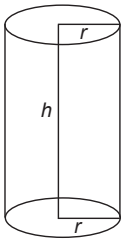
Volume and Surface Area of Solids 20

STUDY NOTES

- A cylinder is a three-dimensional solid that has two parallel bases joined by curved surface at a fixed distance.



- Curved surface area of a cylinder is the area of cylinder which is contained between the two parallel circular bases. It is also called lateral surface area.

  For a cylinder of base radius r and height h.

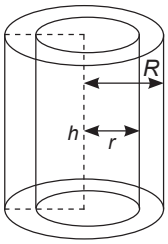
  (i) Curved Surface Area (CSA) = $2\pi rh$ square units.

  (ii) Total Surface Area (TSA) = CSA + area of two circular bases.

  $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$ square units.

  (iii) Volume (V) = $\pi r^2 h$ cubic units.

- Hollow cylinder is a cylinder which is empty from inside and has internal and external radius.



- For a hollow cylinder of external radius R, internal radius r and height h, we have

  (i) Thickness of the cylinder = R – r

  (ii) Area of cross section = $\pi(R^2 - r^2)$

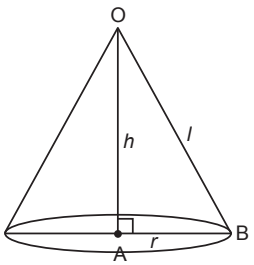
  (iii) Curved surface area (CSA) = $2\pi h(R + r)$ square units

  (iv) Total surface area TSA = $2\pi(Rh + rh + R^2 - r^2)$ square units

  (v) Volume (V) = $\pi h(R^2 - r^2)$ cubic units.

- When a right angled triangular lamina OAB is revolved about OA, it generates a cone. The point O is called the vertex of the cone. The length OA is called the height of the cone. The length AB is called the radius of the cone. The length OB is called the slant height of the cone.

  Slant height = $\sqrt{\text{radius}^2 + \text{height}^2} \Rightarrow l = \sqrt{r^2 + h^2}$



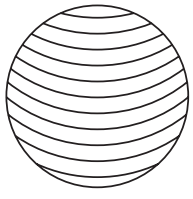
  For a cone of radius r, height h and slant height l:

  (i) Volume (V) = $\frac{1}{3}\pi r^2 h$ cubic units

  (ii) Curved surface area (CSA) = $\pi rl = \pi r\left(\sqrt{r^2 + h^2}\right)$ square units.

  (iii) Total surface area (TSA) = $\pi rl + \pi r^2 = \pi r(l + r)$ square units.

- Objects like football, volleyball, throw-ball, etc., are said to have to shape of a sphere.



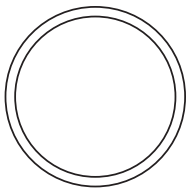
- When a circular lamina is revolved about any of its diameters, then the solid generated is called a sphere.

- For a solid sphere of radius = r, we have:

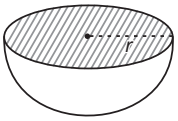
  (i) Volume of the sphere = $\left(\frac{4}{3}\pi r^3\right)$ cubic units,

  (ii) Surface area of the sphere = $(4\pi r^2)$ sq. units.

- The solid enclosed between two concentric spheres is called a spherical shell.
- For a spherical shell with External Radius = R and Internal Radius = r, we have:
  (i) Thickness of shell = (R – r) units;
  (ii) Volume of the material = $\frac{4}{3}\pi(R^3 - r^3)$ cubic units.



