Chapter 6

System of Particles and Rotational Motion

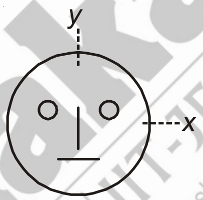
Solutions

SECTION - A

Objective Type Questions (One option is correct)

[Dynamics of Rotational Motion]

1. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is $6m$. The coordinates of the centres of the different parts are : outer circle (0, 0), left inner circle ($-a$, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, $-a$). The y-coordinate of the centre of mass of the ink in this drawing is

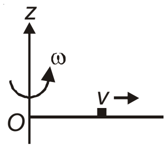


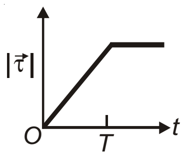
(1) $\frac{a}{10}$ (2) $\frac{a}{8}$ (3) $\frac{a}{12}$ (4) $\frac{a}{3}$

Sol. Answer (1)

$$y_{\text{cm}} = \frac{2ma - ma}{10m} = \frac{a}{10}$$

2. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot?



(1)


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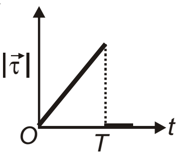

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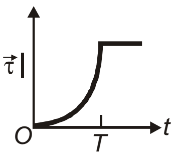

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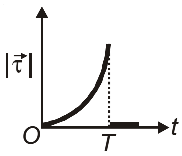


Sol. Answer (2)

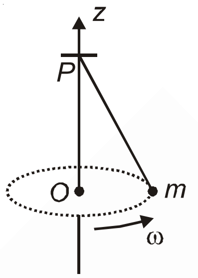
$$\tau = \frac{d}{dt}(I\omega) = \frac{d}{dt}\left(\frac{ML^2}{3} + mx^2\right)\omega = 2mx\frac{dx}{dt}\omega$$

Now, $x = vt$

$\Rightarrow \quad \tau \propto t$

Finally torque becomes zero.

3. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



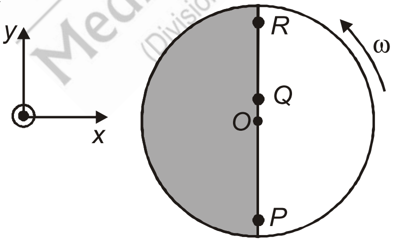
(1) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time
(2) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant
(3) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time
(4) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

Sol. Answer (3)

The torque about O is zero, but about P is non-zero.

As $\vec{\tau} = \frac{d\vec{L}}{dt}$ thus $\vec{L}_P$ must be a variable.

4. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then



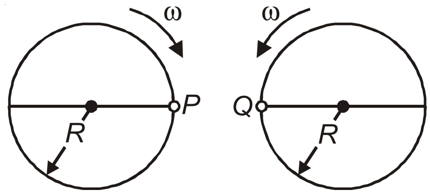
(1) P lands in the shaded region and Q in the unshaded region
(2) P lands in the unshaded region and Q in the shaded region
(3) Both P and Q land in the unshaded region
(4) Both P and Q land in the shaded region

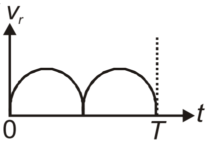
Sol. Answer (3)

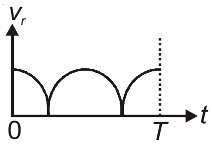
Horizontal component (towards right), of the velocity of P at any time t will always be greater than that of any point on the disc below it. Therefore it will land in unshaded region.

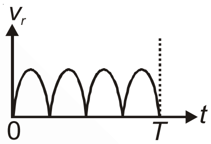
Similarly, horizontal component towards left of Q is always less than that of the point below it and it will also land in unshaded region.

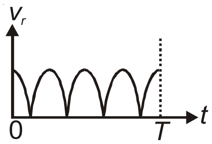
5. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by



(1) 

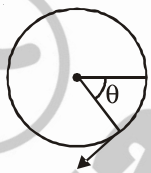
(2) 

(3) 

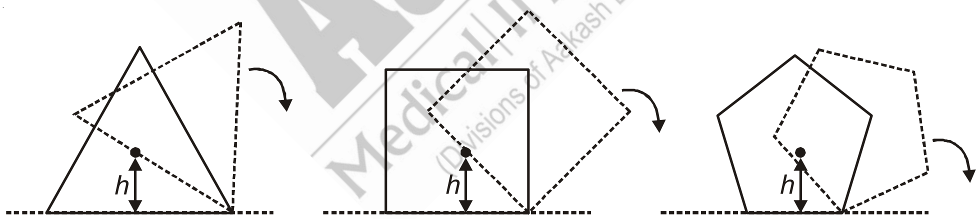
(4) 

Sol. Answer (1)

$v_r = 2v|\sin\theta|$



6. Consider regular polygons with number of sides n = 3, 4, 5.... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as



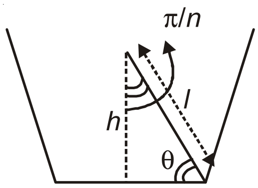
(1) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

(2) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

(3) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

(4) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

Sol. Answer (4)



$\theta = \frac{\pi}{2} - \frac{\pi}{n}$

$\sin\theta = \cos\frac{\pi}{n}$

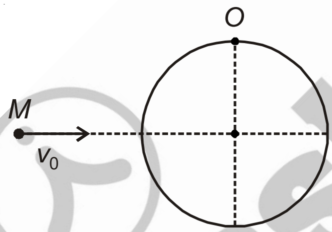
$l = \frac{h}{\sin\theta}$

$\Delta = l - h$

$= h\left[\frac{1}{\sin\theta} - 1\right]$

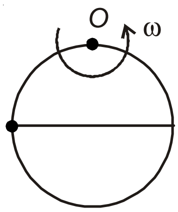
$= h\left[\frac{1}{\cos\frac{\pi}{n}} - 1\right]$

7. Uniform disc of mass M and radius R can rotate about an axes perpendicular to plane of disc passing through O. A point mass M moving with velocity v_0 collides with disc as shown. If, collision is perfectly inelastic then angular speed of disc just after collision will be



(1) $\frac{v_0}{7R}$ (2) $\left(\frac{2v_0}{R}\right)$ (3) $\left(\frac{v_0}{R}\right)$ (4) $\left(\frac{2v_0}{7R}\right)$

Sol. Answer (4)



$\overrightarrow{L_i^o} = \overrightarrow{L_f^o}$

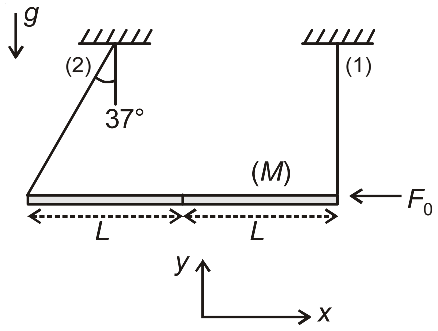
$Mv_0R(+\hat{k}) = I\omega(+\hat{k})$

$I = \frac{3}{2}MR^2 + 2MR^2 = \frac{7}{2}MR^2$

$Mv_0R = \left(\frac{7}{2}MR^2\right)\omega$

$\Rightarrow \omega = \left(\frac{2v_0}{7R}\right)$

8. Uniform rod of mass M, length $2L$ is suspended with the help of two ideal strings as shown. Complete arrangement is in equilibrium in verticle x-y plane. Find value of F_0 so that string 1 remains vertical (assuming rod is horizontal).



(1) $\frac{3Mg}{10}$ (2) $\frac{3Mg}{8}$ (3) $\frac{Mg}{2}$ (4) $\frac{2Mg}{5}$

Sol. Answer (2)

$F_0 = \left(\frac{3T_2}{5}\right)$ ...(i)

$T_1 + \frac{4T_2}{5} = Mg$ ...(ii)

$\vec{\tau}_A = 0$

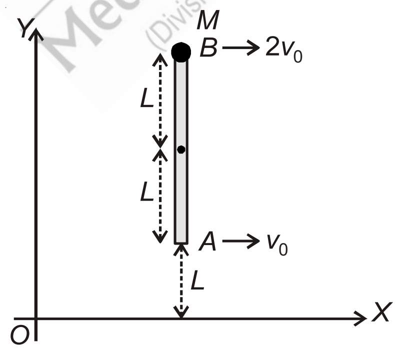
$\Rightarrow$ $(Mg)L = T_1 \times 2L$

$\Rightarrow$ $T_1 = \left(\frac{Mg}{2}\right)$ ...(iii)

$\Rightarrow$ From (i), (ii) and (iii)

$F_0 = \left(\frac{3Mg}{8}\right)$

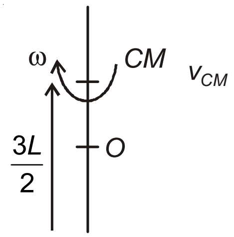
9. Ends A and B of a rigid uniform rod of mass M parallel to Y-axis have velocities $\overrightarrow{v_A} = v_0\hat{i}, \overrightarrow{v_B} = 2v_0\hat{i}$ as shown. A point mass M is also attached at the end B of rod, then the angular momentum of system about point O will be



(1) $\frac{55}{6}Mv_0L(-\hat{k})$ (2) $\frac{55}{3}Mv_0L(-\hat{k})$

(3) $\frac{49}{6}Mv_0L(-\hat{k})$ (4) $\frac{35}{3}Mv_0L(-\hat{k})$

Sol. Answer (1)



$\omega = \left(\frac{v_0}{2L}\right)$

$v_{CM} = \left(\frac{7v_0}{4}\right)$

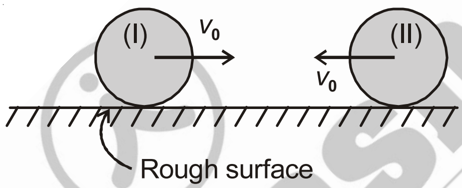
$\overrightarrow{L_0} = I_{CM}\omega(-\hat{k}) + 2Mv_{CM}\left(\frac{5L}{2}\right)(-\hat{k})$

