CHAPTER-21
CAPACITANCE

CAPACITOR AND CAPACITANCE

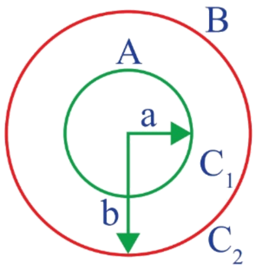
A capacitor consists of two conductors carrying charges of equal magnitude and opposite sign. The capacitance C of any capacitor is the ratio of the charge Q on either conductor to the potential difference V between them **C = Q/V** The capacitance depends only on the geometry of the conductors.

Capacitance of an Isolated Spherical Conductor

$C = 4\pi \in_0 \in_r R$ in a medium $C = 4\pi \in_0 R$ in air.

This sphere is at infinite distance from all the conductors.

Spherical Capacitor:

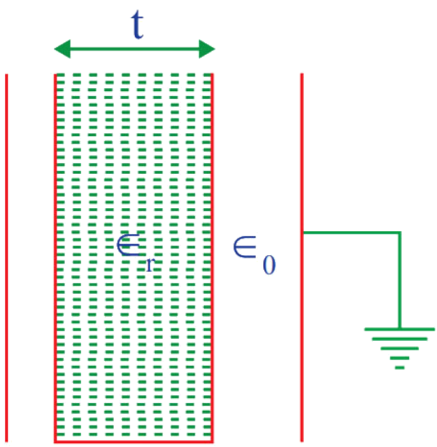


It consists of two concentric spherical shells as shown in figure. Here capacitance of region between the two shells is C_1 and that outside the shell is C_2. We have $C_1 = \frac{4\pi \in_0 ab}{b-a}$ and $C_2 = 4\pi \in_0 b$.

Parallel Plate Capacitor

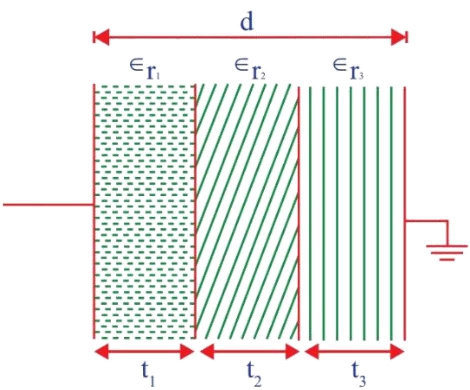
(i) **Uniform Di-Electric Medium:** If two parallel plates each of area A & separated by a distance d are charged with equal & opposite charge Q, then the system is called a parallel plate capacitor & its capacitance is given by, $C = \frac{\in_0 \in_r A}{d}$ in a medium, $C = \frac{\in_0 A}{d}$ with air as medium. This result is only valid when the electric field between plates of capacitor is constant.

(ii) **Medium Partly Air:** $C = \frac{\epsilon_0 A}{d-\left(t-\frac{t}{\epsilon_r}\right)}$



When a di-electric slab of thickness t & relative permittivity ϵ_r is introduced between the plates of an air capacitor, then the distance between the plates is effectively reduced by $\left(t-\frac{t}{\epsilon_r}\right)$ irrespective of the position of the dielectric slab.

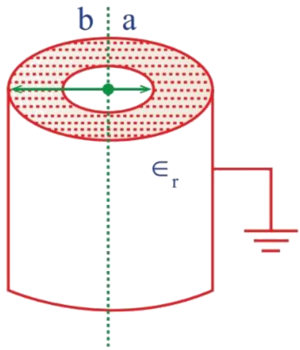
(iii) **Composite Medium:**



$$C = \frac{\epsilon_0 A}{\frac{t_1}{\epsilon_{r1}} + \frac{t_2}{\epsilon_{r2}} + \frac{t_3}{\epsilon_{r3}}}$$

Cylindrical Capacitor:

It consists of two co-axial cylinders of radii a & b, the outer conductor is earthed. The di-electric constant of the medium filled in the space between the cylinders is ϵ_r.

