1. **(b)** $f(x) = \dfrac{x+2}{|x+2|}$

$f(x) = \begin{cases} -1, & x < -2 \\ 1, & x > -2 \end{cases}$ $\therefore$ Range of f (x) is {–1, 1}.

2. **(c)** f (x) = 2 sin x + sin 2x

f ' (x) = 2 cos x + 2 cos 2x = 2 (cos x + cos 2x)

$\therefore$ f ' (x) = 0 $\Rightarrow$ $2\cos^2 x + \cos x - 1 = 0$

$\cos x = \dfrac{-1 \pm 3}{4} = -1, \dfrac{1}{2}$; $\therefore$ $x = \pi, \dfrac{\pi}{3}$

Now, $f(0) = 0, f\left(\dfrac{3\pi}{2}\right) = -2$

$f(\pi) = 0, f\left(\dfrac{\pi}{3}\right) = 2\dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{2}$

$\therefore$ difference between greatest value and least value

$= \dfrac{3\sqrt{3}}{2} + 2$

3. **(c)** Given that *a*, *b*, *c* are in G.P.

So, $b^2 = ac$ … (i)

$x = \dfrac{a+b}{2}$ … (ii)

$y = \dfrac{b+c}{2}$ … (iii)

Now $\dfrac{x}{a} + \dfrac{c}{y} = \dfrac{2a}{a+b} + \dfrac{2c}{b+c} = \dfrac{2(ab+bc+2ca)}{ab+ac+b^2+bc}$

$= \dfrac{2(ab+bc+2ca)}{(ab+ac+ac+bc)} = 2$ $\quad \left[\because b^2 = ac\right]$

4. **(b)** $I=\int_0^{\pi}\frac{x}{1+\cos^2 x}\,dx=\int_0^{\pi}\frac{\pi-x}{1+\cos^2(\pi-x)}\,dx$

$$=\int_0^{\pi}\frac{\pi-x}{1+\cos^2 x}$$

$$=\int_0^{\pi}\frac{\pi}{1+\cos^2 x}\,dx-\int_0^{\pi}\frac{x}{1+\cos^2 x}\,dx=\pi\int_0^{\pi}\frac{dx}{1+\cos^2 x}-I$$

$$2I=\pi\int_0^{\pi}\frac{1}{1+\cos^2 x}\,dx=2\pi\int_0^{\frac{\pi}{2}}\frac{1}{1+\cos^2 x}\,dx$$

$$=2\pi\int_0^{\frac{\pi}{2}}\frac{\sec^2 x\,dx}{2+\tan^2 x}$$ Put $\tan x=t$ $\quad\sec^2 x\,dx=dt$

$$2I=2\pi\int_0^{\infty}\frac{dt}{2+t^2}=2\pi.\frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{t}{\sqrt{2}}\right)_0^{\infty}$$

$$=\sqrt{2}\pi\left[\tan^{-1}\infty-\tan^{-1}0\right]=\sqrt{2}\pi\left[\frac{\pi}{2}-0\right]=\frac{\pi^2}{\sqrt{2}}$$

$$\therefore I=\frac{\pi^2}{2\sqrt{2}}$$

5. **(b)** Let $z=1-t+i\sqrt{t^2+t+2}$

We know that $z=x+iy$

$$x+iy=1-t+i\sqrt{t^2+t+2}$$

compare real and imaginary part, we get

$x=1-t \qquad t=1-x$

and $y=\sqrt{t^2+t+2}$

$$y^2=t^2+t+2$$

$$y^2=(1-x)^2+(1-x)+2$$

$$y^2=1+x^2-2x+1-x+2$$

$$y^2=x^2-3x+4$$

$$y^2=\left(x-\frac{3}{2}\right)^2+\frac{7}{4}$$

Which is a hyperbola.

6. **(a)** This is the linear equation of the form $\frac{dy}{dx}+Py=Q$, where P = tan x and $Q=x^m\cos x$

Now integrating factor $(\text{I.F.})=e^{\int Pdx}=e^{\int \tan dx}$

$$=e^{\log\sec x}=\sec x$$

Thus sol. is given by, $y.e^{\int Pdx}=\int Q.e^{\int Pdx}dx+c$

$$\Rightarrow y.\sec x=\int x^m.\cos x.\sec x dx+c$$

$$\Rightarrow y\sec x=\frac{x^{m+1}}{m+1}+c$$

$$\Rightarrow (m+1)\,y=x^{m+1}\cos x+c\,(m+1)\cos x.$$

7. **(c)** $\lim\limits_{x\to2}\frac{e^{3x-6}-1}{\sin(2-x)}=\lim\limits_{x\to2}\frac{e^{-3(2-x)}-1}{\sin(2-x)}$

