1 MARK

1. Simplify sin (x – y) cos y + cos (x – y) siny

(1) 1 (2) sin x

(3) cos x (4) sinx $\cos^2 y$

2. If sin2x sin3x = cos2x cos3x, then one value for x is

(1) 18° (2) 30° (3) 36° (4) 45°

3. $\dfrac{\sin 10° + \sin 20°}{\cos 10° + \cos 20°}$ equals

(1) tan10°+ tan20° (2) tan30°

(3) 1/2 (tan10°+ tan20°) (4) tan15°

4. If A = 20° and E = 25°, then the value of (1 + tan A) (1 + tan E) is

(1) $\sqrt{3}$ (2) 2

(3) $1+\sqrt{2}$ (4) 2(tanA + tanB)

5. If θ is an acute angle and $\sin\frac{1}{2}\theta = \sqrt{\frac{x-1}{2x}}$, then tanθ equals -

(1) x (2) 1/x

(3) $\dfrac{\sqrt{x-1}}{x+1}$ (4) $\sqrt{x^2-1}$

6. Let x = cos36° – cos72°. Then x equals -

(1) 4/3 (2) 1/2 (3) $3-\sqrt{6}$ (4) $2\sqrt{3}-3$

7. If θ is an acute angle and sin2θ = a, then sinθ + cosθ equals

(1) $\sqrt{a+1}$ (2) $\left(\sqrt{2}-1\right)a+1$

(3) $\sqrt{a+1}-\sqrt{a^2-a}$ (4) $\sqrt{a+1}+\sqrt{a^2-a}$

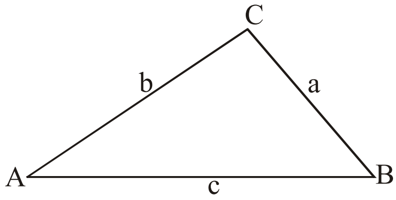
8. If b > 1, sin x > 0. cos x > 0 and $\log_b \sin x = a$, then $\log_b \cos x$ equals :-

(1) $2\log_b(1-b^{a/2})$ (2) $\sqrt{1-a^2}$

(3) b^{a^2} (4) $\frac{1}{2}\log_b(1-b^{2a})$

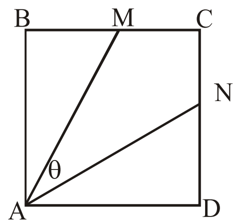
2 MARKS

1. In ΔABC, we have $\angle C = 3\angle A$, a = 27 and c = 48. What is b ?



(1) 33 (2) 35 (3) 37 (4) 39

2. ABCD is a square and M and N are the midpoints of BC and CD respectively. Then sin θ =



(1) $\dfrac{\sqrt{5}}{5}$ (2) $\dfrac{3}{5}$

(3) $\dfrac{\sqrt{10}}{5}$ (4) $\dfrac{4}{5}$

3. If sin x = 3 cos x then what is sin x cos x ?

(1) 1/6 (2) 1/5 (3) 2/9 (4) 3/10

4. Line segments drawn from the vertex opposite the hypotenuse of a right triangle to the point trisection the hypotenuse have lengths sin x and cos x, where x is a real number such that $0 < x < \frac{\pi}{2}$. The length of the hypotenuse is :-

(1) $\dfrac{4}{3}$ (2) $\dfrac{3}{2}$ (3) $\dfrac{3\sqrt{5}}{5}$ (4) $\dfrac{2\sqrt{5}}{3}$

5. If θ is a constant such that 0 < θ < π and $x + \frac{1}{x} = 2\cos\theta$ then for each positive integer n. $x^n + \frac{1}{x^n}$ equals

(1) 2cosθ (2) $2^n\cos\theta$

(3) $2\cos^n\theta$ (4) 2cosnθ

1 MARK

1. Ans. (2)

$\because$ sinAcosB + cosAsinB=sin(A+B)

$\Rightarrow$ sin(x–y)cosy + cos(x–y)siny=sin(x–y+y)

= sinx

2. Ans. (1)

$\Rightarrow$ cos2x·cos3x – sin2x·sin3x = 0

$\Rightarrow$ cos(2x + 3x) = 0

$\Rightarrow$ cos5x = 0

$5x = (2n+1)\frac{\pi}{2}$

For n = 0

$5x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{10} = 18°$

3. Ans. (4)

$$\frac{\sin 10° + \sin 20°}{\cos 10° + \cos 20°} = \frac{2\sin 15° \cos 5°}{2\cos 15° \cos 5°}$$