$I_{cm} = \frac{4ML^2}{12} + (M)\left(\frac{L^2}{4}\right) + M\left(\frac{L^2}{4}\right)$

$= \frac{5}{6}ML^2$

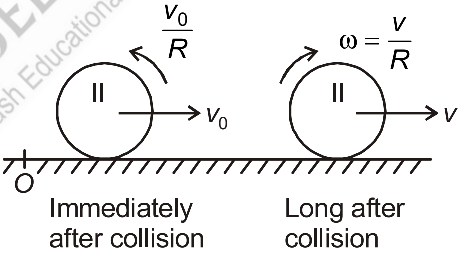
$\Rightarrow \; L_0 = \frac{55}{6}Mv_0L$

10. Two identical solid spheres are rolling without slipping with velocity v_0 over a rough horizontal surface collides elastically as shown. The final speed of centre of mass of sphere (II) after sufficient long time will be



(1) v_0 (2) $\frac{v_0}{2}$ (3) $\frac{3v_0}{7}$ (4) $\frac{4v_0}{7}$

Sol. Answer (3)



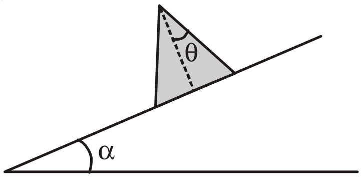
$\overrightarrow{L_i^o} = \overrightarrow{L_f^o}$

$Mv_0R(-\hat{k}) + \left(\frac{2}{5}MR^2\right)\left(\frac{v_0}{R}\right)(+\hat{k})$

$= MvR(-\hat{k}) + \frac{2}{5}MR^2\left(\frac{v}{R}\right)(-\hat{k})$

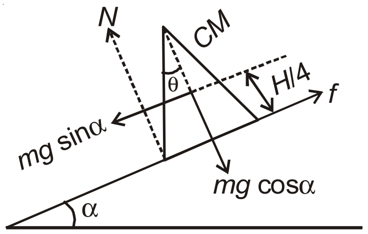
$\Rightarrow \; v = \frac{3}{7}v_0$

11. A right circular cone of semi-vertex angle θ is resting on a rough inclined plane. As angle of inclination α of inclined plane is increased gradually, then cone topples before it slides. Coefficient of friction (μ) follows the relation given by



(1) $\mu > 4\tan\theta$ (2) $\mu < 4\tan\theta$ (3) $\mu = \frac{4}{3}\tan\theta$ (4) $\mu = \frac{2}{3}\tan\theta$

Sol. Answer (1)



Condition of no sliding

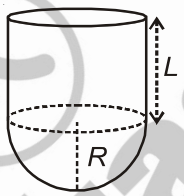
$\mu > \tan\alpha$

Condition of toppling

$(mg \sin\alpha)\dfrac{H}{4} > (mg \cos\alpha)H \tan\theta$

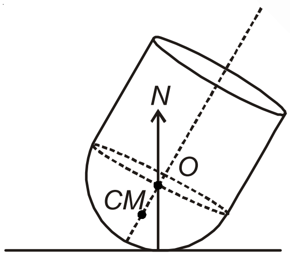
$\tan\alpha > 4\tan\theta$

12. Consider a rigid body which consist of uniform hemispherical shell of radius R and uniform cylindrical shell of height L and radius R (material of hemispherical shell and cylindrical shell are same). Find maximum value of L so that complete rigid body remains in stable equilibrium in the position shown (neglect friction).



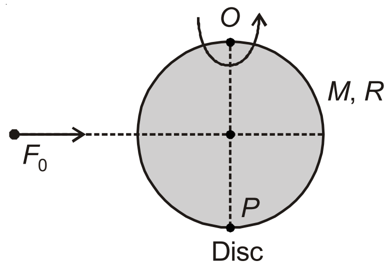
(1) $\dfrac{3R}{2}$ (2) R (3) $\dfrac{R}{3}$ (4) $4R$

Sol. Answer (2)



For stable equilibrium CM of combined body must be below point O, when length of cylinder is increased then CM of combined system will shift towards point O and for stable equilibrium must not cross point O.

13. Horizontal force F_0 is applied on a uniform circular disc of mass M and radius R which can rotate in vertical x–y plane about an axes passing through O as shown. What is the acceleration of bottom most point P on the disc at this instant?



(1) $\frac{F_0}{M}$ (2) $\frac{4F_0}{3M}$ (3) $\frac{2F_0}{3M}$ (4) $\frac{F_0}{3M}$

Sol. Answer (2)

$\tau_0 = I_0\alpha$

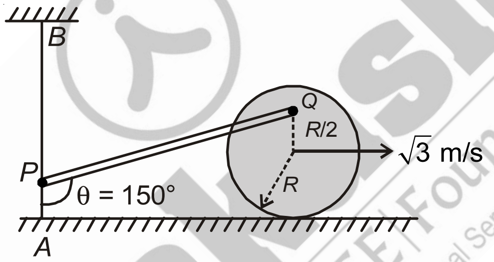
$\Rightarrow (F_0)R = \left(\frac{3}{2}MR^2\right)\alpha$

$\Rightarrow \alpha = \left(\frac{2F_0}{3MR}\right)$

$a_P = 2R\alpha$

$\Rightarrow a_P = (2R)\left(\frac{2F_0}{3MR}\right) = \left(\frac{4F_0}{3M}\right)$

14. Consider the shown diagram. A fixed vertical smooth rod AB is kept in front of a uniform disc of mass m and radius R which is rolling without slipping on a rough horizontal surface. Velocity of centre of disc is $\sqrt{3}$ m/s. A rod PQ of length 2 m is connected with disc at Q, (point Q is at a vertical distance of $\frac{R}{2}$ from centre of disc) by pin joint and other end of rod PQ can freely slide over smooth vertical rod AB. At this instant rod PQ makes an angle $\theta = 150°$ with rod AB as shown. Find the angular velocity of rod at this instant.



(1) $\frac{3}{2}$ rad/s (2) $\frac{1}{2}$ rad/s (3) $\frac{1}{4}$ rad/s (4) $\frac{3}{4}$ rad/s

Sol. Answer (1)

$V_P \cos 30° = \left(\sqrt{3}\times\frac{3}{2}\right)\times\cos 60°$

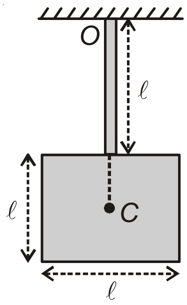
$\Rightarrow V_P = \frac{3}{2}$ m/s

$\therefore \omega = \frac{V_P \sin 30° + V_Q \sin 60°}{\ell}$

$= \frac{\frac{3}{2}\times\frac{1}{2}+\frac{3\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}}{2}$

$= \frac{3}{2}$ rad/s

15. Consider a uniform square plate (mass = 1 kg, side length l = 1 m) which is tied to a uniform rod of same mass and same length l. Whole rigid body can rotate in a vertical plane about an axis parallel to the top edge of the plate as shown. A particle of mass m = 1 kg moving with velocity v_0 collide perpendicularly to the centre of plate. If collision is perfectly inelastic and particle sticks to plate after collision, then find value of v_0 so that after collision centre of mass of the system just reaches the level of the hinge.



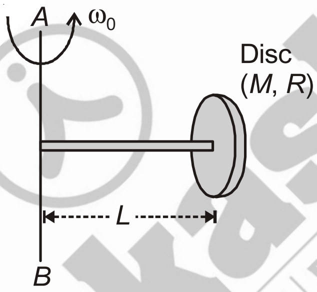
(1) $\sqrt{\frac{213g}{27}}$ (2) $\sqrt{\frac{13g}{27}}$ (3) $\sqrt{\frac{413g}{27}}$ (4) $\sqrt{\frac{23g}{27}}$

Sol. Answer (3)

Apply conservation of angular momentum and conservation of mechanical energy after collision.

16. A uniform circular disc of mass M (= 1 kg) and radius R (= 1 m) is fixed to vertical axis AB with the help of a rod of length L as shown. The disc rolls on a horizontal surface and is free to rotate about its centre. If there is no slipping at the point of contact of disc and horizontal surface then kinetic energy of the disc will be

(given ω_0 = 2 rad/s, L = 1 m)



(1) 3 J (2) 6 J (3) $\left(\frac{7}{2}\right)$ J (4) $\left(\frac{5}{2}\right)$ J

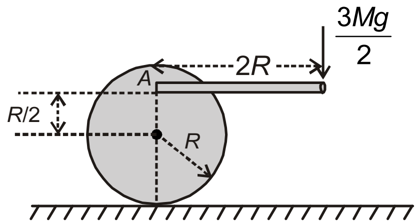
Sol. Answer (1)

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$$

$$\omega = \frac{\omega_0 L}{R}$$

$$KE = \frac{3}{4}M\omega_0^2L^2 = 3\text{ J}$$

17. A uniform circular disc of mass M and radius R is placed on a rough horizontal surface. A light rigid rod of length $2R$ is fixed to the disc at point A as shown in figure, and force $\frac{3Mg}{2}$ is applied at the other end. If μ_{min} is the minimum value of coefficient of friction between disc and horizontal surface to just start pure rolling, then



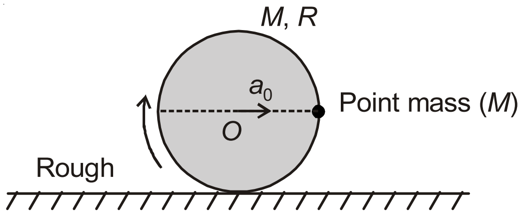
(1) μ_{min} = 0.2 (2) μ_{min} = 0.8 (3) μ_{min} = 0.5 (4) μ_{min} = 0.6

Sol. Answer (2)

For minimum μ, friction must be at its maximum value

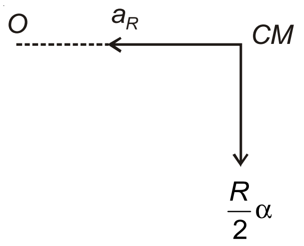
$\mu_{min} = 0.8$

18. A uniform circular disc of mass M and radius R is having a point mass M at its circumference is performing rolling without slipping over a rough horizontal surface as shown. Acceleration of point O is a_0 towards right. If N is the normal reaction on disc by the horizontal surface at this instant then



(1) $N = 2Mg$ (2) $N = Mg$ (3) $N < 2Mg$ (4) $N > 2Mg$

Sol. Answer (3)



a_{cm} is vertically downwards, hence $N < 2mg$

SECTION - B

Objective Type Questions (More than one options are correct)

1. A block of mass m is connected to another block of mass M by a massless spring of spring constant K. Blocks are kept on a smooth horizontal plane. Initially both the blocks are at rest and spring is unstressed. Force F is applied on M to right as shown in the figure then which of the following is correct?