Put 2 – x = t ⇒ x = 2 – t

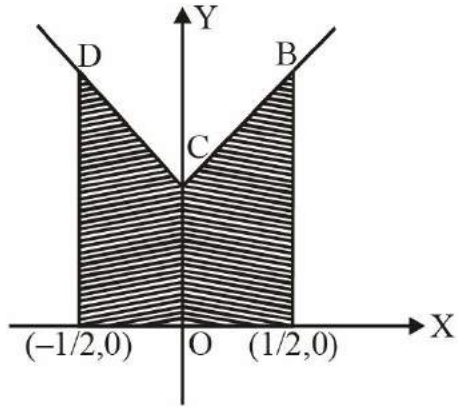
$$\lim_{t\to0}\frac{e^{-3t}-1}{\sin t}\qquad\left(\frac{0}{0}\text{ Form}\right)$$

$$=\lim_{t\to0}\frac{(-3).e^{-3t}}{\cos t}=-3$$ [By L-Hospital's rule]

8. **(c)** The given lines are,

y – 1 = x, x $\geq$0; y – 1 = – x, x < 0 y = 0; $x = -\frac{1}{2}, x < 0$; $x = \frac{1}{2}, x \ge 0$

so that the area bounded is as shown in the figure.



Required area

$$= 2\int_0^{1/2}(1+x)\,dx = 2\left(x+\frac{x^2}{2}\right)_0^{1/2} = 2\left(\frac{1}{2}+\frac{1}{8}\right) = \frac{5}{4}$$

9. **(b)** $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow 2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x$

$\Rightarrow \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$

$\Rightarrow \sin 2x = \cos 2x \left[\text{as} \cos x \ne \frac{3}{2}\right]$

$\Rightarrow \tan 2x = 1 \Rightarrow 2x = n\pi + \frac{\pi}{4} \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}.$

10. **(c)** Let $\vec{a} + 2\vec{b} = t\vec{c}$ and $\vec{b} + 3\vec{c} = s\vec{a}$, where t and s are scalars.

Adding, we get

$\vec{a} + 3\vec{b} + 3\vec{c} = t\vec{c} + s\vec{a} \Rightarrow \vec{a} + 2\vec{b} + 6\vec{c} = t\vec{c} + s\vec{a} - \vec{b} + 3\vec{c}$

$= t\vec{c} + (\vec{b} + 3\vec{c}) - \vec{b} + 3\vec{c} = (t+6)\vec{c}$

$\left[\text{using } s\,\vec{a} = \vec{b} + 3\vec{c}\right]$

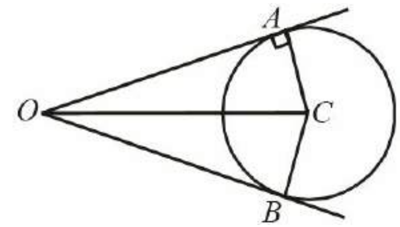
$= \lambda\vec{c}$, where $\lambda = t + 6$

11. **(b)** Area of quadrilateral = 2 [area of $\Delta\, OAC$]

$= 2.\frac{1}{2}OA.AC = \sqrt{S_1}.\sqrt{g^2 + f^2 - c}$

Point is (0, 0) $\Rightarrow S_1 = c$,

$\therefore$ Area = $\sqrt{c(g^2 + f^2 - c)}$



12. **(d)** If (α, β, γ) be the image, then mid point of (α, β, γ) and (–1, 3, 4) must lie on $x - 2y = 0$

$$\frac{\alpha - 1}{2} - 2\left(\frac{\beta + 3}{2}\right) = 0$$

$$\alpha - 1 - 2\beta - 6 = 0 \qquad \alpha - 2\beta = 7 \qquad \text{..... (1)}$$

Also line joining (α, β, γ) and (–1, 3, 4) should be parallel to the normal of the plane $x - 2y = 0$

$$\therefore \frac{\alpha + 1}{1} = \frac{\beta - 3}{-2} = \frac{\gamma - 4}{0} = \lambda$$

$$\Rightarrow \alpha = \lambda - 1, \beta = -2\lambda + 3, \gamma = 4 \qquad \text{..... (2)}$$

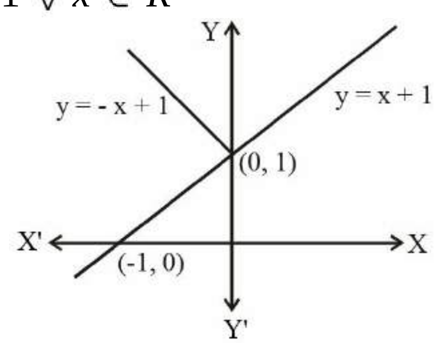
From (1) and (2)

$$\alpha = \frac{9}{5}, \ \beta = -\frac{13}{5}, \gamma = 4$$

None of the option matches.

