Chapter 26

THREE DIMENSIONAL GEOMETRY

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. If α, β, γ are the angles which a directed line makes with the positive directions of the co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to :

(a) 1 (b) 2

(c) 3 (d) $\frac{3}{2}$

2. A line OP through origin O is inclined at 30° and 45° to OX and OY respectively. The angle at which it is inclined to OZ is :

(a) 90° (b) $\cos^{-1}\left(\frac{-1}{4}\right)$

(c) 60° (d) such line does not exist

3. ABC is triangle $A = (2, 3, 5), B = (-1, 3, 2)$ and $C = (\lambda, 5, \mu)$. If the median through A is equally inclined to the axes then :

(a) $\lambda = \mu = 5$ (b) $\lambda = 5, \mu = 7$

(c) $\lambda = 7, \mu = 10$ (d) $\lambda = 0, \mu = 0$

4. A line passes through the point (6, – 7, – 1) and (2, – 3, 1). The direction cosines of the line so directed that the angle made by it with positive direction of x-axis is acute, are

(a) $\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ (b) $-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

(c) $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$ (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

5. The perpendicular distance of a corner of a unit cube from a diagonal not passing through it is :

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{3}$

(c) $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$

6. If $P(x, y, z)$ is a point on the line segment joining $Q(2, 2, 4)$ and $R(3, 5, 6)$ such that the projections of $\overrightarrow{OP}$ on the axes are $\frac{13}{5}, \frac{19}{5}, \frac{26}{5}$ respectively, then P divides QR in the ratio:

(a) 1 : 2 (b) 3 : 2

(c) 2 : 3 (d) 1 : 3

7. The equation of motion of a point in space is $x = 2t$, $y = -4t$, $z = 4t$ where t measured in hour and the co-ordinates of moving point in kilometers. The distance of the point from the starting point O (0, 0, 0) in 10 hours is

(a) 20 km (b) 40 km

(c) 60 km (d) 55 km

8. The co-ordinates of the foot of the perpendicular drawn from the point A (1, 0, 3) to the join of the points B (4, 7, 1) and C (3, 5, 3) are :

(a) $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ (b) $(5, 7, 17)$

(c) $\left(\frac{5}{7}, -\frac{7}{3}, \frac{17}{3}\right)$ (d) $\left(-\frac{5}{3}, \frac{7}{3}, -\frac{17}{3}\right)$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ		

9. A line makes angles $\alpha, \beta, \lambda, \delta$ with the four diagonals of a cube, then the value of

$\cos^2\alpha + \cos^2\beta + \cos^2\lambda + \cos^2\delta$ is :

(a) 1 (b) 2

(c) $\frac{4}{3}$ (d) 4

10. If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two mutually perpendicular lines, then the direction cosines of the line perpendicular to both the lines are :

(a) l_1l_2, m_1m_2, n_1n_2

(b) $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$

(c) $\frac{l_1}{l_2}, \frac{m_1}{m_2}, \frac{n_1}{n_2}$

(d) $m_1n_2 + m_2n_1, n_1l_2 + n_2l_1, l_1m_2 + l_2m_1$

11. If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two concurrent lines, then the direction cosines of the line bisecting the angles between them are proportional to

(a) l_1l_2, m_1m_2, n_1n_2

(b) $\frac{l_1}{l_2}, \frac{m_1}{m_2}, \frac{n_1}{n_2}$

(c) $l_1 + l_2, m_1 + m_2, n_1 + n_2$

(d) $m_1n_2 + m_2n_1, n_1l_2 + n_2l_1, l_1m_2 + l_2m_1$

12. The line $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular if :

(a) $aa' + bb' + 1 = 0$ (b) $ab' + a'b + 1 = 0$

(c) $aa' + bb' + cc' = dd'$ (d) $aa' + cc' + 1 = 0$

13. The centre of the sphere which touches the lines $y = x, z = c$ and $y = -x, z = -c$ lies on

(a) $xy + 2cz = 0$ (b) $yz + 2cx = 0$

(c) $zx + 2cy = 0$ (d) none of these

14. The image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is :

(a) (1, 2, 3) (b) (1, 3, 5)

(c) (0, 1, 2) (d) (1, 0, 7)

15. A mirror and a source of light are situated at the origin *O* and at a point on *OX* respectively. A ray of light from the source strikes the mirror and is reflected. If the *DR*s of the normal to the plane of mirror are 1, –1, 1, then *DC*s for the reflected ray are

(a) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (b) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

(c) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

16. The equation of plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, – 7) is :

(a) $x + y + z = 1$ (b) $x + y + z = 2$

(c) $x + y + z = 0$ (d) none of these

17. A plane meets the co-ordinate axes in A, B, C such that the centroid of ΔABC is the point (p, q, r). The equation of the plane is :

(a) $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1$ (b) $\frac{x}{2p} + \frac{y}{2q} + \frac{z}{2r} = 1$

(c) $\frac{x}{3p} + \frac{y}{3q} + \frac{z}{3r} = 1$ (d) $\frac{3x}{p} + \frac{3y}{q} + \frac{3z}{r} = 1$

18. The Cartesian equation of the plane $\vec{r} = (1 + \lambda - \mu)\vec{i} + (2 - \lambda)\vec{j} + (3 - 2\lambda + 2\mu)\vec{k}$ is

(a) $2x + y = 5$ (b) $2x - y = 5$

(c) $2x + z = 5$ (d) $2x - z = 5$

19. A variable plane which remains at a constant distance 3*p* from the origin, cut the co-ordinate axes at *A*, *B*, *C*. The locus of the centroid of ΔABC is .

(a) $x^{-1} + y^{-1} + z^{-1} = p^{-1}$

(b) $x^{-2} + y^{-2} + z^{-2} = p^{-2}$

(c) $x^{-1} + y^{-1} + z^{-1} = 3p^{-1}$

(d) $x + y + z = p$

MARK YOUR RESPONSE					
	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ
	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ
	19. ⓐⓑⓒⓓ				

20. The distance between the line

$r = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane

$\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{10}{3}$

(c) $\frac{10\sqrt{3}}{9}$ (d) none of these

21. The equation of the plane through the line of intersection of planes $ax+by+cz+d=0$, $a'x+b'y+c'z+d'=0$ and parallel to the line $y=0, z=0$ is :

(a) $(ab'-a'b)x+(bc'-b'c)y+(ad'-a'd)=0$

(b) $(ab'-a'b)x+(bc'-b'c)y+(ad'-a'd)z=0$

(c) $(ab'-a'b)y+(ac'-a'c)z+(ad'-a'd)=0$

(d) none of these

22. The distance of the point (1, 0, –3) from the plane $x-y-z=9$ measured parallel to the line $\frac{x-2}{2}=\frac{y+2}{3}=\frac{z-6}{-6}$ is

(a) 6 (b) 7

(c) $\frac{7}{2}$ (d) $\frac{8}{3}$

23. The area of the triangle whose vertices are (0, 0, 0), (3, 4, 7), and (5, 2, 6) is :

(a) $\frac{3}{2}\sqrt{65}$ (b) $\frac{1}{3}\sqrt{65}$

(c) $\frac{3}{\sqrt{74}}$ (d) $\frac{3}{2}\sqrt{74}$

24. The plane $ax+by=0$ is rotated through an angle α about its line of intersection with the plane $z = 0$. The equation of the plane in new position is :

(a) $ax+by \pm z\sqrt{a^2+b^2}\tan\alpha = 0$

(b) $(ax+by)\sqrt{a^2+b^2} \pm z\tan\alpha = 0$

(c) $\tan\alpha(ax+by) \pm z\sqrt{a^2+b^2} = 0$

(d) $ax+by \pm \sqrt{a^2+b^2}z = \tan\alpha$

25. If a variable plane forms a tertrahedron of constant volume $64k^3$ with the co-ordinate planes, then the locus of the centroid of the tetrahedron is

(a) $x^3+y^3+z^3=6k^3$

(b) $xyz=6k^3$

(c) $x^2+y^2+z^2=4k^2$

(d) $x^{-2}+y^{-2}+z^{-2}=4k^{-2}$

26. If P be any point on the plane $lx+my+nz=p$ and Q be a point on the line OP such that $OP.OQ=p^2$. The locus of the point Q is

(a) $lx+my+nz-p=x^2+y^2+z^2$

(b) $lx+my+nz=p(x^2+y^2+z^2)$

(c) $p(lx+my+nz)=x^2+y^2+z^2$

(d) $x^2+y^2+z^2=p^2$

27. The angle between any two diagonals of a cube is

(a) $\cos^{-1}\frac{1}{2}$ (b) $\cos^{-1}\frac{1}{3}$

(c) $\cos^{-1}\frac{1}{4}$ (d) $\frac{\pi}{2}$

28. Through a point $P(h, k, l)$ a plane is drawn at right angles to OP to meet the co-ordinate axes in A, B and C. If $OP = p$, then the area of ΔABC is :

(a) $\frac{p^2hk}{l^2}$ (b) $\frac{p^3l}{3hk}$

(c) $\frac{p^2t^2}{2hk}$ (d) $\frac{p^5}{2hkl}$

29. Equation of the sphere with centre in the positive octant which passes through the circle $x^2+y^2=4, z=0$ and is cut by the plane $x+2y+2z=0$ in a circle of radius 3 is :

(a) $x^2+y^2+z^2-6x-4=0$

(b) $x^2+y^2-6y-4=0$

(c) $x^2+y^2+z^2-6z-4=0$

(d) $x^2+y^2-6x-6y-4=0$

MARK YOUR RESPONSE					
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ
	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ

30. The locus of the point, the sum of squares of whose distances from the planes : $x-z=0$, $x-2y+z=0$ and $x+y+z=0$ is 36 is :

(a) $x^2+y^2+z^2=6$

(b) $x^2+y^2+z^2=36$

(c) $x^2+y^2+z^2=216$

(d) $x^{-2}+y^{-2}+z^{-2}=\frac{1}{36}$

31. The smaller radius of the sphere passing through (1, 0, 0), (0, 1, 0) and (0, 0, 1) is :

(a) $\sqrt{\frac{3}{5}}$ (b) $\sqrt{\frac{3}{8}}$

(c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{5}{12}}$

32. A plane passes through a fixed point (a, b, c). The locus of the foot of the perpendicular to it from the origin is a sphere of radius :

(a) $\sqrt{a^2+b^2+c^2}$ (b) $\frac{1}{2}\sqrt{a^2+b^2+c^2}$

(c) $a^2+b^2+c^2$ (d) none of these

33. The line, $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy=c^2$, $z=0$ if c is equal to

(a) ± 1 (b) $\pm\frac{1}{3}$

(c) $\pm\sqrt{5}$ (d) none of these

34. The direction cosines of a line satisfy the relation $\lambda(\ell+m)=n$, $mn+nl+lm=0$. The value of λ, for which the two lines are perpendicular to each other is

(a) 1 (b) 2

(c) 1/2 (d) –1

35. O is the centre, AB and BC are two diagonals of the adjacent faces of a rectangular box. If angles AOB, BOC and COA are θ, ϕ and ψ respectively, then $\cos\theta+\cos\phi+\cos\psi$ is equal to

(a) 1 (b) 0 (c) – 1 (d) $\frac{3}{2}$

36. Projection of the line $x+y+z-3=0=2x+3y+4z-6$ on the plane $z=0$ is

(a) $\frac{x}{-2}=\frac{y-6}{1}=\frac{z}{0}$ (b) $\frac{x}{1}=\frac{y-6}{-2}=\frac{z}{0}$

