Exponents and Powers

Exponential Form of Numbers

Many a times, we come across very large numbers, especially in Science. For example, the speed of light in air is approximately 300000000 m/s. As we advance to higher classes, we will also be required to solve questions, in which we will have to perform calculations involving very large numbers. Now, the speed of light has digit 3 followed by eight 0s. If we mistakenly add or remove even one 0 from this, then our calculations will be wrong.

For this reason, large numbers are often written in exponential form, which is less confusing.

We may also come across some cases where the base is a negative number. For example, let us consider $(-2)^5$. Is its value same as that of 2^5?

$(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Thus, the values of $(-2)^5$ and 2^5 are not equal.

Now, let us look at the values of $(-2)^6$ and 2^6.

$(-2)^6 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 64$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

In this case, the values of $(-2)^6$ and 2^6 are equal. Why is this so?

If the exponent of a negative base is odd, then the value of the exponential form is negative. However, if the exponent of a negative base is even, then the value of the exponential form is positive.

Let us now find the values of $(-1)^5$ and $(-1)^6$.

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

$(-1)^6 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1$

Thus, we can conclude that

$$\mathbf{(-1)^{\text{odd number}} = -1}$$

$$\mathbf{(-1)^{\text{even number}} = 1}$$

We already know how to prime factorize numbers. For example, the prime factorization of 1800 can be done as

1800 = 2 × 2 × 2 × 3 × 3 × 5 × 5

Now, let us try and express it using exponents.

We can break this expression into smaller groups as

$$1800 = \underbrace{2 \times 2 \times 2}_{\text{Product of three 2s}} \times \underbrace{3 \times 3}_{\text{Product of two 3s}} \times \underbrace{5 \times 5}_{\text{Product of two 5s}}$$

Now, $2 \times 2 \times 2 = 2^3$, $3 \times 3 = 3^2$, and $5 \times 5 = 5^2$

Thus, we can restate the prime factorization of 1800 as

1800 = 2 × 2 × 2 × 3 × 3 × 5 × 5

$= 2^3 \times 3^2 \times 5^2$

Now, if we have to write a negative number, say −1800, in exponential form then we proceed as follows:

−1800 = (−1) × (1800)

−1800 = (−1) × 2 × 2 × 2 × 3 × 3 × 5 × 5

$-1800 = (-1) \times 2^3 \times 3^2 \times 5^2$

Similarly, the prime factorizations of the numbers 1000 and −432 can be done as follows:

$1000 = 2^3 \times 5^3$

$-432 = (-1) \times 2^4 \times 3^3$

Let us look at some examples:

Number	Expanded form	Exponential form	Base and exponent
10000	10 × 10 × 10 × 10	10^4	base 10, exponent 4
$\frac{1}{243}$	$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$	$\left(\frac{1}{3}\right)^5$	base $\frac{1}{3}$, exponent 5
64	2 × 2 × 2 × 2 × 2 × 2	2^6	base 2, exponent 6
64	(−2) × (−2) × (−2) × (−2) × (−2) × (−2)	$(-2)^6$	base −2, exponent 6
−32	(−2) × (−2) × (−2) × (−2) × (−2)	$(-2)^5$	base −2, exponent 5

Let us solve some more examples to understand the concept better.

Example 1:
Write the following in exponential form.

a. Minus nine to the power of six

b. One fourth to the power of five

c. Three square to the power of five

Solution:

a. Minus nine to the power of six = $(-9)^6$

b. One fourth to the power of five = $\left(\frac{1}{4}\right)^5$

c. Three square to the power of five = $(3^2)^5$

Example 2:
Write the base and the exponent for the following.

a. $\left(\frac{1}{3}\right)^2$
b. $(-2.5)^5$

Solution:
a. $\left(\frac{1}{3}\right)^2$

Here, base = $\frac{1}{3}$, exponent = 2

b. $(-2.5)^5$

Here, base = –2.5, exponent = 5

Example 3:
Expand the following expressions.
a. 5^4
b. $(3^2)^3$
c. $(-2)^2$

Solution:
a. $5^4 = 5 \times 5 \times 5 \times 5$

b. $(3^2)^3 = 3^2 \times 3^2 \times 3^2$

c. $(-2)^2 = (-2) \times (-2)$

Example 4:
Write the exponents for the base given.

a. −125 with (–5) as base
b. 16 with (–2) as base

Solution:
a.
$-125 = (-5) \times (-5) \times (-5) = (-5)^3$
Thus, The exponent is $(-5)^3$.

b.
$16 = (-2) \times (-2) \times (-2) \times (-2) = (-2)^4$
Thus, the exponent is $(-2)^4$.

