**1.** **(b)** The first equation can be written as

$$x\sin a + y\times 2\sin a\cos a + z\sin a(3-4\sin^2 a)$$

$$= 2\times 2\sin a\cos a\cos 2a$$

$$\Rightarrow x + 2y\cos a + z(3+4\cos^2 a - 4)$$

$$= 4\cos a(2\cos^2 a - 1)\ as \sin a \neq 0$$

$$\Rightarrow\ 8\cos^3 a - 4z\cos^2 a - (2y+4)\cos a + (z-x) = 0$$

$$\Rightarrow\ \cos^3 a - \frac{z}{2}\cos^2 a - \frac{y+2}{4}\cos a + \frac{z-x}{8} = 0$$

which shows that cos a is root of the equation

$$t^3 - \frac{z}{2}t^2 - \frac{y+2}{4}t + \frac{z-x}{8} = 0$$

Similarly from second and third equations we can varify that cos $b$ and cos $c$ are the roots of the above equation

**2.** **(a)** $$u^2 = a^2 + b^2 + 2\sqrt{\begin{matrix}(a^4+b^4)\cos^2\theta\sin^2\theta \\ +a^2b^2(\cos^4\theta+\sin^4\theta)\end{matrix}} \quad \ldots\text{(i)}$$

Now $(a^4+b^4)\cos^2\theta\sin^2\theta + a^2b^2(\cos^4\theta+\sin^4\theta)$

$$= (a^4+b^4)\cos^2\theta\sin^2\theta + a^2b^2(1-2\cos^2\theta\sin^2\theta)$$

$$= (a^4+b^4-2a^2b^2)\cos^2\theta\sin^2\theta + a^2b^2$$

$$= (a^2-b^2)^2.\frac{\sin^2 2\theta}{4} + a^2b^2 \quad \ldots\text{(ii)}$$

$$\because 0 \le \sin^2 2\theta \le 1$$

$$\Rightarrow 0 \le (a^2-b^2)^2\frac{\sin^2 2\theta}{4} \le \frac{(a^2-b^2)^2}{4}$$

$$\Rightarrow a^2b^2 \le (a^2-b^2)^2\frac{\sin^2 2\theta}{4} + a^2b^2$$

$$\le (a^2-b^2)^2.\frac{1}{4} + a^2b^2 \quad \text{....(iii)}$$

$\therefore$ from (i), (ii) and (iii)

Minimum value of $u^2 = a^2+b^2+2\sqrt{a^2b^2} = (a+b)^2$

Maximum value of $u^2$

$$= a^2+b^2+2\sqrt{\left(a^2-b^2\right)^2.\frac{1}{4}+a^2b^2}$$

$$= a^2+b^2+\frac{2}{2}\sqrt{(a^2+b^2)^2} \ = 2(a^2+b^2)$$

$\therefore$ Max value – Min value

$$= 2(a^2+b^2)-(a+b^2) = (a-b)^2$$

**3.** **(c)** We have

$$\sum_{m=1}^{6}\text{cosec}\left[\theta+\frac{(m-1)\pi}{4}\right]\text{cosec}\left[\theta+\frac{m\pi}{4}\right] = 4\sqrt{2}$$

$$\Rightarrow\ \sum_{m=1}^{6}\frac{\sin\frac{\pi}{4}}{\sin\left[\theta+\frac{(m-1)\pi}{4}\right]\sin\left[\theta+\frac{m\pi}{4}\right]} = 4$$

$$\Rightarrow\ \sum_{m=1}^{6}\frac{\sin\left[\left(\theta+\frac{m\pi}{4}\right)-\left(\theta+\frac{(m-1)\pi}{4}\right)\right]}{\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)} = 4$$

$$\Rightarrow\ \sum_{m=1}^{6}\frac{\left[\begin{matrix}\sin\left(\theta+\frac{m\pi}{4}\right)\cos\left(\theta+\frac{(m-1)\pi}{4}\right)\\ -\cos\left(\theta+\frac{m\pi}{4}\right)\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\end{matrix}\right]}{\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)} = 4$$

