Chapter 12

Conic Sections-I

Solutions (Set-1)

Very Short Answer Type Questions :

1. Find the centre and radius of the circle $(x + 1)^2 + (y + 3)^2 = 16$.

Sol. The given equation is $(x + 1)^2 + (y + 3)^2 = 16$

i.e., $[x - (-1)]^2 + [y - (-3)]^2 = (4)^2$

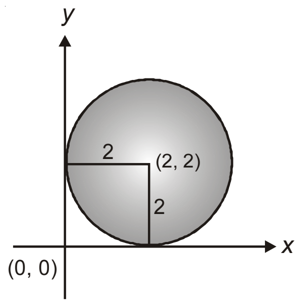
which is of the form $(x - h)^2 + (y - k)^2 = r^2$

where, (h, k) = centre and r = radius

$\therefore$ The centre and radius of the given circle are (–1, –3) and 4 units respectively.

2. A circle with radius r is touching both the axes and the abscissa of its centre is 2. Find the radius of the circle and ordinate of the centre.

Sol. It is given that the circle is touching both the axes.



$\Rightarrow$ $h = k = r$

where (h, k) = centre and r = radius

We are given that $h = 2$

$\Rightarrow$ $k = 2$ and $r = 2$

$\therefore$ Required radius = 2 units and ordinate of the centre = 2.

3. Find the equation of the circle with centre at (2, 3) and diameter as 8 units.

Sol. Let the equation of the circle be $(x - h)^2 + (y - k)^2 = r^2$

where (h, k) = centre and r = radius

$\therefore$ The required equation of the circle with centre at (2, 3) and radius 4 units (half of diameter *i.e.*, $\frac{8}{2} = 4$) is

$(x - 2)^2 + (y - 3)^2 = (4)^2$

i.e., $x^2 + y^2 - 4x - 6y - 3 = 0$

4. Find the centre and radius of the circle $(x - 1)^2 + (y + 2)^2 = 4$.

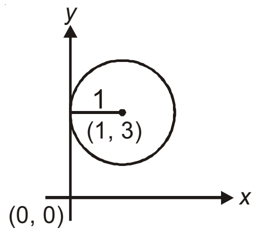
Sol. The given equation is $(x - 1)^2 + (y + 2)^2 = 4$

or, $[x - 1]^2 + [y - (-2)]^2 = (2)^2$

which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 1$, $k = -2$ and $r = 2$

Thus the centre of the given circle is (1, –2) while its radius is 2 units.

5. Find the equation of the circle which touches y-axis and whose centre is (1, 3).



Sol. Since, the centre of the circle is (1, 3) and is touching the y-axis, therefore the radius of the circle is given by the abscissa of the co-ordinates of the centre.

$\Rightarrow$ radius = 1 unit

$\therefore$ The required equation of the circle is $(x-1)^2 + (y-3)^2 = (1)^2$

i.e., $x^2 + y^2 - 2x - 6y + 9 = 0$

Short Answer Type Questions :

6. If the circle passes through the points (0, 0), (3, 0) and (0, 4), then find its radius.

Sol. Let the required equation of the circle be $(x-h)^2 + (y-k)^2 = (r)^2$

Since the circle passes through the points (0, 0), (3, 0) and (0, 4),

we have,

$(0-h)^2 + (0-k)^2 = r^2$

$(3-h)^2 + (0-k)^2 = r^2$

$(0-h)^2 + (4-k)^2 = r^2$

$\Rightarrow$ $h^2 + k^2 = r^2$ ... (i)

$h^2 + k^2 - 6h + 9 = r^2$ ... (ii)

and $h^2 + k^2 + 16 - 8k = r^2$ ... (iii)

On putting the value of (i) in (ii) and (iii), we get

$r^2 - 6h + 9 = r^2$ and $r^2 + 16 - 8k = r^2$

$\Rightarrow$ $6h = 9$ and $8k = 16$

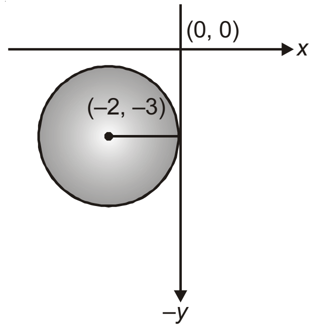
$\Rightarrow$ $h = \frac{3}{2}$ and $k = 2$

Now, $r^2 = h^2 + k^2$

$\Rightarrow$ $r^2 = \left(\frac{3}{2}\right)^2 + (2)^2 = \frac{9}{4} + 4 = \frac{25}{4}$

$\Rightarrow$ $r = \frac{5}{2}$, which is the required radius.

7. Find the equation of the circle which touches the y-axis and whose centre is (–2, –3).



Sol. The circle with centre (–2, –3) is touching the y-axis *i.e.*, the line whose equation is $x = 0$.

$\Rightarrow$ $x = 0$ is tangent to the required circle.

Thus, the perpendicular distance of y-axis from the centre *i.e.*, (–2, –3) = radius

$\Rightarrow$ Radius = 2 units

$\therefore$ The required equation of the circle with centre (–2, –3) and radius 2 is

$[x-(-2)]^2 + [y-(-3)]^2 = (2)^2$

i.e., $(x+2)^2 + (y+3)^2 = (2)^2$

8. Find the equation of the circle concentric with the circle $x^2 + y^2 - 8x + 14y + 1 = 0$ and has half of its area.

Sol. The given equation of the circle is $x^2 + y^2 - 8x + 14y + 1 = 0$

which is of the form $x^2 + y^2 + 2gx + 2fy + c = 0$

where $2g = -8$, $2f = 14$ and $c = 1$

$\Rightarrow$ $g = -4$ and $f = 7$

$\therefore$ Centre of the given circle = (4, –7) and radius = $\sqrt{g^2+f^2-c} = \sqrt{16+49-1} = 8$ units.

It is given that the required circle is concentric with the given circle.

$\Rightarrow$ The centre of the required circle is (4, –7)

Also, it has half the area of the given circle.

Now, area of the given circle = πr^2

$= \pi \times 8 \times 8$

$= 64\pi$

Let the radius of the required circle be r_1.

$\therefore$ $\pi r_1^2 = \dfrac{64\pi}{2}$

$\Rightarrow$ $r_1^2 = 32$

i.e., $r_1 = 4\sqrt{2}$

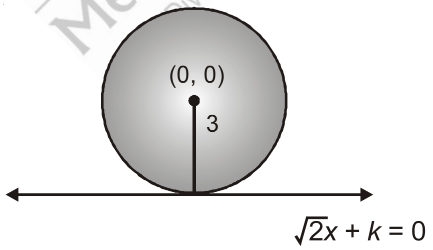
$\therefore$ Equation of the required circle with centre at (4, –7) and radius $4\sqrt{2}$ is

$[x - 4]^2 + [y - (-7)]^2 = \left(4\sqrt{2}\right)^2$

i.e., $(x - 4)^2 + (y + 7)^2 = 32$

9. If the line $\sqrt{2}x + k = 0$ touches the circle $x^2 + y^2 = 9$, then find the value of k.

Sol. As per the given information, the line $\sqrt{2}x + k = 0$ is tangent to the circle $x^2 + y^2 = 9$, whose centre lies at the origin and radius is 3 units.



$\therefore$ The perpendicular distance of $\sqrt{2}x + k = 0$ from the centre of the circle

i.e., (0, 0) is equal to its radius *i.e.*, 3

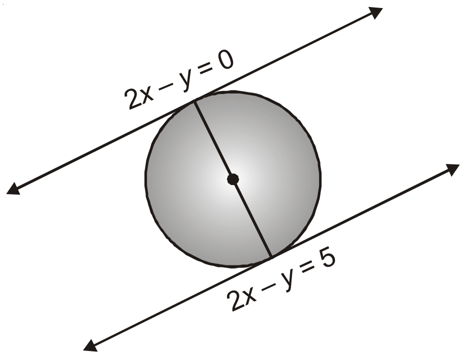
$\Rightarrow$ $\dfrac{|\sqrt{2}(0)+0+k|}{\sqrt{(\sqrt{2})^2+0}} = 3$

$\Rightarrow$ $|k| = 3\sqrt{2}$

$\Rightarrow$ $k = \pm 3\sqrt{2}$

10. If the lines $2x - y = 0$ and $2x - y = 5$ are tangents to the circle, then find the diameter of the circle.

Sol. It is given that the lines $2x - y = 0$ and $6x - 3y = 15$ are tangents to a circle.