Example 5:

If 400 can be prime factorized as $2^x \times 5^2$, then what is the value of *x*?

Solution:

Let us first perform the prime factorization of 400.

2	400
2	200
2	100
2	50
5	25
5	5
	1

$\therefore$ 400 = 2 × 2 × 2 × 2 × 5 × 5 = $2^4 \times 5^2$

Now, comparing $2^4 \times 5^2$ with $2^x \times 5^2$, we observe that $2^4 = 2^x$

Thus, the value of x is 4.

Example 6:

What is the value of the expression $2^2 \times (-3)^3 \times (-5)^2$?

Solution:

$2^2 \times (-3)^3 \times (-5)^2 = (2 \times 2) \times [(-3) \times (-3) \times (-3)] \times [(-5) \times (-5)]$

$= 4 \times (-27) \times 25$

$= -2700$

Example 7:

How can the number 3960 be written as a product of the powers of its prime factors?

Solution:

3960 = 2 × 2 × 2 × 3 × 3 × 5 × 11 = $2^3 \times 3^2 \times 5 \times 11$

Example 8:

Express 500 using base 2 and exponents.

Solution:

$$\begin{aligned}500 &= 256 + 128 + 64 + 32 + 16 + 4 \\ &= 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^2 \\ &= 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0\end{aligned}$$

Comparing and Ordering Numbers with Exponents

We know how to compare positive and negative integers. For example, among the numbers 32, 64, 27, and 25, the smallest number is 25, while the greatest number is 64.

Similarly, among the numbers −128, 16, −81, and 125, the smallest number is −128, while the greatest number is 125.

Let us now try and extend this knowledge to compare numbers written in exponential forms.

What if we were asked to write 2^5, 2^6, 3^3, and 5^2 in ascending order? We first have to find the number that represents each exponential form, and then compare these numbers. Let us see how.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

$3^3 = 3 \times 3 \times 3 = 27$

$5^2 = 5 \times 5 = 25$

Now, we can easily compare the numbers 32, 64, 27, and 25, just as we did in the beginning.

We know that $25 < 27 < 32 < 64$

$\therefore 5^2 < 3^3 < 2^5 < 2^6$

Let us solve some more examples to understand the concept better.

Let us now find the greatest and the least numbers among $(-2)^7$, 2^4, $(-3)^4$, and 5^3.

$(-2)^7 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -128$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3) = 81$

$5^3 = 5 \times 5 \times 5 = 125$

As seen here, the greatest number is 125, while the smallest number is −128.

Thus, the greatest and the least numbers are 5^3 and $(-2)^7$ respectively.

We can also compare numbers given in exponential forms without simplifying them.

Let us solve some more examples to understand the concept better.

Example 1:

Arrange 5^4, $(-4)^6$, and 6^3 in decreasing order.

Solution:

$5^4 = 5 \times 5 \times 5 \times 5 = 625$

$(-4)^6 = (-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) = 4096$

$6^3 = 6 \times 6 \times 6 = 216$

Now, $4096 > 625 > 216$

$\therefore (-4)^6 > 5^4 > 6^3$

Thus, the given numbers can be arranged in descending order as $(-4)^6$, 5^4, 6^3.

Example 2:

Arrange $(-5)^4$, $(-6)^3$, 6^3, and 5^5 in increasing order.

Solution:

$(-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 625$

$(-6)^3 = (-6) \times (-6) \times (-6) = -216$

$6^3 = 6 \times 6 \times 6 = 216$

$5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$

Now, $-216 < 216 < 625 < 3125$

$\therefore (-6)^3 < 6^3 < (-5)^4 < 5^5$

Thus, the given numbers can be arranged in ascending order as $(-6)^3$, 6^3, $(-5)^4$, 5^5.

Example 3:

Find the greater number out of 1.5×10^6 and 3.9×10^5.

Solution:

$1.5 \times 10^6 = 1.5 \times 1000000 = 1500000$

$3.9 \times 10^5 = 3.9 \times 100000 = 390000$

Since $1500000 > 390000$,

$1.5 \times 10^6 > 3.9 \times 10^5$

Law of Multiplying Powers with the Same Base

Suppose we have to simplify the expression $(3^2 \times 3^3)$.

One way of doing this is

$$3^2 \times 3^3 = \underbrace{3 \times 3}_{\text{2 times}} \times \underbrace{3 \times 3 \times 3}_{\text{3 times}} = \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{\text{5 times}} = 3^5$$

$\therefore 3^2 \times 3^3 = 3^5$

If you notice carefully, the sum of the exponents on the left hand side equals the exponent on the right hand side. Also, the bases are same on both the sides.