(c) $\frac{x}{1}=\frac{y-6}{2}=\frac{z}{0}$ (d) none of these

37. The volume of tetrahedron included between the plane $3x+4y-5z-60=0$ and coordinate planes is

(a) 600 cube units (b) 300 cube units

(c) 3600 cube units (d) 1200 cubic units

38. The equations $ax+a^2y+z=0$, $bx+b^2y+z=0$ and $cx+c^2y+z=0$ have only solution (0, 0, 0).
If the coefficients a, b, c are in G. P., then the common ratio of G.P. can not be equal to

(a) – 1 (b) 2

(c) – 3 (d) – 2

39. Let $A(1,-1,\ 1)$ and $B(-1,1,-1)$ be the vertices of triangle ABC such that $\angle A=\angle B$. The locus of the vertex C is

(a) $x+2y+z=0$ (b) $2x+y-z=0$

(c) $x-y+z=0$ (d) $x-2y+z=0$

40. The locus of a point which moves so that its distance from the line $x=y=-z$ is twice its distance from the plane $x-y+z=1$ is

(a) $x^2+y^2+z^2-5yz+3zx-3xy-4x+4y-4z+2=0$

(b) $x^2+y^2+z^2+5yz-3zx+3xy-4x-4y-4z+2=0$

(c) $x^2+y^2+z^2=2$

(d) none of these

41. If lines $x=y=z$, $x=\frac{y}{2}=\frac{z}{3}$ and third line passing through (1, 1, 1) form a triangle of area $\sqrt{6}$ units then point of intersection of third line with second line will be

(a) $(1,2,3)$ (b) $(2,4,6)$

(c) $\left(\frac{4}{3},\frac{8}{3},\frac{12}{3}\right)$ (d) $(-1,-2,-3)$

42. A rectangle $ABCD$ of dimension r and $2r$ is folded along diagonal BD such that planes ABD and CBD are perpendicular to each other, then the distance AC' (in new position is)

(a) $\sqrt{3}\,r$ (b) $\sqrt{85}\,r$

(c) $\frac{\sqrt{85}}{5}r$ (d) $\sqrt{\frac{17}{5}}\,r$

MARK YOUR RESPONSE					
	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ
	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ
	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ		

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

Consider a right pyramid on a square base ABCD of side 2α. The height of the vertex V above the base is a. Attempt following questions for this figure.

1. The line of the greatest slope in any of the triangular plane is inclined the base at the angle

(a) 30° (b) 45°
(c) 60° (d) 75°

2. The shortest distance of the vertex A from the edge BV is

(a) $\frac{2\sqrt{6}}{3}a$ (b) $2\sqrt{3}\,a$
(c) $\frac{\sqrt{6}}{2}a$ (d) a

3. The angle between two adjacent triangular faces is

(a) 90° (b) 75°
(c) 120° (d) 60°

PASSAGE-2

A tetrahedron is a three dimensional figure bounded by four non coplanar triangular planes. So, a tetrahedron has four non-coplanar points as its vertices.

Suppose a tetrahedron has points A, B, C, D as its vertices, which have coordinates (x_1, y_1, z_1) (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) respectively in a rectangular three-dimensional space. Then the coordiantes of its centroid are

$\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$. The circumcentre of the tetrahedron is the centre of a sphere pasing through its vertices. So, this is a point equidistant from each of the vertices of tetrahedron.

Let a tetrahedron has three of its vertices represented by the points (0, 0, 0), (6, –5, –1) and (–4, 1, 3) and its centroid lies at the point (1, –2, 5). Now answer the following questions.

4. The coordinate of the fourth vertex of the tetrahedron is

(a) (2, –4, 18) (b) (1, –2, 13)
(c) (–2, 4, –2) (d) (1, –1, 1)

5. The equation of the triangular plane of tetrahedron that contains the given vertices is

(a) $x-2y+z=0$ (b) $5x-3y-2z=0$
(c) $x+y+z=0$ (d) $x+2y+3z=0$

6 The coordinates of the centre of the sphere circumsbring the tetrahedron is

(a) $\left(\frac{8}{7}, \frac{47}{7}, 8\right)$ (b) $\left(\frac{8}{7}, -\frac{45}{7}, -8\right)$
(c) $\left(-\frac{8}{7}, \frac{45}{7}, 8\right)$ (d) $\left(\frac{8}{7}, -\frac{45}{7}, 8\right)$

PASSAGE-3

If the projections of three points A, B, C on a given plane are A', B', C', then $\Delta A'B'C' = \cos\theta(\Delta ABC)$, where θ is the angle between the planes ABC and A'B'C' (i.e., the angle that the positive direction of a normal to one makes with the positive direction of a normal to the other). In general, if A_0 is the area of any plane curve and A is the area of its projection on any given plane, then $A = \cos\theta\, A_0$.

7. Suppose AB is a diameter of a circle and P is a plane through AB making an angle θ with the plane of the circle. If diameter of the circle be $2a$, then the eccentricity of the curve of projection of the circle on P is

(a) $\sin\theta$ (b) $\frac{2a\sin\theta}{1+a}$
(c) $\frac{a\cos\theta}{1+a}$ (d) $1+\sin^2\theta$

8. A plane makes intercepts OA, OB, OC whose measures are a, b, c on the axes OX, OY, OZ. The area of the triangle ABC is

(a) $\sqrt{a^4+b^4+c^4-a^2b^2-b^2c^2-c^2a^2}$
(b) $\frac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}$
(c) $a^2+b^2+c^2-bc-ca-ab$
(d) $\frac{1}{4}\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ		

C — REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. **Statement-1 :** The shortest distance between the skew lines $\bar{r} = \bar{a} + \alpha\bar{b}$ and $\bar{r} = \bar{c} + \beta\bar{d}$ is $\dfrac{|[\bar{a} - \bar{c}\ \bar{b}\ \bar{d}]}{|\bar{b} \times \bar{d}|}$

Statement-2 : Two lines are skew lines if there exists no plane passing through them.

2. A right pyramid has its base a square ABCD and vertex V. A variable point P is taken on the edge VB and Q is the mid point of BC.

Statement-1 : The sum AP + PQ is the least if A, P and Q are collinear, when the plane VBQ is rotated about the edge VB to be in the same plane with the plane VAP

Statement-2 : When the planes VAP and VBQ form a single plane, AP and PQ are common perpendicular to edge VB.

3. In a three dimensional coordinate system P, Q and R are images of a point $A(a, b, c)$ in xy-plane, yz-plane and zx-plane respectively. G is the centroid of the triangle PQR and O is the origin.

Statement 1 : The points A, G and O are collinear.

Statement 2 : G is the midpoint of A and O.

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

D — MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. Let PM be the perpendicular from the point (1, 2, 3) to x–y plane. If OP makes an angle θ with the positive direction of the z-axis and OM makes and angle ϕ with the positive direction of x-axis, where O is the origin then (θ and ϕ are acute angles)

(a) $\tan\theta = \dfrac{\sqrt{5}}{3}$
(b) $\sin\theta\sin\phi = \dfrac{2}{\sqrt{14}}$
(c) $\tan\phi = 2$
(d) $\cos\theta\cos\phi = \dfrac{1}{\sqrt{14}}$

2. The value(s) of λ, for which the triangle with vertices (6, 10, 10), (1, 0, –5) and (6,–10,λ) will be a right angled triangle is / are :

(a) 1
(b) $\dfrac{70}{3}$
(c) 35
(d) 0

3. The direction ratios of lines intersecting the line $\dfrac{x-3}{2} = \dfrac{y-3}{1} = \dfrac{z}{1}$ at angles 60° are

(a) 1, 1, 2
(b) 1, 2, –1
(c) 1, –1, 2
(d) 1, –2, 1

4. Two coplanar lines having d.c.'s l_1, m_1, n_1 and l_2, m_2, n_2 are inclined at an angle θ. The d.c.'s of the lines bisecting the angle between them are

(a) $\dfrac{l_1 + l_2}{2\cos\frac{\theta}{2}}, \dfrac{m_1 + m_2}{2\cos\frac{\theta}{2}}, \dfrac{n_1 + n_2}{2\cos\frac{\theta}{2}}$

(b) $\dfrac{l_1 - l_2}{2\cos\frac{\theta}{2}}, \dfrac{m_1 - m_2}{2\cos\frac{\theta}{2}}, \dfrac{n_1 - n_2}{2\cos\frac{\theta}{2}}$

(c) $\dfrac{l_1 + l_2}{2\sin\frac{\theta}{2}}, \dfrac{m_1 + m_2}{2\sin\frac{\theta}{2}}, \dfrac{n_1 + n_2}{2\sin\frac{\theta}{2}}$

(d) $\dfrac{l_1 - l_2}{2\sin\frac{\theta}{2}}, \dfrac{m_1 - m_2}{2\sin\frac{\theta}{2}}, \dfrac{n_1 - n_2}{2\sin\frac{\theta}{2}}$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	

5. If $OABC$ is a tetrahedron such that $OA^2 + BC^2 = OB^2 + CA^2 = OC^2 + AB^2$ then
(a) OA is perpendicular to BC
(b) OB is perpendicular to CA
(c) OC is perpendicular to AB
(d) AB is perpendicular to BC

6. The equation of plane bisecting the angle between the planes $x = 0$ and $z = 0$ and passing through (1, 2, 3) is
(a) $x + y - 4 = 0$ (b) $x + z - 4 = 0$
(c) $x - z + 2 = 0$ (d) $x - y + 2 = 0$

MARK YOUR RESPONSE	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ			

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. **Observe the following columns :**

Column I	**Column II**
(A) The vector equations of a line parallel to the vector $\bar{b}$ and passing through the point A whose position vector is $\bar{a}$ is	p. $\bar{r} = \alpha\bar{a} + \beta\bar{b}$
(B) The vector equation of a line passing through the points whose position vectors are $\bar{a}$, $\bar{b}$ is	q. $\bar{r} = (1-\alpha-\beta)\bar{a} + \alpha\bar{b} + \beta\bar{c}$
(C) The vector equation of the plane passing through origin and parallel to vectors $\bar{a}$, $\bar{b}$ is	r. $\bar{r} = \bar{a} + \alpha(\bar{b} - \bar{a})$
(D) The vector equation of the plane passing through points with P.V.s $\bar{a}$, $\bar{b}$ and $\bar{c}$ is	s. $\bar{r} = a + \alpha\bar{b}$
	t. $\bar{r} = \alpha(\bar{a} - \bar{b}) + \bar{b}$, Where α, β are scalars

2. **Observe the following columns :**

Column I	**Column II**
(A) If the coordinates of the mid-points of the sides BC, CA, AB of ΔABC are $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ respectively then $\dfrac{AB^2 + BC^2 + CA^2}{a^2 + b^2 + c^2}$ is equal to	p. 3/4
(B) The distance of the image of the point (1, –2, 3) in the plane $x - y + z = 5$ from the origin is equal to	q. –3
(C) If θ be the angle between a diagonal of a cube and an edge of the cube intersecting the diagonal then tanθ is equal to	r. 8
(D) If the equation $px^2 + y^2 + qz^2 + 2yz + zx + 3xy = 0$ represents a pair of mutually perpendicular planes then q is equal to	s. $\sqrt{2}$
	t. $5\sqrt{2}$

MARK YOUR RESPONSE

1.

	p	q	r	s	t
A	○	○	○	○	○
B	○	○	○	○	○
C	○	○	○	○	○
D	○	○	○	○	○

2.

	p	q	r	s	t
A	○	○	○	○	○
B	○	○	○	○	○
C	○	○	○	○	○
D	○	○	○	○	○

3. **Observe the following columns :**

Column I		**Column II**
(A) If the plane $ax - by + cz = d$ contains the line $\dfrac{x-a}{a} = \dfrac{y-2d}{b} = \dfrac{z-c}{c}$, then $\dfrac{b}{d}$ is equal to	p.	0
(B) The distance of the point (1, –2, 3) from the plane $x - y + z - 5 = 0$ measured parallel to $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z-1}{-6}$ is equal to	q.	1
(C) If the straight lines $\dfrac{x-2}{1} = \dfrac{y-3}{1} = \dfrac{4-z}{k}$ and $\dfrac{x-1}{k} = \dfrac{y-4}{2} = \dfrac{z-5}{1}$ intersect then k is equal to	r.	2
(D) If a line makes an angle θ with *x* and *y*-axis then cot θ can be equal to	s.	$\dfrac{1}{3}$
	t.	–3

MARK YOUR RESPONSE

3.