$$\Rightarrow\ \sum_{m=1}^{6}\left[\cot\left(\theta+\frac{(m-1)\pi}{4}\right)-\cot\left(\theta+\frac{m\pi}{4}\right)\right] = 4$$

$$\Rightarrow\ \left[\cot\theta-\cot\left(\theta+\frac{\pi}{4}\right)\right]+\left[\cot\left(\theta+\frac{\pi}{4}\right)-\cot\left(\theta+\frac{2\pi}{4}\right)\right]$$

$$+\ldots+\left[\cot\left(\theta+\frac{5\pi}{4}\right)-\cot\left(\theta+\frac{6\pi}{4}\right)\right] = 4$$

$$\Rightarrow\ \cot\theta-\cot\left(\theta+\frac{3\pi}{2}\right) = 4 \Rightarrow\ \cot\theta+\tan\theta = 4$$

$$\Rightarrow\ \cos^2\theta+\sin^2\theta = 4\sin\theta\cos\theta$$

$$\Rightarrow\ \sin 2\theta = \frac{1}{2} \Rightarrow\ 2\theta = \frac{\pi}{6}\text{or}\frac{5\pi}{6}\ \Rightarrow\ \theta = \frac{\pi}{12}\text{or}\frac{5\pi}{12}$$

**4.** **(c)** $\sqrt{3}$ sec x + cosec x + 2 (tan x – cot x) = 0

$$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\ = \cos^2 x\ \sin^2 x$$

$$\Rightarrow \cos\left(x-\frac{\pi}{3}\right)\ = \cos 2x$$

$$\Rightarrow x - \frac{\pi}{3}\ = 2n\pi \pm 2x$$

$$\Rightarrow x = \frac{2n\pi}{3} + \frac{\pi}{9}\ \text{or}\ x = -2n\pi - \frac{\pi}{3}$$

For $x \in S$, $n = 0 \Rightarrow x = \frac{\pi}{9}, -\frac{\pi}{3}$

$n = 1 \Rightarrow x = \frac{7\pi}{9}$

$n = -1 \Rightarrow x = \frac{-5\pi}{9}$

$\therefore$ Sum of all values of $x = \frac{\pi}{9} - \frac{\pi}{3} + \frac{7\pi}{9} - \frac{5\pi}{9} = 0$

**5.** **(a)** We are given that

$(\cot\alpha_1).(\cot\alpha_2)\ ....\ (\cot\alpha_n) = 1$

$\Rightarrow (\cos\alpha_1)(\cos\alpha_2)\ ....\ (\cos\alpha_n)$

$= (\sin\alpha_1)(\sin\alpha_2)\ ....(\sin\alpha_n)$ ....(i)

Let $y = (\cos\alpha_1)(\cos\alpha_2)\ ....\ (\cos\alpha_n)$ (to be max.)

Squaring both sides, we get

$y^2 = (\cos^2\alpha_1)(\cos^2\alpha_2)\ ....\ (\cos^2\alpha_n)$

$= \cos\alpha_1 \sin\alpha_1 \cos\alpha_2 \sin\alpha_2\ ....\ \cos\alpha_n \sin\alpha_n$ (Using (i))

$= \frac{1}{2^n}[\sin 2\alpha_1 \sin 2\alpha_2\ ....\ \sin 2\alpha_n]$

As $0 \le \alpha_1, \alpha_2, .....\alpha_n \le \pi/2$

$\therefore 0 \le 2\alpha_1, 2\alpha_2, .....2\alpha_n \le \pi$

$\Rightarrow 0 \le \sin 2\alpha_1, \sin 2\alpha_2, .....\sin 2\alpha_n \le 1$

$\therefore y^2 \le \frac{1}{2^n}.1 \Rightarrow y \le \frac{1}{2^{n/2}}$

$\therefore$ Max. value of $y$ is $1/2^{n/2}$.