= tan15°

4. Ans. (2)

$$\tan(A+E) = \frac{\tan A + \tan E}{1 - \tan A \cdot \tan E}$$

$$\Rightarrow \tan(20°+25°) = \frac{\tan A + \tan E}{1 - \tan A \cdot \tan E}$$

$$\Rightarrow \tan 45° = 1 = \frac{\tan A + \tan E}{1 - \tan A \cdot \tan E}$$

$\Rightarrow$ 1 – tanA·tanE = tanA + tanE

$\Rightarrow$ 1 = tanA + tanE + tanA·tanE

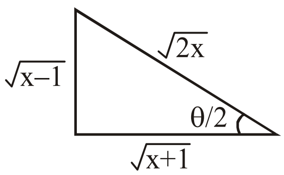
Adding 1 on both sides:

$\Rightarrow$ 1+1=(1+tanA) + tanE(1+tanA)

$\Rightarrow$ 2 = (1+tanA)·(1+tanE)

5. Ans. (4)

Given $\sin\frac{\theta}{2} = \frac{\sqrt{x-1}}{\sqrt{2x}}$



$$\Rightarrow \tan\frac{\theta}{2} = \frac{\sqrt{x-1}}{\sqrt{x+1}}$$

$$\Rightarrow \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{2\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)}{1-\left(\frac{x-1}{x+1}\right)}$$

$$\Rightarrow \tan\theta = \frac{2\sqrt{x-1}}{(x+1)-(x-1)} \cdot \frac{x+1}{\sqrt{x+1}}$$

$$\Rightarrow \tan\theta = \frac{2\sqrt{x-1}\cdot\sqrt{x+1}}{2}$$

$$\Rightarrow \tan\theta = \sqrt{x^2-1}$$

6. Ans. (2)

x = cos36° – cos72°

$$x = -2\sin\left(\frac{36°+72°}{2}\right)\sin\left(\frac{36°-72°}{2}\right)$$

$$x = -2\sin\left(\frac{108°}{2}\right)\sin\left(\frac{-36°}{2}\right)$$

$\Rightarrow$ x = +2sin54°sin18°

$$x = 2\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)$$

$$\Rightarrow x = \frac{5-1}{8} = \frac{1}{2}$$

7. Ans. (1)

Let x = sinθ + cosθ

$\Rightarrow x^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta$

$\Rightarrow x^2 = 1 + \sin 2\theta = 1 + a$

$\Rightarrow x = \sqrt{a+1}$

8. Ans. (4)

$\log_b \sin x = a$

$\Rightarrow \sin x = b^a \Rightarrow \sin^2 x = b^{2a}$

$\Rightarrow 1 - \cos^2 x = b^{2a}$

$\Rightarrow \cos^2 x = 1 - b^{2a}$

taking log with base b on both sides:

$\Rightarrow \log_b \cos^2 x = \log_b(1-b^{2a})$

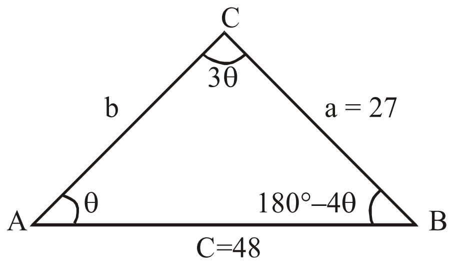
$\Rightarrow 2\log_b \cdot \cos x = \log_b(1-b^{2a})$

$$\Rightarrow \log_b \cos x = \frac{1}{2}\log_b(1 - b^{2a})$$

2 MARK

1. Ans. (2)

Let $\angle A=\theta \Rightarrow \angle C = 3\angle A = 3\theta$



Applying sine rule :

$$\frac{27}{\sin\theta}=\frac{48}{\sin 3\theta}=\frac{b}{\sin(180^\circ-4\theta)}$$

$\Rightarrow$ $27\sin 3\theta = 48\sin\theta$

$\Rightarrow$ $9(3\sin\theta - 4\sin^3\theta) = 16\sin\theta$

$\Rightarrow$ $9(3-4\sin^2\theta) = 16 \Rightarrow 27 - 36\sin^2\theta = 16$

$\Rightarrow$ $36\sin^2\theta = 27 - 16 = 11$

$$\Rightarrow \sin^2\theta=\frac{11}{36}$$

Also $\dfrac{27}{\sin\theta}=\dfrac{b}{\sin(180^\circ-4\theta)}$

$$\Rightarrow \frac{27}{\sin\theta}=\frac{b}{\sin 4\theta}$$

$$\Rightarrow \frac{27}{\sin\theta}=\frac{b}{2\sin 2\theta\cos 2\theta}$$

$$\Rightarrow \frac{27}{\sin\theta}=\frac{b}{2(2\sin\theta\cdot\cos\theta)(1-2\sin^2\theta)}$$