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.

0	●	0	0
1	1	1	1
2	2	2	●
3	3	3	3
4	4	4	4
5	5	5	5
●	6	6	6
7	7	7	7
8	8	8	8
9	9	●	9

1. The length of shortest distance between the two lines $\dfrac{x+3}{-4} = \dfrac{y-6}{3} = \dfrac{z}{2}$ and $\dfrac{x+2}{-4} = \dfrac{y}{1} = \dfrac{z-7}{1}$ is :

2. The shortest distance between the z-axis and the line $x + y + 2z - 3 = 0$, $2x + 3y + 4z - 4 = 0$ is :

3. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then the value of $a^2 + b^2 + c^2 + 2abc$ is :

4. *P* is a point and *PM*, *PN* are perpendiculars from *P* to the ZX and *XY* planes respectively. If *OP* makes angles θ, α, β, γ with the plane *OMN* and the *XY*, *YZ*, *ZX* plane respectively then $\sin^2\theta(\operatorname{cosec}^2\alpha + \operatorname{cosec}^2\beta + \operatorname{cosec}^2\gamma)$ is equal to

5. P is a point on the plane ax + by + cz = d. A point Q is taken on the line OP such OP. OQ = d^2. If the locus of Q satisfies $\dfrac{d(ax+by+cz)}{x^2+y^2+z^2} = k$ then *k* is equal to

MARK YOUR RESPONSE

1. 2. 3. 4. 5.

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Answerkey

A — SINGLE CORRECT CHOICE TYPE

1	(b)	**11**	(c)	**21**	(c)	**31**	(c)	**41**	(b)
2	(d)	**12**	(d)	**22**	(b)	**32**	(b)	**42**	(c)
3	(c)	**13**	(a)	**23**	(a)	**33**	(c)		
4	(a)	**14**	(d)	**24**	(a)	**34**	(b)		
5	(c)	**15**	(d)	**25**	(b)	**35**	(c)		
6	(b)	**16**	(c)	**26**	(c)	**36**	(b)		
7	(c)	**17**	(c)	**27**	(b)	**37**	(a)		
8	(a)	**18**	(c)	**28**	(d)	**38**	(a)		
9	(c)	**19**	(b)	**29**	(c)	**39**	(c)		
10	(b)	**20**	(c)	**30**	(b)	**40**	(a)		

B — COMPREHENSION TYPE

1	(b)	**3**	(c)	**5**	(c)	**7**	(a)
2	(a)	**4**	(a)	**6**	(d)	**8**	(b)

C — REASONING TYPE

1	(b)	**2**	(c)	**3**	(c)

D — MULTIPLE CORRECT CHOICE TYPE

1	(a, b, c)	**3**	(b, c)	**5**	(a, b, c)
2	(b, d)	**4**	(a, d)	**6**	(b,c)

E — MATRIX-MATCH TYPE

1. A-s; B-r; C-p; D-q

2. A-r, B-t, C-s, D-p,q

3. A-r, B-q, C-p,t, D-p,q,s

F — NUMERIC/INTEGER ANSWER TYPE

1	9	**2**	2	**3**	1	**4**	1	**5**	1

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(b)** The *DC*s of the line are

$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma.$

Since, $l^2 + m^2 + n^2 = 1.$

$\therefore\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow\ 1 - \sin^2\alpha + 1 - \sin^2\beta + 1\sin^2\gamma = 1$

$\Rightarrow\ \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

2. **(d)** Let l, m, n be the direction cosines of the given vector, then $l^2 + m^2 + n^2 = 1$

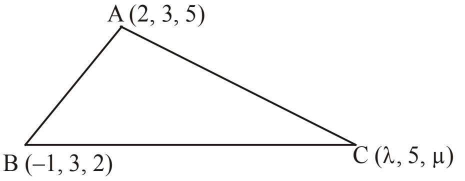
If $l = \cos 30° = \frac{\sqrt{3}}{2},\ m = \cos 45° = \frac{1}{\sqrt{2}}$, then

$\frac{3}{4} + \frac{1}{2} + n^2 = 1 \Rightarrow n^2 = -\frac{1}{4}$, which is not possible. So, such a line cannot exist.

3. **(c)** Mid point of *BC* is $\left(\frac{\lambda - 1}{2}, 4, \frac{2 + \mu}{2}\right)$

Direction ratios of median through *A* are

$\frac{\lambda - 1}{2} - 2, 4 - 3, \frac{2 + \mu}{2} - 5$, i.e., $\frac{\lambda - 5}{2}, 1, \frac{\mu - 8}{2}$



The median is equally inclined to axes, so, the direction ratios must be equal, so

$\frac{\lambda - 5}{2} = 1 = \frac{\mu - 8}{2} \Rightarrow \lambda = 7,\ \mu = 10$

4. **(a)** Let l, m, n be the *DC*s of the given line. Then as it makes an acute angle with x-axis, therefore $l > 0$. The line passes through $(6, -7, -1)$ and $(2, -3, 1)$, therefore its *DR*s are $6 - 2, -7 + 3, -1 - 1$ or $4, -4, -2$

Hence *DC*s of the given line are $\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$

5. **(c)** Let the edges *OA*, *OB*, *OC* of the unit cube along *OX*, *OY*, *OZ* respectively. Since

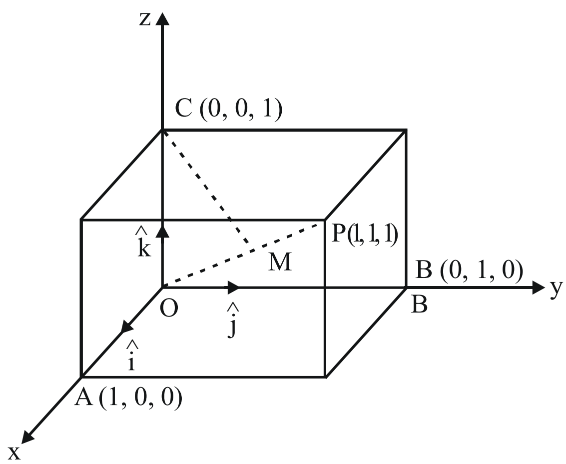
$OA = OB = OC = 1$ unit

$\therefore\ \overrightarrow{OA} = i,\ \overrightarrow{OB} = \hat{j},\ \overrightarrow{OC} = \hat{k}$

Let *CM* be perpendicular from the corner *C* on the diagonal *OP*. The vector equation of *OP* is; $\vec{r} = \lambda(\hat{i} + \hat{j} + \hat{k})$

$\therefore$ *OM* = projection of $\overrightarrow{OC}$ on $\overrightarrow{OP} = \overrightarrow{OC} \cdot \overrightarrow{OP} =$

$\hat{k} \cdot \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} = \frac{1}{\sqrt{3}}$



Now $OC^2 = OM^2 + CM^2$

$\Rightarrow CM^2 = |OC^2| - OM^2 = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow CM = \sqrt{\frac{2}{3}}$

6. **(b)** Since $\overrightarrow{OP}$ has projections $\frac{13}{5}, \frac{19}{5}$ and $\frac{26}{5}$ on the co-ordinate axes, therefore $\overrightarrow{OP} = \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k}$

Suppose *P* divides the join of *Q* (2, 2, 4) and *R* (3, 5, 6) in the ration $\lambda : 1$. Then the position vector of *P* is

$\left(\frac{3\lambda + 2}{\lambda + 1}\right)\hat{i} + \left(\frac{5\lambda + 2}{\lambda + 1}\right)\hat{j} + \left(\frac{6\lambda + 4}{\lambda + 1}\right)\hat{k}$

$\therefore\ \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k}$

$= \left(\frac{3\lambda + 2}{\lambda + 1}\right)\hat{i} + \left(\frac{5\lambda + 2}{\lambda + 1}\right)\hat{j} + \left(\frac{6\lambda + 4}{\lambda + 1}\right)\hat{k}$

$$\Rightarrow \frac{3\lambda+2}{\lambda+1}=\frac{13}{5}, \frac{5\lambda+2}{\lambda+1}=\frac{19}{5}, \frac{6\lambda+4}{\lambda+1}=\frac{26}{5}$$

$$\Rightarrow \lambda=\frac{3}{2}$$

7. **(c)** Eliminating 't' from the given equations, we get the equation of the path,

$$\frac{x}{2}=\frac{y}{-4}=\frac{z}{4} \text{ or } \frac{x}{1}=\frac{y}{-2}=\frac{z}{2}$$

Thus, the path of the point represents a straight line passing through the origin. For $t = 10$ hour we have : $x = 20, y = -40, z = 40$

and $|\vec{r}| = |\overrightarrow{OM}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{400+1600+1600} = 60$ km

8. **(a)** Let D be the foot of perpendicular and let it divides BC in the ratio $\lambda : 1$. Then the co-ordinates of

D are $\frac{3\lambda+4}{\lambda+1}, \frac{5\lambda+7}{\lambda+1}, \frac{3\lambda+1}{\lambda+1}$.

Now $\overrightarrow{AD} \perp \overrightarrow{BC} = \overrightarrow{AD}.\overrightarrow{BC} = 0$

$$\Rightarrow -(2\lambda+3)-2(5\lambda+7)-4=0 \Rightarrow \lambda=-\frac{7}{4}$$

$\therefore$ Co-ordinates of $D \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$.