**6.** **(b)** $\alpha < \beta < \gamma < \delta$ and $\sin\alpha = \sin\beta = \sin\gamma = \sin\delta = k$

$\Rightarrow \beta = \pi - \alpha,\ \gamma = 2\pi + \alpha,\ \delta = 3\pi - \alpha$

So that the given expression is equal to

$$4\sin\frac{\alpha}{2} + 3\sin\left(\frac{\pi - \alpha}{2}\right) + 2\sin\frac{2\pi + \alpha}{2} + \sin\frac{3\pi - \alpha}{2}$$

$$= 4\sin\frac{\alpha}{2} + 3\cos\frac{\alpha}{2} - 2\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}$$

$$= 2\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right) = 2\sqrt{1 + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = 2\sqrt{1+k}$$

**7.** **(a,b,c,d)** We have, $1 + \sec\theta = \frac{1+\cos\theta}{\cos\theta} = \frac{2\cos^2\frac{\theta}{2}}{\cos\theta}$,

similarly for others.

$$f_n(\theta) = \tan\frac{\theta}{2}.\frac{2\cos^2\frac{\theta}{2}}{\cos\theta}.\frac{2\cos^2\theta}{\cos 2\theta}.....\frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}$$

$$= \tan\frac{\theta}{2}.2^{n+1}\frac{[\cos\theta.\cos 2\theta.......\cos 2^{n-1}\theta]\cos^2\frac{\theta}{2}}{\cos 2^n\theta}$$

$$= \frac{\sin\theta}{\cos 2^n\theta}.2^n.\frac{\sin 2^n\theta}{2^n\sin\theta} = \tan 2^n\theta$$

$$\therefore f_2\left(\frac{\pi}{16}\right) = \tan\left(4.\frac{\pi}{16}\right) = 1,.$$

$$f_3\left(\frac{\pi}{32}\right) = \tan\left(8.\frac{\pi}{32}\right) = 1$$

Similarly others are also true.

**8.** **(a,c)** $\sin\beta = \frac{12}{13} \Rightarrow \cos\beta = \pm\frac{5}{13}$

according as $\tan\beta > 0$ or $< 0$

$\therefore 5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)$

$= 5[\sin\alpha\cos\beta + \cos\alpha\sin\beta]$
$\quad -12[\cos\alpha\cos\beta - \sin\alpha\sin\beta]$

$= (5\cos\beta + 12\sin\beta)\sin\alpha$
$\quad + (5\sin\beta - 12\cos\beta)\cos\alpha$

$$= \left(\frac{25}{13} + \frac{144}{13}\right)\sin\alpha + \left(\frac{60}{13} - \frac{60}{13}\right)\cos\alpha$$

$= 13\sin\alpha$ if $\tan\beta > 0$

$\Rightarrow \{(5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)\}\operatorname{cosec}\alpha = 13$

If $\tan\beta < 0$ then $5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)$

$$= \frac{119}{13}\sin\alpha + \frac{120}{13}\cos\alpha$$

$\Rightarrow [5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)]\operatorname{cosec}\alpha$

$$= \frac{119}{13} + \frac{120}{13}\cot\alpha$$

**9.** **(b, c, d)** From the first relation we have

$a[\sin(\theta+\phi) - \sin(\theta-\phi)] = b[\sin(\theta-\phi) + \sin(\theta+\phi)]$

$\Rightarrow 2a\sin\phi\cos\theta = 2b\sin\theta\cos\phi$

$\Rightarrow a\tan\phi = b\tan\theta \Rightarrow$ (b) is correct

$$\Rightarrow \frac{2a\tan\frac{\phi}{2}}{1-\tan^2\frac{\phi}{2}}=\frac{2b\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