(1) Acceleration of the centre of mass is $\dfrac{F}{M+m}$

(2) At the instant of maximum elongation of the spring, acceleration of centre of mass is zero.

(3) At maximum extension of the spring acceleration of centre of mass is $\dfrac{F}{(M+m)}$

(4) At maximum extension, velocity of both the blocks will be same

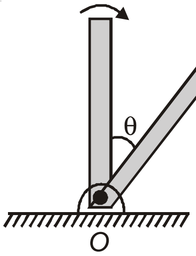
Sol. Answer (1, 3, 4)

Irrespective of what is the extension or compression in the spring, as net external force is F, acceleration of centre of mass is $\dfrac{F}{M+m}$.

At maximum extension, relative velocity between the blocks is zero or velocity of both the blocks is same. Net external force = F at maximum extension

$\Rightarrow \quad a = \dfrac{F}{M+m}$

2. A rod of mass m and length L is pivoted at the bottommost point O and can freely rotate about the point O. The rod is disturbed from the vertical position so that it starts rotating about O. When it makes an angle θ with the vertical



(1) The angular acceleration of rod will be $\frac{3}{2L} g \sin\theta$

(2) Acceleration of centre of mass of rod can be in vertical direction

(3) Net acceleration of centre of mass of rod can't be in horizontal direction

(4) The tangential acceleration of centre of mass of rod will be $\frac{3}{2} g \sin\theta$

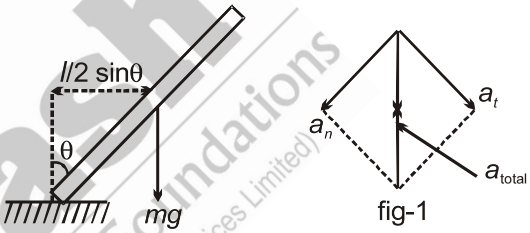
Sol. Answer (1, 2, 3)

Net Torque = $mg\frac{l}{2}\sin\theta$

$= I \times \alpha$

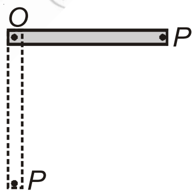
$$\alpha = \frac{mgl\frac{\sin\theta}{2}}{\frac{ml^2}{3}} = \frac{3g\sin\theta}{2l}$$

$$a_t = \alpha \times \frac{l}{2} = \frac{3g\sin\theta}{4}$$



from figure -1 we can see that acceleration can never be in the horizontal direction but it may be in vertical direction for a particular value of θ.

3. A rod of mass m and length L is pivoted at a point O and kept in horizontal position as shown in figure. Now it is released from this position so that it can rotate freely about the point O in downward direction. When it becomes vertical



(1) Angular velocity of rod will be $\sqrt{\frac{6g}{L}}$

(2) Angular velocity of rod will be $\sqrt{\frac{3g}{L}}$

(3) The velocity of point P will be $\sqrt{3gL}$

(4) The velocity of point P will be $\sqrt{6gL}$

Sol. Answer (2, 3)

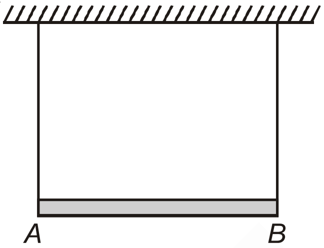
From conservation of energy

$$\frac{1}{2} \times \frac{mL^2}{3} \times \omega^2 = mg \times \frac{L}{2}$$

$$\omega^2 = \frac{3g}{L} \Rightarrow \omega = \sqrt{\frac{3g}{L}}$$

$$v_P = \omega \times L = \sqrt{\frac{3g}{L}} \times L = \sqrt{3gL}$$

4. A horizontal rod of length L is tied to two vertical strings symmetrically as shown in figure. One of the strings at point B is cut at time $t = 0$ and rod starts rotating about other end A in downward direction. Then



(1) At $t = 0$, angular acceleration of rod about A is $\frac{3g}{2L}$

(2) At $t = 0$, angular acceleration of rod about centre of mass of rod is $\frac{3g}{2L}$

(3) At $t = 0$, acceleration of centre of mass of rod is $\frac{3g}{4}$ in downward direction

(4) The tension in the other string through A at $t = 0$ is $\frac{mg}{4}$ (m is the mass of rod)

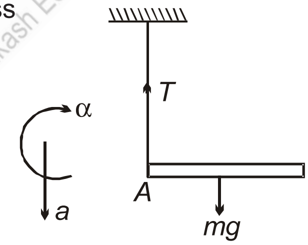
Sol. Answer (1, 2, 3, 4)

Taking torque about point A.

We get, $mg \times \frac{l}{2} = \frac{ml^2}{3} \times \alpha$

$$\alpha = \frac{3g}{2l}$$

when we analyze the motion of rod about its centre of mass



$$a = \frac{mg - T}{m}$$

$$T \times \frac{l}{2} = \frac{ml^2}{12} \times \alpha$$

$$\alpha = \frac{6T}{ml}$$

Since net acceleration of A is zero.

$$a - \alpha \times \frac{l}{2} = 0 \Rightarrow a = \alpha \times \frac{l}{2}$$

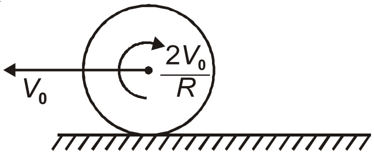
$$\frac{mg - T}{m} = \frac{6T}{ml} \times \frac{l}{2}$$

$$4T = mg \Rightarrow T = \frac{mg}{4}$$

So, $\alpha = \frac{6}{ml} \times \frac{mg}{4} = \frac{3g}{2l}$

$$\Rightarrow \quad a = \alpha \times \frac{l}{2} = \frac{3g}{4}$$

5. A ring of mass m and radius R is placed on a rough horizontal surface (co-efficient of friction μ) with velocity of C.M. V_0 and angular speed $\frac{2V_0}{R}$ as shown in figure. Initially the ring is rolling with slipping but attains pure rolling motion after some time t. Then



(1) After time t, the ring is moving rightward with angular speed $\frac{V_0}{2R}$

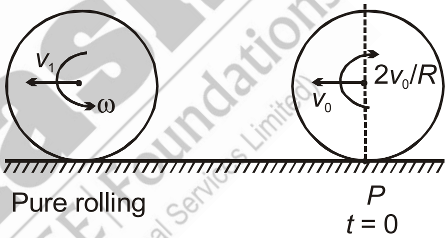
(2) After time t, the ring is moving leftward with angular speed $\frac{3V_0}{2R}$

(3) $t = \frac{V_0}{2\,\mu g}$

(4) $t = \frac{3V_0}{2\,\mu g}$

Sol. Answer (1, 4)

Using angular momentum conservation about point P.



$$mv_0R - mR^2 \times \frac{2v_0}{R} = mv_1 \times R + mR^2 \times \frac{v_1}{R}$$

$$v_1 = -\frac{v_0}{2}$$

Negative sign indicates that velocity is towards right

and $\omega = \frac{v_1}{R} = -\frac{v_0}{2R}$

using $v_1 = v_0 - \mu g \times t$

$$-\frac{v_0}{2} = v_0 - \mu gt \Rightarrow t = \frac{3v_0}{2\mu g}$$

6. A solid sphere of mass m and radius R is released from top of an incline having co-efficient of friction μ and making an angle of 45° with the horizontal. Choose the correct alternative(s).

(1) The force of friction acting on the sphere is $\frac{\mu mg}{\sqrt{2}}$ if $\mu < 0.25$

(2) The force of friction acting on the sphere is $\frac{\sqrt{2}}{7}mg$ if $\mu > 0.3$

(3) Work done by force of friction is zero if $\mu > 0.3$

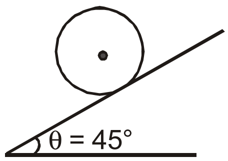
(4) Work done by force of friction is non-zero if $\mu > 0.3$

Sol. Answer (1, 2, 3)

The minimum value of friction and co-efficient of friction for a body to perform rolling over the inclined plane is

$$f_r = \frac{mg\sin\theta}{1+\frac{mR^2}{I}}$$

$$\mu = \frac{\tan\theta}{1+\frac{mR^2}{I}}$$



μ required for solid sphere to perform pure rolling over the inclined plane is,

$$\frac{1}{1+\frac{5}{2}} = \frac{2}{7} \approx 0.286$$

If μ < 0.286 body does not perform pure rolling and the sphere slips. So

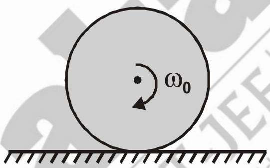
$$f_r = \mu mg\cos\theta = \frac{\mu mg}{\sqrt{2}}$$

If μ > 0.286, body performs pure rolling so, friction is

$$\frac{\frac{mg}{\sqrt{2}}}{1+\frac{mR^2}{2/5mR^2}} = \frac{\sqrt{2}mg}{7}$$

In case of pure rolling contact point does not move so work done by force of friction is zero.

