(MATHS) INVERSE TRIGONOMETRIC FUNCTIONS

DPP – 07

CLASS –12th

TOPIC – PROPERTIES OF ITFS

Q.1 Prove that $\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\tan^{-1}\left(\frac{2}{9}\right)$

Q.2 Prove that $\tan^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)=\pi$

Q.3 Prove the following result:

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\sin^{-1}\frac{1}{\sqrt{5}}$$

Q.4 Find the value of $\tan^{1}\left(\frac{x}{y}\right)-\tan^{1}\left(\frac{x-y}{x+y}\right)$

Q.5 Solve the equation for x:

$$\tan^{-1}2x+\tan^{-1}=n\pi+\frac{3\pi}{4}$$

Q.6 Solve the equation for x:

$$\tan^{-1}\ x+1\ +\tan^{-1}\ x-1\ =\tan^{-1}\frac{8}{31}$$

Q.7 Solve the equation for x:

$$\tan^{-1}\ x-1\ +\tan^{-1}x+\tan^{-1}\ x+1\ =\tan^{-1}3x$$

Q.8 Solve the following equations for x:

$$\tan^{-1}\left(\frac{1-x}{1+x}\right)-\frac{1}{2}\tan^{-1}x=0,\ \text{where } x>0$$

Q.9 Solve the equation for x:

$$\cot^{-1}x-\cot^{-1}\ x+2\ =\frac{\pi}{12},\ \text{where } x>0$$

Q.10 Solve the equation for x:

$$\tan^{-1}\ x+2\ +\tan^{-1}\ x-2\ =\tan^{-1}\left(\frac{8}{79}\right),\ x>0$$

Q.11 Solve the equation for x:

$$\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{4},\ 0<x<\sqrt{6}$$

Q.12 Solve the following equations for x:

$$\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$$

Q.13 Solve the following equations for x:

$$\tan^{-1}\ 2+x\ +\tan^{-1}\ 2-x\ =\tan^{-1}\frac{2}{3},\ \text{where } x<-\sqrt{3}\ \propto,\ x>\sqrt{3}$$

SOLUTION

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Sol.1 $\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\tan^{-1}\left(\frac{2}{9}\right)$

LHS $=\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)$

$=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\times\frac{1}{13}}\right)$ $\left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$=\tan^{-1}\left(\frac{\frac{13+7}{91}}{\frac{91-1}{91}}\right)$

$=\tan^{-1}\left(\frac{20}{91}\times\frac{91}{90}\right)$

$=\tan^{-1}\left(\frac{2}{9}\right)$

$=$ RHS

Sol.2 $\sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)=\pi$

LHS $=\sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)$

$=\tan^{-1}\left(\frac{\frac{12}{13}}{\sqrt{1-\left(\frac{12}{13}\right)^2}}\right)+\tan^{-1}\left(\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}\right)+\tan^{-1}\left(\frac{63}{16}\right)$

$\left\{\text{Since } \sin^{-1}x=\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\cos^{-1}x=\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right\}$

$=\tan^{-1}\left(\frac{\frac{12}{13}}{\frac{5}{13}}\right)+\tan^{-1}\left(\frac{\frac{3}{5}}{\frac{4}{5}}\right)+\tan^{-1}\left(\frac{63}{16}\right)$

$=\tan^{-1}\left(\frac{12}{5}\right)+\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{63}{16}\right)$

$=\pi+\tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}\right)+\tan^{-1}\left(\frac{63}{16}\right)$

$\left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } x>0, y>0 \text{ and } xy>0\right\}$

$$= x + \tan^{-1}\left(\frac{\frac{63}{20}}{-\frac{16}{20}}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= x + \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= x - \tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$\left\{\text{Since } \tan^{-1}(-x) = -\tan^{-1} x\right\}$$

$= x$

Hence,

$$\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$$

Sol.3 $\tan^{-1} 1/4 + \tan^{-1} 2/9 = \frac{\tan^{-1}(1/4 + 2/9)}{(1 - 1/4 \times 2/9)}$

$$= \tan^{-1}\frac{\left(\frac{17}{36}\right)}{\left(\frac{34}{36}\right)}$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

