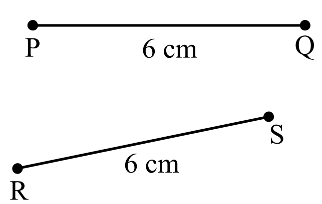
CONGRUENCE OF TRIANGLES

CONTENTS

➢ CONGRUENT FIGURES

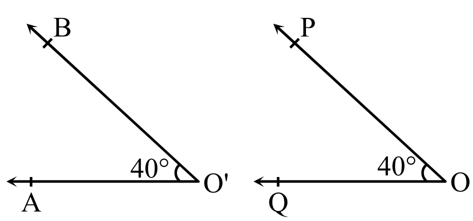
Two figures/objects are said to be congruent if they are exactly of the same shape and size. The relationship between two congruent figures is called congruence. We use the symbol $\cong$ for 'congruent to'.

1. Congruence among line segments. Two line segments are congruent if they have the same length.



Thus, line segment PQ $\cong$ line segment RS as PQ = RS = 6 cm.

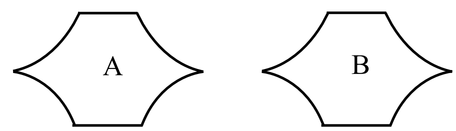
2. Congruence of Angles. Two angles are congruent if they have the same measure.



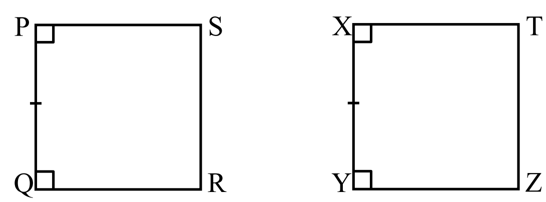
Thus, $\angle$AO'B $\cong$ $\angle$QOP,

as m $\angle$AO'B = m $\angle$QOP = 40°.

3. Congruence of plane figures. Two plane figures A and B are congruent as they superpose each other. We can write it as figure A $\cong$ figure B.

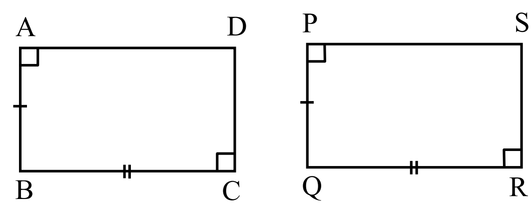


4. Congruence of squares. Two squares are congruent if they have same side length.



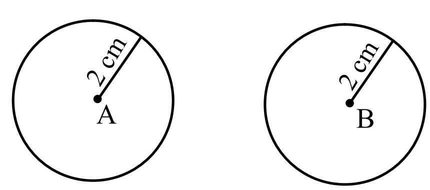
Square PQRS $\cong$ Square XYZT as PQ = XY.

5. Congruence of rectangles. Two rectangles are said to be congruent if they have the same length and breadth.



Rectangle ABCD $\cong$ Rectangle PQRS as AB = PQ and BC = QR.

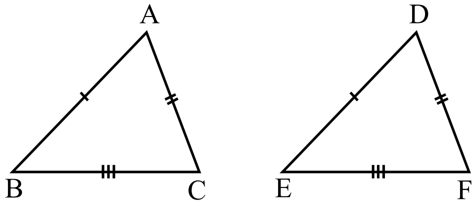
6. Congruence of circles. Two circles are congruent if they have the same radius.



Circle A $\cong$ Circle B, as radius of A = radius of B = 2 cm.

CONGRUENCE OF TRIANGLES

Two triangles are congruent if they are copies of each other, and when superposed they cover each other exactly.



ΔABC and ΔDEF have the same size and shape. They are congruent. So we would express this as $\Delta ABC \cong \Delta DEF$. This means that, when we place ΔDEF on ΔABC, D falls on A, E falls on B and F falls on C, also $\overline{DE}$ falls along $\overline{AB}$, $\overline{EF}$ falls along $\overline{BC}$ and $\overline{DF}$ falls along $\overline{AC}$.

Corresponding angles are : $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$.

Corresponding vertices are : A and D, B and E, C and F.

Corresponding sides are : $\overline{AB}$ and $\overline{DE}$, $\overline{BC}$ and $\overline{EF}$, $\overline{AC}$ and $\overline{DF}$.

Hence, three sides and three angles are the six matching parts for the congruence of triangles.

❖ **EXAMPLES** ❖

Ex.1 Write the correspondence between the vertices, sides and angles of the triangles XYZ and MLN, if $\Delta XYZ \cong \Delta MLN$.

Sol. By the order of letters, we find that

$X \leftrightarrow M$, $Y \leftrightarrow L$ and $Z \leftrightarrow N$

$\therefore$ XY = ML, YZ = LN, XZ = MN

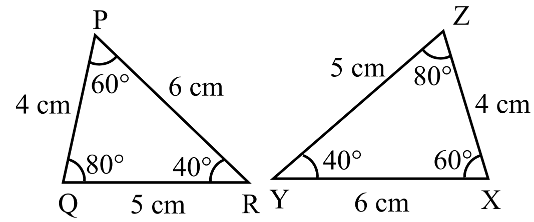
Also $\angle X = \angle M$, $\angle Y = \angle L$ and $\angle Z = \angle N$.

Ex.2 In following pairs of triangles, find the correspondence between the triangles so that they are congruent.

In ΔPQR : PQ = 4 cm, QR = 5 cm, PR = 6 cm, $\angle P = 60°$, $\angle Q = 80°$, $\angle R = 40°$.

In ΔXYZ : XY = 6 cm, ZY = 5 cm, XZ = 4 cm, $\angle X = 60°$, $\angle Y = 40°$, $\angle Z = 80°$

Sol. Let us draw the triangles and write the measures of their corresponding parts along with them.



From the above figures, we note that

PQ = XZ, QR = YZ, PR = XY

and $\angle P = \angle X$, $\angle Q = \angle Z$, $\angle R = \angle Y$

$\therefore$ $P \leftrightarrow X$, $Q \leftrightarrow Z$ and $R \leftrightarrow Y$

Hence, $\Delta PQR \cong \Delta XZY$

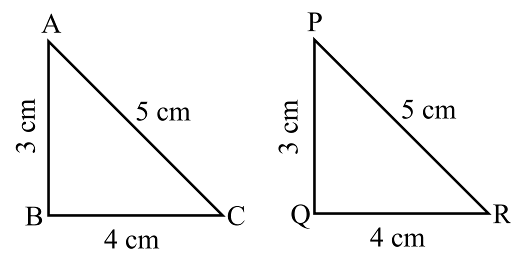
CRITERIA FOR CONGRUENCE OF TRIANGLES

1. SSS Congruence Criteria (Condition)

Two triangles are congruent, if three sides of one triangle are equal to the corresponding three sides of the other triangle.

❖ **EXAMPLES** ❖

Ex.3 Two triangles, ABC and PQR have been drawn such that AB = 3 cm, BC = 4 cm and AC = 5 cm. Also PR = 5 cm, QR = 4 cm and PQ = 3 cm.



Examine the congruence of triangles by method of superposition. Also verify the congruence by equality of six corresponding elements of the triangles.

Sol. Trace a copy of a ΔABC and super-impose it on ΔPQR. We find that the triangles cover each other exactly, so that $A \leftrightarrow P$, $B \leftrightarrow Q$ and $C \leftrightarrow R$ i.e., $\Delta ABC \cong \Delta PQR$.

Also measure the angles of the triangles and fill the information in the following table :

Triangle ABC	Triangle PQR	Difference
$\angle A =$	$\angle P =$	$\angle A - \angle P =$
$\angle B = 90°$	$\angle Q = 90°$	$\angle B - \angle Q = 0$

We find that in all cases the difference is either zero or very close to zero, which may be treated as zero.