Please note that the slope of both the lines is same.

⇒ Both the lines *i.e.,* $2x - y = 0$ and $6x - 3y = 15$ are parallel.

⇒ The diameter of the circle is given by the distance between the given parallel lines.

Distance between two parallel lines = $\dfrac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$

where $c_1 = 0$ and $c_2 = -5$

[∵ $6x - 3y = 15$ can be written as $2x - y - 5 = 0$]

$= \dfrac{|-5-0|}{\sqrt{(2)^2 + (-1)^2}}$ *i.e.,* $\dfrac{5}{\sqrt{5}} = \sqrt{5}$ = diameter of the circle.

11. Find the centre and radius of the circle $x^2 + y^2 - 4x + 2y - 4 = 0$.

Sol. The given equation of the circle is $x^2 + y^2 - 4x + 2y - 4 = 0$

which is of the form $x^2 + y^2 + 2gx + 2fy + c = 0$

where $2g = -4$, $2f = 2$ and $c = -4$

⇒ $g = -2$, $f = 1$ and $c = -4$

∴ Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{(-2)^2 + (1)^2 + 4}$

$= \sqrt{4 + 1 + 4}$ = 3 units

and, centre = $(-g, -f) = (2, -1)$

12. Find the equation of the circle which passes through the origin and cuts off intercepts 6 and 8 from the positive parts of the axes respectively.

Sol. Let the required equation of the circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through the origin

⇒ $(0 - h)^2 + (0 - k)^2 = r^2$

⇒ $h^2 + k^2 = r^2$

It is given that the circle makes intercepts 6 and 8 on the positive side of the co-ordinate axes respectively. This means that the circle passes through (6, 0) and (0, 8).

$\therefore$ $(6 - h)^2 + (0 - k)^2 = h^2 + k^2$ [$\because$ $r^2 = h^2 + k^2$]

and $(0 - h)^2 + (8 - k)^2 = h^2 + k^2$

$\Rightarrow$ $36 + h^2 - 12h + k^2 = h^2 + k^2$

and $h^2 + 64 + k^2 - 16k = h^2 + k^2$

$\Rightarrow$ $36 - 12h = 0$

$\Rightarrow$ $h = 3$

And, $64 - 16k = 0$

$\Rightarrow$ $k = 4$

Now, $r^2 = h^2 + k^2$

$\Rightarrow$ $r^2 = (3)^2 + (4)^2 = 25$

$\Rightarrow$ $r = 5$

$\therefore$ The required equation of the circle is $(x - 3)^2 + (y - 4)^2 = (5)^2$

13. If the co-ordinates of one end of a diameter of the circle $x^2 + y^2 - 6x - 7 = 0$ is (7, 0), then find the co-ordinates of the other end of the diameter.

Sol. The given equation of the circle is $x^2 + y^2 - 6x - 7 = 0$

which is of the form $x^2 + y^2 + 2gx + 2fy + c = 0$

where, $2g = -6$ and $2f = 0$

$\Rightarrow$ $g = -3$ and $f = 0$

$\therefore$ The centre of the circle is (3, 0).

The co-ordinates of the one end of the diameter is given as (7, 0).

Let the co-ordinates of the other end be (x, y)

We know that, the centre divides the diameter in two equal parts.

i.e., The centre (3, 0) is the mid-point of the two end points of the diameter *i.e.*, (7, 0) and (x, y)

$\Rightarrow$ $\dfrac{x+7}{2} = 3$ and $\dfrac{y+0}{2} = 0$

$\Rightarrow$ $x = -1$ and $y = 0$

$\therefore$ The co-ordinates of the other end of the diameter are (–1, 0).

Long Answer Type Questions :

14. Find the equation of the circle which passes through the points (3, 2) and (1, 4) and the centre lies on the straight line $x + y = 5$.

Sol. Let the required equation of the circle be $(x - h)^2 + (y - k)^2 = r^2$

where, (h, k) = centre and r = radius.

As per the given information, $h + k = 5$ [$\because$ Centre lies on the line $x + y = 5$] and the point (3, 2) and (1, 4) lie on the circle.

$\Rightarrow$ $(3 - h)^2 + (2 - k)^2 = r^2$,

$(1 - h)^2 + (4 - k)^2 = r^2$

i.e., $h^2 + k^2 - 6h - 4k + 13 = r^2$ ... (i)

$h^2 + k^2 - 2h - 8k + 17 = r^2$ ... (ii)

Equating (i) and (ii), we get

$-6h - 4k + 13 = -2h - 8k + 17$

$\Rightarrow \quad -4h + 4k - 4 = 0$

$\Rightarrow \quad h - k + 1 = 0$ ... (iii)

Also, $h + k = 5$

i.e., $h + k - 5 = 0$ ... (iv)

Adding equations (iii) and (iv), we get

$2h - 4 = 0$

$\Rightarrow \quad h = 2$

$\therefore \quad k = 3$

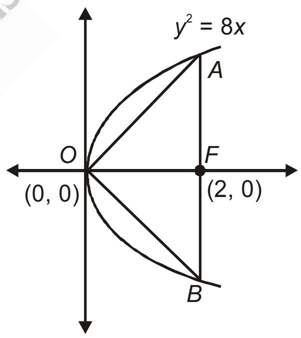
$\Rightarrow \quad (3-2)^2 + (2-3)^2 = r^2$ $[\because\ r^2 = (3-h)^2 + (2-k)^2]$

$\Rightarrow \quad r^2 = 2$

$\therefore$ The required equation of the circle is $(x-2)^2 + (y-3)^2 = 2$.

15. A rod of fixed length l slides along the co-ordinate axes in the first quadrant. Find the locus of mid-point of the rod.

Sol. It is given that the vertices of ΔOAB are the vertex and the ends of the latus rectum of the parabola $y^2 = 8x$.



Now, $y^2 = 8x$ is of the form $y^2 = 4ax$, where $4a = 8 \Rightarrow a = 2$ units

We know that $AF = BF = 4$ units and $OF = 2$ units

$\therefore$ The co-ordinates of the point A and B are (2, 4) and (2, –4) respectively.

In ΔOAF, we have

$(OA)^2 = (OF)^2 + (AF)^2$

$= (2)^2 + (4)^2 = 20$

$\Rightarrow \quad OA = 2\sqrt{5}$ units

Since the given parabola is symmetric about x-axis, therefore $OA = OB = 2\sqrt{5}$ units

$\therefore$ Perimeter of $\Delta OAB = (2\sqrt{5} + 2\sqrt{5} + 8)$

$= 8 + 4\sqrt{5}$

$= 4(2 + \sqrt{5})$ units

16. A circle has radius 4 units and its centre lies on the line $y = 3$. If it passes through the point (6, 3), then find the equation of the circle.

Sol. Let the equation of the circle be $(x-h)^2 + (y-k)^2 = r^2$ ... (i)

where (h, k) = centre and r = radius

It is given that $r = 4$ units

Thus equation (i) reduces to $(x-h)^2 + (y-k)^2 = 16$

Also, (h, k) lies on $y = 3$

$\Rightarrow \quad k = 3$

Since the circle passes through the point (6, 3), we have

$(6-h)^2 + (3-k)^2 = 16$

Since $k = 3$, the above equation reduces to $(6 - h)^2 = 16$

$\Rightarrow \quad h^2 + 36 - 12h - 16 = 0$

$\Rightarrow \quad h^2 - 12h + 20 = 0$

$\Rightarrow \quad (h - 2)(h - 10) = 0$

$\Rightarrow \quad h = 2, 10$

$\therefore$ The required equation of the circle is

(i) $(x - 2)^2 + (y - 3)^2 = (4)^2$ and

(ii) $(x - 10)^2 + (y - 3)^2 = (4)^2$

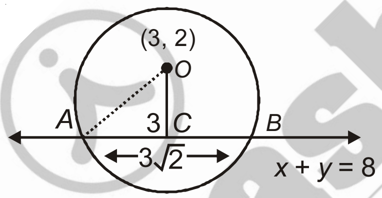
17. Find the equation of a circle whose centre is (3, 2) and which cuts off a chord of length $3\sqrt{2}$ units on the line $x + y = 8$.

Sol. We know that the perpendicular from the centre to the chord of the circle bisects the chord.