Therefore, can we write $3^2 \times 3^3$ as $3^{2+3} = 3^5$?

Let us take another example. Let us simplify the expression $(5^4 \times 5^6)$.

$$5^4 \times 5^6 = \underbrace{5 \times 5 \times 5 \times 5}_{\text{4 times}} \times \underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}_{\text{6 times}} = \underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}_{\text{10 times}} = 5^{10}$$

$\therefore 5^4 \times 5^6 = 5^{10}$

Looking at both the examples, one might be tempted to say for the second example that

$5^4 \times 5^6 = 5^{4+6} = 5^{10}$

In fact, when two numbers with the same base are multiplied, then their product also has the same base and its exponent equals the sum of the exponents of the two numbers.

Thus, we can say that

If a is any non-zero integer, and p and q are any two whole numbers, then $a^p \times a^q = a^{p+q}$

This is one of the many laws of exponents

Let us go through the following examples to understand the concept better.

Example 1:

In a class, there are 64 students. Each student was given 4 apples. Rajesh was asked to calculate the total number of apples that were distributed. Rajesh answered that 2^8 apples were distributed. Was he correct?

Solution:

Total number of students = 64

Number of apples given to each student = 4

∴ Total number of apples distributed = Number of students × Number of apples given to each student

$= 64 \times 4 = (2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2) = 2^6 \times 2^2$

$= 2^{6+2}$ $\left\{a^p \times a^q = a^{p+q}\right\}$

$= 2^8$

Thus, Rajesh was correct.

Example 2:

What is the value of the expression $3^5 \times 3^3$?

Solution:

$3^5 \times 3^3$

$= 3^{5+3}$ $\left(a^p \times a^q = a^{p+q}\right)$

$= 3^8$

Example 3:

If a is a non-zero integer, then how can the expression $a^{10} \times a^2$ be simplified?

Solution:

Here, both the numbers have the same base a. In order to find their product, we have to add their exponents.

$\therefore a^{10} \times a^2 = a^{10+2} = a^{12}$

Law of Dividing Powers with the Same Base

Suppose we have to divide 2^8 by 2^3 i.e., we have to simplify the expression $\frac{2^8}{2^3}$.

$$\frac{2^8}{2^3} = \frac{2\times2\times2\times2\times2\times2\times2\times2}{2\times2\times2}$$

Now, three 2s in the denominator will cancel out with three 2s in the numerator. Thus, the remaining expression is

$$\frac{2^8}{2^3} = \frac{2\times2\times2\times2\times2\times2\times2\times2}{2\times2\times2} = 2\times2\times2\times2\times2 = 2^5$$

Note that the divisor and the dividend have the same base and the difference between their exponents equals the exponent of the quotient.

Thus, instead of carrying out this lengthy calculation, we could have simply stated

$$2^8 \div 2^3 = 2^{8-3} = 2^5$$

We can generalise this as

If a is any non-zero integer, and p and q are any two whole numbers ($p > q$), then $a^p \div a^q = a^{p-q}$

Let us solve some examples to understand the concept better.

Example 1:

What is the value of the expression $5^9 \div 5^7$?

Solution:

$$5^9 \div 5^7 = 5^{9-7} \qquad \left(a^p \div a^q = a^{p-q}\right)$$
$$= 5^2 = 5 \times 5 = 25$$

Example 2:

What is the value of the expression $\dfrac{2^5 \times 3^2 \times 2^3 \times 3^7}{3^8 \times 2^6}$?

Solution:

$$\frac{2^5 \times 3^2 \times 2^3 \times 3^7}{3^8 \times 2^6} = \frac{2^5 \times 2^3 \times 3^2 \times 3^7}{3^8 \times 2^6} = \frac{2^{5+3} \times 3^{2+7}}{3^8 \times 2^6} \qquad \left(a^p \times a^q = a^{p+q}\right)$$
$$= \frac{2^8 \times 3^9}{3^8 \times 2^6} = 2^{8-6} \times 3^{9-8} \qquad \left(a^p \div a^q = \frac{a^p}{a^q} = a^{p+q}\right)$$
$$= 2^2 \times 3^1 = 4 \times 3 = 12$$

Law of Taking Power of a Power

We know that 3^2 means the number 3 multiplied with itself twice i.e., $3^2 = 3 \times 3 = 9$

Now, can we extend this knowledge to find the value of $(3^2)^2$? What does this expression imply? It means the number 3^2 is multiplied with itself twice.

