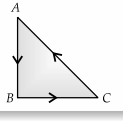
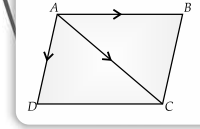
Vectors

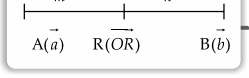
- Scalars and vectors
  - Quantities which have only magnitude and no direction is called scalar
  - Quantities which have magnitude as well as direction is called vector.
- Representation
  - $A \xrightarrow[\overrightarrow{AB}]{\vec{a}} B$
- Magnitude and direction of vector
  - Magnitude of vector $\overrightarrow{AB}$ is $|\overrightarrow{AB}|$
  - Any line which can be described by a vector called its direction vector.
- Position vector
  - Position vector of a point P(x,y,z) is $x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude is OP$(r) = \left|\sqrt{x^2+y^2+z^2}\right|$, where O(0,0,0) be the origin. Position vector of P (2, 3, 5) is $2\hat{i} + 3\hat{j} + 5\hat{k}$ and its magnitude is $\left|\sqrt{2^2+3^2+5^2}\right| = \sqrt{38}$ unit
- Direction cosines and direction ratios
  - Direction cosines are generally donated by letters l, m and n i.e., $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.
  - $\cos\alpha = \frac{x}{|\vec{r}|}$, $\cos\beta = \frac{y}{|\vec{r}|}$ and $\cos\gamma = \frac{z}{|\vec{r}|}$
  - $\Rightarrow l = \frac{x}{|\vec{r}|}, m = \frac{y}{|\vec{r}|}$, and $n = \frac{z}{|\vec{r}|}$
  - $\Rightarrow x = l|\vec{r}|, y = m|\vec{r}|$ and $z = n|\vec{r}|$
  - $|\vec{r}| = \left|\sqrt{x^2+y^2+z^2}\right|$ unit
  - Numbers $l|\vec{r}|, m|\vec{r}|$ and $n|\vec{r}|$ are proportional to the direction cosines are called direction ratios of vector $\vec{r}$
  - For e.g.: If $\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3k$, then $|\vec{r}| = |\sqrt{1+4+9}| = \sqrt{14}$ unit
    Direction ratios are (1, 2, 3)
    and direction cosines are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$
- Types of Vectors
  - (i) Zero vector (initial and terminal points coincide)
  - (ii) Unit vector (magnitude is unity)
  - (iii) Coinitial vectors (same initial points)
  - (iv) Collinear vectors (parallel to the same line)
  - (v) Equal vectors (same magnitude and direction)
  - (vi) Negative of a vector (same magnitude and opposite direction)
- Triangle law of vector addition
  - In ΔABC, we have $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, where $\overrightarrow{AC}$ is resultant vector.
  - The vector sum of the three sides of a triangle taken in order is 0 *i.e.*, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$.



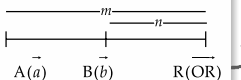
- Parallelogram law of vector addition
  - The vector sum of two coinitials vector is given by diagonal of the parallelogram whose adjacent sides are given vectors, if $\overrightarrow{AB}$, $\overrightarrow{AC}$ are given vectors, then $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$



- Addition of vectors
- Components of vectors
  - In $\overrightarrow{OP}(\vec{r}) = x\hat{i} + y\hat{j} + z\hat{k}$
  - $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the *X*-axis, *Y*-axis and *Z*-axis
  - x, y and z are called scalar components of $\vec{r}$
  - $x\hat{i}, y\hat{j}$ and $z\hat{k}$ are called vector components of $\vec{r}$
- Multiplication of a vector by a scalar
  - If λ is multiplied to vector $\overrightarrow{AB}$, then the magnitude is $|\lambda|$ times that of vector $\overrightarrow{AB}$ and direction remains same if $\lambda > 0$, opposite $\overrightarrow{AB}$ if $\lambda < 0$.
- Vector joining of two points
  - $\overrightarrow{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
  - $P_1P_2 = \left|\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}\right|$ units, the distance between two points.
- Section formulae (position vector of a point dividing a line segment in a given ratio)
  - Internally
    - $\overrightarrow{OR} = \frac{m\vec{b} + n\vec{a}}{m+n}$



  - Externally
    - $\overrightarrow{OR} = \frac{m\vec{b} - n\vec{a}}{m-n}$



- Dot (or scalar) product of two vectors
  - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ where $0 \le \theta \le \pi$
- Cross (or vector) product of two vectors
  - $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, such that $0 \le \theta \le \pi$ where, $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, such that $\vec{a}, \vec{b}$ and $\hat{n}$ form a right handed system.
- Properties of cross product of two vectors
  - If $\vec{a}$ and $\vec{b}$ are Adjacent side of 11gm then $\vec{a} \times \vec{b} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$
  - Area of parallelogram $= |\vec{a} \times \vec{b}|$ units^2
  - Area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$ units^2

Trace the Mind Map

‣ First Level ‣ Second Level ‣ Third Level