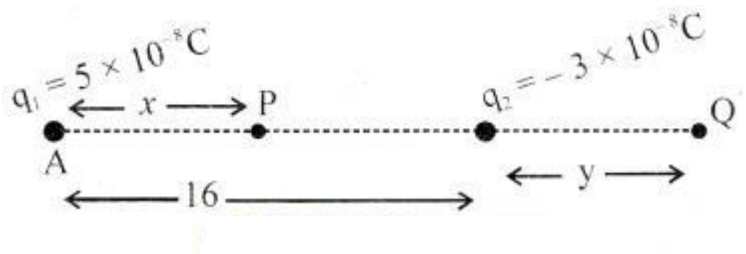
Chapter 2 Electrostatic Potential and Capacitance

Question 1. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.
Solution: On the line there are two locations where potential becomes zero.



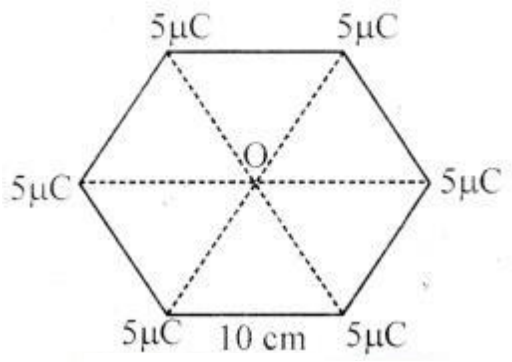
For P,
5×10−8x = 3×10−816−x
$5(16 - x) = 3x$
$80 - 5x = 3x$
$8x = 80$
$\therefore x = 10cm$
For Q,

$$\frac{5\times10^{-8}}{x} = \frac{3\times10^{-8}}{16-x}$$

$5y = 48 + 3y$
$\therefore 2y = 48$
$y = 24$ cm
i.e., at $(16 + 24) = 40$ cm from A

Question 2. A regular hexagon of side 10 cm has a charge 5μC at each of its vertices. Calculate the potential at the centre of the hexagon.
Solution:
If ‘q‘ is the charge,

potential at 'O' is $9 \times 10^9 \times \frac{q}{a}$



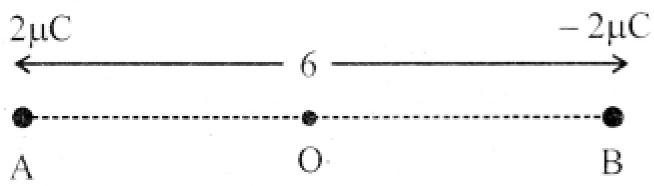
where 'a' is the side of the hexagon.

$$\therefore \text{Total potential} = \frac{6 \times q \times 9 \times 10^9}{a}$$

$$= \frac{9 \times 10^9 \times 6 \times 5 \times 10^{-6}}{10 \times 10^{-2}}$$

$$= 2.7 \times 10^6 \text{ V or } 2.7 \text{ MV}$$

Question 3. Two charges 2 μC and -2 μC are placed at points A and B 6 cm apart.
(a) Identify an equipotential surface of the system.
(b) What is the direction of the electric field at every point on this surface?
Solution:



(a) For the given system of the two charges, the equipotential surface will be a plane normal to the line AB joining the two charges and passing through its mid-point O. At any point on this plane, the potential is zero.

(b) The electric field is in a direction from point A to point B i.e. From the positive charge to negative charge and normal to the equipotential surface.

Question 4. The spherical conductor of radius 12 cm has a charge of 1.6 x 10^{-7} C distributed uniformly on its surface. What is the electric field (a) inside the sphere, (b) just outside the sphere (c), at a point 18 cm from the centre of the sphere?
Solution: R = 12 x 10^{-2} m, Q = 1.6 x 10^{-7}C
(a) Zero

(b) $E = \frac{9\times10^{9}\times1.6\times10^{-7}}{(12\times10^{-2})^{2}}$
$= \frac{9\times10^{9}\times1.6\times10^{-7}}{12\times12\times10^{-4}}$
$= 10^{5}\ NC^{-1}$

(c) Here $R = 18 \times 10^{-2}$m
Hence $E = \frac{9\times10^{9}\times1.6\times10^{-7}}{18\times18\times10^{-4}}$
$= \frac{4}{9} \times 10^{5} \approx 4.4 \times 10^{4}\ NC^{-1}$