So we have $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

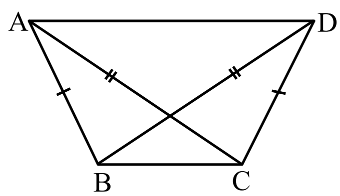
Because all sides (given) and all the angles (observed) of ΔABC are equal to the corresponding sides and angles of triangle PQR. $\therefore$ $\Delta ABC \cong \Delta PQR$

Ex.4 ABC and DBC are two triangles drawn on a common base BC such that AB = DC and DB = AC on the same side of BC. (See figure)

Are ΔADB and ΔDAC congruent ?

If yes, state the corresponding parts. Which condition did you use to establish the congruence ?

Sol. In ΔADB and ΔDAC, we have



AB = DC (Given)

BD = CA (Given)

and AD = AD (Common side)

$\therefore$ $\Delta ADB \cong \Delta DAC$

Also, $A \leftrightarrow D$, $D \leftrightarrow A$ and $B \leftrightarrow C$

Since, the three corresponding equal parts are the sides of the triangles, therefore, SSS congruence condition is used to prove the congruence.

2. SAS Congruence Criteria (Condition)

When two sides and the included angle of one triangle is equal to the corresponding sides and the included angle of another triangle, the two triangles are congruent. This, condition of congruence is known as side-angle-side congruence. In short we write SAS condition.

❖ EXAMPLES ❖

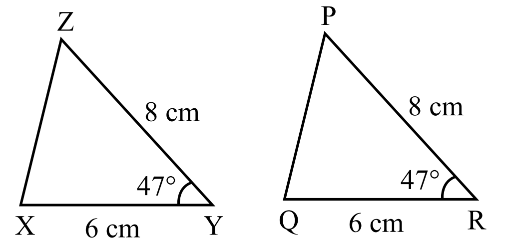
Ex.5 Given below are measures of some parts of two triangles.

Examine whether the two triangles are congruent or not by using the given information.

In ΔXYZ : XY = 6 cm, YZ = 8 cm, $\angle Y = 47°$

In ΔPQR : QR = 6 cm, PR = 8 cm, $\angle R = 47°$

Sol. Let us make a rough sketch of the triangles before examining their congruence.



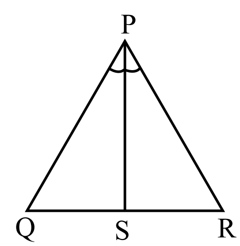
Clearly, here XY = QR = 6 cm, ZY = PR = 8 cm and $\angle Y = \angle R = 47°$ (included angles). Thus, by SAS congruence criteria $\Delta XYZ \cong \Delta QRP$.

Ex.6 Triangle PQR is isosceles with PQ = PR. Line segment PS bisects $\angle P$ and meets the side QR at point S.

(i) Is $\Delta PSQ \cong \Delta PSR$?

(ii) Can we say that QS = SR ?

Sol. In ΔPSR and ΔPSQ, the three pairs of equal parts (two sides and one angle) are as follows:



PQ = PR (given)

PS = PS (common)

and $\angle QPS = \angle RPS$ (PS bisects $\angle P$)

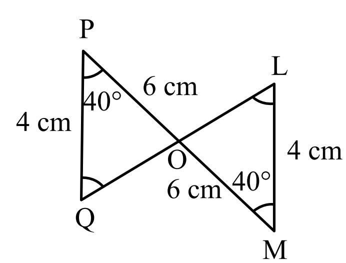
So (i) Yes, $\Delta PSQ \cong \Delta PSR$

(ii) Yes, QS = SR (corresponding sides of congruent triangles).

Ex.7 In adjoining figure, prove that

$\Delta POQ \cong \Delta MOL$

Sol. In ΔPOQ and ΔMOL, we have



$\angle P = \angle M = 40°$

PO = OM = 6 cm

and PQ = ML = 4 cm (given)

Thus, by SAS congruence criteria

$\Delta POQ \cong \Delta MOL$

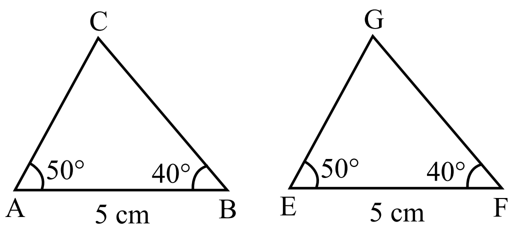
3. ASA Congruence Criteria (Condition)

Two triangles are congruent, if two angles and the included side of one is equal to the corresponding angles and side of the other.

❖ EXAMPLES ❖

Ex.8 In the following pair of triangles figure, the measure of some parts are given. Verify if the two triangles are congruent.

Sol.



In triangles, ABC and EFG.

Given, AB = EF = 5 cm

$\angle A = \angle E = 50°$

$\angle B = \angle F = 40°$

Therefore, by ASA congruence condition

$\Delta ABC \cong \Delta EFG$

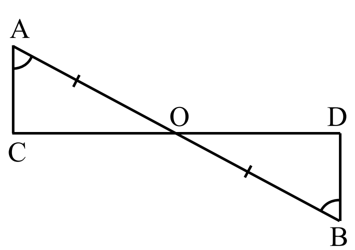
Ex.9 In figure, AO = BO and $\angle A = \angle B$.

(i) Is $\angle AOC = \angle BOD$? Why ?

(ii) Is $\Delta AOC \cong \Delta BOD$ by ASA congruence condition ?

(iii) State the three facts you have used to answer (ii).

(iv) Is $\angle ACO = \angle BDO$?



Sol. (i) Yes, $\angle AOC = \angle BOD$

[Vertically opposite angles]

(ii) In ΔAOC and ΔBOD, we have

$\angle AOC = \angle BOD$

[Vertically opposite angles]

AO = BO [Given]

$\angle OAC = \angle DBO$ [Given]

Therefore, by ASA congruence condition, we have

$\Delta AOC \cong \Delta BOD$

(iii) AO = BO, $\angle A = \angle B$ and $\angle AOC = \angle BOD$

(iv) Yes, since $\Delta AOC \cong \Delta BOD$

4. RHS Congruence Criteria (Condition)

Two right triangles are congruent, if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

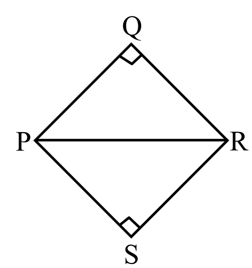
❖ EXAMPLES ❖

Ex.10 In figure, PQ = PS, $PQ \perp QR$ and $PS \perp RS$.

(i) Is $\Delta PQR \cong \Delta PSR$? Why ?

(ii) Is QR = RS ? Why ?

Sol.



In ΔPQR and ΔPSR, we have

PQ = PS (given)

$\angle PQR = \angle PSR$ (both are right angles)

PR = PR (common side)

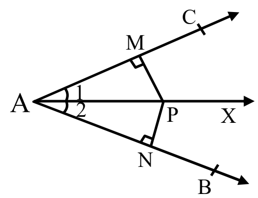
(i) $\therefore$ By RHS congruence condition, we have

$\Delta PQR \cong \Delta PSR$

(ii) Yes, QR = RS, because they are corresponding parts of congruent triangles.

Ex.11 AX is the bisector of $\angle BAC$, P is any point on AX. Prove that the perpendicular drawn from P to AB and AC are equal.

Sol. **Given :** An angle BAC bisected by AX. From any point P on AX, PM and PN are perpendiculars drawn to AB and AC respectively.



To Prove : PM = PN

Proof : In ΔAMP and ΔANP

$\angle M = \angle N$ [Each 90°]

$\angle 1 = \angle 2$

[AX is bisector of $\angle BAC$]

AP = AP [Common]

$\Delta AMP \cong \Delta ANP$

[By AAS congruence condition]

PM = PN

[Corresponding parts of congruent triangles]

Ex.12 Complete the following statements :

(i) Two line segments are congruent if_____.

(ii) Among two congruent angles, one has a measure of 70°, the measure of the other angle is _____ .

(iii) When we write $\angle A = \angle B$, we actually mean —— .

(iv) Two circles C_1 and C_2 are congruent, then their radii will be _____ .

Sol. (i) Two line segments are congruent if **they have the same length.**

(ii) Among two congruent angles, one has a measure of 70°, the measure of the other angle is 70°.