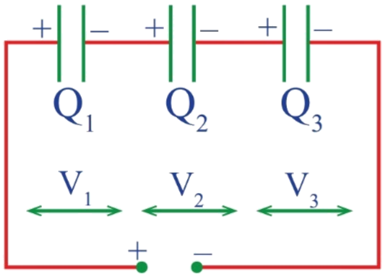


The capacitance **per unit length** is $C = \frac{2\pi \epsilon_0 \epsilon_1}{\ell n\left(\frac{b}{a}\right)}$

Combination of Capacitors:

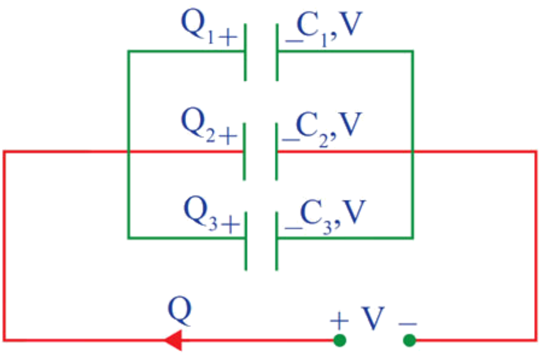
(i) **Capacitors in Series:** In this arrangement all the capacitors when uncharged get the same charge Q but the potential difference across each will differ (if the capacitance are unequal).

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots \ldots \ldots \ldots \ldots + \frac{1}{C_n}$$



(ii) **Capacitors in Parallel:** In this arrangement all the capacitors when uncharged get the same potential difference (V) but charge across each will differ (if the capacitance are unequal).

$$C_{eq.} = C_1 + C_2 + C_3 + \dots\dots\dots + C_n.$$



Energy Stored in a Charged Capacitor

Capacitance C, Charge Q & potential difference V; then energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

This energy is stored in the electrostatic field set up in the di-electric medium between the conducting plates of the capacitor.

Heat Produced in Switching in Capacitive Circuit

Heat = Work done by battery – Energy absorbed by capacitor

Work done by battery to charge a capacitor

$W = CV^2 = QV = Q^2/C$=Double the amount of energy stored in the capacitor

Sharing of Charges: When two charged conductors of capacitance C_1 & C_2 at potential V_1 & V_2 respectively are connected by a conducting wire, the charge flows from higher potential conductor to lower potential conductor, until the potential of the two condensers become equal. The common potential (V) after sharing of charges;

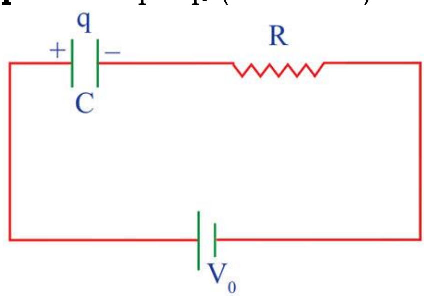
$$V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{q_1+q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2}$$

Charges after sharing $q_1 = C_1 V$ & $q_2 = C_2 V$. In this process energy is lost in the connecting wire as heat.

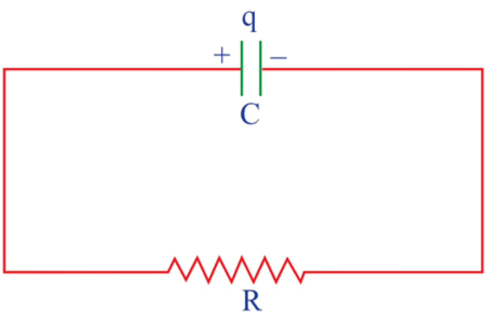
This loss of energy is $(U_{initial} - U_{final}) = \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$

Attractive force between capacitor plate: $F = \left(\frac{\sigma}{2\,\epsilon_0}\right)(\sigma A) = \frac{Q^2}{2\,\epsilon_0\,A}$

Charging of a capacitor: $q = q_0\,(1 - e^{-t/RC})$ where $q_0 = CV_0$



Discharging of a capacitor: $q = q_0e^{-t/RC}$



Key Tips

- The energy of a charged conductor resides outside the conductor in its electric field, where as in a condenser it is stored within the condenser in its electric field.
- The energy of an uncharged condenser = 0.
- The capacitance of a capacitor depends only on its size & geometry & the dielectric between the conducting surface.
- The two adjacent conductors carrying same charge can be at different potential because the conductors may have different sizes and means different capacitance.
- On filling the space between the plates of a parallel plate air capacitor with a dielectric, capacity of the capacitor is increased because the same amount of charge can be stored at a reduced potential.
- The potential of a grounded object is taken to be zero because capacitance of the earth is very large.