From the second relation replacing

$\tan\frac{\theta}{2}=\frac{1}{a}[b\tan\frac{\phi}{2}+c]$ we have

$$\frac{a\tan\frac{\phi}{2}}{1-\tan^2\frac{\phi}{2}}=\frac{\left(b(b\tan\frac{\phi}{2}+c)\right)}{a\left[1-\left\{\frac{1}{a}(b\tan\frac{\phi}{2}+c\right\}^2\right]}$$

$$\Rightarrow \tan\frac{\phi}{2}\left[a^2-(b\tan\frac{\phi}{2}+c)^2\right]$$

$$=b\left(b\tan\frac{\phi}{2}+c\right)\left(1-\tan^2\frac{\phi}{2}\right)$$

$$\Rightarrow \tan\frac{\phi}{2}(a^2-b^2-c^2)=bc\left(1+\tan^2\frac{\phi}{2}\right)$$

$$\Rightarrow \frac{2\tan\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}=\frac{2bc}{a^2-b^2-c^2}$$

$$\Rightarrow \sin\phi=\frac{2bc}{a^2-b^2-c^2}$$

Similarly we get $\sin\theta=\frac{2ac}{a^2-b^2+c^2}$

**10. (b, c, d)** Given : $\frac{\tan 3A}{\tan A}=k$ …(1)

$$\Rightarrow \frac{\tan 3A-\tan A}{\tan A}=k-1\Rightarrow\frac{\sin 2A}{\cos 3A\sin A}=k-1$$

$$\Rightarrow \frac{2\cos A}{\cos 3A}=k-1\Rightarrow\frac{\cos A}{\cos 3A}=\frac{k-1}{2}$$

$\Rightarrow$ (a) is incorrect

Again $\frac{\tan 3A}{\tan A}=k\Rightarrow\frac{\sin 3A}{\cos 3A}.\frac{\cos A}{\sin A}=k$

$$\Rightarrow \frac{\sin 3A}{\sin A}=k.\frac{2}{k-1}=\frac{2k}{k-1}$$

$$\Rightarrow \frac{3\sin A-4\sin^3 A}{\sin A}=\frac{2k}{k-1}$$

$$\Rightarrow 3-4\sin^2 A=\frac{2k}{k-1} \text{ or } 4\sin^2 A=\frac{k-3}{k-1}$$

$$\Rightarrow 0<\frac{k-3}{k-1}<4 \; [\sin A\neq 0 \text{ or } 1]$$

Now, $\frac{k-3}{k-1}>0$ is $k<1$ or $k>3$ …(ii)

and $\frac{k-3}{k-1}<4$

$\Rightarrow \frac{3k-1}{k-1}>0\Rightarrow k<\frac{1}{3}$ or $k>1$ ...(iii)

(ii) and (iii) simultaneously hold if $k<\frac{1}{3}$ or $k>3$

**11. (4)**

We have

$$\left(a\tan\beta-\sqrt{a^2-1}\tan\alpha\right)^2+\left(\sqrt{a^2+1}\tan\beta-\sqrt{a^2-1}\tan\gamma\right)^2$$

$$+\left(a\tan\gamma-\sqrt{a^2+1}\tan\alpha\right)^2\geq 0$$

$$\Rightarrow \{a^2+a^2-1+a^2+1\}(\tan^2\alpha+\tan^2\beta+\tan^2\gamma)$$

$$-\left\{a\tan\alpha+\sqrt{a^2-1}\tan\beta+\sqrt{a^2+1}\tan\gamma\right\}^2\geq 0$$

$$\Rightarrow \tan^2\alpha+\tan^2\beta+\tan^2\gamma\geq\frac{4a^2}{3a^2}\Rightarrow 3\sum\tan^2\alpha\geq 4$$