9. **(c)** Let the cube be shown in the figure. Where, four diagonals are OP, AL, BM and CN.
and $A(a, 0, 0)$, $B(0, a, 0)$, $C(0, 0, a)$, $L(0, a, a)$, $M(a, 0, a)$, $N(a, a, 0)$ and P (a, a, a) hence, direction cosines of OP are

$$\frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}$$

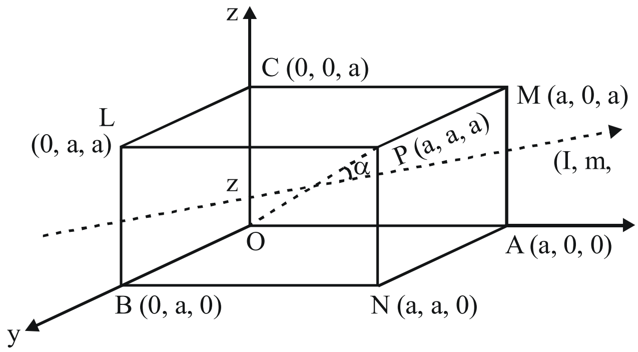
$$= \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Similarly,

The direction cosines of AL are $-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

The direction cosines of BM are $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

The direction cosines of CN are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$



If the direction cosines of the line be l, m, n then

$$\cos\alpha=\frac{l+m+n}{\sqrt{3}}, \cos\beta=\frac{-l+m+n}{\sqrt{3}},$$

$$\cos\gamma=\frac{l-m+n}{\sqrt{3}}, \cos\delta=\frac{l+m-n}{\sqrt{3}}$$

$$\therefore \cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$$

$$=\frac{1}{3}\{(l+m+n)^2+(-l+m+n)^2$$

$$+(l-m+n)^2+(l+m-n)^2\}$$

$$=\frac{1}{3}[4(l^2+m^2+n^2)]=\frac{4}{3}$$

10. **(b)** Let l, m, n be the direction cosines of the line perpendicular to both the given lines.

$\therefore \quad ll_1+mm_1+nn_1=0$

$ll_2+mm_2+nn_2=0$

Solving them, we get

$$\Rightarrow \frac{l}{m_1n_2-m_2n_1}=\frac{m}{n_1l_2-n_2l_1}=\frac{n}{l_1m_2-l_2m_1}=k$$

$\therefore l=k(m_1n_2-m_2n_1);$

$m=k(n_1l_2-n_2l_1); n=k(l_1m_2-l_2m_1)$

Squaring and adding; we have $l^2+m^2+n^2$

$$=k^2\{(m_1n_2-m_2n_1)^2+(n_1l_2-n_2l_1)^2+(l_1m_2-l_2m_1)^2$$

$\Rightarrow l = k^2(\sin^2\theta)$

where θ is the angle between the given lines as we know ;

$$\sin\theta=\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}$$

where a_1, b_1, c_1; a_1, b_2, c_2 are direction cosines.

$\Rightarrow 1=k^2.1$ (as $\theta=90°$ given) $\Rightarrow k=\pm 1$

Hence, direction cosines of a line perpendicular to both of them are : $m_1n_2-m_2n_1, n_1l_2-n_2l_1, l_1m_2-l_2m_1$

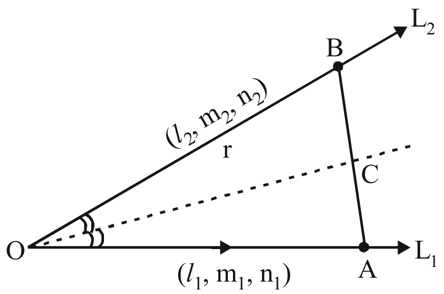
11. (c) Let the lines be OA and OB taking two points A and B such that $OA = OB = r$

Let C be the mid-point of AB then OC is the bisector of the angle AOB

Now, co-ordinates of A are $(l_1 r, m_1 r, n_1 r)$ co-ordinate of B are $(l_2 r, m_2 r, n_2 r)$

$\therefore$ The co-ordinates of C are

$$\frac{(l_1+l_2)r}{2}, \frac{(m_1+m_2)r}{2}, \frac{(n_1+n_2)r}{2}$$



Hence, the direction cosines of OC are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2$

12. (d) The equations of straight lines can be rewritten as

$$x = ay + b, z = cy + d \quad \Rightarrow \quad \frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$$

and $x = a'y + b'$, $z = c'y + d'$

$$\Rightarrow \frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'}$$

The above lines are perpendiuclar if $aa' + 1.1 + c.c' = 0 \Rightarrow aa' + cc' + 1 = 0$

13. (a) Let equation of sphere be

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

The line $y = x, z = c$ intersects it at the point where

$$2x^2 + 2(u+v)x + (c^2 + 2wc + d) = 0$$

Since the line touch the sphere, the roots of this equation are coincident.

$$\therefore \quad 4(u+v)^2 - 8(c^2 + 2wc + d) = 0 \text{.......(1)}$$

Similarly for second line

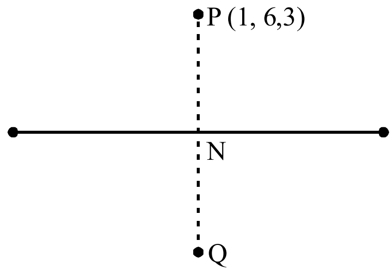
$$4(u-v)^2 - 8(c^2 - 2wc + d) = 0 \text{.......(2)}$$

Subtracting (2) from (1), we get $4uv - 8wc = 0$

$\Rightarrow uv = 2wc$

$\therefore$ Locus of centre $(-u, -v, -w)$ is $xy = -2cz$

14. (d) Any point on the given line is $(\lambda, 1+2\lambda, 2+3\lambda)$. Let it represents the co-ordinates of foot N of perpendicular from $P(1, 6, 3)$ then direction ratios of PN are $\lambda - 1, 2\lambda - 5, 3\lambda - 1$, now PN is perpendicular to the given line



So, $1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0 \Rightarrow \lambda = 1$.

So co-ordinates of N are (1, 3, 5)

If $Q(x_1, y_1, z_1)$ be the image of P, then N is mid point of PQ

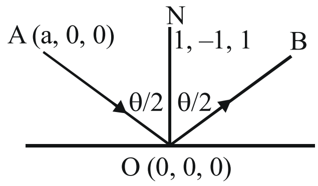
$$\therefore 1 = \frac{x_1+1}{2}, 3 = \frac{y_1+6}{2}, 5 = \frac{z_1+3}{2}$$

$\Rightarrow x_1 = 1, y_1 = 0, z_1 = 7$

15. (d) Let the source of light be situated at A (a, 0, 0), where $a \neq 0$. Let OA be the incident ray and OB the reflected ray. ON is the normal to the mirror at O

$$\angle AON = \angle NOB = \frac{\theta}{2} \text{ (say)}$$

DRs of OA are a, 0, 0 and so its DCs are 1, 0, 0



DCs of ON are $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$$\therefore \cos\frac{\theta}{2} = \frac{1}{\sqrt{3}}$$

Let l, m, n be the DCs of the reflected ray OB. Then

$$\frac{\ell+1}{2\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}}, \quad \frac{m+0}{2\cos\frac{\theta}{2}} = -\frac{1}{\sqrt{3}} \text{ and } \frac{n+0}{2\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow l = \frac{2}{3} - 1, \ m = \frac{-2}{3}, \ n = \frac{2}{3}$$

$$\Rightarrow l = -\frac{1}{3}, \ m = -\frac{2}{3}, \ n = \frac{2}{3}$$

Hence, DCs of the reflected ray, are $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

16. **(c)** The equation of the plane containing the line $\frac{x+1}{-3.}=\frac{y-3}{2}=\frac{z+2}{1}$ is

$a(x+1)+b(y-3)+c(z+2)=0$ ...... (1)

where $-3a+2b+c=0$ ...... (2)

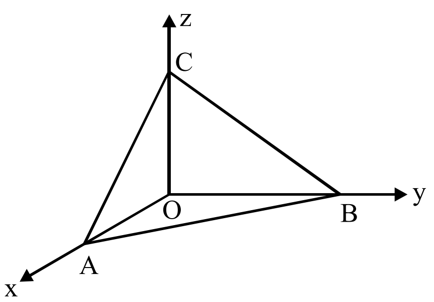
This passes through (0, 7, – 7) $\therefore$

$a+4b-5c=0$ ...... (3)

From (2) and (3) $\frac{a}{-14}=\frac{b}{-14}=\frac{c}{-14}$ or $\frac{a}{1}=\frac{b}{1}=\frac{c}{1}$.

So the required plane is $x+y+z=0$

17. **(c)** Let the equation of the plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. Then the co-ordinates of *A, B* and *C* are respectively (*a*, 0, 0), (0, *b*, 0) and (0, 0, *c*). So the centroid of the triangle *ABC* is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$.



The co-ordinates of centroid are given (*p*, *q*, *r*)

$\therefore$ $a=3p$, $b=3q$, $c=3r$. So, the equation of the required plane is $\frac{x}{3p}+\frac{y}{3q}+\frac{z}{3r}=1$

18. **(c)** We have, $\vec{r}=(1+\lambda-\mu)\hat{i}+(2-\lambda)\hat{j}+(3-2\lambda+2\mu)\hat{k}$

$\Rightarrow \vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-\hat{j}-2\hat{k})+\mu(-\hat{i}+2\hat{k})$

Which is a plane passing through $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and parallel to the vectors $\vec{b}=\hat{i}-\hat{j}-2\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{k}$

Therefore it is $\perp$ to the vector $\vec{n}=\vec{b}\times\vec{c}=2\hat{i}-\hat{k}$

Hence its vector equation is $(\vec{r}-\vec{a}).\vec{n}=0$

$\Rightarrow \vec{r}.\vec{n}=\vec{a}.\vec{n} \Rightarrow \vec{r}(-2\hat{i}-\hat{k})=-2-3$

$\Rightarrow \vec{r}(2\hat{i}+\hat{k})=5$

So, the Cartesian equation is

$(x\hat{i}+y\hat{j}+z\hat{k}).(2\hat{i}+\hat{k})=5$ or $2x+z=5$

19. **(b)** If the equation of the plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, then it cuts axes at $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$. Given

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\pm 3p$$

$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{9p^2}$ .... (1)

Let the centroid be (α,β,λ) then

$\alpha=\frac{a}{3}, \beta=\frac{b}{3}, \gamma=\frac{c}{3}$

Then, from (1) $\frac{1}{9\alpha^2}+\frac{1}{9\beta^2}+\frac{1}{9\gamma^2}=\frac{1}{9p^2}$

$\Rightarrow$ locus of (α,β,γ) is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

20. **(c)** The given line is $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ or $\vec{r}=\vec{a}+\lambda\vec{b}$ where $\vec{a}=2\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+4\hat{k}$.

The given plane is $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ or $\vec{r}\cdot\vec{n}=d$.

Where $\vec{n}=(\hat{i}+5\hat{j}+\hat{k})$.

Since $\vec{b}\cdot\vec{n}=(\hat{i}-\hat{j}+4\hat{k}).(\hat{i}+5\hat{j}+\hat{k})=1-5+4=0$.

Therefore , the line is parallel to the plane. Thus, the distance between the line and the plane is equal to the length of the $\perp$ from a point $\vec{a}=2\hat{i}-2\hat{j}+3\hat{k}$ on the line to the given plane .