- When a plane through the centre of a sphere cuts it into two equal parts, then each part is called a hemisphere.
- For a Hemisphere of Radius r, we have:
  (i) Volume = $\frac{2}{3}\pi r^3$ cubic units;
  (ii) Curved surface area = $2\pi r^2$ sq. units;
  (iii) Total surface area = $(2\pi r^2 + \pi r^2) = 3\pi r^2$ sq.units



QUESTION BANK

A. Multiple Choice Questions [1 Mark]

Choose the correct option:

1. If the volume of a right circular cone of height 9 cm is 48 π cm^3, then diameter of its base is :
(a) 6 cm (b) 7 cm (c) 8 cm (d) 10 cm

2. A hemispherical bowl has a radius of 3.5 cm. The volume of water it would contain would be:
(a) 86.5 cm^3 (b) 89.8 cm^3 (c) 92.5 cm^3 (d) 93.5 cm^3

3. The height of a right circular cone is 12 cm. If its volume be 100 π cm^3, then its slant height is:
(a) 12 cm (b) 13 cm (c) 15 cm (d) 16 cm

4. A conical flask of base radius r and height h is full of milk. The milk is now poured into a cylindrical flask of radius $2r$. What is the height to which the milk will rise in the flask?
(a) $\frac{h}{2}$ (b) $\frac{h}{6}$ (c) $\frac{h}{9}$ (d) $\frac{h}{12}$

5. The radius of base and the height of a righrt circular cylinder are each increased by 20%. The volume of the cylinder will increase by:
(a) 72.8% (b) 80% (c) 82% (d) 92%

6. A sphere of radius R has volume equal to that of a cone of radius R, the height of the cone is:
(a) R (b) 2R (c) 3R (d) 4R

7. A sphere and a cube have the same surface area. The ratio of their volumes is:
(a) $\sqrt{6} : \sqrt{\pi}$ (b) $\sqrt{3} : \pi$ (c) $\pi : \sqrt{2}$ (d) $\sqrt{5} : \pi$

8. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm. with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is :
(a) 0.21 cm^3 (b) 0.42 cm^3 (c) 1.33 cm^3 (d) 0.33 cm^3

9. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is:
(a) $4\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$

10. A right circular cylinder of radius r cm and height h cm ($h > r$) just encloses a sphere of diameter :
(a) r cm (b) $2r$ cm (c) h cm (d) $2h$ cm

11. During conversion of a solid from one shape to another, the volume of the new shape will:
(a) increase (b) decrease (c) remain unaltered (d) be doubled

12. Volume of two spheres are in the ratio 64 : 27. The ratio of their surface areas is:
(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

13. A solid metallic sphere of diameter 21 cm is melted and recast into a number of smaller cones, each of diameter 7 cm and height 3 cm. The number of cones so formed is :

(a) 121 (b) 123 (c) 125 (d) 126

14. A hemispherical bowl of internal diameter 30 cm contains some liquid. This liquid is to be filled into cylindrical shaped bottles, each of diameter 5 cm and height 6 cm. The number of bottles necessary to empty the bowl is:

(a) 30 (b) 40 (c) 55 (d) 60

15. The base radius and height of a right circular solid cone are 2 cm and 8 cm respectively. It is melted and recast into spheres of diameter 2 cm each. The number of spheres so formed is:

(a) 6 (b) 8 (c) 10 (d) 12

16. In a cylinder, if radius is halved and height is doubled, the volume will be:

(a) same (b) doubled (c) halved (d) four times

17. The radius of a sphere is $3r$, then its volume will be:

(a) $\frac{4}{3}\pi r^3$ (b) $36\ \pi r^3$ (c) $\frac{8\pi r^3}{3}$ (d) $\frac{32}{3}\pi r^3$

18. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is:

(a) 4.2 cm (b) 2.1 cm (c) 2.4 cm (d) 1.6 cm

19. In a cylinder, radius is doubled and height is halved, curved surface area will be:

(a) halved (b) doubled (c) same (d) four times

20. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is:

(a) $2\pi r(l + r)$ (b) $\pi r\left(l + \frac{r}{4}\right)$ (c) $\pi r(l + r)$ (d) $2\pi rl$

21. The radius of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is :

(a) 10 : 17 (b) 20 : 27 (c) 17 : 27 (d) 20 : 37

22. The circumference of the base of a cylindrical vessel is 132 cm and its height is 50 cm. How many litres of water can it hold?