Question 5. A parallel plate capacitor with air between the plates has a capacitance of 8 pF (lpF = 10^{-12}F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?
Solution: C = 8pF,

$C = \frac{\varepsilon_0 A}{d}$
$C' = \frac{\varepsilon_r \varepsilon_0 A}{\frac{d}{2}}$
$= 2\varepsilon_r$
$\therefore C' = 2 \times 6 \times 8 \times 10^{-12}\ F = 96\ pF$

Question 6. Three capacitors each of capacitance 9 pF are connected in series.
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?
Solution: C = 9 pF

(a) $C_{eff} = \frac{C}{3} = 3pF$

(b) Potential difference = $\frac{120}{3}$ = 40 V

Question 7. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.
(a) What is the total capacitance of the combination?
(b) Determine the charge on each capacitor if the combination is connected to a 100V supply. Sol.
Solution ; $C_1 = 2pF$, $C_2 = 3pF$, $C_3 = 4pF$
(a) 9 pF
(b) $Q_1 = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10}C$
$Q_2 = 3 \times 10^{-10}C$
$Q_3 = 4 \times 10^{-10}C$

Question 8. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\ m^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?
Solution: $A = 6 \times 10^{-3}\ m^2$
$d = 3\ mm = 3 \times 10^{-3}m$

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$

$$= 17.70 \times 10^{-12}$$

$$= 17.7\ pF$$

$$Q = CV = 17.7 \times 10^{-12} \times 100$$

$$= 17.7 \times 10^{-9}C$$

$$= 1.77\ nC$$

Question 9. Explain what would happen if in the capacitor given in Exercise.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,
(a) while the voltage supply remained connected.
(b) after the supply was disconnected.
Solution:

a. $C = \dfrac{\varepsilon_0 A}{\left(d - t + \dfrac{t}{\varepsilon_r}\right)} = \dfrac{8.85 \times 10^{-12} \times 6 \times 10^{-3} \times 6}{3 \times 10^{-3}} = 106.2\ \text{pF} = 0.106\ \text{nF}$

b. $V = \dfrac{Q}{C} = \dfrac{C_0 V}{C} = \dfrac{\varepsilon_0 A}{d} \times \dfrac{V}{\dfrac{\varepsilon_0 \varepsilon_r A}{d}} = \dfrac{100}{6} = 16.67\ \text{V}$

Question 10. A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Solution: C = 12 pF, V = 50V

$E = \frac{1}{2}CV^2$

$= \frac{1}{2} \times 10^{-12} \times 50 \times 50$

$= 150 \times 10^{-10}$

$= 1.5 \times 10^{-8}$ J

Question 11. A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Solution: C_1 = 600 pF, V_1 = 200V C_2 = 600 pF, ΔE = ?

$E_1 = \frac{1}{2}C_1 V_1^2$

$= \frac{1}{2} \times 600 \times 10^{-12} \times 200 \times 200$

$= 12 \times 10^{-6}$J

$$E_2 = \frac{1}{2}\frac{Q^2}{C_1 + C_2} = \frac{1}{2} \times \frac{(C_1V_1)^2}{C_1 + C_2} = \frac{1}{2} \times \frac{(600 \times 10^{-12} \times 200)^2}{1200 \times 10^{-12}}$$

$$= \frac{1}{2} \times \frac{(600)^2 \times (10^{-12})^2 \times 200 \times 200}{1200 \times 10^{-12}} = \frac{600 \times 600 \times 10^{-12} \times 100}{6} = 6 \times 10^{-6}\text{J}$$

$\Delta E = 6 \times 10^{-6}$J

Question 12. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of – 2 x 10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Solution: q = 8 mC = 8 x 10^{-3}C

$q_0 = -2 \times 10^{-9} C$

Work done = change in PE

$$U_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} = \frac{9\times10^9 \times 8\times10^{-3}\times(-2)\times10^{-9}}{3\times10^{-2}}$$

$$U_2 = \frac{9\times10^9 \times 8\times10^{-3}\times -2\times10^{-9}}{4\times10^{-2}}$$

$$U_2 - U_1 = \frac{9\times10^9 \times 8\times10^{-3}\times 2\times10^{-9}}{10^{-2}}\left\{\frac{1}{3}-\frac{1}{4}\right\} = 9\times 8\times 2\times 10^{-1}\times\frac{1}{12} = 1.2\text{ J}$$