$\therefore (3^2)^2 = 3^2 \times 3^2 = 9 \times 9 = 81 = 3^4$

Looking at this expression, it seems that we can write $(3^2)^2$ as $3^{2 \times 2}$. However, let us take another example to be sure.

Let us try and simplify the expression $(2^2)^3$. This expression means the number 2^2 multiplied with itself three times.

$\therefore (2^2)^3 = 2^2 \times 2^2 \times 2^2 = 4 \times 4 \times 4 = 64 = 2^6$

Thus, we could have also written $\left(2^2\right)^3$ as $2^{2 \times 3} = 2^6$

Looking at these two examples, we can conclude that

If *a* is any non-zero integer and *p* and *q* are any two whole numbers, then

$\left(a^{p}\right)^{q} = a^{p \times q}$

Let us look at some more examples to understand the concept better.

Example 1:

Find the value of

(i) $\left\{(-2)^{2}\right\}^{2}$

(ii) $\left\{(-2)^{2}\right\}^{3}$

(iii) $\left\{(-2)^{3}\right\}^{2}$

(iv) $\left\{(-2)^{3}\right\}^{3}$

Solution:

(i) $\left\{(-2)^{2}\right\}^{2} = (-2)^{2\times 2} = (-2)^{4} = (-2)\times(-2)\times(-2)\times(-2) = 16$

(ii) $\left\{(-2)^{2}\right\}^{3} = (-2)^{2\times 3} = (-2)^{6} = (-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2) = 64$

(iii) $\left\{(-2)^{3}\right\}^{2} = (-2)^{3\times 2} = (-2)^{6} = (-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2) = 64$

$\therefore \left\{(-2)^{3}\right\}^{2} = \left\{(-2)^{2}\right\}^{3}$

(iv) $\left\{(-2)^{3}\right\}^{3} = (-2)^{3\times 3} = (-2)^{9}$

$$= (-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2) = -512$$

Example 2:

Find the value of *y* for the expression $\left\{\left(2^3\right)^2\right\}^2 = \left(y^4\right)^3$.

Solution:

$$\left\{\left(2^3\right)^2\right\}^2 = \left(2^3\right)^{2\times 2} \qquad \left[\left(a^p\right)^q = a^{p\times q}\right]$$

$$= \left(2^3\right)^4 = 2^{3\times 4} \qquad \left[\left(a^p\right)^q = a^{p\times q}\right]$$

$$= \left(2^4\right)^3 \qquad \left[a^{p\times q} = \left(a^p\right)^q\right]$$

Comparing $\left(2^4\right)^3$ with $\left(y^4\right)^3$, we obtain the value of *y* as 2.

Example 3:

What is the value the expression $\dfrac{\left(2^2\right)^3\times 2^4\times 3^5}{\left(3^2\right)^2\times 2^3\times 2^5}$?

Solution:

$$\frac{\left(2^2\right)^3\times 2^4\times 3^5}{\left(3^2\right)^2\times 2^3\times 2^5} = \frac{2^{2\times 3}\times 2^4\times 3^5}{3^{2\times 2}\times 2^3\times 2^5} \qquad \left[\left(a^p\right)^q = a^{p\times q}\right]$$

$$= \frac{2^6\times 2^4\times 3^5}{3^4\times 2^3\times 2^5} = \frac{2^{6+4}\times 3^5}{3^4\times 2^{3+5}} \qquad \left[a^p\times a^q = a^{p+q}\right]$$

$$= \frac{2^{10}\times 3^5}{3^4\times 2^8} = 2^{10-8}\times 3^{5-4} \qquad \left[\frac{a^p}{a^q} = a^{p-q}\right]$$

$$= 2^2\times 3^1 = 4\times 3 = 12$$

Law of Multiplying Powers with the Same Exponent

Suppose we have to compare the numbers $4^5 \times 7^5$ and 28^5.

We can express $4^5 \times 7^5$ as

$4^5 \times 7^5 = (4 \times 4 \times 4 \times 4 \times 4) \times (7 \times 7 \times 7 \times 7 \times 7)$

$= (4 \times 7) \times (4 \times 7) \times (4 \times 7) \times (4 \times 7) \times (4 \times 7)$

$= (4 \times 7)^5$

$= 28^5$

Thus, we see that the expression $4^5 \times 7^5$ equals the number 28^5. Note that the two numbers that we multiplied had different bases but the same exponent. Their product also has the same exponent and its base is given by the product of the two bases.

Let us take another expression $3^4 \times 5^4$.