It is given that $AB = 3\sqrt{2}$

$\Rightarrow \quad CA = \dfrac{3}{\sqrt{2}}$ ...(1)

To determine the equation of the circle, first we need to find *OA.*



Now, in right ΔOCA,

$(OC)^2 + (CA)^2 = (OA)^2$ ... (2)

and $OC = \dfrac{|3(1) + 2(1) - 8|}{\sqrt{(1)^2 + (1)^2}}$ [Perpendicular distance of the line $x + y = 8$ from the centre *i.e.*, (3, 2)]

$\Rightarrow \quad OC = \dfrac{3}{\sqrt{2}}$ ...(3)

From (1), (2) and (3), we get

$$\frac{9}{2} + \frac{9}{2} = (OA)^2$$

$\Rightarrow \quad OA = 3$ units = radius of the circle

Centre = (3, 2)

Thus the required equation of the circle is $(x - 3)^2 + (y - 2)^2 = (3)^2$

$\Rightarrow \quad (x^2 + 9 - 6x) + (y^2 + 4 - 4y) = 9$

$\Rightarrow \quad x^2 + y^2 - 6x - 4y + 4 = 0$

18. If the lines $2x + y - 6 = 0$ and $4x - 5y + 16 = 0$ are the diameters of a circle of area 154 sq. units, then find the equation of the circle.

Sol. It is given that the lines $2x + y - 6 = 0$ and $4x - 5y + 16 = 0$ are the diameters of a circle.

The point of intersection of these two lines will give the centre of the circle.

Solving the above equations, we get $x = 1$ and $y = 4$

Thus (1, 4) = centre of the circle

It is given that $\pi r^2 = 154$

$\Rightarrow \quad r^2 = \frac{154}{22} \times 7$

$\Rightarrow \quad r^2 = 49$

$\Rightarrow \quad r = 7$

$\therefore$ The required equation of the circle is $(x - 1)^2 + (y - 4)^2 = (7)^2$

19. Find the equation of the circle which passes through the points (2, –3), (–6, –3) and (–2, 1).

Sol. Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through the points (2, –3), (–6, –3) and (–2, 1)

We have,

$(2 - h)^2 + (-3 - k)^2 = r^2$ ... (i)

$(-6 - h)^2 + (-3 - k)^2 = r^2$ ... (ii)

$(-2 - h)^2 + (1 - k)^2 = r^2$ ... (iii)

Equating (i) and (ii), we get

$(2 - h)^2 + (-3 - k)^2 = (-6 - h)^2 + (-3 - k)^2$

$\Rightarrow \quad (2 - h)^2 = (6 + h)^2$

$\Rightarrow \quad (2 - h)^2 - (6 + h)^2 = 0$

$\Rightarrow \quad (2 - h + 6 + h)(2 - h - 6 - h) = 0$

$\Rightarrow \quad 8(-2h - 4) = 0$

$\Rightarrow \quad h = -2$

Equating (i) and (iii), we get

$(2 - h)^2 + (-3 - k)^2 = (-2 - h)^2 + (1 - k)^2$

Putting $h = -2$ in the above equation, we get

$16 + (3 + k)^2 = (1 - k)^2$

$\Rightarrow \quad 16 + (9 + k^2 + 6k) - (1 + k^2 - 2k) = 0$

$\Rightarrow \quad (16 + 9 - 1) + (k^2 - k^2) + (6k + 2k) = 0$

$\Rightarrow \quad 24 + 8k = 0$

$\Rightarrow \quad k = -3$

Putting, the values of h and k in (i), we get

$[2 - (-2)]^2 + [-3 - (-3)]^2 = r^2$

$\Rightarrow \quad r^2 = 16 + 0$

$\Rightarrow \quad r = 4$

Therefore, equation of the required circle is

$[x - (-2)]^2 + [y - (-3)]^2 = (4)^2$

$\Rightarrow \quad (x + 2)^2 + (y + 3)^2 = (4)^2$

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Chapter 12

Conic Sections-I

Solutions (Set-2)

[Equation of Circles in Different Form]

1. The equation of diameter of a circle $x^2 + y^2 + 2x - 4y = 4$, that is parallel to $3x + 5y = 4$ is

(1) $3x + 5y = 7$ (2) $3x - 5y = 7$

(3) $3x + 5y = -7$ (4) $3x - 5y = -7$

Sol. Answer (1)

Equation of diameter $3x + 5y = \lambda$

Centre (–1, 2) lie on the diameter

$-3 + 10 = \lambda \Rightarrow \lambda = 7$

Diameter $3x + 5y = 7$

2. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as the diameter is

(1) $x^2 + y^2 + x - y = 0$ (2) $x^2 + y^2 + x + y = 0$

(3) $x^2 + y^2 - x - y = 0$ (4) $x^2 + y^2 - x + y = 0$

Sol. Answer (3)

Equation of circle $S + \lambda L = 0$

$$x^2 + y^2 - 2x + \lambda(x - y) = 0$$

$$x^2 + y^2 + (\lambda - 2)x - \lambda y = 0 \quad \ldots(1)$$

Centre $\left(\frac{2-\lambda}{2}, \frac{\lambda}{2}\right)$ lie on the line $y = x$

$$\frac{2-\lambda}{2} = \frac{\lambda}{2} \Rightarrow \lambda = 1$$

$\therefore$ From (1), $x^2 + y^2 - x - y = 0$

3. If x_1, x_2 are the roots of the equation $x^2 + bx + c = 0$ and y_1 and y_2 are the roots of $y^2 + qy + r = 0$ then the equation of the circle having (x_1, y_1) and (x_2, y_2) as ends of diameter is

(1) $x^2 + y^2 + bx + qy + c - 2r = 0$
(2) $x^2 + y^2 + bx + qy + 2c + r = 0$
(3) $x^2 + y^2 + bx + qy + c + r = 0$
(4) $x^2 + y^2 - bx - qy - c - r = 0$

Sol. Answer (3)

$x_1 + x_2 = -b,\ x_1x_2 = c$

$y_1 + y_2 = -q,\ y_1y_2 = r$

Circle is

$$x^2 - x(x_1 + x_2) + y^2 - y(y_1 + y_2) + x_1x_2 + y_1y_2 = 0$$

$$x^2 + y^2 + bx + qy + c + r = 0$$

4. The shortest distance of the point $P(-7, 2)$ from the circle $x^2 + y^2 - 10x - 14y - 151 = 0$ is (in units)

(1) 4
(2) 3
(3) 2
(4) 1

Sol. Answer (3)

$S_1 \equiv (-7)^2 + (2)^2 - 10(-7) - 14(2) - 151 < 0$

Point P lie inside the circle

Centre of circle = C (5, 7), P (–7, 2)

Radius of circle = 15

$CP = \sqrt{144 + 25} = 13$

Shortest distance of circle from $P = r - CP = 15 - 13 = 2$

5. The number of normals from any point to a circle cannot be

(1) 0
(2) 1
(3) 2
(4) 3

Sol. Answer (1)

Number of normals cannot be zero.

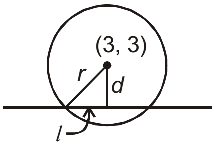
6. The length of intercept on the straight line $3x + 4y - 1 = 0$ by the circle $x^2 + y^2 - 6x - 6y - 7 = 0$ is

(1) $2\sqrt{2}$
(2) 6
(3) $4\sqrt{2}$
(4) $\sqrt{2}$

Sol. Answer (2)

$$d = \frac{3 \times 3 + 4 \times 3 - 1}{\sqrt{9 + 16}} = 4$$

$$2\sqrt{5^2 - 4^2} = 6$$



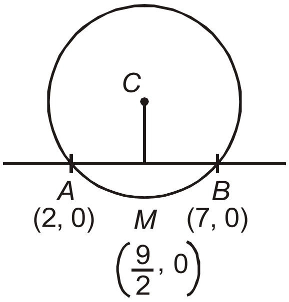
7. Circles are drawn through the point (2, 0) to cut intercept of length 5 units on x-axis. If their centre lies in the first quadrant, then their equation is

(1) $x^2 + y^2 - 9x + 2ky + 14 = 0,\ k \in R^+$
(2) $3x^2 + 3y^2 + 27x - 2ky + 42 = 0,\ k \in R^+$
(3) $x^2 + y^2 - 9x - 2ky + 14 = 0,\ k \in R^+$
(4) $x^2 + y^2 - 2kx - 9y + 14 = 0,\ k \in R^+$

Sol. Answer (3)