7. A disc of mass m and radius R rotating with angular speed ω_0 is placed on a rough surface (co-efficient of friction = μ). Then



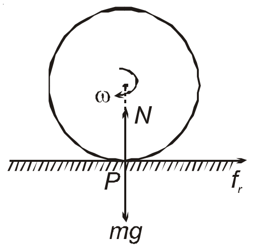
(1) The angular momentum of disc is conserved about centre of disc

(2) The angular momentum of disc is conserved about point of contact of disc

(3) Initially the force of friction acting on the disc is μmg leftward

(4) Initially the force of friction acting on the disc is μmg rightward

Sol. Answer (2, 4)



Since all the forces are passing through the point P so torque of all the forces about P is zero so angular momentum of the disc remains conserved about point P and

$f_r = \mu.N = \mu mg$

8. An uniform rod of length l and mass $2m$ rests on a smooth horizontal table. A point of mass m moving horizontally at right angle to the rod with velocity v collides with one end of the rod and sticks to it, then

(1) Angular velocity of the system after collision is $\dfrac{v}{l}$

(2) Angular velocity of the system after collision is $\dfrac{v}{2l}$

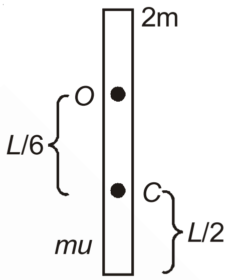
(3) The loss in kinetic energy of whole system in collision is $\dfrac{mv^2}{6}$

(4) The loss in kinetic energy of whole system as in collision is $\dfrac{7mv^2}{24}$

Sol. Answer (1, 3)

Centre of mass of the system is $L/6$ from O.

From conservation of the angular momentum,



$$mv\frac{L}{3}=\left(\frac{2ML^2}{12}+2m\times\frac{L^2}{36}+\frac{ML^2}{9}\right)\omega$$

$$\frac{mvL}{3}=\left(\frac{2}{12}+\frac{2}{36}+\frac{1}{9}\right)mL^2\omega=\left(\frac{6+2+4}{36}\right)\omega$$

$$\Rightarrow\ \frac{v}{3}=\frac{\omega L}{3}$$

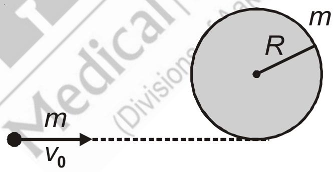
$$\frac{v}{L}=\omega$$

Loss in K.E. $= \dfrac{1}{2}\ mu^2 - \dfrac{1}{2}\ \dfrac{ML^2}{3}\ w^2 - \dfrac{1}{2}3m\dfrac{u^2}{9}$

$$=\frac{1}{2}mu^2\left[1-\frac{1}{3}-\frac{1}{3}\right]$$

$$=\frac{1}{2}mu^2\left[1-\frac{2}{3}\right]=\frac{1}{6}mu^2$$

9. A disc of mass m and radius R is kept on a smooth horizontal surface with its plane parallel to the surface. A particle of same mass m travelling with speed v_0 collides with the stationary disc and gets embedded into it as shown in the figure. Then



(1) Angular momentum of (disc + particle) system can be conserved only about centre of the disc

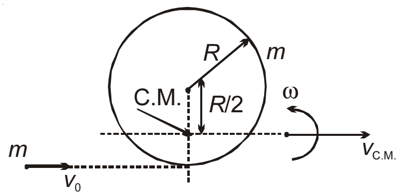
(2) The speed of centre of mass of the system is $\dfrac{v_0}{2}$ after collision

(3) The angular speed of the system after collision is $\dfrac{v_0}{2R}$ about C.M. of the disc + particle system

(4) The K.E. of the system is conserved during the collision

Sol. Answer (2, 3)

Using conservation of linear momentum



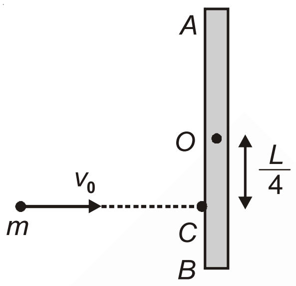
$mv_0 = \{m + m\}\ v_{c.m.}$

$$V_{c.m.}=\frac{V_0}{2}$$

Using conservation of angular momentum about C.M.

$$mv_0 \times \frac{R}{2} = \left\{\frac{mR^2}{2} + m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2\right\}\omega \Rightarrow \omega = \frac{v_0}{2R}$$

10. A rod AB of mass M and length L is kept on a smooth horizontal surface. A particle of mass m moving with speed v_0 collides with the stationary rod (shown in figure) at point C, $\frac{L}{4}$ distance from centre of rod. After collision the particle becomes stationary but rod translates with speed v and rotates anticlockwise with angular speed ω about the centre of the rod O. Following conservation of angular momentum, which of the following statements is correct for the (rod + particle) system?



(1) About point O, $mv_0\frac{L}{4} = \frac{ML^2}{12}.\omega$

(2) About point C, $0 = Mv.\frac{L}{4} - \frac{ML^2}{12}.\omega$

(3) About point A, $mv_0.\frac{3L}{4} = Mv.\frac{L}{2} + \frac{ML^2}{12}.\omega$

(4) About point B, $mv_0.\frac{L}{4} = Mv.L + \frac{ML^2}{12}.\omega$

Sol. Answer (1, 2, 3)

Using conservation of angular momentum about different point.

About- *O*

$$mv_0 \times \frac{L}{4} = \frac{M}{12}l^2 \times \omega$$

About- *C*

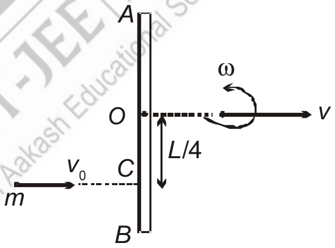
$$O = Mv\frac{L}{4} - \frac{Ml^2}{12} \times \omega$$

About-*A*

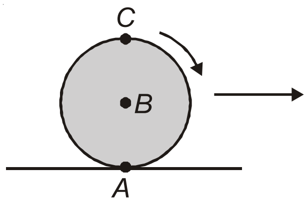
$$mv_0 \times \frac{3l}{4} = Mv \times \frac{L}{2} + \frac{ML^2}{12} \times \omega$$

About-*B*

$$mv_0 \times \frac{L}{4} = Mv \times \frac{L}{2} - \frac{ML^2}{12} \times \omega$$



11. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,



(1) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

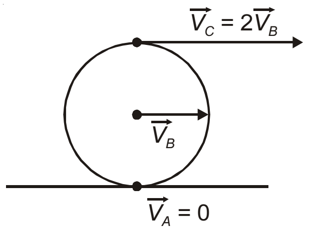
(2) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(3) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(4) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

Sol. Answer (2, 3)

The motion can be considered to be pure rotation about point *A*, velocities will be as shown



$\Rightarrow \quad \vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

and $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

12. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(1) Linear momentum of the system does not change in time

(2) Kinetic energy of the system does not change in time

(3) Angular momentum of the system does not change in time

(4) Potential energy of the system does not change in time

Sol. Answer (1)

According to law of conservation of linear momentum if $\vec{F} = 0$ linear momentum of the system does not change in time.

13. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses?

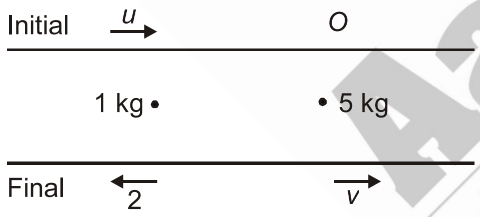
(1) Total momentum of the system is 3 kg ms^{-1}

(2) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(3) Kinetic energy of the centre of mass is 0.75 J

(4) Total kinetic energy of the system is 4 J

Sol. Answer (1, 3)



By conservation of momentum,

$1u = -2 + 5v$

By coefficient of restitution equation,

$$\frac{v+2}{u} = 1$$

Solving we get,

$\boxed{u = 3\text{ m/s}}$ $\quad \boxed{v = 1\text{ m/s}}$

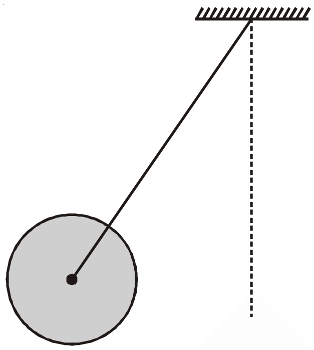
Momentum = 1 × 3 = 3 kg m/s

Momentum of 5 kg block after collision = 5 × 1 = 5 kg ms^{-1}

Kinetic energy of centre of mass $= \frac{1}{2}(1+5)\left(\frac{1\times 3}{6}\right)^2$ = 0.75 J

Total kinetic energy $= \frac{1}{2} \times 1 \times 3^2 = 4.5$ J

14. A metal rod of length '*L*' and mass '*m*' is pivoted at one end. A thin disk of mass '*M*' and radius '*R*' (< *L*) is attached at its center to the free end of the rod. Consider two ways the disc is attached. (case *A*): The disc is not free to rotate about its center and (case *B*): The disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



(1) Restoring torque in case *A* = Restoring torque in case *B*

(2) Restoring torque in case *A* < Restoring torque in case *B*

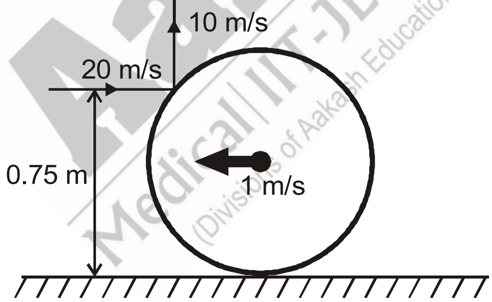
(3) Angular frequency for case *A* > Angular frequency for case *B*

(4) Angular frequency for case *A* < Angular frequency for case *B*

Sol. Answer (1, 4)

$$\omega_B = \sqrt{\frac{mgl}{ml^2}};\omega_A = \sqrt{\frac{mgl}{ml^2+\frac{1}{2}mr^2}}$$

15. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(1) The ring has pure rotation about its stationary CM

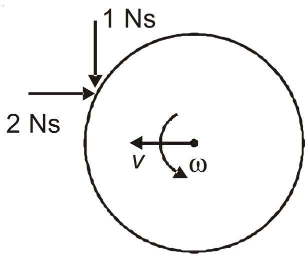
(2) The ring comes to a complete stop

(3) Friction between the ring and the ground is to the left

(4) There is no friction between the ring and the ground

Sol. Answer (1, 3)

The data is incomplete, if we assume that the friction is not impulsive during impact then the solution is as follows



$2 = –2 \times v – (–2 \times 1)$

$\Rightarrow \quad v = 0$

Thus centre of mass becomes stationary.