(a) 34.39 litres (b) 70.50 litres (c) 69.30 litres (d) 67.67 litres

23. The diameter of a cone is 28 cm and slant height is 10.5 cm. Then, its lateral surface area is:

(a) 480 cm^2 (b) 490 cm^2 (c) 462 cm^2 (d) 466 cm^2

24. The diameter of a cone is 10 cm and vertical height is 12 cm. Then, the area of curved surface area is:

(a) 205.15 cm^2 (b) 214.16 cm^2 (c) 204.28 cm^2 (d) 241.20 cm^2

25. The lateral surface area of a cone is $60\ \pi cm^2$. If the slant height of the cone be 8 cm, then the diameter of its base is :

(a) 25 cm (b) 18 cm (c) 12 cm (d) 15 cm

A. Answers

1. (c)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (d)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (b)
11. (c)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (b)
21. (b)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (d)					

B. Short Answer Type Questions [3 Marks]

1. 2.2 dm^3 of copper is to be drawn into a cylindrical wire of diameter 0.50 cm. Find the length of the wire.

Sol. Volume of cylinder = 2.2 dm^3

$\Rightarrow \pi r^2 h = 2.2\ dm^3 = 2200\ cm^3$

$\Rightarrow \frac{22}{7} \times 0.25 \times 0.25 \times h = 2200\ cm^3$

$\Rightarrow h = \frac{2200 \times 7}{22 \times 0.25 \times 0.25} = \frac{2200 \times 7 \times 100 \times 100}{22 \times 25 \times 25} = 16 \times 7 \times 100 = 11200\ cm = 112\ m.$

2. A well of diameter 1 m is dug 7 m deep. The earth taken out of it is completely used for making a conical body of scare-crow. The base diameter of the body is 2 m. Find the height of scare-crow.

Sol. Volume of conical scare-crow = Volume of well earth taken out

$\Rightarrow \frac{1}{3}\pi r_2^2 h_2 = \pi r_1^2 h_1$

$\Rightarrow 1^2 \times h_2 = 3 \times \frac{1}{2}^2 \times 7 \Rightarrow h_2 = 3 \times \frac{1}{4} \times 7 \text{ m} = \frac{21}{4} \text{ m} = 5.25 \text{ m}.$

3. The total surface area of a right circular cone of slant height 13 cm is 90 π cm^2. Calculate:

(a) Its radius in cm.

(b) Its volume in cm^3, in terms of π

Sol. (a) Total surface area of right circular cone = 90 π cm^2

$\Rightarrow \pi r (r + l) = 90\ \pi \Rightarrow r (r + 13) = 90 \Rightarrow r^2 + 13r - 90 = 0 \Rightarrow r^2 + 18r - 5r - 90 = 0$

$\Rightarrow r (r + 18) - 5 (r + 18) \Rightarrow (r - 5)(r + 18) = 0$

$\therefore \quad r = 5$ cm

(b) $h = \sqrt{l^2 - r^2} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$ cm

Volume of right circular cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 5 \times 5 \times 12\ cm^3 = 100\ \pi\ cm^3$

4. If the volumes of two cylinders are in the ratio 128 : 75, then find the ratio of their curved surface areas. It is given that the ratio of their heights is 2 : 3.

Sol. $\frac{V_1}{V_2} = \frac{128}{75}$ and $\frac{h_1}{h_2} = \frac{2}{3}$

$\Rightarrow \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{128}{75} \Rightarrow \frac{r_1^2}{r_2^2} \times \frac{2}{3} = \frac{128}{75} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{128 \times 3}{2 \times 75} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{64}{25} \Rightarrow \frac{r_1}{r_2} = \frac{8}{5}.$

$\frac{\text{C.S.A of one cylinder}}{\text{C.S.A of other cylinder}} = \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{8}{5} \times \frac{2}{3} = \frac{16}{15} = 16 : 15.$

5. How many metres of cloth 5 m wide will be required to make a conical tent the radius of whose base is 7m and whose height is 24 m?

Sol. $r = 7$ m and $h = 24$ m

So, $l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = 25$ m.

Curved surface area of the tent = $\pi r l$

$= \frac{22}{7} \times 7 \times 25\ m^2 = 550\ m^2.$

So, required length of cloth $= \frac{550}{5}$ m = 110 m.