$AB = 5$

$A(2, 0)$ $\therefore$ $B(7, 0)$



Middle point $M\left(\frac{9}{2}, 0\right)$

$\therefore$ Centre is $\left(\frac{9}{2}, k\right)$ and radius = $AC = \sqrt{\left(\frac{5}{2}\right)^2 + k^2}$

Equation of circle $\left(x - \frac{9}{2}\right)^2 + (y-k)^2 = \left(\frac{5}{2}\right)^2 + k^2$

$$x^2 + y^2 - 9x - 2yk + 14 = 0$$

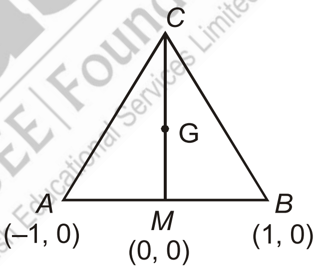
8. Two vertices of an equilateral triangle are (–1, 0) and (1, 0) and its third vertex lies above the x-axis. The equation of circumcircle is

(1) $x^2 + y^2 - \frac{2y}{\sqrt{3}} - 1 = 0$ (2) $x^2 + y^2 - \frac{y}{\sqrt{3}} - 1 = 0$ (3) $x^2 + y^2 - \frac{2y}{3} - 1 = 0$ (4) $x^2 + y^2 + x + y = 0$

Sol. Answer (1)

$CM = \frac{\sqrt{3}}{2}(AB) = \sqrt{3}$

$\therefore$ $C(0, \sqrt{3})$



Centroid $G\left(0, \frac{\sqrt{3}}{3}\right)$ = circumcentre $\left(0, \frac{1}{\sqrt{3}}\right)$

Radius $r = AG = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$

Equation of circumcircle,

$$x^2 + \left(y - \frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3}$$

$$x^2 + y^2 - \frac{2y}{\sqrt{3}} - 1 = 0$$

9. The centre of a circle passing through the origin and cutting of intercepts 3 and 4 on the x and y-axes is

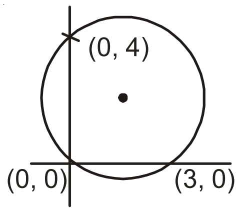
(1) $\left(2, \frac{3}{2}\right)$ (2) $\left(\frac{3}{2}, \frac{3}{2}\right)$ (3) $\left(\frac{3}{2}, 2\right)$ (4) (2, 2)

Sol. Answer (3)

Equation of a circle is

$x^2 + y^2 - 3x - 4y = 0$

Centre is $\left(\frac{3}{2}, 2\right)$



10. The co-ordinates of a point P, which lies on the circle $x^2 + y^2 - 4x + 4y + 7 = 0$ in such a way that OP is minimum, are

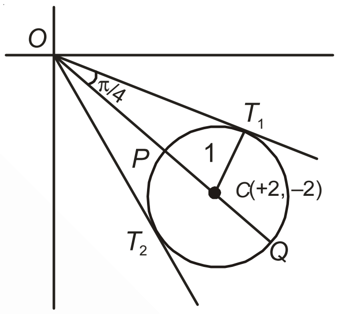
(1) $\left(2+\frac{1}{\sqrt{2}}, -2+\frac{1}{\sqrt{2}}\right)$ (2) $\left(2-\frac{1}{\sqrt{2}}, -2+\frac{1}{\sqrt{2}}\right)$ (3) $\left(-2, -2+\frac{1}{\sqrt{2}}\right)$ (4) $\left(\frac{1}{\sqrt{2}}, -2+\frac{1}{\sqrt{2}}\right)$

Sol. Answer (2)

Let $S : x^2 + y^2 - 4x + 4y + 7 = 0$

$\Rightarrow (x - 2)^2 + (y + 2)^2 = 1$

$\therefore \angle COT_1 = -\frac{\pi}{4}$



$\Rightarrow \angle POT_1 = -\frac{\pi}{4}$

$\therefore OP = OC - CP$

$= \sqrt{4+4} - 1 = 2\sqrt{2} - 1$

As we know that,

$z = r(\cos\theta + i\sin\theta)$

$= |z|(\cos\theta + i\sin\theta)$

$= |OP|\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$= |OP|\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$

$= \frac{|OP|}{\sqrt{2}} - i.\frac{OP}{\sqrt{2}}$

$= \left(\frac{2\sqrt{2}-1}{\sqrt{2}}\right) - i.\left(\frac{2\sqrt{2}-1}{\sqrt{2}}\right)$

$= \left(\frac{2\sqrt{2}-1}{\sqrt{2}}, -\frac{2\sqrt{2}-1}{\sqrt{2}}\right)$

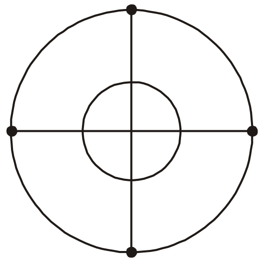
$= \left(2-\frac{1}{\sqrt{2}}, -2+\frac{1}{\sqrt{2}}\right)$

11. The number of points $(a + 1, a)$ where $a \in I$, lying inside the region bounded by the circles $x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2x - 15 = 0$ is

(1) 2 (2) 3 (3) 4 (4) 6

Sol. Answer (3)

Let $S_1 : x^2 + y^2 - 2x - 1 = 0$



$\Rightarrow (x - 1)^2 + y^2 = 1$

and $S_2 : x^2 + y^2 - 2x - 15 = 0$

$\Rightarrow (x - 1)^2 + y^2 = 4^2$

Number of required points are 4, there are (2, 1), (3, 2) (–2, –1), (–2, –3)

12. Four distinct points $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ and $\left(d, \frac{1}{d}\right)$ are lie on a circle, where a, b, c, $d \neq 0$, then the value of $abcd$ is

(1) 2 (2) 1 (3) 3 (4) 4

Sol. Answer (2)

Let the equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Since $\left(a, \frac{1}{a}\right)$ lies on the circle, we get

$$a^2 + \frac{1}{a^2} + 2g.a + 2f.\frac{1}{a} + c = 0$$

$\Rightarrow$ $a^4 + 2ga^3 + 2fa + ca^2 + 1 = 0$

$\Rightarrow$ $a^4 + 2ga^3 + ca^2 + 2fa + 1 = 0$

$\therefore$ $\Sigma abcd = 1$

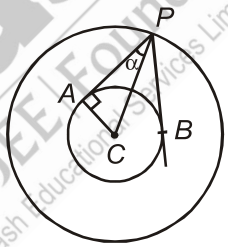
[Tangent to Circle]

13. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\theta + (g^2 + f^2)\cos^2\theta = 0$, then the angle between the tangents is

(1) $\frac{\theta}{4}$ (2) $\frac{\theta}{2}$ (3) θ (4) 2θ

Sol. Answer (4)

Circles are concentric and centre is $C(-g, -f)$



$R_1 = \sqrt{g^2 + f^2 - c}$

$R_2 = \sqrt{g^2 + f^2 - c\sin^2\theta - (g^2 + f^2)\cos^2\theta}$

$R_2 = \sin\theta\sqrt{g^2 + f^2 - c} = \sin\theta \cdot R_1$

$\sin\alpha = \frac{AC}{PC} = \frac{R_2}{R_1} = \sin\theta$

$\therefore$ $\alpha = \theta$

$2\alpha = 2\theta$

$\therefore$ $\angle APB = 2\theta$

14. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + a = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + b = 0$ is

(1) $\sqrt{b-a}$ (2) $\sqrt{a-b}$ (3) $\sqrt{a+b}$ (4) $\sqrt{ab}$

Sol. Answer (1)

Length of tangent from any point on the circle S_1 to the circle S_2 = $\sqrt{S_2 - S_1} = \sqrt{b-a}$

15. If the line $y = 3x + c$ is a tangent to $x^2 + y^2 = 4$ then the value of c is

(1) ±4 (2) $\pm 2\sqrt{10}$ (3) $\pm 10\sqrt{2}$ (4) $\pm\sqrt{10}$

Sol. Answer (2)

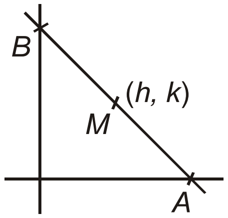
$c = \pm a\sqrt{1+m^2} = \pm 2\sqrt{1+9} = \pm 2\sqrt{10}$