(iii) When we write $\angle A = \angle B$, we actually mean **m $\angle$A = m $\angle$B.**

(iv) Two circles C_1 and C_2 are congruent, then their radii will be **equal**.

Ex.13 If $\Delta ABC \cong \Delta FED$ under the correspondence $ABC \leftrightarrow FED$, write all the corresponding congruent parts of the triangles.

Sol. As $\Delta ABC \cong \Delta FED$

So, $\angle A \leftrightarrow \angle F$, $\angle B \rightarrow \angle E$, $\angle C \leftrightarrow \angle D$.

$\overline{AB} \leftrightarrow \overline{FE}$, $\overline{BC} \leftrightarrow \overline{ED}$, $\overline{AC} \leftrightarrow \overline{FD}$.

Ex.14 If $\Delta DEF \cong \Delta BCA$, write the part(s) of ΔBCA that correspond to

(i) $\angle E$ (ii) $\overline{EF}$

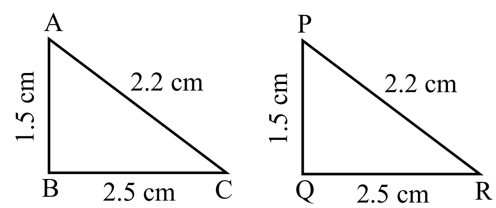
(iii) $\angle F$ (iv) $\overline{DF}$

Sol. If $\Delta DEF \cong \Delta BCA$, then $D \leftrightarrow B$, $E \leftrightarrow C$, $F \leftrightarrow A$

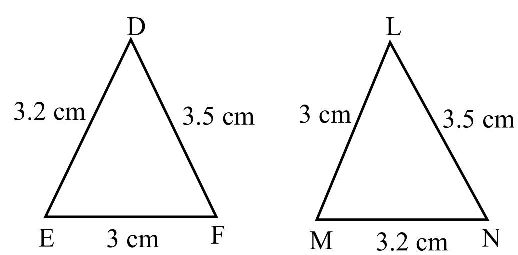
(i) $\angle E = \angle C$ (ii) $\overline{EF} = \overline{CA}$

(iii) $\angle F = \angle A$ (iv) $\overline{DF} = \overline{BA}$

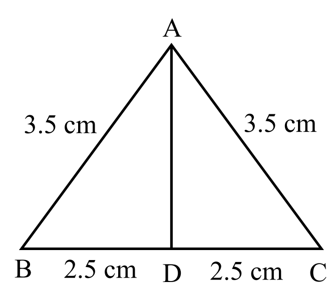
Ex.15 In the figures given below, lengths of the sides of the triangles are indicated. By applying SSS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



(i)



(ii)



(iii)

Sol. (i) In ΔABC and ΔPQR

$AB = PQ = 1.5$ cm

$BC = QR = 2.5$ cm

$CA = RP = 2.2$ cm

$\therefore\ \Delta ABC \cong \Delta PQR$ (by SSS)

(ii) $DE \neq LM$, $EF \neq MN$

So, $\Delta DEF \ncong \Delta LMN$.

(iii) In ΔADB and ΔADC

$AD = AD$ (common)

$AB = AC = 3.5$ cm

$BD = CD = 2.5$ cm

$\Delta ADB \cong \Delta ADC$ (by SSS)

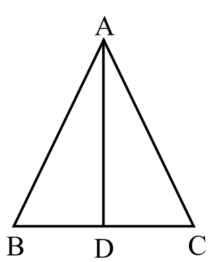
Ex.16 In figure, AB = AC and D is the mid point of $\overline{BC}$.

(i) State the three pairs of equal parts in ΔADB and ΔADC.

(ii) Is $\Delta ADB = \Delta ADC$? Give reason

(iii) Is $\angle B = \angle C$? Why ?

Sol. (i) In ΔADB and ΔADC



$AB = AC$ (given)

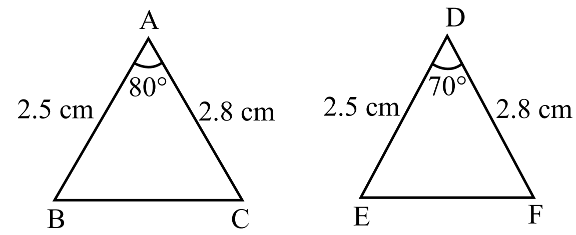
$AD = AD$ (common)

$BD = DC$ (Θ D is mid point of BC)

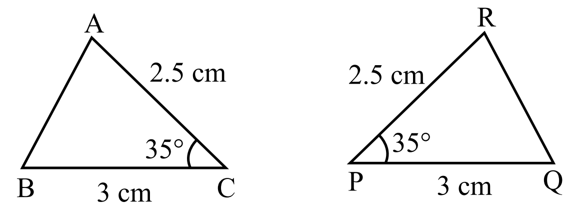
(ii) $\Delta ADB \cong \Delta ADC$ (by SSS property)

(iii) Yes, $\angle B = \angle C$ (by corresponding parts of congruent triangles)

Ex.17 In figures, measures of some parts of the triangles are indicated. By applying SAS congruence rule, state the pairs of congruent triangles, if any, in each case. In case of congruent triangles, write them in symbolic form.



(i)



(ii)

Sol. (i) In ΔABC and ΔDEF

As $AB = DE = 2.5$ cm (Θ $80° \neq 70°$)

$\angle A \neq \angle D$

$AC = DF = 2.8$ cm

So, $\Delta ABC \ncong \Delta DEF$

(ii) In ΔACB and ΔRPQ

$AC = RP = 2.5$ cm

$\angle C = \angle P = 35°$

$CB = PQ = 3$ cm

$\therefore\ \Delta ACB \cong \Delta RPQ$ (by SAS)

Ex.18 In figure, $\overline{AB}$ and $\overline{CD}$ bisect each other at O.

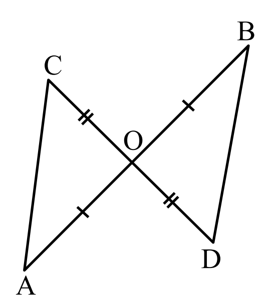
(i) State the three pairs of equal parts in two triangles AOC and BOD.

(ii) Which of the following statements are true

(a) $\Delta AOC \cong \Delta DOB$

(b) $\Delta AOC \cong \Delta BOD$?

Sol.



(i) $AO = OB$

CO = OD

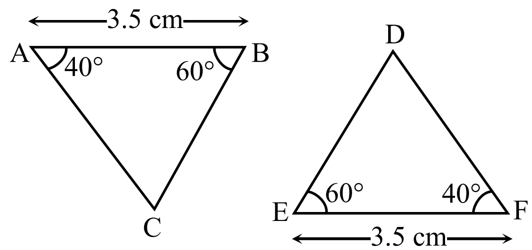
∠AOC = ∠BOD

(vertically opposite angles)

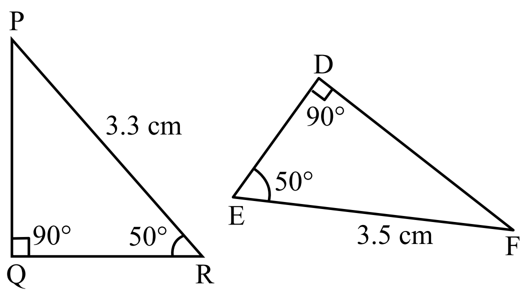
(ii) ΔAOC ≅ ΔBOD (by SAS)

Hence, (b) is true

Ex.19 In figures, measures of some parts are indicated. By applying ASA congruence rule, state which pairs of triangles are congruent. In case of congruence, write the result in symbolic form.



(i)



(ii)

Sol. (i) In ΔABC and ΔEFD.

∠A = ∠F (40° each)

AB = EF (3.5 cm each)

and ∠B = ∠E (60° each)

∴ ΔABC ≅ ΔFED (by ASA)

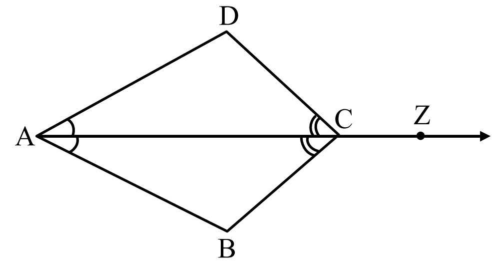
(ii) ΔPQR ≇ ΔDEF

as ∠Q = ∠D = 90°

∠E = ∠R = 50°

PR ≠ EF (Θ 3.3 cm ≠ 3.5 cm)

Ex.20 In figure, ray AZ bisect ∠DAB as well as ∠DCB.