Question 13. A cube of side ‘b’ has a charge ‘q’ at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Solution:

$$\frac{\sqrt{b^2+2b^2}}{2}$$

$= \frac{b}{2}\sqrt{3}$ is the centre

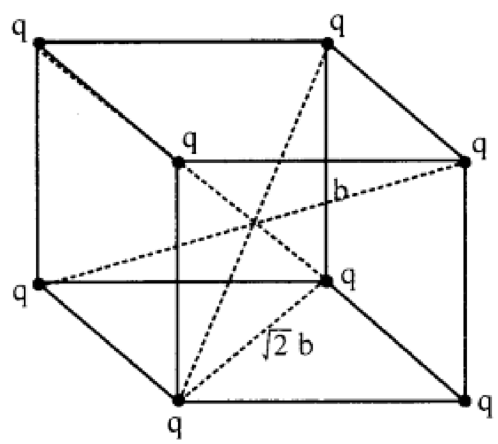
Distance of centre of the cube from one vertex is $\frac{b}{2}\sqrt{3}$.

Hence potential due to one charge

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\frac{b}{2}\sqrt{3}}$$

$$\text{Total potential} = \frac{8\times q}{4\pi\varepsilon_0\frac{b}{2}\sqrt{3}}$$

$$= \frac{2\times8\times q}{4\pi\varepsilon_0 b\sqrt{3}} = \frac{4q}{\sqrt{3}\,b\pi\varepsilon_0}$$



Field = zero

Question 14. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field
(a) at the mid-point of the line joining the two charges, and
(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Answer: $q_1 = 1.5\ \mu C$
$q_2 = 2.5\ \mu C$
$r = 30cm$
(a) Potential at the midpoint $= V_1 + V_2$

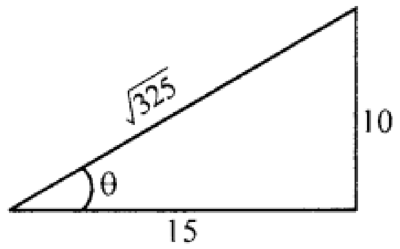
$$= \frac{9\times10^9\times1.5\times10^{-6}}{15\times10^{-2}} + \frac{9\times10^9\times2.5\times10^{-6}}{15\times10^{-2}} = \frac{9\times10^9\times10^{-6}}{15\times10^{-2}}\{1.5+2.5\}$$

$$= \frac{3}{5}\times10^5\times4 = 2.4\times10^5\ V$$

Field $= E_1 - E_2$

$$= \frac{9\times10^9}{(15\times10^{-2})^2}[(2.5\times10^{-6})-(1.5)\times10^{-6}] = \frac{9\times10^9\times10^{-6}\times1}{15\times15\times10^{-4}} = 4\times10^5\ Vm^{-1}$$

(b)



$$V = \frac{9\times10^9\times10^{-6}(4)}{\sqrt{325}\times10^{-2}} \qquad (\sqrt{325}\approx 18) \cong \frac{36}{18}\times10^5 = 2\times10^5\ \text{V}$$

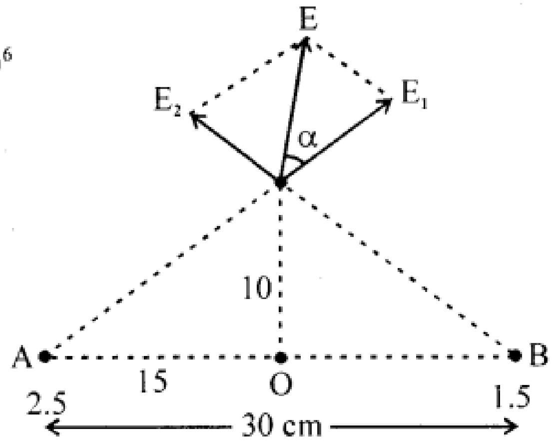
$$E = \sqrt{E_1^2+E_2^2+2E_1E_2\cos(180-2\theta)} = \frac{9\times10^9\times10^{-6}}{325\times10^{-4}}\ \sqrt{2.25+6.25-7.5\cos 2\theta}$$
$$= 6.6\times10^5\ \text{Vm}^{-1}$$

$$E_1 = \frac{9\times10^9\times2.5\times10^{-6}}{325\times10^{-4}} = \frac{4.5\times10^7}{65} = \frac{9}{13}\times10^6$$

$$E_2 = \frac{9\times10^9\times1.5\times10^{-6}}{325\times10^{-4}} = \frac{27}{65}\times10^6$$