6. How long would it take to fill a conical vessel whose diameter of base is 20 cm and depth is 21 cm, if it is filling through a pipe of diameter 5 mm, at the rate of 10 m/minute.

Sol. Diameter of conical vessel = 20 cm $= \frac{20}{100}$ m = 0.2 m $\Rightarrow r = \frac{0.2}{2} = 0.1$ m

Depth = 21 cm $= \frac{21}{100}$ m = 0.21 m

Time taken to fill vessel $= \frac{\text{Volume of conical vessel}}{\text{Volume of water through pipe in one minute}}$

$= \frac{\frac{1}{3}\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{\frac{1}{3}\pi \times 0.1 \times 0.1 \times 0.21}{\pi \times 0.0025 \times 0.0025 \times 10} = \frac{\frac{1}{3}\pi \times 1 \times 1 \times 21 \times 10000}{25 \times 25 \times 10}$

$= \frac{56}{5} \times \frac{2}{2} = \frac{112}{10} = \frac{11 \times 2 \times 60}{10} = 11.20$ min or 11 min 12 sec.

7. An iron sphere of radius 5 cm has been melted and recast into smaller spheres each of radius 2.5 cm. How many smaller spheres are made?

Sol. Radius of iron sphere (r_1) = 5 cm
Radius of smaller sphere (r_2) = 2.5 cm

$$\text{Number of smaller sphere made} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{5\times5\times5}{2.5\times2.5\times2.5} = 8.$$

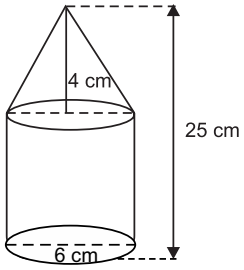
8. A cone and a hemisphere have equal bases and equal volume. Find the ratio of their heights.

Sol. $$\frac{\text{Volume of cone}}{\text{Volume of hemisphere}} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{2}{3}\pi r_1^3}$$

$$\Rightarrow \quad \frac{1}{1} = \frac{h_1}{2r_1} \Rightarrow \frac{2}{1} = \frac{h_1}{h_2} \Rightarrow h_1 : h_2 = 2 : 1 \ [\because h_2 = r_2]$$

C. Long Answer Type Questions [4 Marks]

1. The given solid figure is a cylinder surmounted by a cone. The diameter of the base of the cylinder is 6 cm. The height of the cone is 4 cm and the total height of the solid is 25 cm. Take $\pi = \frac{22}{7}$. Find the :



(a) volume of the solid
(b) curved surface area of the solid
Give your answers correct to the nearest whole number.

Sol. Radius of cone = radius of cylinder
$r = r_1 = r_2 = 3$ cm
Height of cylinder (h_1) = 21 cm
Height of cone (h_2) = 4 cm

(a) Volume of solid = Volume of cylinder + Volume of cone

$$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$$

$$= \pi r^2 \left[h_1 + \frac{1}{3}h_2\right]$$

$$= \frac{22}{7}\times3\times3\left[21+\frac{1}{3}\times4\right] = \frac{22}{7}\times3\times3\left[\frac{63+4}{3}\right] = \frac{22}{7}\times3\times67 = \frac{4422}{7} = 631.71 \text{ cm}^3 = 632 \text{ cm}^3.$$

(b) Cuvered surface area of solid = C.S.A of cylinder + C.S.A of cone

$$= 2\pi rh + \pi r\, l. \ [l = \sqrt{r^2+h^2} = \sqrt{9+16} = 5 \text{ cm}]$$

$$= \pi r\,[2h_1 + l]$$

$$= \frac{22}{7} \times 3\,[2 \times 21 + 5]$$

$$= \frac{22}{7}\times3\times47 = \frac{22\times141}{7} = \frac{3102}{7} = 443 \text{ cm}^2.$$

2. A solid metallic sphere of radius 6 cm is melted and made into a solid cylinder of height 32 cm. Find the:
(a) radius of the cylinder
(b) curved surface area of the cylinder (Take $\pi = 3.1$)

Sol. Radius of sphere (r_1) = 6 cm

Radius cylinder r_2 = ?