(i) State the three pairs of equal parts in ΔBAC and ΔDAC.

(ii) Is ΔBAC ≅ ΔDAC ? Give reasons.

(iii) Is AB = AD ? Justify your answer.

(iv) Is CD = CB ? Give reasons.

Sol. (i) In ΔBAC and ΔDAC,

∠BAC = ∠DAC [Θ AZ bisects ∠DAB]

AC = AC (common)

∠BCA = ∠DCA [Θ AZ bisects ∠DCB]

(ii) Yes, ΔBAC ≅ ΔDAC (by ASA)

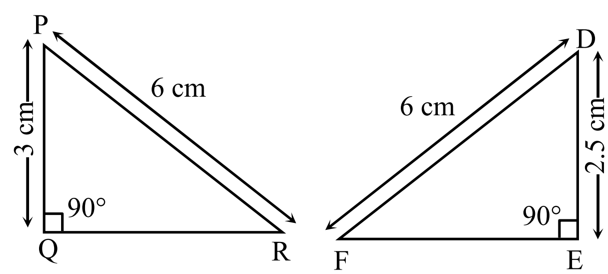
(iii) Yes, AB = AD

(corresponding parts of congruent triangles)

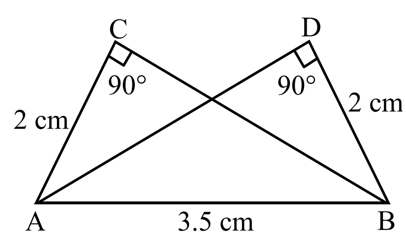
(iv) Yes, CD = CB

(corresponding parts of congruent triangles)

Ex.21 In figure, measures of some parts of triangles are given. By applying R.H.S. congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



(i)



(ii)

Sol. (i) In ΔPQR and ΔDEF

as $PR = DF = 6$ cm

$\angle Q = \angle E = 90°$

But $PQ \neq DE$ (as 3 cm $\neq$ 2.5 cm)

So, $\Delta PQR \ncong \Delta DEF$

(ii) In ΔCAB and ΔDBA

$\angle C = \angle D = 90°$ each

$AB = AB = 3.5$ cm

$CA = DB = 2$ cm

$\Delta CAB \cong \Delta DBA$ (by RHS)

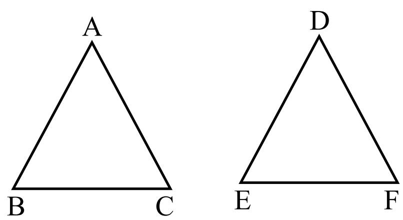
Ex.22 Which congruence criterion do you use in the following ?

(a) Given $AC = DF$

$AB = DE$

$BC = EF$

So, $\Delta ABC \cong \Delta DEF$

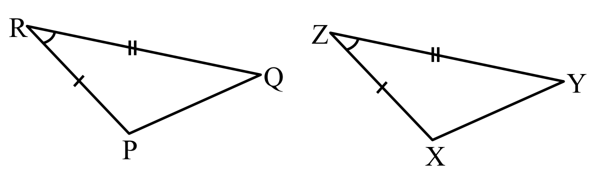


(b) Given $RP = ZX$

$\angle PRQ = \angle XZY$

$RQ = ZY$

So, $\Delta PQR \cong \Delta XYZ$

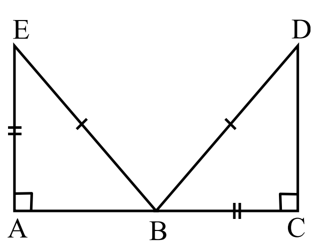


(c) Given $EB = DB$

$AE = BC$

$\angle A = \angle C = 90°$

So, $\Delta ABE \cong \Delta CDB$



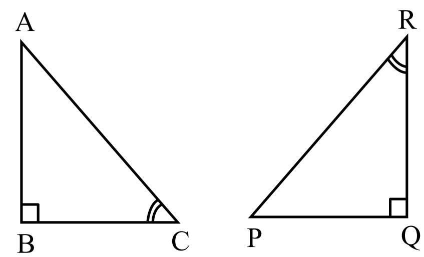
Sol. (a) $\Delta ABC \cong \Delta DEF$ (by SSS)

(b) $\Delta PQR \cong \Delta XYZ$ (by SAS)

(c) $\Delta EAB \cong \Delta DCB$ (by RHS)

Ex.23 If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use ?

Sol. To prove $\Delta ABC \cong \Delta PQR$,



We need one additional pair of corresponding parts which is

$BC = QR$

As, if $\angle ABC = \angle PQR$ (90° each)

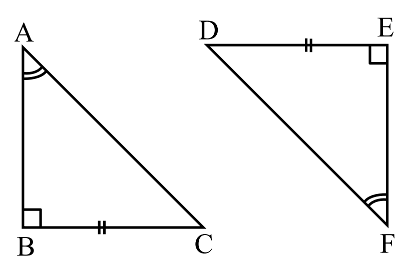
$BC = QR$

$\angle ACB = \angle PRQ$ (given)

$\Delta ABC \cong \Delta PQR$ (by ASA)

Ex.24 Explain why $\Delta ABC \cong \Delta FED$.

Sol. In ΔABC and ΔFED



$\angle A = \angle F$ (given)

$\angle B = \angle E$ (90° each) (given)

$\Rightarrow \angle C = \angle D$ ....(i)

(third $\angle C$ = third $\angle D$)

So, now $\angle B = \angle E = 90°$

$BC = DE$ (given)

$\angle C = \angle D$ [From (i)]

$\Delta ABC = \Delta FED$ (by ASA)

IMPORTANT POINTS TO BE REMEMBERED

(1) Two figures are congruent, if they have the same shape and size.

(2) Two line segments say $\overline{AB}$ and $\overline{CD}$ are congruent if they have equal lengths, we write this as $AB \cong CD$.

(3) Two squares are congruent if measure of their side is same.

(4) Two rectangles are congruent if they have the same length and breadth.

(5) Two circles are congruent if they have same radius.

(6) Two triangles are congruent if the three sides and three angels of one triangle are equal to the corresponding sides and angles of the other triangle.

(7) Two triangles are congruent if three sides of one triangle are equal to corresponding three sides of another triangle (SSS congruence condition).

(8) Two triangles are congruent if two sides and the included angle of one triangle are equal to corresponding sides and included angle of the other triangle (SAS congruence condition). 'Triangle' can be denoted as 'Δ'.

(9) Two triangles are congruent if two angles and included side of one triangle are equal to the corresponding angles and included side of the other (ASA congruence condition).

(10) Two right triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and corresponding side of other triangle.

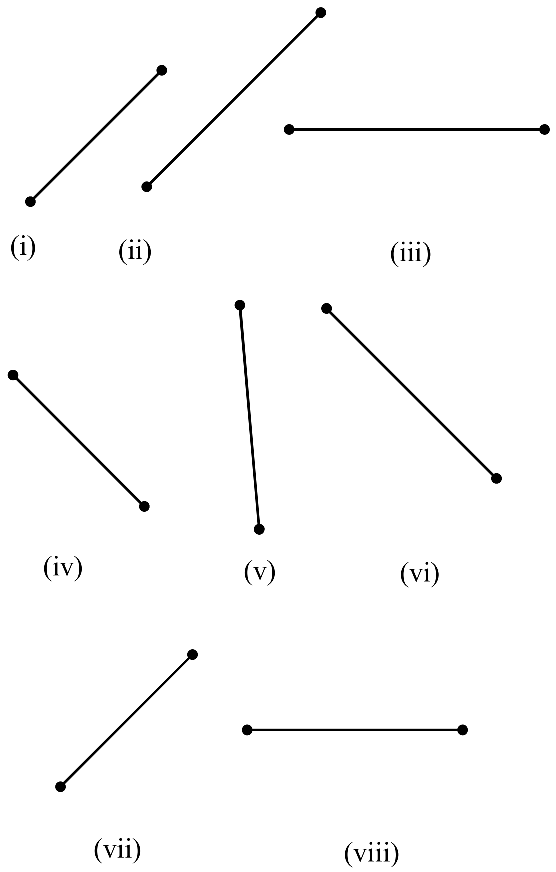
(11) Two congruent figures are equal in area, but the figures having equal area may not be congruent.