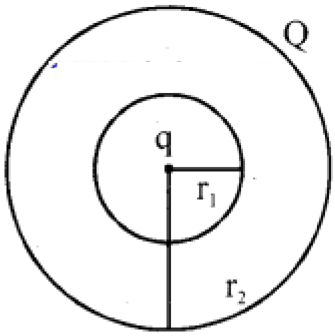
$$\tan\alpha = \frac{E_2\sin\theta}{E_1+E_2\cos\theta}$$

$$\tan\alpha = \frac{\frac{27}{65}\times\sin113^\circ}{\frac{9}{13}+\frac{27}{65}\times\cos113^\circ} = \frac{0.382363}{0.53} = 35.8$$



∴ Inclination of resultant is 69.3° from AB

Question 15. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.
(a) A charge 'q' is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?
(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.
Solution:



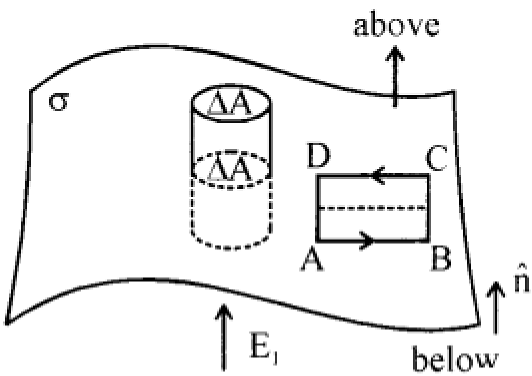
(a) $\frac{-q}{4\pi r_1^2}, \frac{Q+q}{4\pi r_2^2}$

(b) Yes. By Gauss' law

Question 16. (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $(E_2 - E_1).\hat{n} = \frac{\sigma}{\varepsilon_0}$, where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of n is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is $\frac{\sigma\hat{\mathbf{n}}}{\varepsilon_0}$.

(b) Show that the tangential component of the electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]

Solution: (a) In figure, a Gaussian surface, in the form of a pill box is shown. It is of area ΔA (on end faces) and of negligible thickness. Let $\overline{E1}$ be the field below and $\overline{E2}$, the field above.



The flux through upper surface = $\overline{E2}\ \overline{\Delta A}$

= $(\overline{E2}.\hat{n})\Delta A$

Similarly flux through lower face = $\overline{E1}\ \overline{\Delta A}$

= $(-\overline{E1},.\hat{n})\Delta A$

Hence by Gauss theorem

$(\overline{E2} - \overline{E1}).\hat{n}\overline{\Delta A}$

= $q\varepsilon 0$

= $\sigma\Delta A\varepsilon 0$ ∴$(\overline{E2} - \overline{E1})\hat{n} = \sigma\varepsilon 0$

Inside a conducting surface, E = 0 ∴$\overline{E2} = \overline{E} = \sigma\hat{n}\varepsilon 0$

(b) Consider a closed path ABCD. If E_1^n and E_2^n are the tangential components, then work done is $E_1^n\ \Delta l + E_2^n\ (-\Delta l) = 0$ (property of electric field)

∴ $E_1^n - E_2^n$ 0r $E_1^n = E_2^n$, showing that tangential components are continuous.

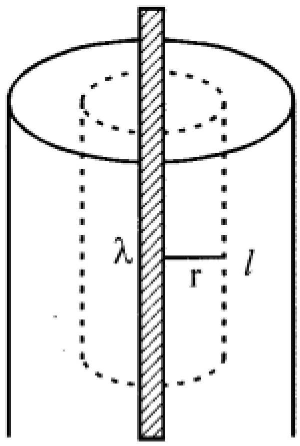
Question 17.