Hence, the required distance

$$=\left|\frac{(2\hat{i}-2\hat{j}+3\hat{k}).(\hat{i}-5\hat{j}+\hat{k})-5}{\sqrt{1+25+1}}\right|$$

$$=\left|\frac{2-10+3-5}{\sqrt{27}}\right|=\frac{10}{3\sqrt{3}}$$

21. **(c)** Equation of a plane through the line of intersection of given planes is

$$ax+by+cz+d+\lambda(a'x+b'y+c'z+d')=0$$

$(a+\lambda a')x+(b+\lambda b')y+(c+\lambda c')z+(d+\lambda d')=0$

It is a parallel to $y=0, z=0$

i.e. x-axis whose direction ratios are 1, 0, 0

$\therefore$ $1(a+\lambda a')+0(b+\lambda b')+0(c+\lambda c')=0$

$\Rightarrow \lambda=-\frac{a}{a'}$

Hence the required plane is

$y(a'b-ab')+z(a'c-ac')+(a'd-ad')=0$

22. (b) Given plane is $x-y-z=9$ ...... (1)

Given line AB is $\frac{x-2}{2}=\frac{y+2}{3}=\frac{z-6}{-6}$ ...... (2)

Eq. of line passing through (1, 0, –3) and parallel to $\frac{x-2}{2}=\frac{y+2}{3}=\frac{z-6}{-6}$ is :

$$\frac{x-1}{2}=\frac{y-0}{3}=\frac{z+3}{-6}=\text{r} \text{ .... (3)}$$

Co-ordinates of any point on (3) may be given as $P(2r+1,\ 3r,\ -6r-3)$

If P is intersection of (1) and (3) then it must lie on (1);

$$(2r+1)-(3r)-(-6r-3)=9$$

$$2r+1-3r+6r+3=9 \Rightarrow \text{r}=1$$

∴ Co-ordinate of P are (3, 3, –9)

∴ Required distance = distance between (1, 0, –3) and $(3,3,-9)=7$

23. (a) Let $O(0,0,0), A(3,4,7),$ and $(5,2,6)$ be the given points.

Area or $\Delta OAB=\frac{1}{2}OA.OB.\sin(\angle AOB)$

Now $OA=\sqrt{3^2+4^2+7^2}=\sqrt{74}$

$OB=\sqrt{5^2+2^2+6^2}=\sqrt{65}$

Also DCs of the line OA and OB are $\frac{3}{\sqrt{74}},\frac{4}{\sqrt{74}},\frac{7}{\sqrt{74}}$

and $\frac{5}{\sqrt{65}},\frac{2}{\sqrt{65}},\frac{6}{\sqrt{65}}$

$\cos\angle AOB$

$$=\frac{3}{\sqrt{74}}\times\frac{5}{\sqrt{65}}+\frac{4}{\sqrt{74}}\times\frac{2}{\sqrt{65}}+\frac{7}{\sqrt{74}}\times\frac{6}{\sqrt{65}}=\sqrt{\frac{65}{74}}$$

∴ $\sin\angle AOB=\sqrt{1-\frac{65}{74}}=\frac{3}{\sqrt{74}}$

∴ Required area $=\frac{1}{2}\times\sqrt{74}\times\sqrt{65}\times\frac{3}{\sqrt{74}}=\frac{3}{2}\sqrt{65}$

24. (a) Given planes are $ax+by=0$ ....(i)

and $z=0$ ....(ii)

∴ Equation of any plane passing through the line of intersection of planes (i) and (ii) may be taken as;

$$ax+by+\lambda z=0 \text{ ......(iii)}$$

The direction cosines of a normal to the plane (iii) are :

$$\frac{a}{\sqrt{a^2+b^2+\lambda^2}},\frac{b}{\sqrt{a^2+b^2+\lambda^2}},\frac{\lambda}{\sqrt{a^2+b^2+\lambda^2}}$$

The direction cosines of a normal to the plane (i) are :

$$\frac{a}{\sqrt{a^2+b^2}},\frac{b}{\sqrt{a^2+b^2}},0$$

Since the angle between the plane (i)and (iii) is α,

$$\cos\alpha=\frac{a.a+b.b+\lambda.0}{\sqrt{a^2+b^2+\lambda^2}\sqrt{a^2+b^2}}=\sqrt{\frac{a^2+b^2}{a^2+b^2+\lambda^2}}$$

$$\Rightarrow \lambda^2\cos^2\alpha=a^2(1-\cos^2\alpha)+b^2(1-\cos^2\alpha)$$

$$\Rightarrow \lambda^2=\frac{(a^2+b^2)\sin^2\alpha}{\cos^2\alpha}$$

$$\Rightarrow \lambda=\pm\sqrt{a^2+b^2}\tan\alpha$$

Putting in (iii) we get, eq. of plane as

$$ax+by\pm z\sqrt{a^2+b^2}\tan\alpha=0$$

25. (b) Let the variable plane intersects the co-ordinate axes at $A(a,0,0), B(0,b,0), C(0,0,c)$.

Then the equation of the plane will be,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \text{ ....(1)}$$

let $P(\alpha,\beta,\gamma)$ be the centroid of tetrahedron *OABC,*

then : $\alpha=\frac{a}{4},\beta=\frac{b}{4},\gamma=\frac{c}{4}$ or $a=4\alpha,b=4\beta,c=4\gamma$

$\Rightarrow$ Volume of tetrahedron $=\frac{1}{3}(\text{Area of }\Delta AOB).OC$

$$\Rightarrow 64k^3=\frac{1}{3}\left(\frac{1}{2}ab\right)c=\frac{abc}{6}$$

$$\Rightarrow 64k^3=\frac{(4\alpha)\cdot(4\beta)\cdot(4\gamma)}{6} \Rightarrow k^3=\frac{\alpha\beta\gamma}{6}$$

∴ Required locus of P (α, β, γ) is $xyz=6k^3$

26. (c) Let P be the point (x_1,y_1,z_1) on the given plane, then

$lx_1+my_1+nz_1=p$ ..... (i)

Let Q be (α,β,γ) O, P, Q are collinear so

$\frac{x_1}{\alpha}=\frac{y_1}{\beta}=\frac{z_1}{\gamma}=k$ (say) ....(ii)

Now $OP.OQ=p^2$

$$\Rightarrow \sqrt{x_1^2+y_1^2+z_1^2}\sqrt{\alpha^2+\beta^2+\gamma^2}=p^2$$

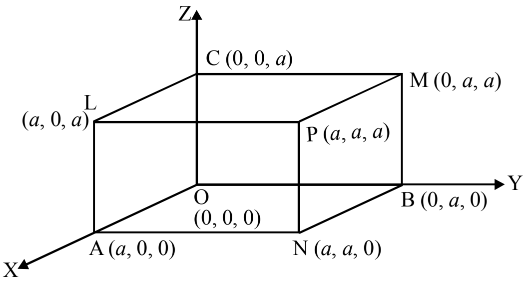
$$\Rightarrow k(\alpha^2+\beta^2+\gamma^2)=p^2 \text{ ...... (iii)}$$

Also from (i)and (ii), $k(l\alpha+m\beta+n\gamma)=p$; From (iii) and (iv), $p(l\alpha+m\beta+n\gamma)=(\alpha^2+\beta^2+\gamma^2)$

∴ Locus of $Q=(\alpha,\beta,\gamma)$ is

$$p(lx+my+nz)=x^2+y^2+z^2$$

27. (b) Take O as one corner of the cube as origin and OA, OB, OC the three edge through it as the axes.
Let $OA = OB = OC = a$



The co-ordinates of various corners are shown in the figure. The four diagonals are AM, BL, CN, and OP

The DRs of diagonal AM are $0-a, a-0, a-0$,
i.e. $-a, a, a$.
If θ is the angle between them, then

$$\cos\theta = \frac{|(-a)(a)+(a)(a)+(a)(a)|}{\sqrt{a^2+a^2+a^2}\sqrt{a^2+a^2+a^2}}$$

$$\Rightarrow \cos\theta = \frac{a^2}{3a^2} = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

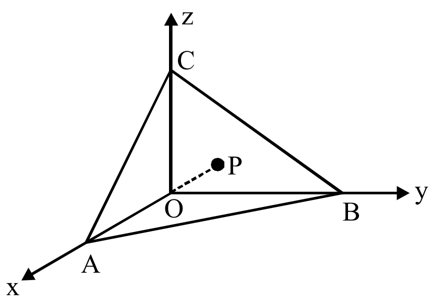
28. (d) Here $OP = \sqrt{h^2+k^2+l^2} = p$

$\therefore$ DRs of OP are :

$$\frac{h}{\sqrt{h^2+k^2+l^2}}, \frac{k}{\sqrt{h^2+k^2+l^2}}, \frac{l}{\sqrt{h^2+k^2+l^2}}$$

or $\frac{h}{p}, \frac{k}{p}, \frac{l}{p}$

Since OP is normal to the plane, therefore, equation of plane is



$$\frac{h}{p}x + \frac{k}{p}y + \frac{l}{p}z = p \quad \text{or} \quad hx + ky + lz = p^2$$

$$\therefore A\left(\frac{p^2}{h}, 0, 0\right), B\left(0, \frac{p^2}{k}, 0\right), C\left(0, 0, \frac{p^2}{l}\right)$$

Now, Area of ΔABC, $\Delta = \sqrt{A_{xy}^2 + A_{yz}^2 + A_{zx}^2}$

Where, A_{xy}^2 is area of projection of ΔABC on xy plane = area of ΔAOB

$$\text{Now, } A_{xy} = \frac{1}{2}\begin{vmatrix} p^2/h & 0 & 1 \\ 0 & p^2/k & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{p^4}{2|hk|}$$

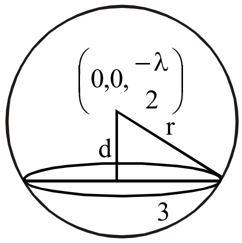
$$\text{Similarly, } A_{yz} = \frac{p^4}{2|kl|} \text{ and } A_{zx} = \frac{p^4}{2|lh|}$$

$$\therefore \Delta^2 = A_{xy}^2 + A_{yz}^2 + A_{zx}^2, \quad \Delta = \frac{p^5}{2hkl}$$

29. (c) Equation of sphere through the given circle is
$x^2 + y^2 + z^2 - 4 + \lambda z = 0$ ...... (1)

Its centre is $\left(0, 0, -\frac{\lambda}{2}\right)$ and radius $\sqrt{0+0+\frac{\lambda^2}{4}+4}$

$\therefore$ d = distance of the plane $x + 2y + 2z = 0$ from the centre of sphere



$$= \frac{\left|0 + 2\times 0 + 2 - \left(\frac{\lambda}{2}\right)\right|}{\sqrt{1+4+4}} = \frac{\lambda}{3}$$

$\therefore$ Since the sphere (1) cuts the plane in a circle of radius 3

$$\therefore r^2 - d^2 = 3^2 \Rightarrow \frac{\lambda^2}{4} + 4 - \frac{\lambda^2}{9} = 9$$

$$\Rightarrow \lambda = \pm 6$$

Since the centre lies in the positive octant, so $\lambda < 0$

$\therefore$ Required equation is $x^2 + y^2 + z^2 - 6z - 4 = 0$

30. (b) Given planes are :
$x - z = 0, x - 2y + z = 0, x + y + z = 0$

Let the point whose locus is required be $P(\alpha, \beta, \gamma)$. According to question

$$\frac{|\alpha - \gamma|^2}{2} + \frac{|\alpha - 2\beta + \gamma|^2}{6} + \frac{|\alpha + \beta + \gamma|^2}{3} = 36$$

or $3(\alpha^2+\gamma^2-2\alpha\gamma)+\alpha^2+4\beta^2+\gamma^2$

$-4\alpha\beta-4\beta\gamma+2\alpha\gamma$

$+2(\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\alpha\gamma)=36\times 6$

or $6\alpha^2+6\beta^2+6\gamma^2=36\times 6$

$\Rightarrow \alpha^2+\beta^2+\gamma^2=36$

Hence, the required equation of locus is : $x^2+y^2+z^2=36$

31. (c) Let the required sphere be

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ .... (1)