13. **(a)** $f(x) = \min\{x + 1, |x| + 1\}$

$\Rightarrow f(x) = x + 1 \ \forall \ x \in R$



Hence, $f(x)$ is differentiable everywhere for all $x \in R$.

14. **(c)** When $x < 0, |x| = -x$

$\therefore$ Equation is $x^2 - x - 6 = 0 \Rightarrow x = -2, 3$

$x < 0,$

$\therefore$ $x = -2$ is the solution.

When $x \geq 0, |x| = x$

∴ Equation is $x^2 + x - 6 = 0 \Rightarrow x = 2, -3$

$x \geq 0,$

∴ $x = 2$ is the solution,

Hence $x = 2$, – 2 are the solutions and their sum is zero.

15. (c) $\int \frac{dx}{4\sin^2 x + 5\cos^2 x} = \int \frac{\sec^2 x dx}{4\tan^2 x + 5} = \frac{1}{4}\int \frac{\sec^2 x dx}{\tan^2 x + \frac{5}{4}}$

Put tan $x = t \Rightarrow \sec^2 x\, dx = dt,$ then it reduces to

$$\frac{1}{4}\int \frac{dt}{t^2 + \left(\frac{\sqrt{5}}{2}\right)^2} = \frac{2}{4\sqrt{5}}\tan^{-1}\left(\frac{2t}{\sqrt{5}}\right) + c$$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c.$$

16. (c) $27^{40} = 3^{120}$

$$3^{119} = (4-1)^{119} = {}^{119}C_0 4^{119} - {}^{119}C_1 4^{118}$$

$$+ {}^{119}C_2 4^{117} - {}^{119}C_3 4^{116} + ... + (-1)$$

∴ $3^{119} = 4k - 1$

∴ $3^{120} = 12k - 3 = 12(k - 1) + 9$

∴ the required remainder is 9.

17. (d) $f(x) = x^{100} + \sin x - 1 \Rightarrow f'(x) = 100x^{99} + \cos x.$

If $0 < x < \pi/2$, then $f'(x) > 0$.

Therefore f(x) is increasing on $(0, \pi/2)$.

If $0 < x < 1$, then $100x^{99} > 0$ and $\cos x > 0$

[∵ x lies between 0 and 1 radian]

$\Rightarrow f'(x) = 100x^{99} + \cos x > 0$

⇒ f(x) is increasing on (0, 1).

If $\pi/2 < x < \pi$, then $100\,x^{99} > 100$[∵ $x > 1$, ∴ $x^{99} > 1$]

$\Rightarrow 100x^{99} + \cos x > 0$

$[\because \cos x \geq -1, \; \therefore 100x^{99} + \cos x > 99]$

$\Rightarrow f'(x) > 0 \Rightarrow f(x)$ is increasing on $(\pi/2, \pi)$.

18. **(b)** Let A denote the event that the stranger succeeds at the k^{th} trial. Then

$$P(A') = \frac{999}{1000} \times \frac{998}{999} \times \ldots\ldots \times \frac{1000-k+1}{1000-k+2} \times \frac{1000-k}{1000-k+1}$$

$$\Rightarrow P(A') = \frac{1000-k}{1000} \Rightarrow P(A) = 1 - \frac{1000-k}{1000} = \frac{k}{1000}$$

19. **(b)** $\lim_{x \to 0^-} f(x) = 0, \qquad f(0) = 0, \lim_{x \to 0^+} f(x) = -4$

f (x) discontinuous at $x = 0$.

and $\lim_{x \to 1^-} f(x) = 1$ and $\lim_{x \to 1^+} f(x) = 1, f(1) = 1$

Hence f (x) is continuous at x = 1.

Also $\lim_{x \to 2^-} f(x) = 4(2)^2 - 3.2 = 10$

f (2) = 10 and $\lim_{x \to 2^+} f(x) = 3(2) + 4 = 10$

Hence f (x) is continuous at x = 2.