Taking angular impulse about centre of mass of ring.

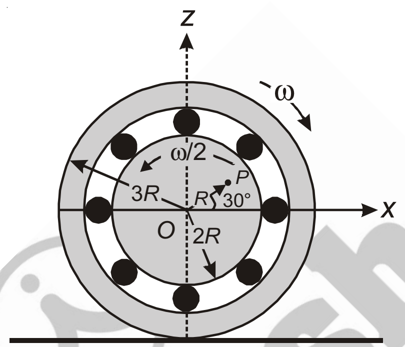
$$1\left(\frac{\sqrt{3}}{2}\times\frac{1}{2}\right)-2\times(0.5)\times\frac{1}{2}=2\times(0.5)^2\left[\omega-\frac{1}{0.5}\right]$$

$$\frac{1.732}{4}-0.5=0.5\omega-1$$

$0.5\omega = 0.5 + 0.433$

$\Rightarrow$ $\omega > 0$ (*i.e.* anticlockwise)

16. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface



(1) The point O has a linear velocity $3R\omega\hat{i}$

(2) The point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(3) The point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(4) The point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

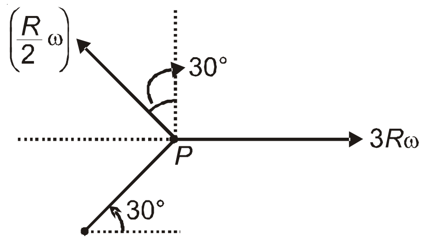
Sol. Answer (1, 2)

As the system is under pure rolling, so

$$V_0=(3R)\omega\hat{i}$$

For point P

$$\vec{V}_P=\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$$



17. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

(1) Both cylinders P and Q reach the ground at the same time

(2) Cylinder P has larger linear acceleration than cylinder Q

(3) Both cylinders reach the ground with same translational kinetic energy

(4) Cylinder Q reaches the ground with larger angular speed

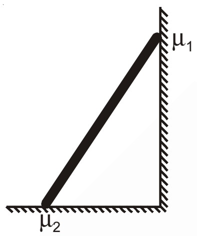
Sol. Answer (4)

$$a_{cm} = \frac{g\sin\theta}{1+\frac{I_{cm}}{mR^2}}$$

$I_{cmP} > I_{cmQ}$

$\Rightarrow$ $a_{cmQ} > a_{cmP}$. Thus Q reaches the ground with larger angular speed.

18. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(1) $\mu_1 = 0$, $\mu_2 \neq 0$ and $N_2\tan\theta = \frac{mg}{2}$

(2) $\mu_1 \neq 0$, $\mu_2 = 0$ and $N_1\tan\theta = \frac{mg}{2}$

(3) $\mu_1 \neq 0$, $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(4) $\mu_1 = 0$, $\mu_2 \neq 0$ and $N_1\tan\theta = \frac{mg}{2}$

Sol. Answer (3, 4)

μ_2 can never be zero for maximum equilibrium.

When $\mu_1 = 0$ we have

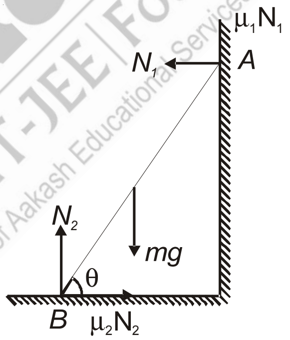
$N_1 = \mu_2 N_2$ .... (i)

$N_2 = m_2 g$ .... (ii)

$\tau_B = 0 \Rightarrow mg\frac{L}{2}\cos\theta = N_1 L\sin\theta$

$\Rightarrow$ $N_1 = \frac{mg\cot\theta}{2}$

$\Rightarrow$ $N_1\tan\theta = \frac{mg}{2}$



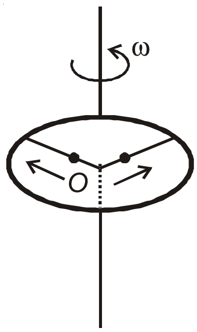
When $\mu_1 \neq 0$ we have

$\mu_1 N_1 + N_2 = mgs$ ... (i)

$\mu_2 N_2 = N_1$ ... (ii)

$\Rightarrow$ $N_2 = \frac{mg}{1+\mu_1\mu_2}$

19. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is



(1) $\frac{2}{3}R$ (2) $\frac{1}{3}R$ (3) $\frac{3}{5}R$ (4) $\frac{4}{5}R$

Sol. Answer (4)

$I_1\omega_1 = I_2\omega_2$

$$\Rightarrow \quad MR^2 \times \omega = \left(MR^2 + \frac{M}{8}\times\frac{9}{25}R^2 + \frac{M}{8}x^2\right)\times\frac{8}{9}\omega$$

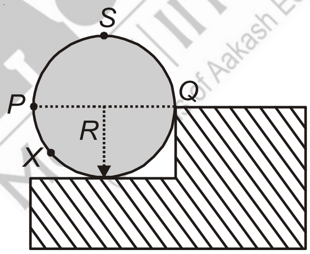
$$\Rightarrow \quad \frac{9}{8}R^2 = R^2 + \frac{9}{200}R^2 + \frac{x^2}{8}$$

$$\Rightarrow \quad \frac{x^2}{8} = \frac{9}{8}R^2 - R^2 - \frac{9}{200}R^2$$

$$\Rightarrow \quad x^2 = \frac{16}{25}R^2$$

$$\Rightarrow \quad x = \frac{4}{5}R$$

20. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?



(1) If the force is applied normal to the circumference at point P then τ is zero

(2) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

(3) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

(4) If the force is applied normal to the circumference at point X then τ is constant

Sol. Answer (1, 4)

Correct options (1, 4) [Treating magnitude of force constant]

For option (1):

Applied force passes through point Q.

So, its torque is zero.

For option (4):

Torque due to applied force at x remains constant.

SECTION - C

Linked Comprehension Type Questions

Comprehension-I

A rod AB of mass m and length l is hinged about the point A. The rod AB is released from the horizontal position. When rod becomes vertical,

1. Find kinetic energy of the rod.

   (1) $\frac{mgl}{4}$ (2) $\frac{3mgl}{4}$ (3) $\frac{5mgl}{2}$ (4) $\frac{mgl}{2}$

2. Find the hinge reaction at point A.

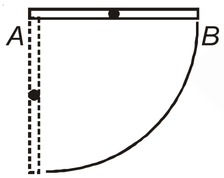
   (1) mg (2) $2\ mg$ (3) $3\ mg$ (4) $\frac{5mg}{2}$

3. Find the angular velocity of the rod.

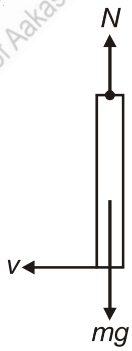
   (1) $\sqrt{\frac{3g}{L}}$ (2) $\sqrt{\frac{2g}{L}}$ (3) $\sqrt{\frac{g}{L}}$ (4) $\sqrt{\frac{g}{2L}}$

Solution of Comprehension-I

1. Answer (4)
2. Answer (4)
3. Answer (1)



Loss in GPE = Gain in K.E.



$$\frac{1}{2}\frac{mL^2}{3}\omega^2 - \frac{mgL}{2}$$

$$m\omega^2 L = 3mg$$

$$N - mg = \frac{m\omega^2 L}{2}$$

$$N = \frac{5mg}{2}$$

$N = 2mg$

Loss in GPE = Gain rotational K.E.

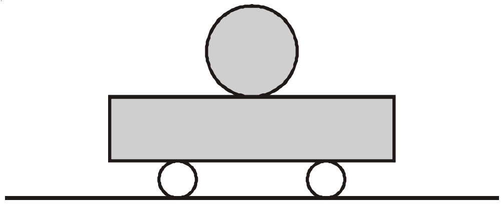
$$\Rightarrow \frac{1}{2}I\omega^2 = \frac{mgl}{2}$$

$$\Rightarrow \frac{ml^2}{3}\omega^2 = mgl$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{l}}$$

Comprehension-II

A solid sphere of mass m and radius R is placed over a plank of same mass m. There is sufficient friction between sphere and plank to prevent slipping.



1. Force of friction between sphere and plank is

(1) $\frac{F}{9}$ (2) $\frac{2F}{9}$ (3) $\frac{F}{3}$ (4) $\frac{2F}{3}$

2. Angular acceleration of sphere is

(1) $\frac{2F}{9mR}$ (2) $\frac{5F}{9mR}$ (3) $\frac{3F}{mR}$ (4) $\frac{9F}{mR}$

3. A Horizontal force F is applied on the plank. What is the maximum value of F, if μ is coefficient of friction between sphere movment and plank and there is no slipping?