Since it passes through (1, 0, 0), (0, 1, 0), and (0, 0, 1)

$\therefore\quad 1+2u+d=0$ ....(2)

$1+2v+d=0$ ......(3)

and $1+2w+d=0$ ....(4)

$\Rightarrow u=v=w=-\frac{1+d}{2}$

If R be the radius of the sphere, then $R^2=u^2+v^2+w^2-d$

$=3\left(\frac{1+d}{2}\right)^2-d=\frac{1}{4}(3d^2+2d+3)$

$=\frac{3}{4}\left[\left(d+\frac{1}{3}\right)^2+\frac{8}{9}\right]$

$\Rightarrow R$ is minimum if $d+\frac{1}{3}=0 \Rightarrow d=-\frac{1}{3}$

$\therefore\quad u=v=w=-\frac{1}{3}$

$\therefore\ R^2=\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{3}=\frac{2}{3} \Rightarrow R=\sqrt{\frac{2}{3}}$

32. (b) Let the foot of the perpendicular from the origin on the given plane be $P(\alpha,\beta,\gamma)$. Since the plane passes through $A\,(a,b,c)$

$\because\ AP\perp OP \Rightarrow \overrightarrow{AP}.\overrightarrow{OP}=0$

$\Rightarrow [(\alpha-a)\hat{i}+(\beta-b)\hat{j}+(\gamma-c)\hat{k}].(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})=0$

Hence , the locus of (α,β,γ) is

$x(x-a)+y(y-b)+z(z-c)=0$

$x^2+y^2+z^2-ax-by-cz=0$ which is a sphere of radius $\frac{1}{2}\sqrt{a^2+b^2+c^2}$

33. (c) We have $z=0$ for the point where the line intersects the curve

Therefore, $\frac{x-2}{3}=\frac{y+1}{2}=\frac{0-1}{-1} \Rightarrow \frac{x-2}{3}=1$ and

$\frac{y+1}{2}=1 \Rightarrow x=5$ and $y=1$

Put these value in $xy=c^2$, we get $5=c^2$

$\Rightarrow c=\pm\sqrt{5}$

34. (b) For $\ell=-m+\frac{n}{\lambda}$, the second relation gives

$mn+n\left(-m+\frac{n}{\lambda}\right)+m\left(-m+\frac{n}{\lambda}\right)=0$

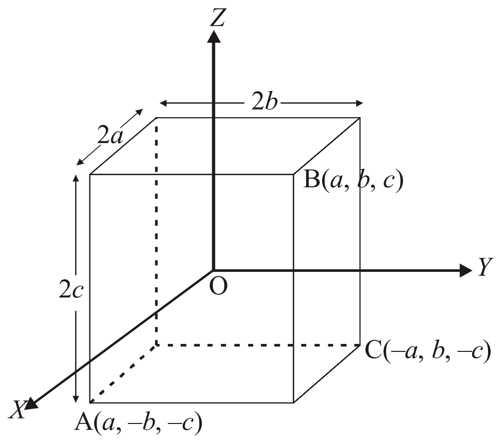
or $n^2+mn-\lambda m^2=0 \Rightarrow \frac{n^2}{m^2}+\frac{n}{m}-\lambda=0.$

If $\frac{n_1}{m_1},\frac{n_2}{m_2}$ are its roots then $\frac{n_1n_2}{m_1m_2}=-\lambda$

$\Rightarrow \frac{\ell_1\ell_2}{-1}=\frac{m_1m_2}{-1}=\frac{n_1n_2}{\lambda}$

Since the lines are perpendicular $\Rightarrow -1-1+\lambda=0$ $\Rightarrow \lambda=2.$

35. (c)



With O as the origin and the coordinates axes parallel to the edges of the box, let the coordinates of A, B, C are $A(a,-b,-c)$, $B(a,b,c)$, $C(-a,b,-c)$.

Hence $\cos\theta+\cos\phi+\cos\psi$

$=\frac{(a^2-b^2-c^2)+(-a^2+b^2-c^2)+(-a^2-b^2+c^2)}{(a^2+b^2+c^2)}$

$=\frac{-(a^2+b^2+c^2)}{(a^2+b^2+c^2)}=-1.$

36. **(b)** A plane containing the given lines is

$2x+3y+4z-6+\lambda(x+y+z-3)=0 \quad \dots(1)$

This plane is perpendicular to plane $z=0$

if $4+\lambda=0 \Rightarrow \lambda=-4$

So, the equation (i) becomes

$-2x-y+6=0 \Rightarrow 2x+y-6=0 \quad \dots(2)$

Equation of the projection will be the line of intersection of plane (2) and the plane z = 0. If the line has d.c. proportional to ℓ, m, n then $2\ell+m=0$ and $n=0$

$\Rightarrow \ell : m : n = 1 : -2 : 0$. Obviously (0, 6, 0) is a point on both the planes, hence lies on the line as well.

$\therefore$ Equation of the line is $\dfrac{x}{1}=\dfrac{y-6}{-2}=\dfrac{z}{0}$

37. **(a)** Equation of plane is $\dfrac{x}{20}+\dfrac{y}{15}+\dfrac{z}{-12}=1.$

Hence the volume of tetrahedron $=\dfrac{1}{6}(20\times15\times12)$

$=600$ cube units.

where 20, 15, – 12 are intercepts on x, y and z axis.

38. **(a)** System has only trivial solution

$$\therefore \Delta=\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}\neq 0$$

$\Rightarrow (a-b)(b-c)(c-a)\neq 0$

$a^3r(1-r)^3(1+r)\neq 0$

$r=0,\pm 1.$

39. **(c)** Let coordinate of C be (α, β, γ).

As $AC=BC \Rightarrow (\alpha-1)^2+(\beta+1)^2+(\gamma-1)^2$

$=(\alpha+1)^2+(\beta-1)^2+(\gamma+1)^2$

$\Rightarrow$ locus of C is $x-y+z=0$

40. **(a)** Let the variable point be $P(\alpha, \beta, \gamma)$.

If foot of perpendicular from P upon the line $x=y=-z$ be $(h, h, -h)$ then

$(h-\alpha)+(h-\beta)-(-h-\gamma)=0 \Rightarrow h=\dfrac{\alpha+\beta-\gamma}{3}$

$\therefore$ Distance of the point from the line

$=\sqrt{(h-\alpha)^2+(h-\beta)^2+(-h-\gamma)^2}$

$=\sqrt{\alpha^2+\beta^2+\gamma^2+h(3h-2\alpha-2\beta+2\gamma)}$

$=\sqrt{\alpha^2+\beta^2+\gamma^2+\left(\dfrac{\alpha+\beta-\gamma}{3}\right)(\alpha+\beta-\gamma-2\alpha-2\beta+2\gamma)}$

$=\sqrt{\alpha^2+\beta^2+\gamma^2-\left(\dfrac{\alpha+\beta-\gamma}{3}\right)^2}$

According to question

$$\alpha^2+\beta^2+\gamma^2-\frac{(\alpha+\beta-\gamma)^2}{3}=\frac{4(\alpha-\beta+\gamma-1)^2}{3}$$

$$\Rightarrow \alpha^2+\beta^2+\gamma^2=\frac{1}{3}[4\alpha^2+4\beta^2+4\gamma^2+4-8\alpha\beta-8\beta\gamma$$
$$+8\alpha\gamma-8\alpha+8\beta-8\gamma+\alpha^2+\beta^2$$
$$+\gamma^2+2\alpha\beta-2\beta\gamma-2\gamma\alpha]$$

$$\Rightarrow 2\alpha^2+2\beta^2+2\gamma^2-6\alpha\beta-10\beta\gamma+6\alpha\gamma-8\alpha+8\beta-8\gamma+4=0$$

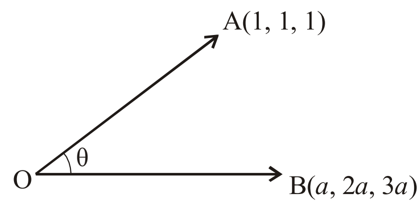
$$\Rightarrow \alpha^2+\beta^2+\gamma^2-3\alpha\beta-5\beta\gamma+3\alpha\gamma-4\alpha+4\beta-4\gamma+2=0$$

$\therefore$ Locus of (α, β, γ) is

$x^2+y^2+z^2-3xy-5yz+3zx-4x+4y-4z+2=0$

41. **(b)** Any point on the second line is $(a, 2a, 3a)$

then $\cos\theta=\dfrac{a+2a+3a}{\sqrt{3}\sqrt{14a^2}}=\pm\dfrac{6}{\sqrt{42}}$



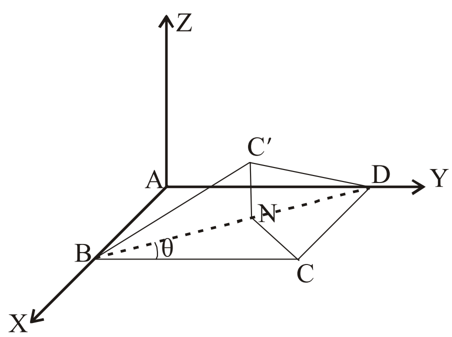
$\therefore \quad \sin\theta=\sqrt{\dfrac{6}{42}}=\dfrac{1}{\sqrt{7}}$

Now area $=\dfrac{1}{2}OA\,OB\sin\theta$

$\Rightarrow \sqrt{6}=\dfrac{1}{2}\sqrt{3}\sqrt{14a^2}\times\dfrac{1}{\sqrt{7}} \Rightarrow a=\pm 2$

So, B is (2, 4, 6) or (–2, –4, –6)

42. **(c)**



Let AB = CD = r and BC = AD = 2r

then $\tan\theta=\dfrac{CD}{BC}=\dfrac{1}{2}$

Also,

$$\sin\theta = \frac{CN}{BC} \Rightarrow \frac{1}{\sqrt{5}} = \frac{CN}{2r} \Rightarrow CN = C'N = \frac{2}{\sqrt{5}}r$$

In xy plane the equation of BD is $\frac{x}{r} + \frac{y}{2r} = 1$

$\Rightarrow 2x + y - 2r = 0$ and C is $(r, 2r)$

$\therefore$ Coordinates of N are given by

$$\frac{x-r}{2} = \frac{y-2r}{1} = \frac{-(2r+2r-2r)}{5}$$

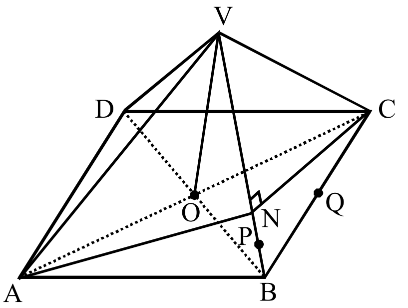
$\therefore \quad x = r - \frac{4r}{5} = \frac{r}{5}$ and $y = 2r - \frac{2r}{5} = \frac{8r}{5}$

$\therefore$ Coordinate of C' in three dimensions are $\left(\frac{r}{5}, \frac{8r}{5}, \frac{2}{\sqrt{5}}r\right)$

$$\therefore \; AC' = \sqrt{\frac{r^2}{25} + \frac{64r^2}{25} + \frac{4r^2}{5}} = \frac{\sqrt{85}r}{5}$$

B COMPREHENSION TYPE

1.(b) Let Q be the mid point of BC. Then VQ and OQ are perpendicular to BC and VQ is the line of greatest slope.