We can simplify this as

$3^4 \times 5^4 = (3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5)$

$= (3 \times 5) \times (3 \times 5) \times (3 \times 5) \times (3 \times 5)$

$= (3 \times 5)^4$

$= 15^4$

In this case also, the product of the two numbers has its base equal to the product of the two bases.

Thus, we can conclude that

If *a* and *b* are any two non-zero integers and *p* is any whole number, then $a^p \times b^p = (a \times b)^p$

Let us solve some more examples to understand the concept better.

Example 1:

If $7^4 \times 3^4 = x^2$, then what is the value of *x*?

Solution:

$7^4 \times 3^4 = (7 \times 3)^4$ $[a^p \times b^p = (a \times b)^p]$

$= 21^4 = 21^{2 \times 2} = (21^2)^2$ [$a^{p \times q} = (a^p)^q$]

$= (441)^2$

Comparing $(441)^2$ with x^2, we obtain the value of x as 441.

Example 2:

Rewrite the expression $2^3 \times 2^5 \times 3^8$ in exponential form with base 6.

Solution:

$2^3 \times 2^5 \times 3^8 = (2^3 \times 2^5) \times 3^8 = 2^{3+5} \times 3^8$ [$a^p \times a^q = a^{p+q}$]

$= 2^8 \times 3^8 = (2 \times 3)^8$ [$a^p \times b^p = (a \times b)^p$]

$= 6^8$

Example 3:

Find the value of the expression $\dfrac{4^2 \times 6^3}{2^3 \times 4 \times 3^2}$.

Solution:

$$\begin{aligned}
\frac{4^2 \times 6^3}{2^3 \times 4 \times 3^2} &= \frac{(2^2)^2 \times (2 \times 3)^3}{2^3 \times 2^2 \times 3^2} \\
&= \frac{2^{2 \times 2} \times (2^3 \times 3^3)}{2^{3+2} \times 3^2} && \left[(a^p)^q = a^{p \times q};\ (a \times b)^p = a^p \times b^p;\ a^p \times a^q = a^{p+q}\right] \\
&= \frac{2^4 \times 2^3 \times 3^3}{2^5 \times 3^2} = \frac{2^{4+3} \times 3^3}{2^5 \times 3^2} && \left[a^p \times a^q = a^{p+q}\right] \\
&= \frac{2^7 \times 3^3}{2^5 \times 3^2} = 2^{7-5} \times 3^{3-2} && \left[\frac{a^p}{a^q} = a^{p-q}\right] \\
&= 2^2 \times 3^1 = 4 \times 3 \\
&= 12
\end{aligned}$$

Law of Dividing Powers with the Same Exponent

Let us begin with the division of 5^4 by 7^4.

$$5^4 \div 7^4 = \frac{5^4}{7^4} = \frac{5\times5\times5\times5}{7\times7\times7\times7} = \frac{5}{7}\times\frac{5}{7}\times\frac{5}{7}\times\frac{5}{7} = \left(\frac{5}{7}\right)^4$$

Similarly, $(-5)^4 \div (-7)^4 = \frac{(-5)^4}{(-7)^4} = \frac{(-5)\times(-5)\times(-5)\times(-5)}{(-7)\times(-7)\times(-7)\times(-7)}$

$$= \left(\frac{-5}{-7}\right)\times\left(\frac{-5}{-7}\right)\times\left(\frac{-5}{-7}\right)\times\left(\frac{-5}{-7}\right) = \left(\frac{-5}{-7}\right)^4 = \left(\frac{5}{7}\right)^4$$

Taking another example, $(-5)^4 \div 7^4 = \frac{(-5)^4}{7^4} = \frac{(-5)\times(-5)\times(-5)\times(-5)}{7\times7\times7\times7}$

$$= \left(\frac{-5}{7}\right)\times\left(\frac{-5}{7}\right)\times\left(\frac{-5}{7}\right)\times\left(\frac{-5}{7}\right) = \left(\frac{-5}{7}\right)^4$$

One thing common in each of the three examples is that although the divisor and the dividend are different, they have the same exponent. The quotient obtained also has the same power as that of the dividend and the divisor.

Therefore, we can generalize that

If *a* and *b* are any two non-zero integers and *p* is any whole number, then $a^p \div b^p = \frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$

Let us solve some more examples to understand the concept better.

Example 1:

Divide 2^{16} by 3^{16}.