Let $\tan^{-1}\left(\frac{1}{2}\right) = \theta$

$\tan = \theta = \frac{1}{2}$

We know that $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

So if opposite side = 1 unit

Adjacent side = 2 unit, then hypotenuse = $\sqrt{5}$ unit

So $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

so $\sin\theta = \frac{1}{\sqrt{5}}$

so $\theta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

Sol.4 We know that, $\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A - B}{1 + AB}\right)$, if $AB > -1$

Consider the given expression $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x - y}{x + y}\right)$:

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{1}\left(\frac{x-y}{x+y}\right)=\tan^{-1}\left(\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1+\left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right)$$

$$=\tan^{-1}\left(\frac{x(x+y)-y(x-y)}{y(x+y)+c(x-y)}\right)$$

$$=\tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)$$

$$=\tan^{-1}(1)$$

$$=\frac{\pi}{4}$$

Sol.5 Given

$$\tan^{-1}2x+\tan^{-1}3x=n\,\pi+\frac{3\pi}{4} \qquad ....(i)$$

$$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-2x\times 3x}\right)=n\,\pi+\frac{3\pi}{4} \qquad \left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } xy<1\right\}$$

$$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right)=nx+\frac{3\pi}{4},6x^2<1$$

$$\Rightarrow \frac{5x}{1-6x^2}=\tan\left(nx+\frac{3\pi}{4}\right),6x^2<1$$

$$\Rightarrow \frac{5x}{1-6x^2}=-1,6x^2<1$$

$$\Rightarrow 5x=-1+6x^2,6x^2<1$$

$$\Rightarrow 6x^2-5x-1=0,x^2<\frac{1}{6}$$

$$\Rightarrow 6x^2-6x+x-1=0,-\frac{1}{\sqrt{6}}<x<\frac{1}{\sqrt{6}}$$

$$\Rightarrow 6x(x-1)+1(x-1)=0,-\frac{1}{\sqrt{6}}<x<\frac{1}{\sqrt{6}}$$

$$\Rightarrow (6x+1)(x-1)=0$$

$$\Rightarrow 6x+1=0 \quad \text{or} \quad x-1=0$$

$$\Rightarrow x=-\frac{1}{6} \text{ or } x=1$$

Since $x=1=\left(-\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$

So, x = 1 is not root of the given equation (i),

Since,

$$x=-\frac{1}{6}=\left(-\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$$

So,

$x=-\frac{1}{6}$ is the root of the given equation (i),

Hence,

$x=-\frac{1}{6}$

Sol.6 Given,

$\tan^{-1}(x+1)+\tan^{-1}(x+1)=\tan^{-1}\frac{8}{31}$ .....(i)

$\Rightarrow \tan^{-1}\left[\frac{(x+1)+(x-1)}{1-(x+1)(x-1)}\right]=\tan^{-1}\frac{8}{31},(x+1)(x-1)<1$

$\left\{\text{Since } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy},\ \text{if } xy<1\right\}$

$\Rightarrow \tan^{-1}\left[\frac{2x}{1-(x^2-1)}\right]=\tan^{-1}\frac{8}{31}, \quad (x^2-1)<1$

$\Rightarrow \tan^{-1}\left[\frac{2x}{1-x^2+1}\right]=\tan^{-1}\frac{8}{31},x^2<2$

$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31},-\sqrt{2}<x<\sqrt{2}$

$\Rightarrow 8x^2+62x-16=0, \quad -\sqrt{2}<x<\sqrt{2}$

$\Rightarrow 4x^2+31x-8=0, \quad -\sqrt{2}<x<\sqrt{2}$

$\Rightarrow 4x(x+8)-1(x+8)=0, \quad -\sqrt{2}<x<\sqrt{2}$

$\Rightarrow (4x-1)(x+8)=0, \quad -\sqrt{2}<x<\sqrt{2}$

$\Rightarrow x=\frac{1}{4}$ or $x=-8$

But, $x=-8\notin(-\sqrt{2},\sqrt{2})$

$\Rightarrow$ x = 8 is not root of the given equation (i)

For $x=\frac{1}{4}\in(-\sqrt{2},\sqrt{2})$

$\Rightarrow x=\frac{1}{4}$ is a root of the equation (i)