20. **(c)** Let $f(x) \neq 2$ be true and $f(y) = 2$, $f(z) \neq 1$ are false

$\Rightarrow \quad f(x) \neq 2, f(y) \neq 2, f(z) = 1$

$\Rightarrow \quad f(x) = 3, f(y) = 3, f(z) = 1$

but then function is many one.

Now, let $f(z) \neq 1$ is true & $f(x) \neq 2$ & $f(y) = 2$ are false

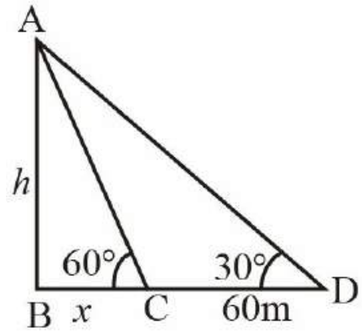
$\Rightarrow \quad f(x) = 2, f(y) \neq 2$ and $f(z) \neq 1$.

$\Rightarrow \quad f(x) = 2, f(z) = 3$ and $f(y) = 1$.

Hence one-one.

21. **(52)** $\tan 30° = \dfrac{h}{x+60}, \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+60}$

$$x + 60 = \sqrt{3}h, x = \sqrt{3}h - 60$$



$$\tan 60° = \frac{h}{x}, x = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}\,h - 60 = \frac{h}{\sqrt{3}}$$

$$\Rightarrow 3h - 60\sqrt{3} = h$$

$$\Rightarrow h = \frac{60\sqrt{3}}{2} = 30\sqrt{3} = 51.96 = 52\,\text{m}$$

22. (896) Required number of ways

$${}^{4}C_1 \times {}^{8}C_5 + {}^{4}C_2 \times {}^{8}C_4 + {}^{4}C_3 \times {}^{8}C_3 + {}^{4}C_4 \times {}^{8}C_2$$

$= 4 \times 56 + 6 \times 70 + 4 \times 56 + 1 \times 28 = 896.$

23. (17) Given that the line L passes through (13, 32).

So, $\frac{13}{5} + \frac{32}{b} = 1 \Rightarrow b = -20$

Now, equation of L is,

$$\frac{x}{5} - \frac{y}{20} = 1 \Rightarrow 4x - y = 20$$

Slope of L is $m_1 = 4$.

Slope of $\frac{x}{c} + \frac{y}{3} = 1$ is,

$$m_2 = -\frac{3}{c}$$

✌ L and K are parallel lines.

$\therefore\ -\frac{3}{c} = 4 \Rightarrow c = -\frac{3}{4}$

So, equation of line K is,

$$-\frac{4x}{3} + \frac{y}{3} = 1 \Rightarrow 4x - y = -3$$

Distance between L and K is, $\left|\frac{20+3}{\sqrt{16+1}}\right| = \frac{23}{\sqrt{17}}$.

24. (0.42) $\left|\frac{z-\alpha}{z-\beta}\right| = k$ ... (i)

where. α and β are constant complex numbers represents circle, if $k \neq 1$ and its radius is

$\left|\frac{k(\alpha-\beta)}{1-k^2}\right|$ ... (ii)

Given $\left|\frac{z-i}{z-(-i)}\right| = 5$

∴ $\alpha = i,\ \beta = -i,\ k = 5$ [by comparing with (i)]

∴ Radius $= \left|\frac{5(i+i)}{1-25}\right| = \frac{5}{12}$ = 0.42

25. (0) ✌ $\sum_{r=1}^{n} (r-1)$ = 1 + 2 + ...+ $(n-1) = \frac{n(n-1)}{2}$

$\sum_{r=1}^{n} (r-1)^2$ = $1^2 + 2^2 + \ldots\ldots\ldots\ldots + (n-1)^2$

$= \frac{n(n-1)(2n-1)}{6}$

$\sum_{r=1}^{n} (r-1)^3$ = $1^3 + 2^3 + \ldots\ldots\ldots\ldots + (n-1)^3 = \frac{n^2(n-1)^2}{4}$ ∴ $\sum_{r=1}^{n} \Delta_r$ =

$$\begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{1}{6}n(n-1)(2n-1) & 2n^2 & 2(2n-1) \\ \frac{1}{4}n^2(n-1)^2 & 3n^3 & 3n(n-1) \end{vmatrix}$$

$$= \frac{n(n-1)}{12}\begin{vmatrix} 6 & n & 6 \\ 2(2n-1) & 2n^2 & 2(2n-1) \\ 3n(n-1) & 3n^3 & 3n(n-1) \end{vmatrix}$$

$= 0$ ($\because$ $C_1 = C_3$)