(1) $\frac{7}{2}\mu\, mg$ (2) $\mu\, mg$ (3) $\frac{9}{2}\mu\, mg$ (4) $\frac{5}{2}\mu\, mg$

Solution of Comprehension-II

1. Answer (2)
2. Answer (2)
3. Answer (3)

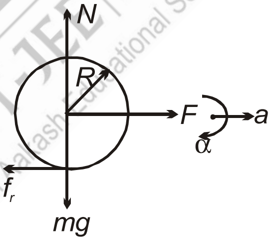
Equation of motion for sphere and planks are

$F - f_r = m \times a$

$f_r \times R = \frac{2}{5} mR^2 \times \alpha$

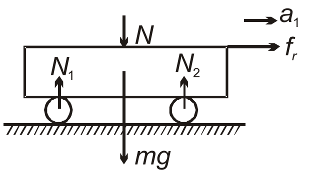
$f_r = m \times a_1$

$a - \alpha R = a_1$



On solving above equation, we get

$$a_1 = \frac{2F}{9m}, a = \frac{7F}{9m} \ \& \ f_r = \frac{2F}{9}, \alpha = \frac{5F}{9mR}$$

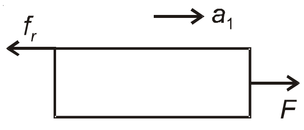


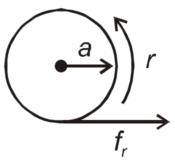
As $f_r \leq \mu.N$

$F - f_r = ma_1$ ...(i)

$a = a_1 - R\alpha$ ...(ii)

$f_r = ma = m(a_1 - R\alpha)$ ...(iii)





$$f_r R = \frac{2}{5} m R^2 \alpha \qquad \text{...(iv)}$$

Solving (i), (ii), (iii) and (iv) we get

$$f_r = \frac{2F}{9}$$

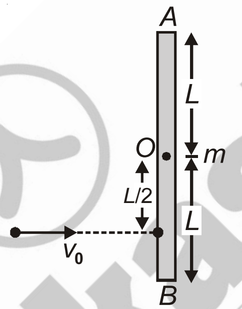
As $f_r \le \mu N$ $\Rightarrow \frac{2F}{9} \le \mu mg$

$\Rightarrow F \le \frac{9\mu mg}{2}$

$\therefore \quad F_{\max} = \frac{9\mu mg}{2}$

Comprehension-III

A rod AB of length $2L$ and mass m is lying on a horizontal frictionless surface. A particle of same mass m travelling along the surface hits the rod at distance $\frac{L}{2}$ from COM with a velocity v_0 in a direction perpendicular to rod and sticks to it.



1. Angular velocity of rod just after collision is

(1) $\frac{6v_0}{11L}$ (2) $\frac{12v_0}{11L}$ (3) $\frac{24v_0}{11L}$ (4) None of these

2. Distance of point P on rod from B which is at rest immediately after collision is

(1) $\frac{11L}{6}$ (2) $\frac{11L}{12}$ (3) $\frac{23L}{11}$ (4) $\frac{5L}{3}$

3. Velocity of P after time $\frac{11\pi L}{3v_0}$ is

(1) $v_0\sqrt{2}$ (2) Zero (3) $2\sqrt{2}\, v_0$ (4) $\frac{v_0}{\sqrt{2}}$

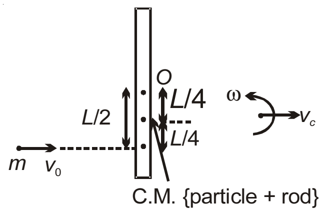
Solution of Comprehension-III

1. Answer (1)
2. Answer (4)
3. Answer (2)

Using conservation of momentum,

$mv_0 = 2mv_c$

$v_c = \frac{v_0}{2}$

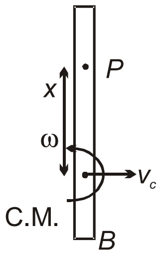


Using conservation of angular momentum about c.m. (particle + rod)

$$mv_0 \times \frac{L}{4} = \left\{ \frac{m\{2L\}^2}{12} + m\left\{\frac{L}{4}\right\}^2 + m\left\{\frac{L}{4}\right\}^2 \right\}\omega$$

$$\Rightarrow \quad \omega = \frac{6v_0}{11L}$$

Further v_p is zero then



$v_c - \omega x = 0$

$$\frac{v_0}{2} - \frac{6v_0}{11L} \times x = 0$$

$$x = \frac{11l}{12}$$

So distance from B is

$$\frac{11L}{12} + \frac{L}{4} + \frac{L}{2} = \frac{5L}{3}$$

Angular displacement during time $\frac{11\pi L}{3v_0}$ is

$$\theta = \frac{6v_0}{11L} \times \frac{11\pi L}{3v_0} = 2\pi$$

$$v_p = v_{c.m.} - \omega x$$

$$v_p = \frac{v_0}{2} - \frac{6v_0}{11L} \times \frac{11L}{12} = \frac{v_0}{2} - \frac{v_0}{2} = 0$$

Comprehension-IV

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

1. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(1) $\frac{2I\omega}{3t}$ (2) $\frac{9I\omega}{2t}$

(3) $\frac{9I\omega}{4t}$ (4) $\frac{3I\omega}{2t}$

Sol. Answer (1)

2. The loss of kinetic energy during the above process is

(1) $\frac{I\omega^2}{2}$ (2) $\frac{I\omega^2}{3}$

(3) $\frac{I\omega^2}{4}$ (4) $\frac{I\omega^2}{6}$

Sol. Answer (2)

SECTION - D

Matrix-Match Type Questions

1. A shell is projected by a cannon at an angle 30° with the vertical for a horizontal range R. It explodes at the highest point of its trajectory. Now match the column I and II.

Column I	**Column II**
(A) Along horizontal	(p) Work done by gravity is/may be positive
(B) On explosion of shell	(q) Chemical energy is converted to kinetic energy.
(C) After explosion	(r) Kinetic energy of centre of mass increases gradually
(D) When the potential energy of center mass decreases	(s) Linear momentum of centre of of mass does not change

Sol. Answer A(s), B(q, s), C(p, r), D(p, r)

Regarding various entries,

(A) External force is gravitational force (vertical) $\Rightarrow$ No change in horizontal linear momentum of centre of mass.

(B) Explosive forces are internal and cannot change linear momentum of centre of mass. But leads to conversion of chemical energy into kinetic energy.

(C) After explosion as centre of mass (un-influenced by explosion) moves down, $W_{gravity}$ = positive, leading to increase in its kinetic energy

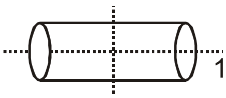
(D) $W_{gravity} = -\Delta U = \Delta k$

U decreases while k increases.

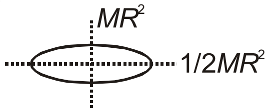
2. Some cases are given in column I and some moment of inertia are given in column II. Match column I with column II.

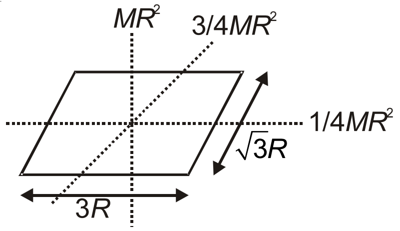
Column I	**Column II**
(A) Moment of inertia of a solid cylinder of mass M, length $2R$ and radius R about an axis passing through its centre	(p) MR^2
(B) Moment of inertia of a ring of mass M and radius R about an axis passing through its centre	(q) 1/2 MR^2
(C) Moment of inertia of a rectangular plate of mass M, length $3R$ and width $\sqrt{3}R$ about an axis passing through its centre	(r) 1/4 MR^2
(D) Moment of inertia of a disc of mass M and radius R about an axis passing through its centre	(s) 3/4 MR^2
	(t) $\frac{7MR^2}{12}$

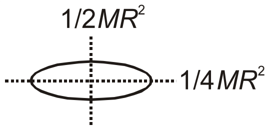
Sol. Answer A(q, t), B(p, q, s, t), C(p, q, r, s, t), D(q, r)

(A)  1/2 MR^2

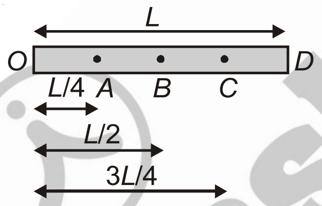
$$\frac{1}{4}MR^2 + \frac{Ml^2}{R} = \frac{7}{12}MR^2$$

(B) 

(C) 

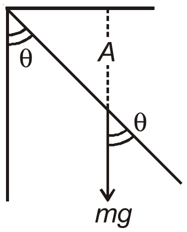
(D) 

3. A uniform rod of length L is free to rotate about an axis passing through O. Initially the rod is horizontal. The rod is released from this position. Match column I with column II.



	Column I		Column II
(A)	Acceleration of A will be $\frac{15\sqrt{10}}{8}$ m/s² when the rod has rotated through an angle	(p)	180°
(B)	Acceleration of B will be $\frac{15}{4}\sqrt{7}\ \text{m/s}^2$ when the rod has rotated through an angle	(q)	150°
(C)	Acceleration of C will be $\frac{45}{8}\sqrt{10}\ \text{m/s}^2$ when the rod has rotated through an angle	(r)	135°
(D)	Acceleration of D will be 15 m/s² when the rod has rotated through an angle	(s)	45°
		(t)	30°

Sol. Answer A(r, s), B(q, t), C(r, s), D(p)



$$a_c = \sqrt{(R\omega^2)^2(R\alpha)^2}$$

$$mgL(1-\cos\theta) = \frac{1}{2}\times\frac{Ml^2}{3}\times\omega^2$$

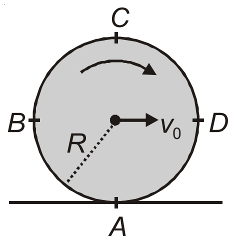
$$\omega^2 = \frac{6g}{l}(1-\cos\theta)$$

$$mg\times\frac{L}{2}\sin\theta = \frac{Ml^2}{3}\times\alpha$$

$$\alpha = \frac{3g}{2l}\sin\theta$$

Put these values we can solve the problem.

4. A wheel is executing pure rolling on a horizontal surface with a speed v_0 ($v_0 = R\omega$). The positions of points A, B, C and D at $t = 0$ are shown in the figure. Match the entries given in column I with those given in column II.