Hence,

$x=\frac{1}{4}$

Sol.7 Given

$\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

$\Rightarrow \tan^{-1}(x-1)+\tan^{-1}(x+1)+\tan^{-1}x=\tan^{-1}3x$

$\Rightarrow \tan^{-1}\left[\frac{(x-1)+(x+1)}{1-(x-1)(x+1)}\right]+\tan^{-1}x=\tan^{-1}3x$

$\left\{\text{Since, } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right), \text{ if } xy<1\right\}$

$\Rightarrow \tan^{-1}\left(\frac{2x}{1-x^2+1}\right)+\tan^{-1}x=\tan^{-1}3x, \quad x^2-1<1$

$\Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right)+\tan^{-1}x=\tan^{-1}3x, \quad x^2<2$

$\Rightarrow \tan^{-1}\left[\frac{\frac{2x}{2-x^2}+x}{1-\left(\frac{2x}{2-x^2}\right)^x}\right]=\tan^{-1}3x, \quad \frac{2x^2}{2-x^2}<1$

$\Rightarrow \tan^{-1}\left[\frac{\frac{2x+2x-x^3}{2-x^2}}{\frac{2-x^2-2x^2}{2-x^2}}\right]=\tan^{-1}3x$

$\Rightarrow \tan^{-1}\left[\frac{4x-x^3}{2-3x^2}\right]=\tan^{-1}3x, \quad 2x^2<2-x^2$

$\Rightarrow \frac{4x-x^3}{2-3x^2}=3x, 3x^2<2$

$\Rightarrow 4x-x^3=6x-9x^3, \quad x^2<\frac{2}{3}$

$\Rightarrow 9x^3-x^3+4x-6x=0$

$\Rightarrow 8x^3-2x=0$

$\Rightarrow 2x\left(4x^2-1\right)=0$

$\Rightarrow x=0, x=\frac{1}{2}, x=-\frac{1}{2}$ all satisfies $x^2<\frac{2}{3}$, so

$x=0, \pm\frac{1}{2}$

Sol.8 $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$

$\Rightarrow \tan^{-1}1-\tan^{-1}x=\frac{1}{2}\tan^{-1}x \qquad \left[\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$

$\Rightarrow \frac{\pi}{4}=\frac{3}{2}\tan^{-1}x$

$\Rightarrow \tan^{-1}x=\frac{\pi}{6}$

$\Rightarrow x=\tan\frac{\pi}{6}$

$\therefore \quad x=\frac{1}{\sqrt{3}}$

Sol.9 Given,

$\cot^{-1}x-\infty^{-1}\left(x+2\right)=\frac{\pi}{12}$, where $x>0$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right)-\tan^{-1}\left(\frac{1}{x+2}\right)=\frac{\pi}{12} \qquad \left\{\text{Since } \cot^{-1}x=\tan^{-1}\frac{1}{x}\right\}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{x}-\frac{1}{x+2}}{1+\frac{1}{x}\times\frac{1}{x+2}}\right)=\frac{x}{12} \qquad \left\{\text{Since, } \tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right\}$$

$$\Rightarrow \left(\frac{\frac{x+2-x}{x(x+2)}}{\frac{x(x+2)+1}{x(x+2)}}\right)=\tan\frac{\pi}{12}$$

$$\Rightarrow \frac{2}{x^2+2x+1}=\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{2}{(x+1)^2}=\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{4}\times\tan\frac{\pi}{3}} \qquad \left\{\text{Since, } \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}\right\}$$

$$\Rightarrow \frac{2}{(x+1)^2}=\frac{\sqrt{3}-1}{1+\sqrt{3}}$$

$$\Rightarrow \frac{2}{(x+1)^2}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \frac{2}{(x+1)^2}=\frac{3-1}{\left(\sqrt{3}+1\right)^2}$$

$$\Rightarrow (x+1)^2-\left(\sqrt{3}+1\right)^2$$

$$\Rightarrow x+1=\pm\left(\sqrt{3}+1\right)$$

$$\Rightarrow x+1=\sqrt{3}+1 \quad \text{or} \quad x+1=\sqrt{3}-1$$

$$\Rightarrow x=\sqrt{3}+1-1 \quad \text{or} \quad x=-\sqrt{3}-2$$

$$\Rightarrow x=\sqrt{3} \quad \text{or} \quad x=-\left(\sqrt{3}+2\right)$$

Given, $x>0$, so

$$x=\sqrt{3}$$

Sol.10 Given,

$$\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\left(\frac{8}{79}\right), x>0$$