**12. (3)**

$$1+\cos\alpha=1+\frac{2\cos\beta-1}{2-\cos\beta}$$

$$=\frac{2-\cos\beta+2\cos\beta-1}{2-\cos\beta}=\frac{1+\cos\beta}{2-\cos\beta}$$

$$\Rightarrow 2\cos^2\frac{\alpha}{2}=\frac{2\cos^2(\beta/2)}{1+2\sin^2(\beta/2)}$$

$$\Rightarrow \cos^2\frac{\alpha}{2}=\frac{\cos^2(\beta/2)}{1+2\sin^2(\beta/2)} \quad ...(1)$$

$$\Rightarrow 1-\cos^2\frac{\alpha}{2}=1-\frac{\cos^2(\beta/2)}{1+2\sin^2(\beta/2)}$$

$$=\frac{1+2\sin^2(\beta/2)-\cos^2(\beta/2)}{1+2\sin^2(\beta/2)}=\frac{3\sin^2\frac{\beta}{2}}{1+2\sin^2\frac{\beta}{2}}$$

$$\Rightarrow \sin^2\frac{\alpha}{2}=\frac{3\sin^2(\beta/2)}{1+2\sin^2(\beta/2)} \quad ...(2)$$

Divide eqs. (2) by (1), we get

$$\tan^2\frac{\alpha}{2}=3\tan^2\frac{\beta}{2} \Rightarrow \frac{\tan(\alpha/2)}{\tan(\beta/2)}=\sqrt{3}$$

$$\Rightarrow \sqrt{3}\frac{\tan(\alpha/2)}{\tan(\beta/2)}=3$$

**13. (0)**

We have $xy+yz+zx=xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

Now, $x\cos\theta=y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)=k$ (say)

then $x=\frac{k}{\cos\theta}, y=\frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)}$ and $z=\frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{k}\left[\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)\right]$$

$$=\frac{1}{k}\left[\cos\theta+\cos\theta\left(\frac{-1}{2}\right)-\sin\theta\left(\frac{\sqrt{3}}{2}\right)+\cos\theta\left(-\frac{1}{2}\right)-\sin\theta\left(-\frac{\sqrt{3}}{2}\right)\right]$$

$$=\frac{1}{k}\left[\cos\theta-\cos\theta-\frac{\sqrt{3}}{2}\sin\theta-\frac{\sqrt{3}}{2}\sin\theta\right]=0$$

$\therefore\ xy+yz+zx=0$

**14. (6)**

Given that $\sin^3 x\sin 3x=\sum_{m=0}^{n}c_m\cos mx$

or $\left(\frac{3\sin x-\sin 3x}{4}\right).\sin x=\sum_{m=0}^{n}c_m\cos mx$

or $\frac{3}{8}.(2\sin 3x\sin x)-\frac{1}{8}.2\sin^2 3x=\sum_{m=0}^{n}c_m\cos mx$

or $\frac{3}{8}.[\cos 2x-\cos 4x]-\frac{1}{8}[1-\cos 6x]=\sum_{m=0}^{n}c_m\cos mx$

or $-\frac{1}{8}+\frac{3}{8}\cos 2x-\frac{3}{8}\cos 4x+\frac{1}{8}\cos 6x=\sum_{m=0}^{n}c_m\cos mx$.

Comparing, we get $n=6$.

**15. (b)** $\because\ P_1=m$

$P_1{}^2=m^2$

$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=m^2$

$\Rightarrow\ \sin\theta\cos\theta=\frac{(m^2-1)}{2}$

Now, from eq. (iii), we get

$P_6=1-3\sin^2\theta\cos^2\theta$

$\Rightarrow\ (1-P_6)=3(\sin\theta\cos\theta)^2=\frac{3(m^2-1)^2}{4}$

$\Rightarrow\ 4(1-P_6)=3(m^2-1)^2$

**16. (a)** Let $\sin^2\theta\cos^2\theta=k$, then from eq. (i), we get

$P_n-P_{n-2}=-kP_{n-4}$.