Column I	Column II
(A) Speed of A with respect to the bottom-most point is $\sqrt{2}v$ at time	(p) $t = \pi/\omega$
(B) Speed of B with respect to the topmost point is zero at time	(q) $t = \pi/2\omega$
(C) Speed of C with respect to the topmost point is $2v$ at time	(r) $t = 3\pi/2\omega$
(D) Speed of D with respect to the bottom-most point is $\sqrt{2}v$ at time	(s) $t = 5\pi/2\omega$
	(t) $t = 3\pi/\omega$

Sol. Answer A(q, r, s), B(q, s), C(p, t), D(p, t)

(A) Speed of A w.r.t. bottom most point is $\sqrt{2v}$ when A is at B or D.

For this rotation required is 90° or $\frac{\pi}{2}$, 270° or $\frac{3\pi}{2}$, or $\frac{5\pi}{2}$.

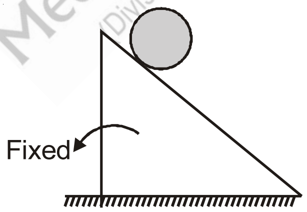
(B) Speed of B w.r.t. topmost point is zero when it coincides with A. This will occur after a rotation of $\frac{\pi}{2}$ or

$$2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$$

(C) For C, rotation is π, 3π, 5π, ...

(D) For D, rotation required is π, 2π, 3π, ...

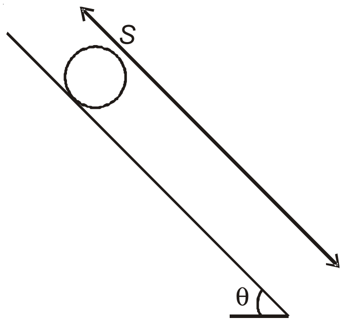
5. A solid sphere hollow sphere disc, ring and hollow cylinder are released from the top of a fixed inclined when all of them have same mass and radius



Column I	Column II
(A) Will reach the bottom first	(p) Ring
(B) Will reach the bottom last	(q) Hollow cylinder
(C) Will have maximum K.E. at the bottom of the inclined plane	(r) Solid sphere
(D) Will have maximum angular velocity at the bottom of the inclined plane	(s) Disc
	(t) All of these

Sol. Answer A(r), B(p, q), C(t), D(r)

$$a = \frac{g\sin\theta}{1+\frac{I}{m^2}}$$



$$a_{ring} = \frac{g\sin\theta}{2}$$

$$a_{\text{hollow cylinder}} = \frac{g\sin\theta}{2}$$

$$a_{\text{solid sphere}} = \frac{5}{7}g\sin\theta$$

$$a_{\text{disc}} = \frac{2}{3}g\sin\theta$$

$$T = \sqrt{\frac{2S}{a}}$$

Loss in GPE = Gain in K.E. (Rotational + Translational)

$\Rightarrow$ K.E. = mgS sin g

Total K.E. at the bottom

K.E.$_{\text{ring}}$ = MU^2R^2

K.E.$_{\text{Hollow cylinder}}$ = MU^2R

K.E. solid sphere = $\frac{7}{10}$ MU^2R

K.E. of the disc = $\frac{3}{4}MU^2R$

SECTION - E

Assertion-Reason Type Questions

1. STATEMENT-1 : The total momentum of a system in C-frame is always zero.

and

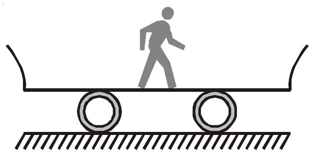
STATEMENT-2 : The total kinetic energy of a system in C-frame is always zero.

Sol. Answer (3)

$\vec{p}$ (in centre of mass frame) = $\Sigma m_i(\vec{v}_i - \vec{v}_{cm}) = \Sigma m_i\vec{v}_i - M\vec{v}_{cm} = \vec{0}$

k (in centre of mass frame) = $\frac{1}{2}\Sigma m_i\,|\vec{v}_i - \vec{v}_{cm}|^2 \neq 0$

2. STATEMENT-1 : A person standing on a stationary trolley placed on rough ground walks for a moment and then stops. The system acquires a net velocity due to this.



and

STATEMENT-2 : In the present situation, law of conservation of linear momentum is violated.

Sol. Answer (3)

As the person walks on trolley, the trolley has tendency to move opposite to the direction of motion of the person. This is opposed by the friction. As friction acts on the trolley, it accelerates the man plus trolley system.

So statement-1 is correct.

But law of conservation of linear momentum is valid in all inertial frames. Statement-2 is, therefore, incorrect.

3. STATEMENT-1 : For a body under translatory as well as rotatory equilibrium, net-torque about any axis is zero.

and

STATEMENT-2 : When the two equation $\Sigma \vec{F_i} = 0$ and $\Sigma(\vec{r_i} \times \vec{F_i}) = 0$ simultaneously hold good, it implies that $\Sigma(\vec{r_i} - \vec{r_0}) \times \vec{F} = 0$.

Sol. Answer (1)

If a rigid body in translational motion only not rotational motion the $\tau_{net} = 0 \Rightarrow \vec{r} \times \vec{F} = 0 \Rightarrow \vec{r_i}\vec{F_i} = 0$. This is applicable about any axis.

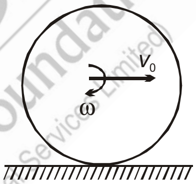
4. STATEMENT-1 : When a rigid body rolls on a rough horizontal ground without slipping, without any external force, the force of friction acting on it is zero.

and

STATEMENT-2 : When there is no slipping at point of contact between two surfaces, no frictional force acts between them.

Sol. Answer (3)

When there is no external force and the body rolls on the horizontal surface then there will be no friction at the point of contact to make v_0 and ω to be constant. But in case of external force friction must act to maintain rolling.

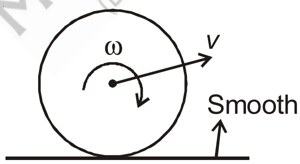


5. STATEMENT-1 : Friction is needed for pure rolling on horizontal surface.

and

STATEMENT-2 : Total work done by friction in pure rolling is zero.

Sol. Answer (4)



If the body have proper angular and linear velocity, then friction is not necessary for pure rolling.

6. STATEMENT-1 : In the absence of any external torque, the angular velocity of a system of particles about the axis of rotation through centre of mass does not change.

and

STATEMENT-2 : When there is no external torque, angular momentum of the system of particles is constant.

Sol. Answer (4)

We know that in absence of any external torque the net angular momentum of a system of particles is constant.

As $L = I\omega$, if I changes then angular velocity also changes.

7. STATEMENT-1 : The relation $\vec{\tau} = \frac{d\vec{L}}{dt}$ is applicable in centre of mass frame, even though the centre of mass is accelerating.

and

STATEMENT-2 : The relation $\vec{\tau} = \frac{d\vec{L}}{dt}$ can be directly applied in a non-inertial frame.

Sol. Answer (3)

In relation $\vec{\tau} = \frac{d\vec{L}}{dt}$ can be applied in non inertial frame but we have to include the torque of pseudo force also.

8. STATEMENT-1 : If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant.

and

STATEMENT-2 : The linear momentum of an isolated system remains constant.

Sol. Answer (4)

9. STATEMENT-1 : Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

and

STATEMENT-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

Sol. Answer (4)

The body with a greater moment of inertia experiences lesser acceleration.

In pure rolling, energy is conserved. So, both objects have same total kinetic energy.

SECTION - F

Integer Answer Type Questions

1. In an atwood machine the two blocks have masses 1 kg and 3 kg. The pulley is massless and frictionless and string is light. The acceleration of the centre of mass (in m/s^2) of this system is 0.5 *x.* Find the value of *x*.

Sol. Answer (5)

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

$$a_{cm} = \left(\frac{3-1}{3+1}\right)^2 g$$

$$a_{cm} = \frac{9}{4} = 2.5 \text{ m/s}^2$$

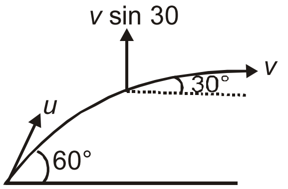
$\therefore \quad x = 5$

2. A body projected with a speed u at an angle of 60° with the horizontal explodes in two equal pieces at a point where its velocity makes an angle of 30° with the horizontal for 1st time. One piece start moving vertically upward with a speed of $\frac{u}{2\sqrt{3}}$ after explosion. What is velocity of one piece with respect to other in the vertical direction just after the explosion?

Sol. Answer (0)

$v \cos 30 = u \cos 60$

$v = \frac{u}{\sqrt{3}}$



Law of conservation of linear momentum

$$mv\sin 30 = \frac{m}{2}\frac{u}{2\sqrt{3}} + \frac{mv'}{2}$$

$$\frac{u}{2\sqrt{3}} = \frac{u}{4\sqrt{3}} + \frac{v'}{2}$$

$$\frac{v'}{2} = \frac{u}{4\sqrt{3}}$$

$$v' = \frac{u}{2\sqrt{3}}$$

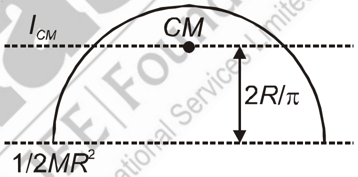
Relative velocity = $\frac{u}{2\sqrt{3}} - \frac{u}{2\sqrt{3}} = 0$

3. The moment of inertia of a uniform semicircular wire of mass M and radius R about an axis passing through its centre of mass and perpendicular to its plane is $x\frac{MR^2}{10}$. Find the value of x. (Take, $\pi^2 = 10$)

Sol. Answer (6)

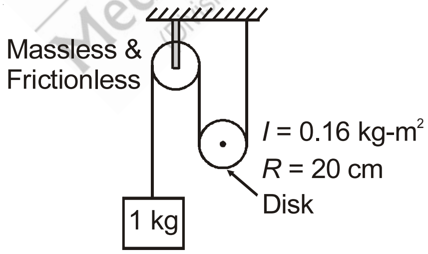
Parallel axis theorem

$$\frac{1}{2}MR^2 = I_{CM} + M\left(\frac{2R}{\pi}\right)^2$$



$$I_{CM} = \left(\frac{1}{2} - \frac{4}{\pi^2}\right)MR^2 = \frac{MR^2}{10}$$

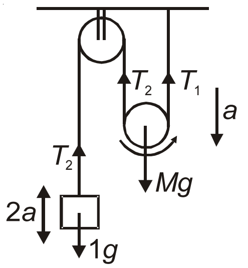
4. In the arrangement shown the fixed pulley is massless and frictionless. If the acceleration of block of mass 1 kg is $\frac{30}{x}$ m/s^2. Find the value of x.