(12) There is no such thing as AAA congruence of two triangles.

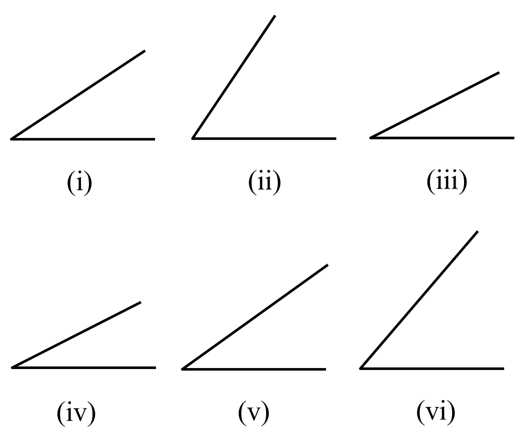
(13) Two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be enlarged copy of the other. (They would be congruent only if they are exact copies of one another).

EXERCISE # 1

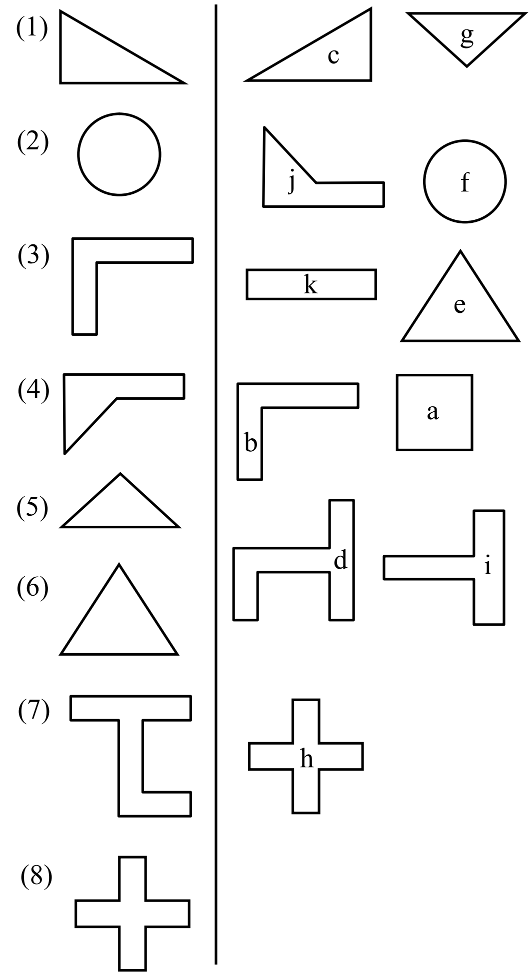
Q.1 Which of the line segments are congruent ? Measure and state.



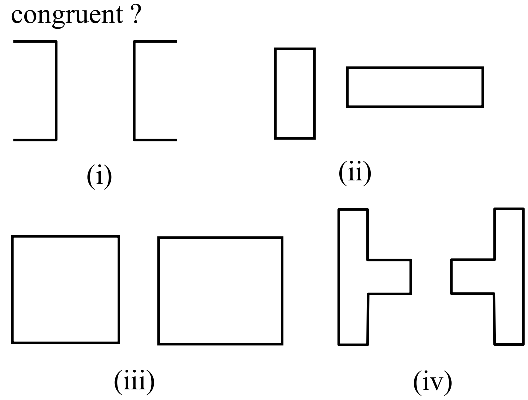
Q.2 Measure each of the angles given below and write which three pairs are congruent :

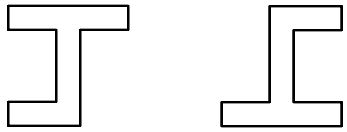


Q.3 Pair each given figure with a congruent figure from the collection at the right.

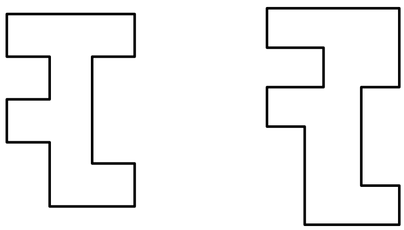


Q.4 Which of the following pairs of figures are congruent ?

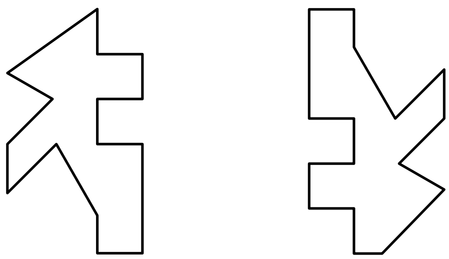




(v)



(vi)



(vii)

Q.5 Two congruent line segments are given. If the length of one segment is 6 cm, find the length of other segment.

Q.6 If $\angle ABC \cong \angle PQR$ and $\angle ABC = 60°$, then find the magnitude of $\angle PQR$.

Q.7 Fill in the blanks :

(i) If two line segments are congruent, they are equal in ……

(ii) If two angles are congruent, they are equal in …….

(iii) Two line segments are congruent if …….

(iv) When we write $\angle P = \angle Q$, it means …….

(v) If two squares are congruent, they are equal in ……..

(vi) If two circles are congruent, they are equal in ……

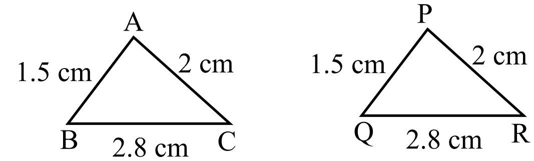
Q.8 If $\Delta ABC \cong \Delta XYZ$, write the parts of ΔXYZ that correspond to :

(i) $\angle B$ (ii) $\overline{YZ}$

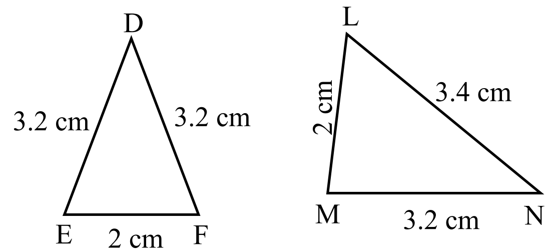
(iii) $\angle C$ (iv) $\overline{AC}$

Q.9 When $\Delta PQR \cong \Delta ABC$ under the correspondence $\Delta PQR \leftrightarrow \Delta ABC$, write all the corresponding congruent parts of the triangles.

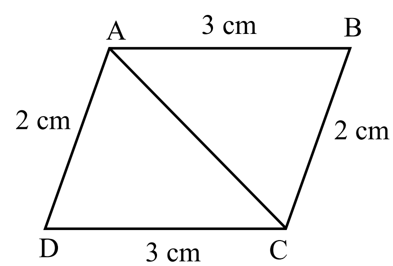
Q.10 In the following pairs of triangles, state which are congruent by applying SSS condition :



(i)



(ii)

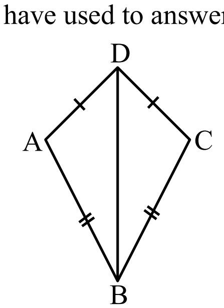


(iii)

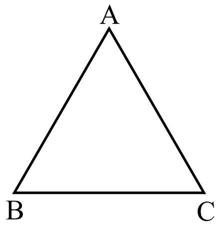
Q.11 In figure, AD = DC and AB = BC.

(i) Is $\Delta ABD \cong \Delta CBD$?

(ii) State the three pairs of matching parts you have used to answer (i).



Q.12 ΔABC is isosceles with $AB = AC$. State if $\Delta ABC \cong \Delta ACB$. If yes, state three relations that you have used to arrive at your answer figure.