Solution:

$$2^{16} \div 3^{16} = \frac{2^{16}}{3^{16}} = \left(\frac{2}{3}\right)^{16}$$

Example 2:

What is the value of the expression $\left(\frac{3}{5}\right)^3 \times \frac{3^2 \times \left(2^2\right)^2 \times 5^4}{3^4 \times 2 \times 8 \times 3 \times 5}$**?**

Solution:

$$\left(\frac{3}{5}\right)^3 \times \frac{3^2 \times \left(2^2\right)^2 \times 5^4}{3^4 \times 2 \times 8 \times 3 \times 5}$$

$$= \frac{3^3}{5^3} \times \frac{3^2 \times 2^{2\times 2} \times 5^4}{3^4 \times 2^1 \times 2^3 \times 3^1 \times 5^1} \qquad \left[\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p};\ \left(a^p\right)^q = a^{p\times q};\ a = a^1\right]$$

$$= \frac{3^{3+2} \times 2^4 \times 5^4}{5^{3+1} \times 3^{4+1} \times 2^{1+3}} \qquad \left[a^p \times a^q = a^{p+q}\right]$$

$$= \frac{3^5 \times 2^4 \times 5^4}{5^4 \times 3^5 \times 2^4}$$

$$= 1$$

Simplification of Expressions Involving Zero as Exponent

Let us try and find the value of the expression $\frac{\left(2^2\right)^2 \times 2}{2^2 \times 8}$.

$$\frac{\left(2^2\right)^2 \times 2}{2^2 \times 8} = \frac{2^{2\times 2} \times 2^1}{2^2 \times 2^3} \qquad \left[\left(a^p\right)^q = a^{p\times q};\ a = a^1\right]$$

$$= \frac{2^4 \times 2^1}{2^2 \times 2^3} = \frac{2^{4+1}}{2^{2+3}} \qquad \left[a^p \times a^q = a^{p+q}\right]$$

$$= \frac{2^5}{2^5}$$

Now, one way of proceeding further is by cancelling the common factors in the numerator and denominator of the fraction. Since the numerator equals the denominator in this case, they cancel each other out and we obtain the value of the expression as 1.

Another way of proceeding further is by applying the law of dividing powers with same bases, which states that $a^p \div a^q = \frac{a^p}{a^q} = a^{p-q}$

$$\therefore \frac{\left(2^2\right)^2 \times 2}{2^2 \times 8} = \frac{2^5}{2^5} = 2^{5-5} = 2^0$$

However, we had previously seen that the value of this expression is 1.

This implies that $2^0 = 1$

In fact, any non-zero integer raised to the power of 0 has its value as 1.

Thus, we can conclude that

If a is any non-zero integer, then $a^0 = 1$

Let us solve some more examples to understand the concept better.

Example 1:

What is the value of the expression $\dfrac{6^3}{2^2 \times 18 \times 3}$?

Solution:

$$\frac{6^3}{2^2 \times 18 \times 3} = \frac{(2 \times 3)^3}{2^2 \times 9 \times 2 \times 3}$$

$$= \frac{2^3 \times 3^3}{2^2 \times 3^2 \times 2^1 \times 3^1} \qquad \left[(a \times b)^p = a^p \times b^p;\ a = a^1\right]$$

$$= \frac{2^3 \times 3^3}{2^{2+1} \times 3^{2+1}} \qquad \left[a^p \times a^q = a^{p+q}\right]$$

$$= \frac{2^3 \times 3^3}{2^3 \times 3^3} = 2^{3-3} \times 3^{3-3} \qquad \left[\frac{a^p}{a^q} = a^{p-q}\right]$$

$$= 2^0 \times 3^0 = 1 \times 1 \qquad \left[a^0 = 1\right]$$

$$= 1$$

Example 2:

What is the value of the expression $\left(\dfrac{3^2}{24}\right)^2 \times \dfrac{4^2 \times 36}{3^4}$?

Solution:

$$\left(\frac{3^2}{24}\right)^2 \times \frac{4^2 \times 36}{3^4} = \left(\frac{3^2}{8 \times 3^1}\right)^2 \times \frac{\left(2^2\right)^2 \times 4 \times 9}{3^4} \qquad \left[a = a^1\right]$$

$$= \left(\frac{3^{2-1}}{2^3}\right)^2 \times \frac{2^{2\times 2} \times 2^2 \times 3^2}{3^4} \qquad \left[\frac{a^p}{a^q} = a^{p-q};\ \left(a^p\right)^q = a^{p\times q}\right]$$

$$= \left(\frac{3^1}{2^3}\right)^2 \times \frac{2^4 \times 2^2 \times 3^2}{3^4} = \left(\frac{3}{2^3}\right)^2 \times \frac{2^{4+2} \times 3^2}{3^4} \qquad \left[a^p \times a^q = a^{p+q}\right]$$