Sol. Answer (4)

For 1 kg

$T_2 - g = 2a$ …(1)



For pulley,

$Mg - (T_1 + T_2) = Ma$ …(2)

$T_1R - T_2R = I$ a

$T_1 - T_2 = Ia / R^2$ …(3)

Solving, $a = \dfrac{(M-2)g}{\left(4+M+\dfrac{I}{R^2}\right)}$

Now, $M = \dfrac{2I}{R^2} = \dfrac{2\times 0.16}{0.04} = 8 \text{ kg}$

$a = \dfrac{60}{16} = \dfrac{15}{4}$

Acceleration of block = 2a = 15/2 m/s^2

5. A solid sphere of radius 2.45 m is rotating with an angular speed of 10 rad/s. When this rotating sphere is placed on a rough horizontal surface then after sometime it starts pure rolling. Find the linear speed of the sphere after it starts pure rolling.

Sol. Answer (7)

Law of conservation of angular momentum

$$\frac{2}{5}mR^2\omega = mvR + \frac{2}{5}mRv$$

$$v = \frac{2}{7}R\omega$$

$$\Rightarrow \quad v = \frac{2}{7}\times 2.45\times 10 = 7 \text{ m/s}$$

6. A body is projected with a speed u at an angle of 30° with the horizontal at t = 0. The angular momentum of the body about the point of projection is $\dfrac{x\sqrt{3}\,mu^3}{64\,g}$ at $t = \dfrac{3u}{4g}$. Find the value of x.

Sol. Answer (9)

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{g}t^2$$

$$\vec{p} = m(\vec{u} + \vec{g}t)$$

$$\vec{L} = \vec{r}\times\vec{p}$$

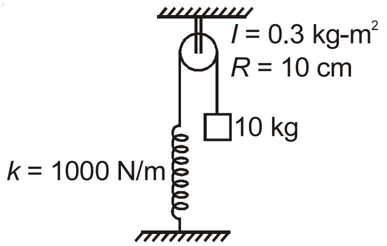
$$\vec{L} = \frac{m}{2}(\vec{u}\times\vec{g})t^2$$

$$L = \frac{mug\sin 120\, t^2}{2}$$

$$L = \frac{mu\times g\times\sqrt{3}\times 9u^2}{4\times 16\times g^2}$$

$$L = \frac{9\sqrt{3}\,mu^3}{64g}$$

7. In the given arrangement the block is released from the position where the spring is unstretched. The speed of the block when it has descended through 2 cm is $\dfrac{x}{10}$. Find the value of x.



Sol. Answer (3)

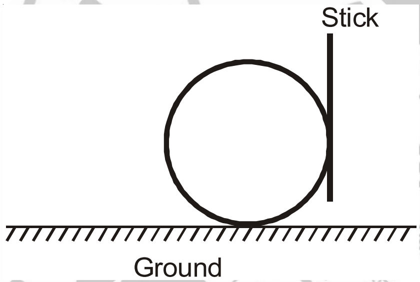
Law of conservation of energy

Loss in gravitational potential energy of block = Gain in K.E. of (block + pulley) + Gain in elastic potential energy of spring

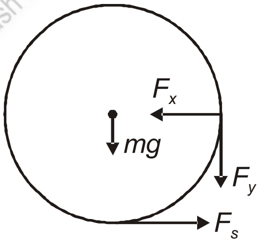
$$10 \times 10 \times 0.02 = \frac{1}{2} \times 10 \times v^2 + \frac{1}{2} \times 0.3 \times \frac{v^2}{0.01} + \frac{1}{2} \times 1000 \times 4 \times 10^{-4}$$

v = 0.3 m/s

8. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is



Sol. Answer (4)



$F_x^2 + F_y^2 = 2^2 = 4$ …(i)

$(F_x - F_y)r = 2mr^2\alpha = 2mra$

$\Rightarrow$ $F_x - Fy = 2ma = 1.2$ …(ii)

Using $F_y = \frac{P}{10} \times F_x$

we get $\boxed{P \simeq 4}$

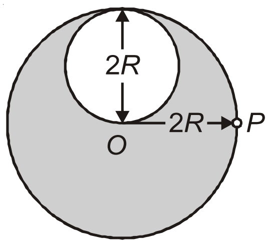
9. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is

Sol. Answer (9)

$$I_{AB} = 2 \times \left\{\frac{2}{5}\frac{md}{4}\right\}^2 + 2 \times \left\{\frac{2}{5}\frac{md^2}{4} + \frac{ma^2}{2}\right\}$$

$= 9 \times 10^{-4}$ kg–m^2

10. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is



Sol. Answer (3)

Let the mass density be σ.

$$I_O = \frac{\sigma\pi(2R)^2(2R)^2}{2} - \frac{3}{2}\sigma\pi(R)^2(R)^2 = \frac{13}{2}\sigma\pi R^4$$

$$I_P = \frac{3}{2}\sigma\pi(2R)^2(2R)^2 - \left[\frac{\sigma\pi R^2(R)^2}{2} + \sigma\pi R^2 \times (\sqrt{5}R)^2\right] = \frac{37}{2}\sigma\pi R^4$$

$$\text{Ratio} = \frac{I_P}{I_O}\ \frac{37}{13} \approx 3$$

11. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is

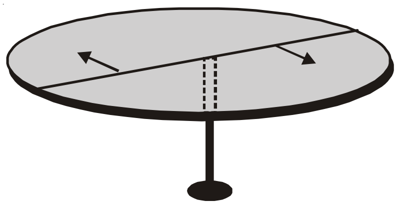
Sol. Answer (8)

By conservation of angular momentum,

$$\frac{1}{2}\times 50\times(0.4)^2\times 10 = \left[\frac{1}{2}\times 50\times(0.4)^2 + 2\times 2\times 6.25\times(0.2)^2\right]\omega$$

$$\Rightarrow \quad \omega = \frac{40}{4+1} = 8 \text{ rad/s}$$

12. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is



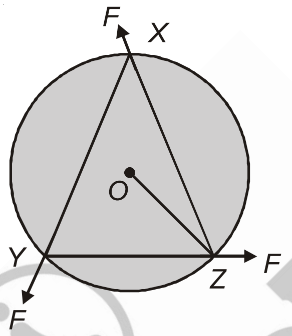
Sol. Answer (4)

$2mvr = I\omega$

$\Rightarrow\ 2 \times 0.05 \times 9 \times 0.25 = \dfrac{0.45 \times (0.5)^2}{2} \times \omega$

$\Rightarrow\ \omega = \dfrac{2 \times 0.05 \times 9 \times 0.25 \times 2}{0.45 \times 0.5 \times 0.5} = 4$

13. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is

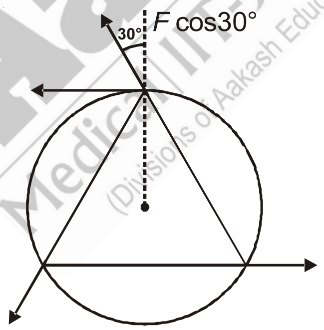


Sol. Answer (2)

$\tau = 3 \times (F\sin 30) \times r$

$= 3 \times 0.5 \times \dfrac{1}{2} \times 0.5$

$= 3 \times \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{3}{8}$



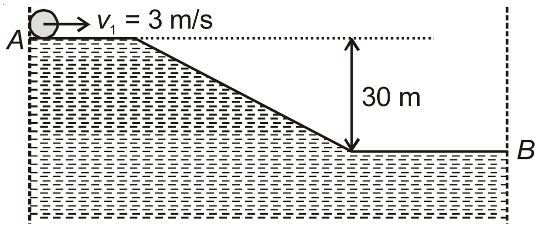
$I = \dfrac{MR^2}{2} = \dfrac{1.5 \times 0.5 \times 0.5}{2}$

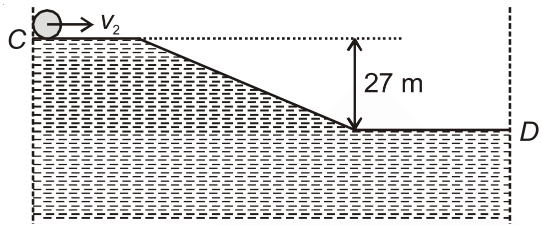
$= \dfrac{15}{10} \times \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{3}{16}$

$\tau = I\alpha \ \Rightarrow\ \alpha = \dfrac{\frac{3}{8}}{\frac{3}{16}} = 2$

$\omega = \omega_0 + \alpha t = 2 \times 1 = 2$

14. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s then v_2 in m/s is (g = 10 m/s^2)





Sol. Answer (7)

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 + mgh = \frac{1}{g}mv'^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega'^2$$

$$\Rightarrow \quad v'^2 = v^2 + \frac{4}{3}gh$$

$$\text{Now, } 3^2 + \frac{4}{3}\times 10(30) = v^2 + \frac{4}{3}\times 10(27)$$

$\Rightarrow \quad v^2 = 49$

$\Rightarrow \quad v = 7$ m/s

15. The densities of two solid spheres A and B of the same radii R vary with radial distance r as

$\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

Sol. Answer (6)

$$I = \int \frac{2}{3}dm\,r^2 = \frac{2}{3}\int \rho r^2\,dr\,r^2$$

$$\frac{I_B}{I_A} = \frac{\int_0^R (\rho_B r^2) r^2 dr}{\int_0^R (\rho_A r^2) r^2 dr} = \frac{6}{10}$$

$\Rightarrow \quad n = 6$

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