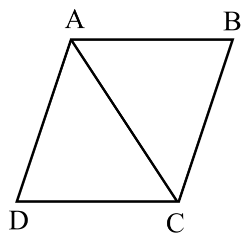


Q.13 In figure, $AB = DC$ and $AD = BC$.

(i) Is $\Delta ABC \cong \Delta CDA$?

(ii) What congruence condition have you used ?

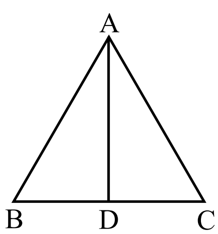
(iii) You have used some fact, not given in the question. What is that ?



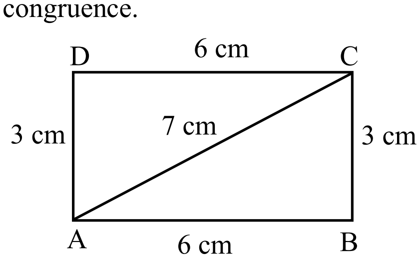
Q.14 In figure, ΔABC is isosceles with $AB = AC$. D is the mid-point of base BC.

(i) Is $\Delta ADB \cong \Delta ADC$?

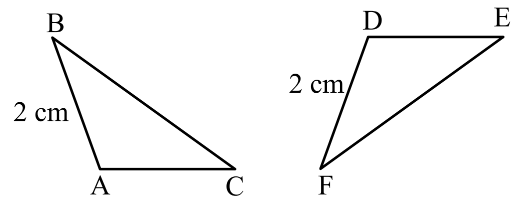
(ii) State the three pairs of matching parts that you have used to arrive at answer (i).



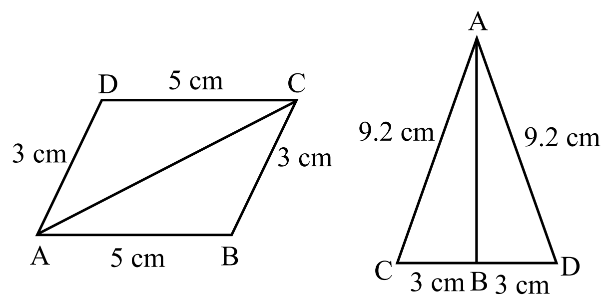
Q.15 In the following figure, a pair of triangles is given. Apply SSS condition to verify their congruence.



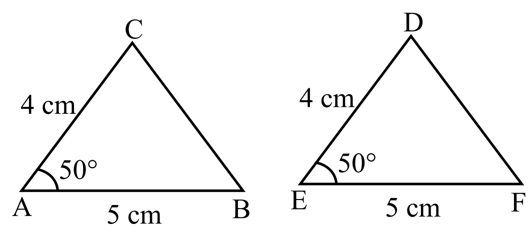
Q.16 In ΔABC and ΔDEF, $AB = DF$ and $BC = EF$. What additional information is required to make the two triangles congruent by SSS congruence condition ?



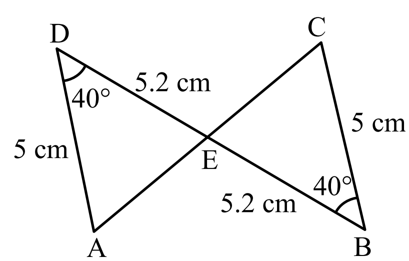
Q.17 In the following pairs of triangles by applying SSS condition, state which are congruent? State the result in symbolic form.



Q.18 Verify the SAS congruence condition in the following pairs of triangles



(i)



(ii)

Q.19 Show that in an isosceles triangle, angles opposite to equal sides are equal.

Q.20 In a ΔABC, $\angle A = 100°$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Q.21 In figure, AB = AD and ∠BAC = ∠DAC.

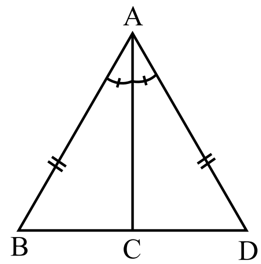
(i) State if the two triangles are congruent.

(ii) State the congruence condition.

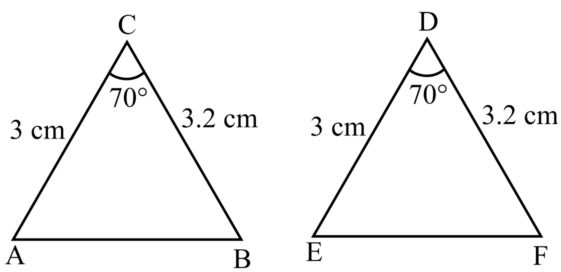
(iii) Complete the following :

∠ABC = …….

∠ACD = …….



Q.22 In the following pair of triangles the measures of some parts are indicated. Use SAS condition to prove their congruence figure.

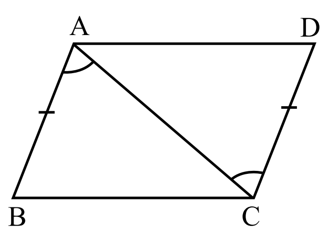


Q.23 In figure, AB || DC and AB = DC

(i) Is ∠BAC = ∠DCA? Why ?

(ii) Is ΔABC ≅ ΔCDA by SAS congruence condition ?

(iii) State the three facts you have used to answer (ii).



Q.24 Which of the following pairs of triangles are congruent :

(i) ΔABC, AB = 10 cm, ∠A = 40°, ∠B = 55°, ΔEFB, EF = 10 cm, ∠E = 40°, ∠F = 55°,

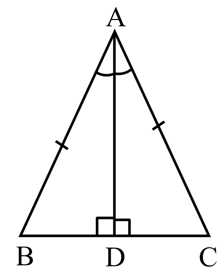
(ii) ΔPQR, PQ = 5 cm, ∠P = 37°, ∠R = 64°, ΔEFG, EF = 5 cm, ∠E = 37°, ∠F = 64°.

Q.25 In figure, AD bisects ∠A and AD ⊥ BC.

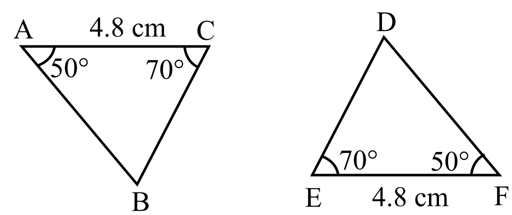
(i) Is ΔADB ≅ ΔADC by ASA congruence condition ?

(ii) If yes, state the three facts you have used to answer (i).

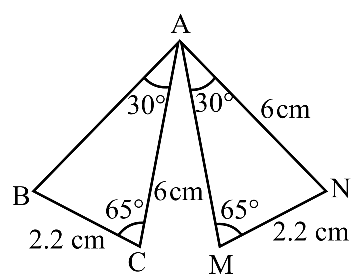
(iii) Is BD = DC ? Why ?



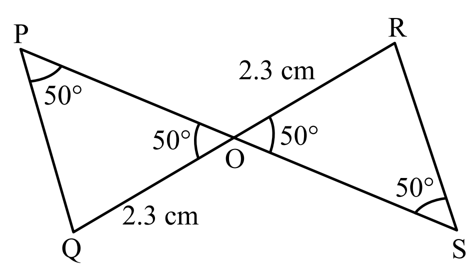
Q.26 In figure, which pairs of triangles are congruent by ASA congruence condition? If congruent, write the congruence of the two triangles in symbolic form.



(i)

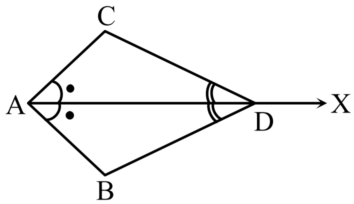


(ii)

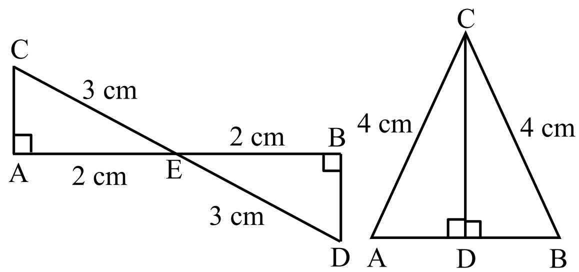


(iii)

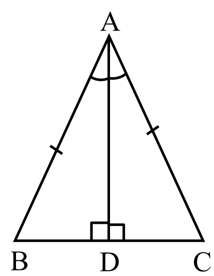
Q.27 In figure, AX bisects ∠BAC and ∠BDC. Find the third pair of corresponding parts to ensure that ΔABD ≅ ΔACD by ASA congruence condition.