Height of cylinder (h) = 32 cm

(a) Volume of cylinder = Volume of sphere

$$\Rightarrow \quad \pi r_2^2 h = \frac{4}{3}\pi r_2^3$$

$$\Rightarrow r_2^2 \times 32 = \frac{4}{3}\times 6\times 6\times 6 \Rightarrow r_2^2 = \frac{4}{3}\times\frac{6\times 6\times 6}{32}$$

$$\Rightarrow r_2^2 = 9 \Rightarrow r_2 = \sqrt{9} \text{ cm} \Rightarrow r_2 = 3 \text{ cm}.$$

(b) C.S.A of cylinder = $2\pi r_2 h$

$= 2 \times 3.1 \times 3 \times 32 = 6.2 \times 96 = 595.2 \text{ cm}^2$.

3. The volume of a conical tent is 1232 m^3 and the area of the base floor is 154 m^2. Calculate the:
(a) radius of the floor.
(b) height of the tent.
(c) length of the canvas required to cover this conical tent, if its width is 2 m.

Sol. Volume of conical tent = 1232 m^3

Area of floor = 154 m^2

(a) Area of floor = 154 $\Rightarrow \pi r^2 = 154 \Rightarrow r^2 = \frac{154}{\pi} = \frac{154\times 7}{22} \Rightarrow r = \sqrt{7\times 7} \Rightarrow r = 7$ m

(b) $$\frac{\text{Volume of conical tent}}{\text{Area of the base}} = \frac{1232}{154}$$

$$\Rightarrow \frac{\frac{1}{3}\pi r^2 h}{\pi r^2} = \frac{1232}{154} \Rightarrow \frac{1}{3}h = \frac{1234}{154} \Rightarrow \frac{1}{3}h = 8$$

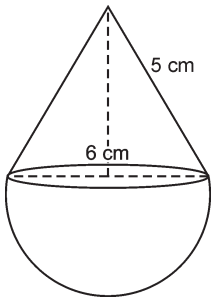
$\Rightarrow h = 24$ m.

(c) $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ m

$$\text{Length of canvas} = \frac{\text{C.S.A of canvas}}{\text{width}}$$

$$= \frac{\pi r l}{2} = \frac{22}{7}\frac{\times 7\times 25}{2} = 275 \text{ m}$$

4. The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate:
(a) the height of the cone.
(b) the volume of the solid.



Sol. (a) Diameter of base = 6 cm

Radius of base = 3 cm

(l) slant height = 5 cm

$l^2 = r^2 + h^2 \Rightarrow 5^2 = 3^2 + h^2$

$\Rightarrow 25 - 9 = h^2 \Rightarrow \sqrt{16} = h \Rightarrow 4 \text{ cm} = h$

Height of cone = 4 cm

Volume of solid = Volume of hemisphere + Volume of cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \pi r^2\left[\frac{2}{3}r + \frac{1}{3}h\right] = \pi r^2\left[\frac{2r+h}{3}\right]$$

$$= \frac{22}{7}\times 3\times 3\left[\frac{2\times 3+4}{3}\right] = \frac{22}{7}\times 9\left[\frac{6+4}{3}\right] = \frac{22}{7}\times 9\times\frac{10}{3} = \frac{660}{7} = 94.29 \text{ cm}^3.$$

5. A plumber fits a pipe of internal radius 10 cm from a tap to a cylindrical tank, which is 5 m in radius and 2 m deep. If the water flows through the pipe at the rate of 3 km/hr, in how much time will tank be filled?

Sol. (Internal radius of pipe) $r_1 = 10$ cm

Radius of cylindrical tank $(r_2) = 5$ m $= 500$ cm

Depth of cylindrical tank $h_2 = 2$ m $= 200$ cm

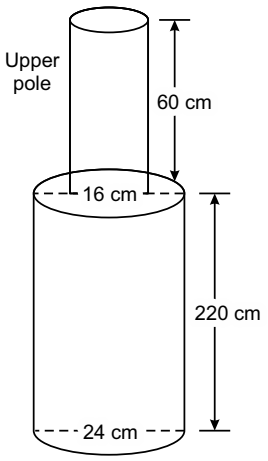
Length of water column made in 1 hour $(h_1) = 3$ km $= 3 \times 1000 \times 100$ cm $= 300000$ cm.