$$= \frac{3^2}{\left(2^3\right)^2} \times \frac{2^6 \times 3^2}{3^4} \qquad \left[\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}\right]$$

$$= \frac{3^{2+2} \times 2^6}{2^{3\times 2} \times 3^4} \qquad \left[\left(a^p\right)^q = a^{p\times q};\ a^p \times a^q = a^{p+q}\right]$$

$$= \frac{3^4 \times 2^6}{2^6 \times 3^4} = 3^{4-4} \times 2^{6-6} \qquad \left[\frac{a^p}{a^q} = a^{p-q}\right]$$

$$= 3^0 \times 2^0 = 1 \times 1 \qquad \left[a^0 = 1\right]$$

$$= 1$$

Writing Numbers In Expanded Form Using Exponents

We know how to write numbers in expanded form. For example, we can write the number 23456 in expanded form as

23456 = 2 × 10000 + 3 × 1000 + 4 × 100 + 5 × 10 + 6

We might sometimes make mistakes while putting the number of zeroes in the multiples of 10. This is where the concept of exponents comes to our rescue and reduces our chances of making mistakes. Let us see how.

Note that we can write 10000 as 10^4, 1000 as 10^3, 100 as 10^2, and 10 as 10^1. Also, digit 6, which occupies the units place, is not multiplied with any multiple of 10.

However, we can write 6 as 6 × 1, which in turn, can be rewritten as 6×10^0.

Thus, we can write the number 23456 in expanded form using exponents as

$23456 = 2 \times 10^4 + 3 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

Similarly, if we have to identify a number from its expanded form, then we have to first write the values of the exponents, and then proceed.

For example, let us find the number, which is expanded as $1 \times 10^4 + 9 \times 10^2 + 5 \times 10^1 + 4 \times 10^0$.

We proceed as

$1 \times 10^4 + 9 \times 10^2 + 5 \times 10^1 + 4 \times 10^0$

$= 1 \times 10000 + 9 \times 100 + 5 \times 10 + 4 \times 1$

$= 10000 + 900 + 50 + 4$

$= 10954$

Thus, the required number is 10954.

Let us look at some more examples to understand the concept better.

Example 1:

Write the number 6504 in expanded form.

Solution:

$6504 = 6000 + 500 + 4$

$= 6 \times 1000 + 5 \times 100 + 4 \times 1$

$= 6 \times 10^3 + 5 \times 10^2 + 4 \times 10^0$

Thus, the expanded form of the number 6504 is $6 \times 10^3 + 5 \times 10^2 + 4 \times 10^0$.

Example 2:

Which number is expanded as $8 \times 10^5 + 9 \times 10^4 + 6 \times 10^1$?

Solution:

$8 \times 10^5 + 9 \times 10^4 + 6 \times 10^1 = 8 \times 100000 + 9 \times 10000 + 6 \times 10$

$= 800000 + 90000 + 60$

$= 890060$

Thus, the number 890060 can be expanded as $8 \times 10^5 + 9 \times 10^4 + 6 \times 10^1$.

Expressing Large Numbers in Standard Form and Vice-versa

The distance between the sun and Earth is approximately 148800000000 m. The speed of light is 300000000 m/s. Now, can you calculate the time taken by light to reach Earth from the sun?

Since light travels at a speed of 300000000 m/s, it travels 300000000 metres in 1 second. Therefore, to calculate the time required by light to reach Earth from the sun, we have to divide the distance between the sun and Earth by the speed of light.

∴ Time taken by light to reach Earth from sun $= \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{148800000000 \text{ m/s}}{300000000 \text{ m}} = 496 \text{ s}$$

$$= \frac{496}{60} \text{ minutes} = 8.3 \text{ minutes (approx.)}$$

This calculation involved two very large numbers. In such cases, we might make mistakes while writing the numbers. If we mistakenly add or subtract even one 0 from any of the numbers, then it will drastically affect our answer.

For example, the speed of light has eight 0s after the digit 3. By mistake, if we write seven 0s after the digit 3, then let us see what the answer will be.

Time taken by light to reach Earth from sun $= \frac{148800000000 \text{ m/s}}{30000000 \text{ m}} = 4960 \text{ s}$

$$= \frac{4960}{60} \text{ minutes} = 83 \text{ minutes (approx.)}$$

Now, note the difference between the two answers i.e., 8.3 minutes and 83 minutes. In order to avoid these mistakes, it is best to first convert such large numbers into their standard forms.