Now, $VO = a$ and $OQ = a \; \therefore \quad \angle VQO = 45°$

2.(a) Let $AN \perp VB$, then AN is the desired length. We have,

$$VB^2 + a^2 + (\sqrt{2}a)^2 = 3a^2 \Rightarrow VB = \sqrt{3}\,a$$

Let, $\angle VBA = \angle VBC = \theta$, then $\cos\theta = \frac{BQ}{VB} = \frac{1}{\sqrt{3}}$

$$\Rightarrow \sin\theta = \sqrt{\frac{2}{3}} \qquad \therefore \; AN = AB\sin\theta = 2a\sqrt{\frac{2}{3}} = \frac{2\sqrt{6}}{3}a$$

3.(c) Because the triangle ABN and CBN are congruent CN is perpendicular to VB and the angle between two adjacent triangular faces is therefore equal to $\angle ANC$.

Now, if $\angle ANO = \alpha$

$$\Rightarrow \sin\alpha = \frac{AO}{AN} = \frac{a\sqrt{2}}{\frac{2\sqrt{3}a}{3}} = \sin^{-1}\frac{\sqrt{3}}{2} = 60°$$

$\therefore \; \angle ANC = 2\alpha = 120°$

4.(a) Here, the three vertices of tetrahedron are $O(0, 0, 0)$; $A(6, -5, -1)$; $B(-4, 1, 3)$ and centroid $G(1, -2, 5)$. Let the fourth vertex be (x_1, y_1, z_1)

$$\therefore \text{centroid} = \left(\frac{x_1+0+6+(-4)}{4}, \frac{y_1+0+(-5)+1}{4}, \frac{z_1+0-1+3}{4}\right)$$

$$\Rightarrow \left(\frac{x_2+2}{2}, \frac{y_1-4}{4}, \frac{z_1+2}{4}\right) = (1, -2, 5)$$

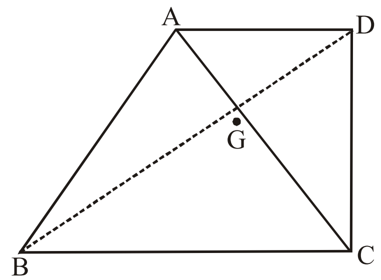
$\Rightarrow x_1 = 2, y_1 = -4, z_1 = 18$

$\therefore$ fourth vertex $(2, -4, 18)$

5.(c) Any plane through $(0, 0, 0)$ is $a(x-0) + b(y-0) + c(z-0)$ If passes through $(6, -5, -1)$ and $(-4, 1, 3)$, Substituting and on solving we get $a = b = c$. So the desired plane is $x + y + z = 0$

6.(d) Now, to find the radious of sphere circumscribing the tetrahedron (let radius be r, and centre be $P(\alpha, \beta, \gamma)$)

$\therefore \quad r^2 = (\alpha-0)^2 + (\beta-0)^2 + (\gamma-0)^2$
$= (\alpha-2)^2 + (\beta+4)^2 + (\gamma-18)^2$
$= (\alpha-6)^2 + (\beta+5)^2 + (\gamma+1)^2$
$= (\alpha+4)^2 + (\beta-1)^2 + (\gamma-3)^2$



Using $r^2 = PA^2 = PB^2 = PC^2$ (P being centre)

$\Rightarrow \; -6\alpha + 5\beta + 6\gamma + 26 = 0$ ...(i)

$8\alpha - 2\beta - 6\gamma + 26 = 0$ ...(ii)

and $-\alpha + 2\beta - 9\gamma + 86 = 0$ ...(iii)

Solving (i), (ii) and (iii), we get

$$\alpha = \frac{8}{7}, \beta = -\frac{45}{7}, \gamma = 8$$

7. (a) The projection will be an ellipse whose major axis is $AB = 2a$.

If its minor axis be b, then $\pi ab = (\cos\theta)\pi a^2$

$\Rightarrow \; b = a\cos\theta$.

$$\therefore \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \sin\theta.$$

8. **(b)** Let the positive direction of the normal to the plane *ABC* from O has direction cosines cosα, cosβ, cosγ. Since Δ*OBC* is the projection of Δ*ABC* on the plane *YOZ*, so

$$\cos\alpha.\Delta = \frac{1}{2}bc.$$

Similarly

$$\cos\beta.\Delta = \frac{1}{2}ca, \cos\gamma.\Delta = \frac{1}{2}ab.$$

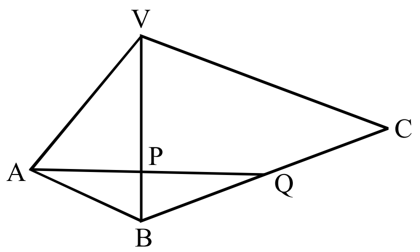
Using $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1,$ we get

$$\Delta = \frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + c^2a^2}.$$

C REASONING TYPE

1. **(b)** Both statements are actually definitions.

2. **(c)** Clearly AP + PQ is least if A, P and Q lie on a straight line when A, P, Q are coplanar but the position of Q is fixed so P need not represent the foot of perpendicular from A upon VB.



3. **(c)** Point A is (a, b, c)

$\Rightarrow$ Points *P*, *Q*, *R* are $(a, b, -c)$, $(-a, b, c)$ and $(a, -b, c)$ respectively.

$\Rightarrow$ centroid of triangle *PQR* is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\Rightarrow$ $G \equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\Rightarrow$ *A*, *O*, *G* are collinear.

D MULTIPLE CORRECT CHOICE TYPE

1. **(a,b,c)** Here, *P* be (x, y, z)

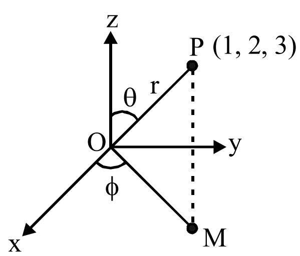
Then,

$x = r\sin\theta.\cos\phi,\ \ y = r\sin\theta\sin\phi,\ z = r\cos\theta$

$\Rightarrow\ 1 = r\sin\theta.\cos\phi,\ 2 = r\sin\theta\sin\phi,\ 3 = r\cos\theta$ ...(i)

$\Rightarrow\ 1^2 + 2^2 + 3^2$

$= r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$



$= r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) + r^2\cos^2\theta$

$= r^2\sin^2\theta + r^2\cos^2\theta = r^2$

$\Rightarrow\ \ r = \pm\sqrt{14}$

∴ From (i), we have

$$\sin\theta\cos\phi = \frac{1}{\sqrt{14}}, \sin\theta\sin\phi = \frac{2}{\sqrt{14}},\quad \cos\theta = \frac{3}{\sqrt{14}}$$

(neglecting –ve sign assuming acute angles)

$$\therefore\ \frac{\sin\theta\sin\phi}{\sin\theta\cos\phi} = \frac{2}{1} \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{5}}{3}$$

$$\Rightarrow\ \tan\phi = 2 \text{ and } \tan\theta = \frac{\sqrt{5}}{3}$$

2. **(b,d)** Let the given points be *A*, *B* and *C* respectively. Then

$AB^2 = 350$,

$AC^2 = 500 - 20\lambda + \lambda^2$ and $BC^2 = 150 + 10\lambda + \lambda^2$

Now $AB^2 + AC^2 = BC^2 \Rightarrow 350 + 500 - 20\lambda + \lambda^2$

$= 150 + 10\lambda + \lambda^2 \Rightarrow \lambda = \frac{70}{3}$

Next, $AB^2 + BC^2 = AC^2$

$\Rightarrow 250 + 150 + 10\lambda + \lambda^2 = 500 - 20\lambda + \lambda^2 \Rightarrow \lambda = 0$

Further, $\mathbf{B}C^2 + AC^2 = AB^2$

$\Rightarrow\ 150 + 10\lambda + \lambda^2 + 500 - 20\lambda + \lambda^2 = 350$

$\Rightarrow \lambda^2 - 5\lambda + 150 = 0$, which have no real solution

$\therefore$ The triangle is right angles for $\lambda = 0, \frac{70}{3}$

3. (b,c) Let the line be $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$, then

$$\begin{vmatrix} 3 & 3 & 0 \\ 2 & 1 & 1 \\ l & m & m \end{vmatrix} = 0 \Rightarrow l - m - n = 0$$

Again, $\frac{2l+m+n}{\sqrt{6(l^2+m^2+n^2)}} = \pm\cos 60^\circ$

$\Rightarrow 5l^2 - m^2 - n^2 + 4mn + 8ln + 8lm = 0$

Eliminating l, we get $2m^2 + 5mn + 2n^2 = 0$

$\Rightarrow m = -2n$ or $m = -\frac{n}{2}$

So, $m = -2n$, $1 = -n$ or $2m = -n$, $l = -m$. The desired direction ratios are 1, 2, –1 and 1, –1, 2.

4. (a,d) Unit vectors along the lines are

$\vec{a} = l_1\vec{i} + m_1\vec{j} + n_1\vec{k}$ *and* $\vec{b} = l_2\vec{i} + m_2\vec{j} + n_2\vec{k}$.

The unit vectors along the angular bisectors are

$\frac{\vec{a}+\vec{b}}{|\vec{a}\pm\vec{b}|}$. Now $|\vec{a}+\vec{b}|^2 = 2 + 2\cos\theta = 4\cos^2\frac{\theta}{2}$

and $|\vec{a}-\vec{b}|^2 = 2 - 2\cos\theta = 4\sin^2\frac{\theta}{2}$.

So, D.C. are $\frac{l_1+l_2}{2\cos\frac{\theta}{2}}, \frac{m_1+m_2}{2\cos\frac{\theta}{2}}, \frac{n_1+n_2}{2\cos\frac{\theta}{2}}$

and $\frac{l_1-l_2}{2\sin\frac{\theta}{2}}, \frac{m_1-m_2}{2\sin\frac{\theta}{2}}, \frac{n_1-n_2}{2\sin\frac{\theta}{2}}$

5. (a,b,c) Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$, then we have

$\vec{a}.\vec{a} + (\vec{b}-\vec{c}).(\vec{b}-\vec{c}) = \vec{b}.\vec{b} + (\vec{c}-\vec{a}).(\vec{c}-\vec{a})$

$\Rightarrow -2\vec{b}.\vec{c} = -2\vec{c}.\vec{a} \Rightarrow (\vec{a}-\vec{b}).\vec{c} = 0$ or $\overrightarrow{BA}.\overrightarrow{OC} = 0$

Hence AB is perpendicular to OC. Similarly, BC is perpendicular to OA and CA is perpendicular to OB.

6. (b, c) The equation of the angle bisector of the given planes is $x = \pm z$

i.e. $x + z = 0$ or $x - z = 0$ ....(1)

Equation of plane parallel to (1) is

$x + z + d = 0$ or $x - z + d_1 = 0$.