16. Locus of middle point of intercept of any tangent with respect to the circle $x^2 + y^2 = 4$ between the axis is

(1) $x^2 + y^2 - x^2y^2 = 0$ (2) $x^2 + y^2 + x^2y^2 = 0$ (3) $x^2 + y^2 - 2x^2y^2 = 0$ (4) $x^2 + y^2 - 3x^2y^2 = 0$

Sol. Answer (1)

Let the tangent is $x\cos\theta + y\sin\theta = 2$



Clearly $A = \left(\frac{2}{\cos\theta}, 0\right), B = \left(0, \frac{2}{\sin\theta}\right)$

$\Rightarrow \quad h = \frac{1}{\cos\theta}, k = \frac{1}{\sin\theta}$

$\Rightarrow \quad x^2 + y^2 = x^2y^2$

17. Two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P. Then the locus of P has the equation

(1) $x^2 + y^2 = 2a^2$ (2) $x^2 + y^2 = 3a^2$ (3) $x^2 + y^2 = 4a^2$ (4) $x^2 + y^2 = 5a^2$

Sol. Answer (1)

Let $P(h, k)$

Equation of line through $P(h, k)$

$y - k = m(x - h)$

$mx - y + k - mh = 0$ ...(i)

Distance of line from centre = radius

$\frac{k - mh}{\sqrt{m^2 + 1}} = a$

$(h^2 - a^2)m^2 - 2mkh + k^2 - a^2 = 0$

Tangents are perpendicular

$\therefore \quad m_1m_2 = -1$

$\frac{k^2 - a^2}{h^2 - a^2} = -1$

$\therefore \quad h^2 + k^2 = 2a^2$

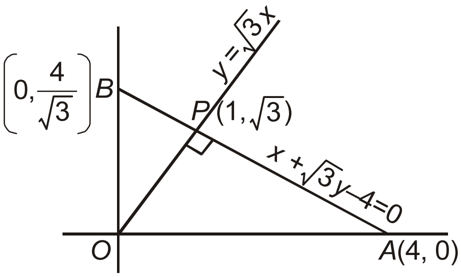
Locus of $P(h, k)$

$x^2 + y^2 = 2a^2$

18. The area of the triangle formed by the +ve x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

(1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) 1

Sol. Answer (1)



Equation of tangent at $P(1, \sqrt{3})$ is

$x + \sqrt{3}y - 4 = 0$ ...(i)

Equation of normal is

$\sqrt{3}x - y + \lambda = 0$

It passes through (1, $\sqrt{3}$) so $\lambda = 0$

Equation of normal is,

$\sqrt{3}x - y = 0$ ...(ii)

Area of the triangle formed by x-axis, tangent at (1, $\sqrt{3}$) and normal at (1, $\sqrt{3}$) is

= Area of ΔOPA

$= \frac{1}{2} \times OP \times AP$

$= \frac{1}{2} \times 2 \times 2\sqrt{3}$

$= 2\sqrt{3}$ sq. units.

19. If $3x + y = 0$ is a tangent to the circle which has its centre at the point (2, –1), then equation of the other tangent to the circle from the origin, is

(1) $x + 3y = 0$ (2) $3x - y = 0$ (3) $x - 3y = 0$ (4) $x + 2y = 0$

Sol. Answer (3)

Centre $C(2, -1)$

Radius of circle = distance of line $3x + y = 0$ from centre

$$r = \frac{6-1}{\sqrt{9+1}} = \frac{5}{\sqrt{10}} = \sqrt{\frac{5}{2}}$$

Eq. of line from origin $y = mx$

Distance of line from centre = r

$$\frac{2m+1}{\sqrt{m^2+1}} = \sqrt{\frac{5}{2}}$$

$$3m^2 + 8m - 3 = 0 \Rightarrow m = -3 \text{ and } \frac{1}{3}$$

$$y = \frac{1}{3}x \Rightarrow x - 3y = 0$$

20. If equation of one tangent drawn from (0, 0) to the circle with centre (2, 4) is $4x + 3y = 0$, then equation of the other tangent from (0, 0) is

(1) $4x - 3y = 0$ (2) $x = 0$ (3) $y = 0$ (4) $x + 4y = 0$

Sol. Answer (3)

Radius of circle = $\frac{8+12}{\sqrt{16+9}} = \frac{20}{5} = 4$

Equation of line through origin $y = mx$

Distance from centre = radius

$$\frac{2m-4}{\sqrt{m^2+1}} = 4 \Rightarrow m(3m+4) = 0$$

$m = -\frac{4}{3}$ already considered

$\therefore$ $m = 0$

$y = 0$

21. The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their points of contact is

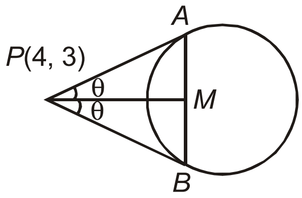
(1) 12 (2) 6 (3) 4 (4) $\frac{192}{25}$

Sol. Answer (4)

Equation of chord of contact AB

$$x \cdot 4 + y \cdot 3 - 9 = 0$$

$$PM = \left|\frac{16+9-9}{\sqrt{25}}\right| = \frac{16}{5}$$



(Length of perpendicular from $P(4, 3)$ to $AB \equiv 4x + 3y - 9 = 0$)

($\because$ Length of tangent is $\sqrt{S_1}$)

$$PA = \sqrt{16+9-9} = 4$$

$$\cos\theta = \frac{PM}{PA} = \frac{4}{5}$$

Area of $\Delta APB = \frac{1}{2}(PA)^2 \cdot \sin 2\theta$ [$\therefore$ Area of $\Delta = \frac{1}{2} \times (PA) \times (PB) \sin \angle APB$]

$$\Delta = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} \cdot 16 \cdot 2 \sin\theta \cdot \cos\theta$$

$$= 16 \cdot \frac{4}{5} \cdot \frac{3}{5}$$

$$= \frac{192}{25}$$

22. If two tangents are drawn from a point to the circle $x^2 + y^2 = 32$ to the circle $x^2 + y^2 = 16$, then the angle between the tangents is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{6}$

Sol. Answer (3)

$S_1 : x^2 + y^2 = 32$

$S_2 : x^2 + y^2 = 16$

$\Rightarrow$ $S_1 = 0$ is the director circle of $S_2 = 0$

$\Rightarrow$ Director circle is the locus of two perpendicular tangents.

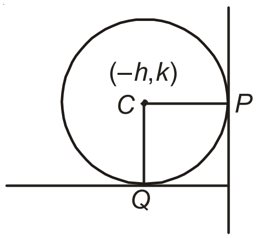
Angle is $\frac{\pi}{2}$.

23. The equation of one of the circles which touch the pair of lines $x^2 - y^2 + 2y - 1 = 0$ is

(1) $x^2 + y^2 + 2x + 1 = 0$ (2) $x^2 + y^2 - 2x + 1 = 0$

(3) $x^2 + y^2 + 2y - 1 = 0$ (4) $x^2 + y^2 - 2y - 1 = 0$

Sol. Answer (3)



Now $x^2 - y^2 + 2y - 1 = 0$

$\Rightarrow x^2 - (y - 1)^2 = 0$

$\Rightarrow (x - y + 1)(x + y - 1) = 0$

$\Rightarrow x - y + 1 = 0,\ x + y - 1 = 0$

Now, $CP = CQ$

$\Rightarrow \left|\frac{h+k-1}{\sqrt{2}}\right| = \left|\frac{h-k+1}{\sqrt{2}}\right|$

$\Rightarrow h + k - 1 = \pm(h - k + 1)$

$\Rightarrow h = 0,\ k = -1$

$\therefore C \equiv (0, -1)$

Now, radius = $CQ = \left|\frac{0-1-1}{\sqrt{2}}\right| = \sqrt{2}$

Equation of circle is

$x^2 + (y + 1)^2 = (\sqrt{2})^2$

$x^2 + y^2 + 2y - 1 = 0$

24. The equations of common tangents to the circles $x^2 + y^2 = 1$ and $(x - 1)^2 + (y - 3)^2 = 4$ are

(1) $3x + 4y - 5 = 0,\ 4x - 3y + 5 = 0$
(2) $3x + 4y - 5 = 0,\ 4x - 3y - 5 = 0$
(3) $3x - 4y + 5 = 0,\ 4x + 3y - 5 = 0$
(4) $3x + 4y + 5 = 0,\ 4x + 3y + 5 = 0$

Sol. Answer (2)

External point = (–1, –3)