The rule to write a number in its standard form is

Place a decimal point after the first digit from the left in the number.
Count the number of digits in the number after the decimal point and remove all the zeroes, if any, which appear at the end of the number.
This number of digits equals the exponent of 10 that has to be multiplied with the decimal.

Using this rule, let us try and express the number 3400000 in standard form.

We start by placing a decimal point after the first digit from the left i.e., 3. Now, there are 6 digits after the decimal point (one 4 and five 0s). Therefore, the exponent of 10 will be 6.

Thus, the number 3400000 can be written in standard form as 3.4×10^6.

Similarly, we can express the numbers 2943 and 50345 in standard form as

$$2943 = 2.943 \times 1000 = 2.943 \times 10^3$$
$$50345 = 5.0345 \times 10000 = 5.0345 \times 10^4$$

Note that when a number is written in standard form, its decimal part is greater than or equal to 1.0 and lesser than 10.0.

The numbers 2943 and 50345 can also be expressed as

$2943 = 29.43 \times 10^2$

$2943 = 294.3 \times 10^1$

$50345 = 50.345 \times 10^3$

$50345 = 0.50345 \times 10^5$

But in each case, the decimal number does not lie between 1.0 and 10.0 (including 1.0). Thus, none of these expressions is the correct standard form of the corresponding number.

Let us again consider the example that we discussed in the beginning. We can now express the distance between the sun and Earth, and the speed of light in standard form as

$148800000000 \text{ m} = 1.488 \times 10^{11} \text{ m}$

$300000000 \text{ m/s} = 3.0 \times 10^8 \text{ m/s}$

Now, let us again find the time taken by light to reach Earth from the sun.

$$\text{Time required} = \frac{1.488 \times 10^{11}\ \text{m/s}}{3.0 \times 10^{8}\ \text{m}}$$

$$= \frac{1.488}{3.0} \times 10^{11-8}\ \text{s} \qquad \left(\frac{a^p}{a^q} = a^{p-q}\right)$$

$$= 0.496 \times 10^{3}\ \text{s}$$

$$= 496\ \text{s}$$

$$= 8.3\ \text{minutes (approx.)}$$

In this way, we minimised the chances of making errors in our calculations. You will encounter various large numbers in Science in higher grades. For this reason, they will often be mentioned in standard form. Standard form is also referred as the scientific notation.

Now, what if we are required to identify a number from its standard form? The rule for this is

First write the exponent of 10 as a number and then multiply it with the decimal part to obtain the required number.

For example, let us consider the standard form 9.005×10^6.

$9.005 \times 10^6 = 9.005 \times 1000000 = 9005000$

Similarly, $3.68 \times 10^4 = 3.68 \times 10000 = 36800$

Extremely big quantities can be expressed as **Mega** meaning 10^6, **Giga** means 10^9 and **Tera** means 10^{12}.

Let us solve some more examples to understand the concept better.

Example 1:

Standard form of a number is 5.67×10^6. Write the number in its normal form.

Solution:

5.67×10^6

$= 5.67 \times 1000000$

$= 5670000$

Thus, 5.67×10^6 is the standard form of number 5670000.

Example 2:

Express the number 643.29×10^3 in standard form.

Solution:

$643.29 \times 10^3 = 643.29 \times 1000$

$= 643290$

$= 6.4329 \times 100000$

$= 6.4329 \times 10^5$

Example 3:

The length and breath of a rectangular paddy field are 150000 cm and 60000 cm respectively. Convert these dimensions into their standard forms and use them to find the area of the paddy field and express it in standard form.

Solution:

Length of the paddy field = 150000 cm = 1.5×10^5 cm

Breadth of the paddy field = 60000 cm = 6.0×10^4 cm

Now, area of the rectangular field = length × breadth

∴ Required area $= \left(1.5\times10^5 \text{ cm}\right)\times\left(6.0\times10^4 \text{ cm}\right)$

$= \left(1.5\times6.0\times10^{5+4}\right) \text{ cm}^2 \qquad \left\{a^p \times a^q = a^{p+q}\right\}$

$= 9\times10^9 \text{ cm}^2$

Example 4:

Express the number represented by $4 \times 10^4 + 3 \times 10^3 + 5 \times 10^1 + 8 \times 10^0$ in standard form.

Solution:

$4 \times 10^4 + 3 \times 10^3 + 5 \times 10^1 + 8 \times 10^0 = 4 \times 10000 + 3 \times 1000 + 5 \times 10 + 8 \times 1$

$= 40000 + 3000 + 50 + 8$

$= 43058$

$= 4.3058 \times 10000$

$= 4.3058 \times 10^4$