 – 18 – 10 = 14 cm.

$s = \frac{42}{2} = 21$ cm

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{(7 \times 3)(3)(11)(7)}$

$= 7 \times 3\sqrt{11} = 21\sqrt{11} \text{ cm}^2 = 21 \times 3.31$

$= 69.51 \text{ cm}^2$.

EXERCISE # 1

A. Short Answer Type Questions

Q.1 Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm. **[NCERT]**

Q.2 The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Find its area. **[NCERT]**

Q.3 An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle. **[NCERT]**

Q.4 Find the perimeter of equilateral triangle whose area is $36\sqrt{3}$ cm^2.

Q.5 The base of a right triangle is 48 cm and its hypotenuse is 50 cm long. Find the area of the triangle.

Q.6 If the height of an equilateral triangle is 6 cm. Then find its area.

Q.7 The area of an equilateral triangle is $81\sqrt{3}$ cm^2. Find its height.

Q.8 Find the area of ΔABC in which BC = 8 cm, AC = 15 cm and AB = 17 cm. Find the length of altitude drawn on AB.

Q.9 If the difference between the semi-perimeter and the sides of a ΔABC are 8 cm, 7 cm and 5 cm respectively. Then find the area of the triangle.

B. Long Answer Type Questions

Q.10 Two parallel side of a trapezium are 60 cm and 77 cm and other sides are 25 cm and 26 cm. Find the area of the trapezium.

Q.11 The sides of a quadrilateral, taken in order are 5, 12, 14 and 15 metres respectively and the angle contained by the first two sides is a right angle. Find its area.

Q.12 Find the area of a cyclic quadrilateral whose sides are 40 cm, 75 cm, 77 cm and 36 cm respectively.

Q.13 Find the ratio of the area of a square to that of the square drawn on its diagonal.

Q.14 The adjacent sides of a parallelogram are 24 cm and 32 cm. If the distance between the longer sides is 17.4 cm, determine the distance between the shorter sides.

Q.15 The lengths of the sides of triangle ABC are in the ratio 4 : 3 : 5, and its perimeter is 144 cm. Find the height corresponding to the longest side.

Q.16 Two parallel sides of a trapezium are 60 cm and 77 cm and other sides are 25 cm and 26 cm. Find the area of the trapezium.

Q.17 A field is in the shape of a trapezium whose parallel sides are 50 m and 15 m. The non-parallel sides are 20 m and 25 m. Prove that the area of the trapezium is $\frac{1300\sqrt{6}}{7}$ m^2.

ANSWER KEY

1. $8\sqrt{30}\,cm^2$	**2.** $1500\sqrt{3}\,m^2$	**3.** 34.83 cm^2	**4.** 36 cm	**5.** 336 cm^2
6. $12\sqrt{3}\,cm^2$	**7.** $9\sqrt{3}\,cm$	**8.** 7.04 cm	**9.** $20\sqrt{14}\,cm^2$	**10.** 1644 cm^2
11. 114 m^2	**12.** 2886 cm^2	**13.** 1 : 2	**14.** 23.2 cm	**15.** 28.8 cm
16. 1644 cm^2	**17.** $\begin{Bmatrix} \text{Area of } \Delta ABC = 22627.41\,m^2 \\ \text{Area of } \Delta ACD = 38400\,m^2 \end{Bmatrix}$			

EXERCISE # 2

A. Very short Answer Type Questions

Q.1 Find the area of a triangle whose sides are respectively 150 cm, 120 cm and 200 cm.

Q.2 In a ΔABC it is given that base = 12 cm and height = 5 cm. Find its area.

Q.3 Find the area of a triangle whose sides are 9 cm, 12 cm and 15 cm.

Q.4 The lengths of three sides of a triangle are 20 cm, 16 cm and 12 cm. Then find the area of the triangle.

Q.5 The base of an isosceles triangle is 6 cm and each of its equal sides is 5 cm. Then find the height of the triangle.

Q.6 Each of the two equal sides of an isosceles right triangle is 10 cm long. Then find its area.

B. Short Answer Type Questions

Q.7 The perimeters of a right triangle is 450 m. If its sides are in the ratio 13 : 12 : 5. Find the area of the triangle.

Q.8 Using Heron's formula find the area of an isosceles triangle whose one of the equal sides is 16 cm and third side is 10 cm.

Q.9 The perimeter of a right triangle is 144 cm and its hypotenuse measures 65 cm. Find the lengths of other sides and calculate its area. Verify the result using Hero's formula.

Q.10 The perimeter of a right triangles is 12 cm and its hypotenuse is of length 5 cm. Fine the other two sides and calculate its area.

C. Long Answer Type Questions

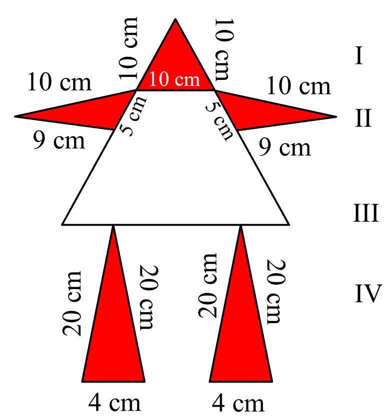
Q.11 The sides of a quadrangular field, taken is order are 26m, 27m, 7m, and 24m respectively. The angle contained by the last two sides is a right angle. Find its Area.

Q.12 An isosceles right triangle has an area 200 cm^2. What is the length of its hypotenuse?

Q.13 The sides of a triangle are of lengths 10 cm, 15 cm and 15 cm. Find the length of the altitude drawn on the side with length 15 cm.

Q.14 Suman made a picture with some white paper and a single coloured paper as shown in figure. White paper is available at her home and free of cost. The cost of coloured paper used is at the rate of 10 p per cm^2. Find the total cost of the coloured paper used.

(Take $\sqrt{3}$ = 1.732 and $\sqrt{11}$ = 3.31)



Q.15 If each of the equal sides of an isosceles triangle measures 2 cm more than its height and the base of the triangle measures 12 cm, then find the area of the triangle.

ANSWER KEY

1. 8966.56 cm^2	**2.** 30 cm^2	**3.** 54 cm^2	**4.** 96 cm^2	**5.** 4 cm
6. 50 cm^2	**7.** 6750 m^2	**8.** $5\sqrt{231}$ cm^2	**9.** 16 cm, 63 cm, 504 cm^2	**10.** 3 cm, 4 cm; 6 cm^2
11. 375.8 m^2	**12.** $20\sqrt{2}$ cm	**13.** 9.42 cm	**14.** 8 cm or 6 cm	**15.** 48 cm^2