$$\Rightarrow \tan^{-1}\left[\frac{(x+2)+(x-2)}{1-(x+2)(x-2)}\right]=\tan^{-1}\frac{8}{79} \qquad \left\{\text{Since, } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$$

$$\Rightarrow \tan^{-1}\left[\frac{2x}{1-x^2+4}\right]=\tan^{-1}\frac{8}{79}$$

$$\Rightarrow \frac{2x}{5-x^2}=\frac{8}{79}$$

$$\Rightarrow 40-8x^2=158x$$

$$\Rightarrow 8x^2+158x-40=0$$

$$\Rightarrow 4x^2+79x-20=0$$

$$\Rightarrow 4x^2+80x-x-20=0$$

$\Rightarrow\ 4x(x+20)-1(x+20)=0$

$\Rightarrow\ (4x-1)(x+20)=0$

$\Rightarrow\ (4x-1)=0 \quad \text{or} \quad x+20=0$

$\Rightarrow\ x=\frac{1}{4} \quad \text{or} \quad x=-20$

Since, x > 0, so

$x=\frac{1}{4}$

Sol.11 Given,

$\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{x}{4}, 0<x<\sqrt{6}$

$\Rightarrow\ \tan^{-1}\left[\frac{\frac{x}{2}+\frac{x}{3}}{1-\frac{x}{2}\times\frac{x}{3}}\right]=\frac{\pi}{4}$ $\quad\left\{\text{Since, } \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$\Rightarrow\ \tan^{-1}\left[\frac{\frac{5x}{6}}{\frac{(6-x^2)}{16}}\right]=\frac{\pi}{4}$

$\Rightarrow\ \tan^{-1}\left(\frac{5x}{6-x^2}\right)=\frac{\pi}{4}$

$\Rightarrow\ \frac{5x}{6-x^2}=\tan\frac{\pi}{4}$

$\Rightarrow\ \frac{5x}{6-x^2}=1$

$\Rightarrow\ 5x=6-x^2$

$\Rightarrow\ x^2+5x-6=0$

$\Rightarrow\ x^2+6x-x-6=0$

$\Rightarrow\ x(x+6)-1(x+6)=0$

$\Rightarrow\ (x+6)(x-1)=0$

$\Rightarrow\ x=-6 \quad \text{or} \quad x=1$

But, $0<x<\sqrt{6}$, so

x=1

Sol.12 $\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$

$\Rightarrow\ \tan^{-1}\left(\frac{\frac{x-2}{x-4}+\frac{x+2}{x+4}}{1-\left(\frac{x-2}{x-4}\right)\left(\frac{x+2}{x+4}\right)}\right)=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{(x-2)(x+4)+(x+2)(x-4)}{(x-4)(x+4)}}{\frac{(x-4)(x+4)-(x-2)(x+2)}{(x-4)(x+4)}}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{(x-2)(x+4)+(x+2)(x-4)}{(x-4)(x+4)-(x-2)(x+2)}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{x^2+2x-8+x^2-2x-8}{(x^2-16)-(x^2-4)}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{2x^2-16}{-12}\right)=\frac{\pi}{4}$$

$$\Rightarrow \left(\frac{x^2-8}{-6}\right)=\tan\frac{\pi}{4}$$

$$\Rightarrow \left(\frac{x^2-8}{-6}\right)=1$$

$$\Rightarrow x^2-8=-6$$

$$\Rightarrow x^2=2$$

$$\therefore x=\pm\sqrt{2}$$

Sol.13 $\tan^{-1}\dfrac{(2+x+2-x)}{\{1-(2+x)(2-x)\}}$

$$\tan^{-1}\frac{4}{(x^2-3)}=\tan^{-1}\frac{2}{3}$$

$$\frac{4}{(x^2-3)}=\frac{2}{3}$$

$$x^2-3=6$$

$$x=3,-3$$