Q.28 In the following pairs of triangles, verify the RHS congruence condition :



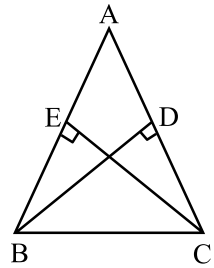
Q.29 If ΔABC is an isosceles triangle such that AB = AC then prove that altitude AD from A on BC bisects BC.



Q.30 In figure, BD and CE are altitudes of ΔABC and BD = CE.

(i) Is ΔBCD ≅ ΔCBE ?

(ii) State the three pairs of matching parts you have used to answer (i).



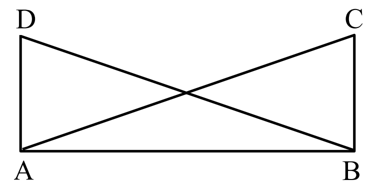
Q.31 In figure, AC = BD and DA ⊥ AB and also CB ⊥ AB. State which of the following statements are true :

(i) ΔABC ≅ ΔABD (ii) ΔABC ≅ ΔADB

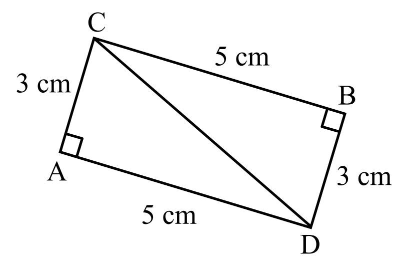
(iii) ΔABC ≅ ΔBAD

State the pairs of matching parts you have used to arrive at the answer.

Now, is it true that AD = BC ?



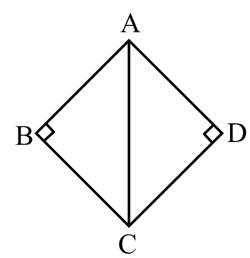
Q.32 In the following pair of right triangles figure, the measures of some parts are given. Verify if the two triangles are congruent by RHS congruence condition.



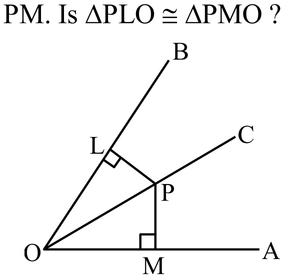
Q.33 In figure, AB = AD, AD ⊥ CD and AB ⊥ BC.

(i) Find the third pair of corresponding part so that ΔABC ≅ ΔADC by RHS congruence condition.

(ii) Is BC = DC ? Why?



Q.34 In figure, PL ⊥ OB and PM ⊥ OA such that PL = PM. Is ΔPLO ≅ ΔPMO ?



ANSWER KEY

1. (i) $\cong$ (vii) ; (ii) $\cong$ (vi) ; (iii) $\cong$ (v) ; (iv) $\cong$ (viii)

2. (i) $\cong$ (v) ; (ii) $\cong$ (vi) ; (iii) $\cong$ (iv)

3. (1) c, (2) f, (3) b, (4) j, (5) g, (6) e, (7) d, (8) h

4. (i), (iv), (v)

5. 6 cm

6. 60°

7. (i) length, (ii) magnitude, (iii) they are equal in length, (iv) $\angle P \cong \angle Q$, (v) sides, (vi) radii

8. (i) $\angle Y$, (ii) $\overline{BC}$, (iii) $\angle Z$, (iv) $\overline{XZ}$

9. $\angle P \cong \angle A$; $\angle Q \cong \angle B$; $\angle R \cong \angle C$; $PQ \cong AB$; $QR \cong BC$; $PR \cong AC$

10. (i), (iii)

11. (i) Yes, (ii) AD = CD, AB = CB and BD = BD

12. Yes ; AB = AC, AC = AB, BC = CB

13. (i) Yes, (ii) AB = CD, BC = DA, SSS congruency, (iii) AC = CA

14. (i) Yes, (ii) AD = AD, AB = AC, DB = DC

16. AC = DE

17. (i) $\Delta ABC \cong \Delta CDA$, (ii) $\Delta ABC \cong \Delta ABD$

20. 40°, 40°

21. (i) Yes, $\Delta ABC \cong \Delta ADC$, (ii) SAS congruence condition , (iii) $\angle ADB$, $\angle ACB$

23. (i) Yes, Alternate angles, (ii) Yes, (iii) AB = CD, AC = CA, $\angle BAC = \angle CDA$

24. (i) $\Delta ABC \cong \Delta EFG$

25. (i) Yes, (ii) $\angle ADB = \angle ADC$ (= 90°), AD = AD, $\angle BAD = \angle CAD$, (iii) Yes, BD = DC (CPCT)

26. (i) $\Delta ACB \cong \Delta FED$

27. AD = AD

30. (i) Yes, (ii) $\angle BDC = \angle CEB$ (= 90°), BC = CB, BD = CE

31. (iii) $\angle ABC = \angle BAD$, AC = BD, AB = BA, Yes, AD = BC (CPCT)

32. Yes

33. (i) AC is common, (ii) Yes, BC = DC (CPCT)

34. Yes

EXERCISE # 2

True/False type Questions (Q. 1 to 10)

Q.1 Two line segments are congruent if they have same length.

Q.2 Two squares are congruent if they have same corresponding angles.

Q.3 Two circles are congruent if they have any radius.

Q.4 Three sides and three angles are the six matching parts for the congruence of triangles.

Q.5 AAA is one of the cases for proving triangles congruent.

Q.6 SAS is one of the cases for proving triangles congruent.

Q.7 SSA is one of the cases for proving triangles congruent.

Q.8 If $\Delta DEF \cong \Delta BCA$, then $EF = CA$.

Q.9 If $\Delta DEF \cong \Delta BCA$, then $\angle F$ is equal to $\angle A$.

Q.10 If $\Delta PQR \cong \Delta ABC$, then their corresponding angles are equal.

Fill in the blanks type Questions(Q. 11 to 25)

Q.11 Two line segments AB and CD are of equal length of 10 cm, M and N are mid points of AB and CD respectively. Is $AM \cong CN$? ——— .

Q.12 Two circles have equal areas. Are these circles congruent? ——— .

Q.13 Area of two congruent square is equal. Is it true? ——— .

Q.14 $\Delta ABC \cong \Delta XYZ$. If $\angle XYZ = 65°$, the measure of $\angle ABC$ is ——— .

Q.15 If all the corresponding angles of two triangles are equal, are these two triangles congruent? ——— .

Q.16 A triangle PQR has each angle of 60°. Another triangle DEF also each angle of 60°. Is $\Delta PQR \cong \Delta DEF$? ——— .

Q.17 $\Delta ABC \cong \Delta DEF$. If $\angle B = 50°$, $\angle C = 70°$, find the measure of $\angle D$ ——— .

Q.18 $\Delta PQR \cong \Delta XYZ$ and $PR = 7$ cm, find XZ ——— .

Q.19 In ΔABC and ΔPQR, $AB = PQ$, $\angle B = \angle Q$ and $AC = PR$. Is $\Delta ABC \cong \Delta PQR$? ——— .

Q.20 If the two sides and an angle of one triangle are respectively equal to two sides and an angle of the other, are the triangles congruent? ——— .

Q.21 Between two congruent line segments, one has a measure of 7 cm, the measure of the other segment is ——— .

Q.22 Two circles C_1 and C_2 are ——— if their radii are equal.

Q.23 If $m\angle A = m\angle B$, we can also mean for it ——— .

Q.24 Two figures are said to be congruent if they have ——— and ——— .