Equation of tangent

$y + 3 = m(x + 1)$

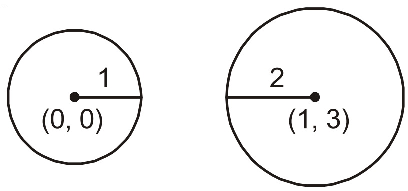
$mx - y + m - 3 = 0$

Distance from (0, 0) is 1

Hence m = not defined, $m = \frac{4}{3}$

Equation of tangent

$\boxed{x = -1}$ $\boxed{4x - 3y - 5 = 0}$



Internal division = $\left(\frac{1}{3}, 1\right)$

Equation of tangent $y - 1 = m\left(x - \frac{1}{3}\right)$

Distance from (0, 0) is 1

Hence, $m = 0$, $m = -\frac{3}{4}$

Equation of tangent $y = 1$ and $3x + 4y - 5 = 0$

25. If the circle $x^2 + y^2 + 4x + 2y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x - 8y - d = 0$, then $c + d$ =

(1) 60 (2) –46 (3) 40 (4) 56

Sol. Answer (2)

Eq. of common chord $S_1 - S_2 = 0$

$6x + 10y + c + d = 0$ ...(i)

Line (i) is a diameter of circle (ii)

$\therefore$ Centre (1, 4) lie on the line (i)

$6 + 40 + c + d = 0$

$c + d = -46$

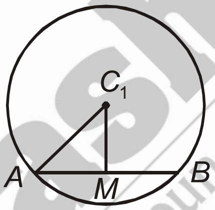
[Analysis of Two Circles and Locus]

26. The length of the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ is

(1) $\frac{9}{2}$ (2) $\frac{3}{2}$ (3) $3\sqrt{2}$ (4) $2\sqrt{2}$

Sol. Answer (4)

Equation of common chord $S_2 - S_1 = 0$



$2x + 1 = 0$

$C_1\left(-1, -\frac{3}{2}\right)$

$r_1 = \sqrt{1 + \frac{9}{4} - 1} = \frac{3}{2}$

$C_1M = \left(\frac{-2+1}{\sqrt{4}}\right) = \frac{1}{2}$

$AM^2 = r^2 - C_1M^2 = \frac{9}{4} - \frac{1}{4} = \frac{8}{4} = 2$

$AM = \sqrt{2}$

$AB = 2\sqrt{2}$

27. The distance between the chords of contact of the tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and from the point (g, f) is

(1) $g^2 + f^2 - c$ (2) $\sqrt{g^2 + f^2 - c}$ (3) $\frac{g^2 + f^2 - c}{\sqrt{g^2 + f^2}}$ (4) $\frac{1}{2}\frac{|g^2 + f^2 - c|}{\sqrt{g^2 + f^2}}$

Sol. Answer (4)

Equation of chord of contact from $P(g, f)$

$T \equiv 0$

$x \cdot g + y \cdot f + g(x + g) + f(y + f) + c = 0$

$2gx + 2fy + g^2 + f^2 + c = 0$ ...(i)

Similarly, equation of chord of contact from origin $O(0, 0)$ is

$$gx + fy + c = 0$$

$$\therefore \quad 2gx + 2fy + 2c = 0 \qquad \ldots\text{(ii)}$$

Distance between line (i) and (ii),

$$= \left|\frac{g^2 + f^2 - c}{\sqrt{4g^2 + 4f^2}}\right| = \frac{|g^2 + f^2 - c|}{2\sqrt{g^2 + f^2}}$$

28. Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = a^2$. The locus of the mid points of the secants intercepted by the given circle is

(1) $2(x^2 + y^2) = hx + ky$

(2) $x^2 + y^2 = hx + ky$

(3) $x^2 + y^2 + hx + ky = 0$

(4) $x^2 + y^2 - hx + ky + 13 = 0$

Sol. Answer (2)

Let mid-point of the chord be $P(x_1, y_1)$.

$\therefore$ Equation of chord $S_1 = T$

$$x_1^2 + y_1^2 - a^2 = xx_1 + yy_1 - a^2$$

This chord passes through (h, k).

$$x_1^2 + y_1^2 = hx_1 + ky_1$$

$\therefore$ Locus of $P(x_1, y_1)$ be $x_1^2 + y_1^2 = hx_1 + ky_1$

29. The equation of circle passing through the point (1, 1) and point of intersection of $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$, is

(1) $x^2 + y^2 - 6x + 4 = 0$

(2) $x^2 + y^2 - 3x + 1 = 0$

(3) $x^2 + y^2 - 4y + 2 = 0$

(4) $x^2 + y^2 - 2x - 2y + 2 = 0$

Sol. Answer (2)

Required circle be $S_1 + \lambda S_2 = 0$

$$x^2 + y^2 - 6x + 8 + \lambda(x^2 + y^2 - 6) = 0$$

$$(1+\lambda)x^2 + (1+\lambda)y^2 - 6x + (8 - 6\lambda) = 0 \qquad \ldots\text{(i)}$$

Circle (i) passes through (1, 1) $\quad \therefore \quad \lambda = 1$

Circle be $x^2 + y^2 - 3x + 1 = 0$

30. A variable chord is drawn through origin to the circle $x^2 + y^2 - 2ax = 0$. Locus of the centre of the circle described on the chord as diameter is

(1) $x^2 + y^2 - ax = 0$

(2) $x^2 + y^2 + ax = 0$

(3) $x^2 + y^2 - ay = 0$

(4) $x^2 + y^2 \;\; ax - ay = 0$

Sol. Answer (1)

Let variable chord be $y = mx$

Equation of a circle passes through the intersection of a circle and a line

$$x^2 + y^2 - 2ax + \lambda(mx - y) = 0$$

$$x^2 + y^2 - (2a - \lambda m)x - \lambda y = 0$$

Centre of this circle $\left(\frac{2a - \lambda m}{2}, \frac{\lambda}{2}\right)$

Lie on the line $y = mx$

$\therefore \quad \frac{\lambda}{2} = m\left(\frac{2a - \lambda m}{2}\right) \quad \ldots(i)$

Let centre be (h, k)

$\therefore \quad h = \frac{2a - \lambda m}{2} \quad \ldots(ii)$

$k = \frac{\lambda}{2} \quad \ldots(iii)$

Eliminate λ and m from eq. (i), (ii) and (iii)

$$h^2 + k^2 - ah = 0$$

$\therefore$ Locus of (h, k)

$$x^2 + y^2 - ax = 0$$

31. If the chord $y = mx + 1$ subtends an angle of measure of 45° at the major segment of the circle $x^2 + y^2 = 1$, then the value of m is

(1) $1 \pm \sqrt{2}$ (2) $-2 \pm \sqrt{2}$ (3) $-1 \pm \sqrt{2}$ (4) ± 1

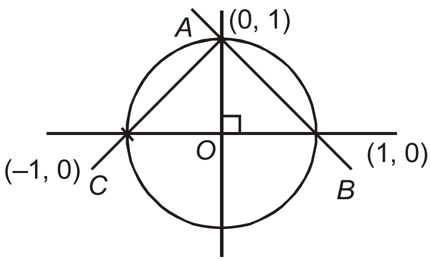
Sol. Answer (4)

Chord subtends 90° at the centre. Chord must pass through (0, 1).

Second point may be (±1, 0).

Chord cuts x-axis at $\left(-\frac{1}{m}, 0\right)$.

$\therefore \quad m = \pm 1$



32. The line $3x - 4y = k$ will cut the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ at distinct points if

(1) $-35 < k < 35$ (2) $-35 < k < 15$ (3) $-15 < k < 15$ (4) $15 < k < 35$

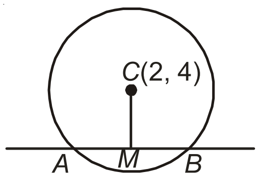
Sol. Answer (2)

Given, $x^2 + y^2 - 4x - 8y - 5 = 0$

$\Rightarrow \quad (x - 2)^2 + (y - 4)^2 = 4 + 16 + 5 = (5)^2$

$\therefore$ Centre = $C(2, 4)$,

Radius = 5



The line *AB* will cut the circle in two distinct points if

Now, *CM* < 5

$\Rightarrow \left|\frac{6-16-k}{\sqrt{25}}\right| < 5$

$\Rightarrow -5 < \frac{-10-k}{5} < 5$

$\Rightarrow -25 < -10 - k < 25$

$\Rightarrow -15 < -k < 35$

$\Rightarrow -35 < k < 15$

33. The equation of the circle, orthogonal to both the circles $x^2 + y^2 + 3x - 5y + 6 = 0$ and $4x^2 + 4y^2 - 28x + 29 = 0$ and whose centre lies on the line $3x + 4y + 1 = 0$ is

(1) $4x^2 + 4y^2 + 2y - 29 = 0$

(2) $4x^2 + 4y^2 + 6y + 5 = 0$

(3) $2x^2 + 2y^2 + 3x + 7y = 0$

(4) $x^2 + y^2 + 3x - 7y + 3 = 0$

Sol. Answer (1)

Let equation of circle be $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

Condition of orthogonality $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Circles S and S_1 are orthogonal

$\therefore \quad 2g\left(\frac{3}{2}\right) + 2f\left(-\frac{5}{2}\right) = c + 6$

$3g - 5f - c = 6 \qquad \ldots(i)$

Circles S and S_2 are orthogonals.