A long charged cylinder of linear charge density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Solution:

Charge density = λ

The flux calculated for a cylindrical surface of radius r, over a length l is E.2πrl.



By Gauss theorem, E.2πrl = $\frac{l\lambda}{\varepsilon_0}$

∴ E = $\frac{\lambda}{2\pi\varepsilon_0}$

or

$\bar{E} = \frac{\lambda}{2\pi r\varepsilon_0}$

Question 18. In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 A.
(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at an infinite separation of the electron from the proton.
(b) What is the minimum work required to free the electron? Given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a).
(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 A separation?
Solution:(a) r = 0.53A =0.53 x 10^{-10}m
Charge of electron = – e
Charge of proton = e

$$\therefore \text{Potential energy} = \frac{1}{4\pi\varepsilon_0}.\frac{e(-e)}{r} = \frac{-e^2}{4\pi\varepsilon_0 r}$$

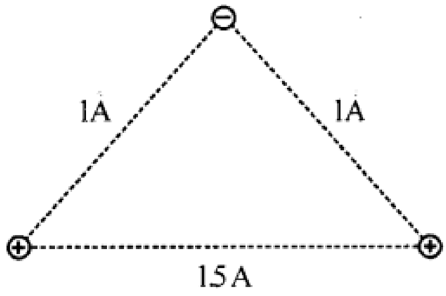
$$V = \frac{-(1.6)^2\times10^{-38}\times9\times10^9}{0.53\times10^{-10}} = \frac{-(1.6)^2\times10^{-38}\times9\times10^9}{0.53\times10^{-10}\times1.6\times10^{-19}}\text{ eV} = \frac{-1.6\times9}{0.53}\text{ eV} = -27.2\text{ eV}$$

(b) Total energy = -27.2 eV + 13.6 eV = -13.6 eV
The work required = 13.6 eV

(c) At 1.06 A distance
PE = -13.6 eV
It is brought to zero by adding + 13.6 eV
∴ PE at 0.53 is -13.6 eV and energy needed is 13.6 eV

Question 19. If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 A, and the electron is roughly 1 A from each proton. Determine the potential energy of the system. Specify your choice of zero potential energy.

Solution:



$$U = U_1 + U_2 + U_3 \qquad U_1 = \frac{9\times10^9 e^2}{1.5\times10^{-10}} \qquad U_2 = U_3.$$

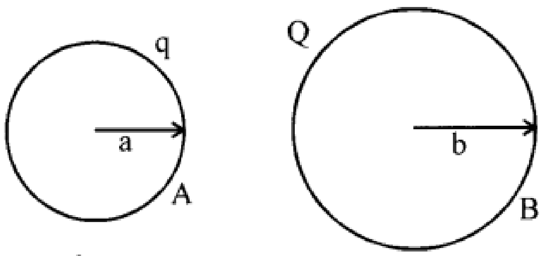
$$\therefore U_2 + U_3 = \frac{9\times10^9 \times -2e^2}{1\times10^{-10}}$$

$$U = \frac{9\times10^9 e^2}{10^{-10}}\left(\frac{1}{1.5} - \frac{2}{1}\right) = \frac{9\times10^9 \times (1.6)^2 \times 10^{-38} \times -2}{10^{-10}\times1.5\times1.6\times10^{-19}} \text{eV}$$

$$= \frac{-9\times1.6\times2}{1.5} = \frac{-9\times16\times4}{30} = -19.2 \text{ eV}$$

Question 20. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why the charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Solution: With arbitrary charges q and Q for A and B respectively,



$$\frac{E_A}{E_B} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}}{\frac{1}{4\pi\varepsilon_0}\frac{Q}{b^2}} = \frac{q}{Q}\cdot\frac{b^2}{a^2}$$

For equipotential condition, $V_A = V_B$

$$\frac{q}{4\pi\varepsilon_0 a} = \frac{Q}{4\pi\varepsilon_0 b}, \quad \therefore \frac{q}{Q} = \frac{a}{b} \qquad \therefore \frac{E_A}{E_B} = \frac{a}{b}\cdot\frac{b^2}{a^2} = \frac{b}{a}$$

E = σε0.

For pointed ends, σ = qA. As A → 0 E becomes extremely large. Flat means A → ∞ , hence E is very small.