Q.25 ——— sides and ——— angles are the six matching parts for the congruence of triangles.

Q.26 State the correspondence between the vertices, sides and angles of the following pairs of congruent triangles :

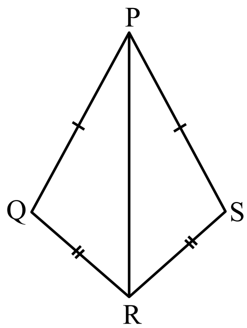
(i) $\Delta XYZ \cong \Delta PQR$

(ii) $\Delta NPM \cong \Delta RQS$

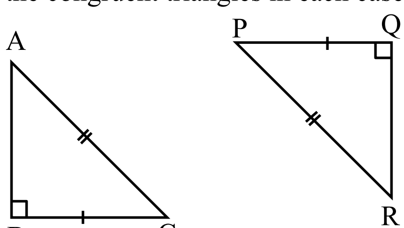
Q.27 Given that $\Delta ABC \cong \Delta RPQ$, $\angle A = 50°$, $\angle B = 60°$, find $\angle P$, $\angle Q$, $\angle R$.

Q.28 In an isosceles ΔABC, $AB = AC$, D and E are two points on the sides AB and AC respectively such that $AD = AE$. Prove that $\Delta ABE \cong \Delta ACD$.

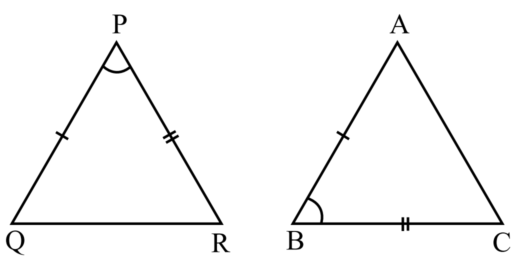
Q.29 In the figure given below, PQ = PS and QR = RS. Find the third pair of corresponding parts that makes $\Delta PQR \cong \Delta PSR$ by SSS congruence condition.



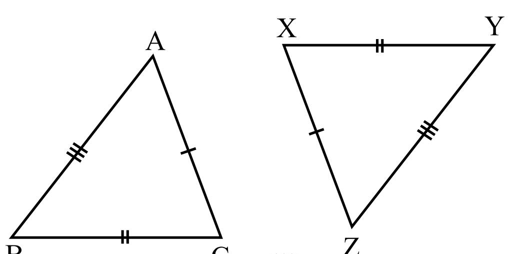
Q.30 Given below are pairs of congruent triangles. State the property of congruence and name the congruent triangles in each case.



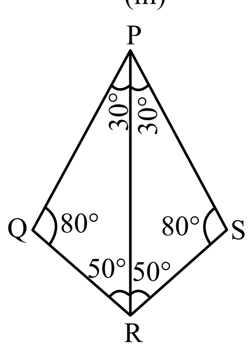
(i)



(ii)

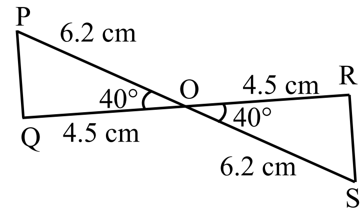


(iii)

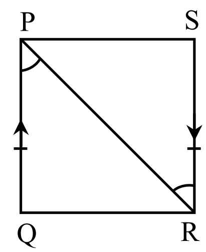


(iv)

Q.31 In the figure below, are the two triangles congruent? If yes, mention the congruence of the two triangles in symbolic form.



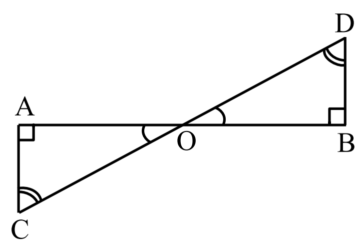
Q.32 In the figure, PQ || SR and PQ = SR



(i) Is $\angle QPR = \angle SRP$? Why ?

(ii) Is $\Delta PQR \cong \Delta RSP$? If yes write congruence condition.

Q.33 In the given figure, $AC \perp AB$, $DB \perp AB$ and AC = DB. Prove that

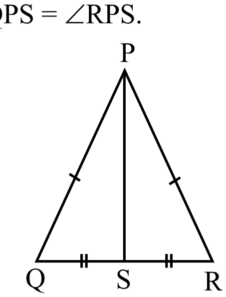


(i) $\Delta OAC \cong \Delta OBD$, write the congruency condition.

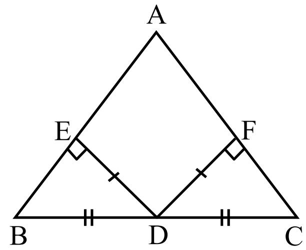
(ii) Is OA = OB? Why ?

Q.34 Prove that in an isosceles triangle, the angles opposite to equal sides are equal.

Q.35 In figure given below, PQ = PR and QS = SR. Prove that $\Delta PSQ \cong \Delta PSR$ and hence show that $\angle QPS = \angle RPS$.

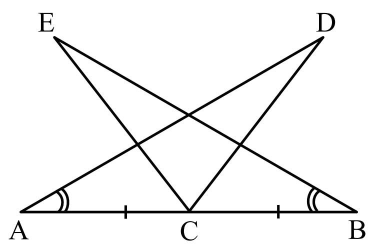


Q.36 In the figure, it is given that ED = DF, BD = DC, DE ⊥ AB and DF ⊥ AC. Prove that AE = AF.



Q.37 Prove that the bisector of the vertical angle of an isosceles triangles bisects the base at right angles.

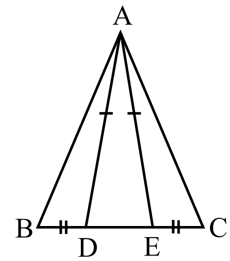
Q.38 In the adjacent figure, C is mid point of AB, ∠BAD = ∠ABE.



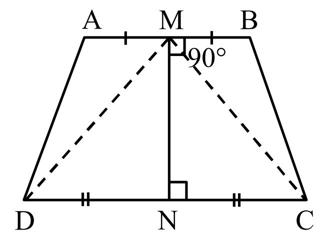
Prove that

(i) ΔDAC ≅ ΔEBC (ii) DA = EB

Q.39 In the adjoining figure, AD = AE, D and E are points on BC such that BD = EC. Prove that AB = AC.



Q.40 In figure, the line segment joining the mid points M and N of opposite sides AB and DC of quadrilateral ABCD is perpendicular to both sides. Prove that the other two sides of the quadrilateral are equal.



ANSWER KEY

1. T	**2.** F	**3.** F	**4.** T	**5.** F	**6.** T	**7.** F
8. T	**9.** F	**10.** T	**11.** Yes	**12.** Yes	**13.** Yes	**14.** 65°
15. No	**16.** No	**17.** 60°	**18.** 7 cm	**19.** Yes	**20.** No	**21.** 7 cm
22. Congruent	**23.** ∠A = ∠B	**24.** Same Shape, Same Size		**25.** 3, 3		

26. (i) X ↔ P, Y ↔ Q, Z ↔ R, $\overline{XY} = \overline{PQ}$, $\overline{YZ} = \overline{QR}$, $\overline{XZ} = \overline{PR}$, ∠X = ∠P, ∠Y = ∠Q, ∠Z = ∠R

(ii) N ↔ R, P ↔ Q, M ↔ S, $\overline{NP} = \overline{RQ}$, $\overline{PM} = \overline{QS}$, $\overline{NM} = \overline{RS}$, ∠N = ∠R, ∠P = ∠Q, ∠M = ∠S

27. ∠R = 50°, ∠P = 60°, ∠Q = 70° **29.** PR = PR (common)

30. (i) RHS, ΔABC ≅ ΔRQP ; (ii) SAS, ΔQPR ≅ ΔABC ; (iii) SSS, ΔABC ≅ ΔZYX ; (iv) ASA; ΔPQR ≅ ΔPSR

31. Yes, by SAS, ΔPOQ ≅ ΔSOR **32.** (i) Yes, alternate angles, (ii) Yes, by SAS

33. (i) Yes, by ASA, (ii) Yes, corresponding parts of congruent triangles