Time taken to fill the tank $= \dfrac{\text{Volume of cylindrical tank}}{\text{Volume of water flows through the pipe}}$ hours

$= \dfrac{\pi r_2^2 h_2}{\pi r_1^2 \times h_1} = \dfrac{500 \times 500 \times 200}{10 \times 10 \times 300000}$

$= \dfrac{50 \times 5 \times 2}{300} = \dfrac{5}{3}$ hrs $= \dfrac{5}{3} \times 60$ min $= 100$ min.

6. Two solid iron poles are lying one over other. The pole at the lower position has height 220 cm and base diameter 24 cm, whereas the pole above it has height of 60 cm, and base diameter 16 cm. Calculate the weight of the pole, if 1 cm^3 of iron weighs 10 g.



Sol. Volume of solid = Volume of lower pole + Volume of upper pole

$= \pi r_1^2 h_1 + \pi r_2^2 h_2 = \pi\left[r_1^2 h_1 + r_2^2 h_2\right] = 3.14\ [12^2 \times 220 + 8^2 \times 60]$

$= 3.14\ [31680 + 3840] = 3.14 \times 35520 = 111532.8\ cm^3$

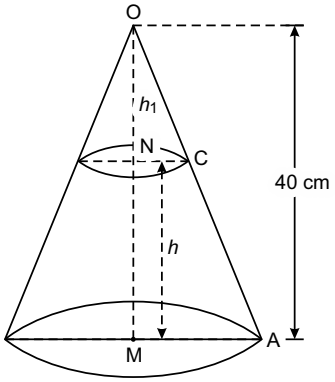
Weight of iron $= \dfrac{111532.8 \times 10}{1000}$ kg $= 1115.328$ kg.

7. A small cone is cut off at the top of a cone by a plane parallel to the base. The volume of the small cone is $\frac{1}{8}$ of the volume of the bigger cone. At what height above the base is the section made, it is given that height of the complete cone is 40 cm?

Sol. Let R be the radius of cone.

r be the radius of smaller cone.

h be the height of frustum and h_1 be the height of small cone



$\Delta ONC \sim \Delta OMA$

$\dfrac{ON}{OM} = \dfrac{NC}{MA} \Rightarrow \dfrac{h_1}{40} = \dfrac{r}{R}$

$\Rightarrow h_1 = \dfrac{1}{R} \times 40$ ...(i)

$\dfrac{\text{Volume of small cone}}{\text{Volume of given cone}} = \dfrac{1}{8}$

$\Rightarrow \dfrac{\frac{1}{3}\pi r^2 h_1}{\frac{1}{3}\pi R^2 40} = \dfrac{1}{8}$

$\Rightarrow \dfrac{r^2}{R^2} \times \dfrac{r}{R} = \dfrac{1}{8}$

$\Rightarrow \left(\dfrac{r}{R}\right)^3 = \dfrac{1}{8} \Rightarrow \dfrac{r}{R} = \dfrac{1}{2}$. ...(ii)

From (i) and (i)

$h_1 = \frac{1}{2} \times 40 \Rightarrow h_1 = 20 \text{ cm} \Rightarrow h = 40 \text{ cm} - 20 \text{ cm} = 20 \text{ cm}$

So, the section has been made at a height of 20 cm from the base.

8. From a solid cylinder whose height is 2.4 cm, and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Sol. Total surface area = C.S.A of cylinder + C.S.A of cone + Area of the base of the cylinder

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi r\ [2h + l + r] \quad \left[l = \sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2} \text{ cm} = 2.5 \text{ cm} \right]$

$= \frac{22}{7} \times 0.7\ [4.8 + 2.5 + 0.7]$

$= 17.6 = 18 \text{ cm}^2.$

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