$\therefore \quad 2g\left(-\frac{7}{2}\right) + 2f\,(0) = c + \frac{29}{4}$

$-7g - c = \frac{29}{4} \qquad \ldots(ii)$

(i)-(ii), $10g - 5f = -\frac{5}{4} \qquad \ldots(iii)$

Centre of S $(-g, -f)$ lie on the line $3x + 4y + 1 = 0$

$\therefore \quad -3g - 4f + 1 = 0 \qquad \ldots(iv)$

Solve equations (iii) and (iv),

$g = 0,\ f = \frac{1}{4},\ c = -\frac{29}{4}$

$\therefore \quad S \equiv x^2 + y^2 + \frac{1}{2}y - \frac{29}{4} = 0$

$4x^2 + 4y^2 + 2y - 29 = 0$

34. The equation of a circle which touches the line $x + y = 5$ at the point (–2, 7) and cuts the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally is

(1) $x^2 + y^2 + 7x - 11y + 38 = 0$ (2) $x^2 + y^2 + 7x + 11y + 38 = 0$

(3) $x^2 + y^2 + 7x - 11y - 38 = 0$ (4) $x^2 + y^2 - 7x - 11y + 39 = 0$

Sol. Answer (1)

Let equation of circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

(–2, 7) lie on the circle

$\therefore$ $-4g + 14f + c = -53$ …(i)

Equation of a line through (–2, 7) and perpendicular to line $x + y = 5$ is

$y - 7 = 1(x + 2)$

$x - y + 9 = 0$

Centre $(-g, -f)$ lie on this line $-g + f + 9 = 0$ …(ii)

Circles S and S_1 are orthogonal

$\therefore$ $2g(2) + 2f(-3) = c + 9$

$4g - 6f - c = 9$ …(iii)

Solve eq. (i) and (iii), $f = \dfrac{-11}{2}$

$g = \dfrac{7}{2},\ c = 38$

$\therefore$ Equation of circle $x^2 + y^2 + 7x - 11y + 38 = 0$.

35. The locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 6y + 4 = 0$ orthogonally is

(1) $8x - 12y + 5 = 0$ (2) $8x + 12y - 5 = 0$ (3) $12x - 8y + 5 = 0$ (4) $3x + 4y + 7 = 0$

Sol. Answer (1)

Let circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ circle S intersect S_1 and S_2 orthogonally

$\therefore$ $2g(2) + 2f(-3) = c + 9$ … (i)

$2g(-2) + 2f(3) = c + 4$ … (ii)

(i) – (ii), $8g - 12f = 5$

Locus of $(-g, -f)$

$-8x + 12y = 5$

$8x - 12y + 5 = 0$

36. If centre of a circle lies on the line $2x - 6y + 9 = 0$ and it cuts the circle $x^2 + y^2 = 2$ orthogonally, then the circle passes through two fixed points

(1) $\left(\frac{1}{2}, \frac{3}{2}\right), \left(\frac{-2}{5}, \frac{6}{5}\right)$ (2) (2, 3), (–2, 6) (3) $\left(\frac{-1}{2}, \frac{3}{2}\right), \left(\frac{-2}{5}, \frac{6}{5}\right)$ (4) (–2, 3) (–2, 6)

Sol. Answer (3)

Centre (–*g*, –*f*) lie on $2x - 6y + 9 = 0$

$\therefore \quad -2g + 6f + 9 = 0$

cuts orthogonally $x^2 + y^2 = 2$

$\therefore \quad 0 = c - 2$

$c = 2$

$\therefore$ Circle be $x^2 + y^2 + 2gx + 2fy + 2 = 0$

Eliminate $x^2 + y^2 + (6f + 9)x + 2fy + 2 = 0$

$(x^2 + y^2 + 9x + 2) + f(6x + 2y) = 0$

This circle passes through the point of intersection of

$x^2 + y^2 + 9x + 2 = 0$ ...(i)

and $6x + 2y = 0$ ...(ii)

Solve these equation intersection points are $\left(-\frac{1}{2}, \frac{3}{2}\right)$ and $\left(-\frac{2}{5}, \frac{6}{5}\right)$.

37. The radical centre of three circles described on the three sides $4x - 7y + 10 = 0$, $x + y - 5 = 0$ and $7x + + 4y - 15 = 0$ of a triangle as diameters is

(1) (2, 3) (2) (2, 1) (3) (3, 2) (4) (1, 2)

Sol. Answer (4)

Here, radical centre = orthocenter

$S_1 = 4x - 7y + 10 = 0$ ...(1)

$S_2 = x + y - 5 = 0$ ...(2)

$S_3 = 7x + 4y - 15 = 0$ ...(3)

Thus, the sides (1) and (3) are perpendicular to each other. So the point of intersection of these two lines will be orthocenter.

$$\therefore \quad \begin{array}{r} 28x - 49y + 70 = 0 \\ 28x + 16y - 60 = 0 \\ - \quad - \quad + \\ \hline -65y + 130 = 0 \end{array}$$

$\Rightarrow \quad y = \frac{130}{65} = 2$

When $y = 2$, $4x - 7.2 + 10 = 0$

$\Rightarrow \quad 4x = 14 - 10 = 4$

$\Rightarrow \quad x = 1$

$\therefore$ Point is (1, 2)

$\Rightarrow$ Orthocentre = (1, 2)

38. The equation of a circle passing through (1, 0) and (0, 1) having the smallest possible radius is

(1) $x^2 + y^2 + x - y = 0$ (2) $x^2 + y^2 - x - y = 0$ (3) $x^2 + y^2 + x + y = 0$ (4) $x^2 + y^2 - x + 2y = 0$

Sol. Answer (2)

Let $S : x^2 + y^2 + 2gx + 2fy + c = 0$ … (1)

(1) passes through (1, 0) and (0, 1),

Then $1 + 2g + c = 0$

$\Rightarrow \quad g = -\frac{1+c}{2} g$

Also, $1 + 2f + c = 0$

$\Rightarrow \quad f = -\frac{1+c}{2}$

Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{\frac{(1+c)^2}{4} + \frac{(1+c)^2}{4} - c}$

$= \sqrt{\frac{c^2 + 2c + 1 + c^2 + 2c + 1 - 4c}{4}}$

$= \sqrt{\frac{2c^2 + 2}{4}}$

$= \sqrt{\frac{c^2 + 1}{2}}$

$\Rightarrow$ For minimum radius $c = 0$

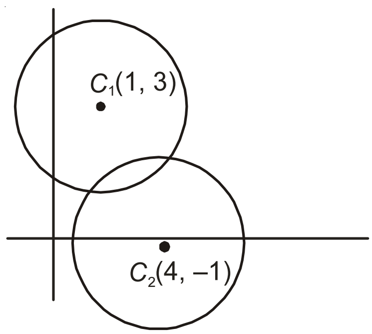
$\therefore \quad f = -\frac{1}{2}, g = -\frac{1}{2}$

Equation of a circle is $x^2 + y^2 - x - y = 0$

39. If two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersects in the distinct points, then

(1) $1 < r < 4$ (2) $-2 < r < 2$ (3) $2 < r < 8$ (4) None of these

Sol. Answer (3)



$S_1 : (x - 1)^2 + (y - 3)^2 = r^2$

$S_2 : x^2 + y^2 - 8x + 2y + 8 = 0$

$\Rightarrow \quad (x - 4)^2 + (y + 1)^2 = 16 + 1 - 8 = 9 = 3^2$

$C_1 : (1, 3)$

$C_2 : (4, -1)$

$C_1C_2 = \sqrt{(1-4)^2 + (3+1)^2} = 5$

$|r - 3| < C_1C_2 < r + 3$

$\Rightarrow$ $|r - 3| < 5 < r + 3$

$\Rightarrow$ $|r - 3| < 5$ $r + 3 > 5$

$\Rightarrow$ $-5 < r - 3 < 5$

$\Rightarrow$ $r > 2$

$\Rightarrow$ $2 < r < 8$

40. The locus of the centre of a circle, which touches externally to the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by

(1) $x^2 - 6x - 10y + 14 = 0$ (2) $x^2 - 10x - 6y + 14 = 0$

(3) $y^2 - 6x - 10y + 14 = 0$ (4) $y^2 - 10x - 6y + 14 = 0$

Sol. Answer (4)

Let centre of circle S_1 be $C_1(a, b)$.