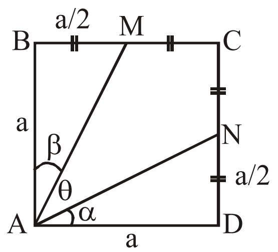
$\Rightarrow$ $b = 4.27\cos\theta(1 - 2\sin^2\theta)$

$$\left(\because \sin\theta=\frac{\sqrt{11}}{6}\Rightarrow \cos\theta=\sqrt{1-\frac{11}{36}}=\frac{5}{6}\right)$$

$$\Rightarrow b = 4\cdot(27)\frac{5}{6}\left(1-2.\frac{11}{36}\right)$$

$\Rightarrow$ $b = 35$

2. Ans. (2)



$$\tan\alpha=\frac{\left(a/2\right)}{a}=\frac{1}{2} \quad \text{and} \quad \tan\beta=\frac{\left(a/2\right)}{a}=\frac{1}{2}$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}=\frac{\frac{1}{2}+\frac{1}{2}}{1-\frac{1}{2}\cdot\frac{1}{2}}$$

$$\tan(\alpha+\beta)=\frac{1}{\frac{3}{4}}=\frac{4}{3}$$

$$\Rightarrow \alpha+\beta=\tan^{-1}\left(\frac{4}{3}\right)$$

$\theta = 90^\circ - (\alpha + \beta)$

$$\theta = 90^\circ - \tan^{-1}\left(\frac{4}{3}\right)$$

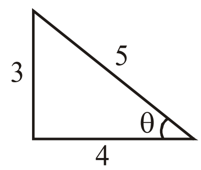
taking tan on both sides :

$$\tan\theta=\tan\left(90^\circ-\tan^{-1}\left(4/3\right)\right)$$

$$\tan\theta=\cot\left(\tan^{-1}\left(4/3\right)\right)$$

$$\left(\because \tan(90^\circ-x)=\cot x\right)$$

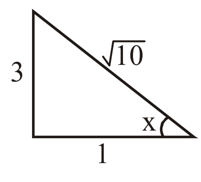
$$\Rightarrow \tan\theta=\cot\left(\cot^{-1}\left(3/4\right)\right)=\frac{3}{4}$$



$\Rightarrow$ $\boxed{\sin\theta=\frac{3}{5}}$

3. Ans. (4)

$\sin x = 3\cos x \Rightarrow \tan x = 3$

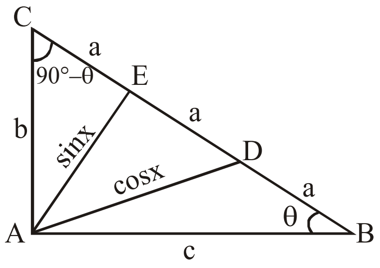


$$\sin x=\frac{3}{\sqrt{10}}$$

$$\cos x=\frac{1}{\sqrt{10}}$$

$$\sin x\cdot\cos x = \frac{3}{\sqrt{10}}\cdot\frac{1}{\sqrt{10}}=\frac{3}{10}$$

4. **Ans. (3)**



In ΔABC :

$$b^2 + c^2 = (3a)^2 \Rightarrow b^2 + c^2 = 9a^2 \quad ...(1)$$

Applying cosine rule in triangle ACE :

$$\cos(90° - \theta) = \frac{b^2 + a^2 - \sin^2 x}{2ab}$$

$$b\cos(90° - \theta) = \frac{b^2 + a^2 - \sin^2 x}{2a} \quad ...(2)$$

Applying cosine rule in triangle ABD :-

$$\cos\theta = \frac{a^2 + c^2 - \cos^2 x}{2ac}$$

$$\Rightarrow \quad c\cos\theta = \frac{a^2 + c^2 - \cos^2 x}{2a} \quad ...(3)$$

Applying projection formula on triangle ABC:

$3a = b \cos C + \cos B$

From (2) and (3)

$$\Rightarrow \quad 3a = \frac{b^2 + a^2 - \sin^2 x}{2a} + \frac{a^2 + c^2 - \cos^2 x}{2a}$$

$\Rightarrow$ $(3a)(2a) = (b^2+c^2) + 2a^2 - 1$

($\because$ $b^2 + c^2 = 9a^2$)

$\Rightarrow$ $6a^2 = 9a^2 + 2a^2 - 1 \Rightarrow 5a^2 = 1$

$$a = \frac{1}{\sqrt{5}}$$

$\Rightarrow$ Hypotenuse = $3a = \dfrac{3}{\sqrt{5}}$

5. **Ans. (4)**

$0 < \theta < \pi$

$$\left(x + \frac{1}{x}\right) = (2\cos\theta) \quad ...(1)$$

One squaring both sides :

$$\left(x + \frac{1}{x}\right)^2 = (2\cos\theta)^2$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = 4\cos^2\theta$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 2(2\cos^2\theta - 1)$$

($\because$ $2\cos^2\theta - 1 = \cos 2\theta$)

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 2\cos 2\theta$$

On cubing equation (1)

$$\Rightarrow \quad \left(x + \frac{1}{x}\right)^3 = (2\cos\theta)^3$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right) = 8\cos^3\theta$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3(2\cos\theta) = 8\cos^3\theta$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta)$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 2\cos 3\theta$$

Similarly $\quad x^n + \dfrac{1}{x^n} = 2\cos n\theta$