It passes through (1, 2, 3)

therefore, $d = -4$ or $d_1 = 2$

so, $(x + z - 4)$ and $(x - z + 2) = 0$

E MATRIX-MATCH TYPE

1. A-s; B-r; C-p; D-q

(A) If position vector of a variable point P on the line be $\vec{r}$ then $\overrightarrow{AP} \parallel \vec{b} \Rightarrow (\vec{r}-\vec{a}) = \alpha\vec{b}$ for some $\alpha \in R$

$\Rightarrow \vec{r} = \vec{a} + \alpha\vec{b}$

(B) If $P(\vec{r})$ be a variable point on the line then $\vec{r} - \vec{a} = \alpha(\vec{b}-\vec{a})$ for some $\alpha \in R \Rightarrow \vec{r} = \vec{a} + \alpha(\vec{b}-\vec{a})$

(C) and (D)

Vector equation of the plane passing through a point of position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$ is $\vec{r} = \vec{a} + \alpha\vec{b} + \beta\vec{c}$, where α and β are real.

If it passes through origin and parallel to $\vec{a}$ and $\vec{b}$ then $\vec{r} = \alpha\vec{a} + \beta\vec{b}$

This plane passing through points with position vectors $\vec{a}, \vec{b}, \vec{c}$ is

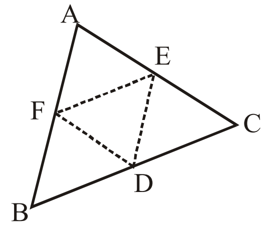
$\vec{r} = (1-\alpha-\beta)\vec{a} + \alpha\vec{b} + \beta\vec{c}$

2. A-r, B-t, C-s, D-p,q

(A) Let D, E, F be the mid points then

$$EF^2 = \frac{BC^2}{4} \Rightarrow BC^2 = 4(b^2 + c^2)$$

Similarly $AB^2 = 4(a^2 + b^2)$ and $AC^2 = 4(a^2 + c^2)$



$$\therefore \frac{AB^2 + BC^2 + CA^2}{a^2 + b^2 + c^2} = 8$$

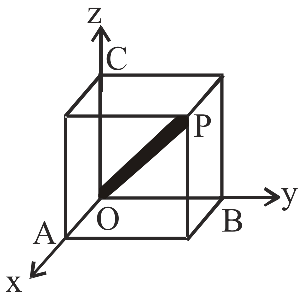
(B) The image (x_1, y_1, z_1) is given by

$$\frac{x_1-1}{1}=\frac{y_1+2}{-1}=\frac{z-3}{1}==-\frac{2}{3}(1-2+3-5)$$

$\Rightarrow (x_1, y_1, z_1) \equiv (3, -4, 5)$

$\therefore$ desired distance = $\sqrt{50}=5\sqrt{2}$

(C) $D.R.$ of edge OA are 1, 0, 0 and $D.R.$ of diagonal OP are 1, 1, 1.



$\therefore \cos\theta=\frac{1}{\sqrt{3}} \Rightarrow \tan\theta=\sqrt{2}$

(D) We have $\begin{vmatrix} p & \frac{3}{2} & \frac{1}{2} \\ \frac{3}{2} & 1 & 1 \\ \frac{1}{2} & 1 & q \end{vmatrix}=0$ and $p+1+q=0$.

$\Rightarrow 4pq-4p-5q+5=0$ and $p+1+q=0$

Eliminating p, we get $q=-3, \frac{3}{4}$.

3. A-r, B-q, C-p,t, D-p,q,s

(A) We have $a^2-b^2+c^2=0$ and $a^2-2bd+c^2=0$

$\Rightarrow b^2=2bd.\ As\ b\neq 0 \Rightarrow \frac{b}{d}=2$.

(B) Equation of line through (1, –2, 3) and parallele to given line is $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}$.

Any point on it is $(2r+1, 3r-2, -6r+3)$.

This point lies on the plane if $r=\frac{1}{7}$. So, the point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$.

Desired distance

$$=\sqrt{\left(\frac{9}{7}-1\right)^2+\left(-\frac{11}{2}+2\right)^2+\left(\frac{15}{7}-3\right)^2}=1.$$

(C) General points on two given lines may be written as $(r_1+2, r_1+3, -kr_1+4)$ and $(kr_2+1, 2r_2+4, r_2+5)$.
If two lines intersect then for some r_1 and r_2
$r_1+2=kr_2+1, r_1+3=2r_2+4$ and $-kr_1+4=r_2+5$

Eliminating r_1 and r_2, $k^2+3k=0 \Rightarrow k=0$ or -3

(D) $\cos^2\theta+\cos^2\theta+\cos^2\gamma=1 \Rightarrow \cos^2\gamma=-\cos 2\theta$

$\cos 2\theta \le 0 \Rightarrow \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \Rightarrow 0 \le \cot\theta \le 1$

F NUMERIC/INTEGER ANSWER TYPE

1. Ans. : 9

Let l, m, n be the direction cosines of the line MN which is perpendicular to each of the given lines

$\therefore -4l+3m+2n=0$ and $-4l+m+n=0$

Solving $\frac{l}{3-2}=\frac{m}{-8+4}=\frac{n}{-4+12}$

$\Rightarrow \frac{l}{1}=\frac{m}{-4}=\frac{n}{8}=\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{1+16+64}}=\frac{1}{9}$

$\Rightarrow l=\frac{1}{9}, m=-\frac{4}{9}, n=\frac{8}{9}$

It is obvious that the points $P(-3, 6, 0)$ and $Q(-2, 0, 7)$ are situated on the given line

$\therefore$ Length of shortest distance

= Projection of PQ on the common perpendicular MN

$\frac{1}{9}\{(-2)-(-3)\}+\left(-\frac{4}{9}\right)[0-6]+\frac{8}{9}[7-0]=9$

2. Ans. 2

The equation of any plane containing the given line is

$(x+y+2z-3)+\lambda(2x+3y+4z-4)=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(2+4\lambda)z-(3+4\lambda)=0$ ..... (1)

If the plane is parallel to z- axis whose direction cosines are 0, 0, 1 ; then the normal to the plane will be perpendicular to z-axis

$\therefore (1+2\lambda)(0)+(1+3\lambda)(0)+(2+4\lambda)(1)=0$

$\Rightarrow \lambda=-\frac{1}{2}$

Put in eq. (1), the required plane is

$$(x+y+2z-3)-\frac{1}{2}(2x+3y+4z-4)=0 \Rightarrow y+2=0$$

...... (2)

$\therefore$ S.D. = distance of any point say (0, 0, 0) on z- axis from plane (2)

$$= \frac{2}{\sqrt{(1)^2}}=2$$

3. Ans.: 1

Given planes are : $x-cy-bz=0$ ..... (1)

$cx-y+az=0$ ...... (2)

$bx+ay-z=0$ ..... (3)

Equation of plane passing through the line of intersection of plane (1)and (2) may be taken as;

$$(x-cy-bz)+\lambda(cx-y+az)=0$$

or $x(1+\lambda c)-y(c+\lambda)-z(-b+a\lambda)=0$ ...... (4)

If plane (3) and (4) are same ; then equation (3) and (4) will be identical

$$\therefore \quad \frac{1+c\lambda}{b}=\frac{-(c+\lambda)}{a}=\frac{-b+a\lambda}{-1}$$

or $\lambda=-\frac{(a+bc)}{(ac+b)}$ and $\lambda=-\frac{(a+bc)}{(1-a^2)}$

$$\therefore \quad \frac{-(a+bc)}{(ac+b)}=-\frac{(ab+c)}{(1-a^2)}$$

$$\Rightarrow a-a^3+bc-a^2bc=a^2bc+ac^2+ab^2+bc$$

$$\Rightarrow 2a^2bc+ac^2+a^3-a=0$$

$$\Rightarrow a(2abc+c^2+b^2+a^2-1)=0$$

$$\Rightarrow a^2+b^2+c^2+2abc=1$$

4. Ans. 1

Let P be (x_1, y_1, z_1). Point M is $(x_1, 0, z_1)$ and N is $(x_1, y_1, 0)$

So normal to plane OMN is $\overrightarrow{OM}\times\overrightarrow{ON}=\vec{x}$ (*say*)

$$i.e. \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & 0 & z_1 \\ x_1 & y_1 & 0 \end{vmatrix} = \hat{i}(-y_1z_1)-\hat{j}(-x_1z_1)+\hat{k}(x_1y_1)$$

$$\text{therefore, } \sin\theta=\frac{-x_1y_1z_1+x_1y_1z_1+x_1y_1z}{\sqrt{x_1^2+y_1^2+z_1^2}\sqrt{\sum x_1^2y_1^2}}$$

$$\text{or } \left(\because \sin=\frac{\vec{n}\times\overrightarrow{OP}}{|n||\overrightarrow{OP}|}\right)$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta=\frac{\sum x_1^2\sum x_1^2y_1^2}{(x_1y_1z_1)^2}$$

$$=\frac{\sum x_1^2}{x_1^2}+\frac{\sum x_1^2}{y_1^2}+\frac{\sum x_1^2}{z_1^2}$$

$$\text{Now, } \sin\alpha=\frac{\overrightarrow{OP}.\hat{k}}{|\overrightarrow{OP}|}=\frac{z_1}{\sqrt{\sum x_1^2}}$$

$$\sin\beta=\frac{x_1}{\sqrt{\sum x_1^2}} \text{ and } \sin\gamma=\frac{y_1}{\sqrt{\sum x_1^2}}$$

Now, $\operatorname{cosec}^2\alpha+\operatorname{cosec}^2\beta+\operatorname{cosec}^2\gamma$

$$=\frac{x_1^2+y_1^2+z_1^2}{x_1^2}+\frac{\sum x_1^2}{y_1^2}+\frac{\sum x_1^2}{z_1^2}=\operatorname{cosec}^2\theta$$

5. Ans. 1

Let P (x_1, y_1, z_1) be a point on $ax+by+cz=d$. Then

$ax_1+by_1+cz_1=$ d ...(1)

Let $OP=r$. Then direction cosines of OP are $\frac{x_1}{r},\frac{y_1}{r},\frac{z_1}{r}$

Equation of line OP is $\dfrac{x-0}{\frac{x_1}{r}}=\dfrac{y-0}{\frac{y_1}{r}}=\dfrac{z-0}{\frac{z_1}{r}}$,

Let Q (α, β, γ) be a point on OP such that OQ = λ. Then coordinates of $Q\left(\frac{x_1\lambda}{r},\frac{y_1\lambda}{r},\frac{z_1\lambda}{r}\right)$. Thus

$$x_1=\frac{\alpha r}{\lambda}, y_1=\frac{\beta r}{\lambda}, z_1=\frac{\gamma r}{\lambda}$$

Now $ax_1+by_1+cz_1=$ d

$$\Rightarrow \frac{r}{\lambda}(a\alpha+b\beta+c\gamma)=d \quad \text{....(ii)}$$

Now $OP.OQ=d^2$

$$\Rightarrow r\lambda=d^2$$

$$\Rightarrow r\lambda=d.\frac{r}{\lambda}(a\alpha+b\beta+c\gamma)$$

$$\Rightarrow (a\alpha+b\beta+c\gamma)d=\lambda^2$$

$$\Rightarrow (a\alpha+b\beta+c\gamma)d=\alpha^2+\beta+\gamma^2$$

So, locus of (α, β, γ) is $(ax+by+cz)\,d=x^2+y^2+z^2$

$$\text{or } \frac{(ax+by+cz)\,d}{x^2+y^2+z^2}=1$$

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