Circle touch y-axis.

$\therefore$ $r_1 = a$

Centre of S_2 is C_2 (3, 3) and $r_2 = 2$

S_1 and S_2 touch externally

$C_1C_2 = r_1 + r_2$

$$\sqrt{(a-3)^2 + (b-3)^2} = a + 2$$

$$\Rightarrow \quad b^2 - 10a - 6b + 14 = 0$$

Locus of $C_1(a, b)$ is $y^2 - 10x - 6y + 14 = 0$

41. The locus of centre of a circle of radius 2 which rolls outside of the circle $x^2 + y^2 + 3x - 6y - 9 = 0$ is

(1) $x^2 + y^2 + 3x - 6y + 5 = 0$ (2) $x^2 + y^2 + 3x - 6y + 31 = 0$

(3) $x^2 + y^2 + 3x - 6y + \frac{29}{4} = 0$ (4) $x^2 + y^2 + 3x - 6y - 31 = 0$

Sol. Answer (4)

Let centre $C_1(h, k)$, $r_1 = 2$

Centre of S_2 is $C_2\left(-\frac{3}{2}, 3\right)$, $r_2 = \sqrt{\frac{9}{4} + 9 + 9} = \sqrt{\frac{81}{4}} = \frac{9}{2}$

Distance between centres = sum of radii

$$\sqrt{\left(h + \frac{3}{2}\right)^3 + (k-3)^2} = 2 + \frac{9}{2}$$

Locus of (h, k)

$$x^2 + y^2 + 3x - 6y - 31 = 0$$

42. The locus of the point of intersection of the tangents to the circle $x = a \cos\theta$, $y = a \sin\theta$ at the points, whose parametric angles differ by $\frac{\pi}{3}$ is

(1) Straight line (2) Ellipse (3) Circle of radius $2a$ (4) Circle of radius $\frac{2a}{\sqrt{3}}$

Sol. Answer (4)

Equation of tangent at A ($a \cos \theta$, $a \sin \theta$) is $x \cos\theta + y \sin\theta = a$ ...(i)

Equation of tangent at B

$$\left[a \cos\left(\theta+\frac{\pi}{3}\right)\cdot a \sin\left(\theta+\frac{\pi}{3}\right)\right]$$

$$x \cos\left(\theta+\frac{\pi}{3}\right)+y \sin\left(\theta+\frac{\pi}{3}\right)=a$$

$$x\left[\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right]+y\left[\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta\right]=a$$

$$x \cos\theta+y \sin\theta+\sqrt{3}\,(-x \sin\theta+y \cos\theta)=2a$$

$$a+\sqrt{3}\,(-x \sin\theta+y \cos\theta)=2a$$

$$-x \sin\theta+y \cos\theta=\frac{a}{\sqrt{3}} \quad \text{...(ii)}$$

Squaring equation (i) and (ii) and adding,

$$x^2+y^2=a^2+\frac{a^2}{3}=\frac{4a^2}{3}$$

Locus is a circle of radius $\frac{2a}{\sqrt{3}}$.

[Miscellaneous]

43. Equation of a circle passing through (2, 8) and touching the lines $4x - 3y - 24 = 0$ and $4x + 3y - 42 = 0$ and having x-coordinates of the centre less than or equal to 8 is

(1) $(x-2)^2+(y+3)^2=25$ (2) $(x-2)^2+(y-3)^2=16$

(3) $(x-2)^2+(y-3)^2=25$ (4) $(x+2)^2+(y-3)^2=25$

Sol. Answer (3)

Let centre of circle be $C(a, b)$.

Circle touch both lines.

$$r=\left|\frac{4a-3b-24}{5}\right|=\left|\frac{4a+3b-42}{5}\right|=\sqrt{(a-2)^2+(b-8)^2}$$

Solve these pair wise $b = 3$ and $a = 2$

$r = 5$

Equation of circle $(x-2)^2+(y-3)^2=25$

44. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the co-ordinate axes, then one vertex of the square is

(1) $(1+\sqrt{2}, -2)$ (2) $(1-\sqrt{2}, -2)$

(3) $(1, -2+\sqrt{2})$ (4) $(2, -1)$

Sol. Answer (4)

Centre (1, –2), $r = \sqrt{2}$

AB and CD are parallel to x-axis.

$\therefore$ Equation of $AB \to y = -3$

Equation of $CD \to y = -1$

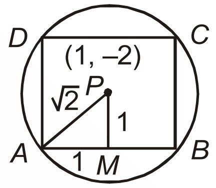
AD and BC are parallel to y-axis.

$\therefore$ Equation of $AD \to x = 0$

Equation of $BC \to x = 2$

$\therefore$ $A(0, -3)$, $B(2, -3)$

$C(2, -1)$, $D(0, -1)$



45. From the origin O tangents OP and OQ are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Then the circumcentre of the triangle OPQ lies at

(1) $\left(\frac{-g}{2}, \frac{-f}{2}\right)$ (2) (g, f) (3) $(-f, -g)$ (4) (f, g)

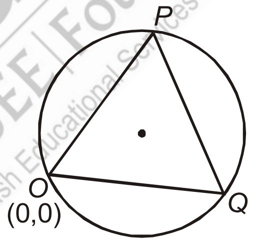
Sol. Answer (1)

Origin (0, 0) lies on the given circle

Circumcircle of ΔOPQ = Given circle

$= x^2 + y^2 + 2gx + 2fy = 0$

Centre $(-g, -f)$



46. Area of a circle in which a chord of length $\sqrt{2}$ makes an angle $\frac{\pi}{2}$ at the centre is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) π (4) 2π

Sol. Answer (3)

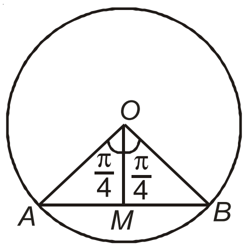
$$\therefore \quad \sin\frac{\pi}{4} = \frac{AM}{OA} = \frac{\frac{1}{2}\times\sqrt{2}}{OA} = \frac{\sqrt{2}}{2OA}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2.OA}$$

$\Rightarrow$ $OA = 1$

$\Rightarrow$ Radius = 1

Area of a circle = $\pi \times (1)^2 = \pi$ sq. units



47. If $3x + b_1y + 5 = 0$ and $4x + b_2y + 10 = 0$ cut the x-axis and y-axis in four concyclic points, then the value of b_1b_2 is

(1) 15 (2) 30 (3) 20 (4) 12

Sol. Answer (4)

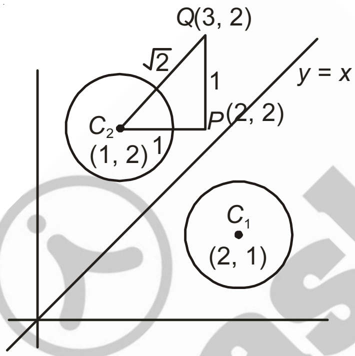
As we know that, if the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ cut x-axis and y-axis in four con-cyclic points, then $a_1a_2 = b_1b_2$.

$\therefore \quad b_1b_2 = 3 \times 4 = 12$

48. The image of the centre of a circle $(x-2)^2 + (y-1)^2 = 9$ w.r.t. the line $y = x$ moves $\sqrt{2}$ units along (N - E) direction, then the co-ordinates of the centre in the new position is

(1) (1, 2) (2) (2, 2) (3) (3, 2) (4) (2, 3)

Sol. Answer (3)



The image of (2, 1) w.r.t. the line $y = x$ is (1, 2).

Now, $P \equiv (2, 2)$

and $Q \equiv (3, 2)$

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