From eq. (ii), $P_4=1-2k$

and from eq. (iii), $P_6=1-3k$

Put $n=10$,

then $P_{10}-P_8=-kP_6=-k(1-3k)$

$\therefore\ P_{10}-P_8=3k^2-k$ …(iv)

and put $n=8$, then $P_8-P_6-kP_4=-k(1-2k)$

$P_8=P_6+2k^2-k$

$=1-3k+2k^2-k$

$\Rightarrow\ P_8=2k^2-4k+1$

From eq. (iv), $P_{10}=5k^2-5k+1$

$\therefore\ 6P_{10}-15P_8+10P_6+7$

$=6(5k^2-5k+1)-15(2k^2-4k+1)+10(1-3k)+7$

$=8$

**17. (b)** $5\sin^2 x+3\sin x\cos x-3\cos^2 x=2(\sin^2 x+\cos^2 x)$

$\Rightarrow\ 3\tan^2 x+3\tan x-5=0$

$\Rightarrow\ \tan x=\frac{-3\pm\sqrt{69}}{6}$

and $\sin^2 x-\cos 2x=2-\sin 2x$

$3\sin^2 x+2\sin x\cos x=3(\sin^2 x+\cos^2 x)$

$\Rightarrow\ \cos x(2\sin x-3\cos x)=0$

Either $\cos x=0$ or $\tan x=\frac{3}{2}\Rightarrow\cos x=\pm\frac{2}{\sqrt{13}}$

Taking $=\alpha=\frac{-3\pm\sqrt{69}}{6}, \tan\beta=\frac{3}{2}$

we get $\tan\alpha+\tan\beta=1\pm\sqrt{69}/6$

**18. (d)** Taking $\tan\alpha=\frac{-3+\sqrt{69}}{6}, \tan\beta=\frac{-3-\sqrt{69}}{6}$

$\cos\gamma=0, \cos\delta=\pm\frac{2}{\sqrt{13}}$

we get $\tan\alpha\tan\beta+\cos\gamma+\cos\delta=-\frac{5}{3}\pm\frac{2}{\sqrt{13}}$

**19. A - p,q,r,s; B - r,s; C - q,r,s; D - q, s**

**(A)** $f(\theta) = (\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$$= \sin^2\theta + \cos^2\theta + \sec^2\theta + \text{cosec}^2\theta + 4$$

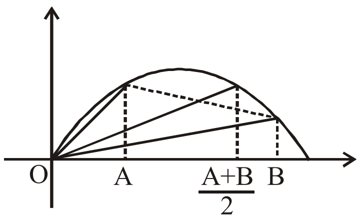
$$= 5 + 1 + \cot^2\theta + 1 + \tan^2\theta = 9 + (\tan\theta - \cot\theta)^2 \ge 9$$

**(B)** $\sin\alpha - \sin\beta = a,\ \cos\alpha + \cos\beta = b$

$$\Rightarrow a^2 + b^2 = 2 + 2\cos(\alpha+\beta) = 4\cos^2\frac{\alpha+\beta}{2} \le 4$$

**(C)** $\dfrac{\sin A + \sin B}{2} \le \sin\left(\dfrac{A+B}{2}\right)$

$$\therefore \sin A + \sin B \le 2\sin\frac{\pi}{4}$$




or $\dfrac{1}{\sqrt{2}}(\sin A + \sin B) \le 1$

**(D)** Let $A = 7\cos x + 6\sin x = 6(2\cos x + \sin x) - 5\cos x$
$= 6 - 5\cos x$

Now, $2\cos x + \sin x = 1 \Rightarrow \sin x = 1 - 2\cos x$

$\Rightarrow \sin^2 x = 1 - \cos^2 x = 1 - 4\cos x + 4\cos^2 x$

$\therefore \cos x = 0$ or $\dfrac{4}{5}$. So, $A = 6$ or $2$

**20. A - q; B - p, s; C - r; D - p**

**(A)** $y = \cos^2\theta + \sin^4\theta = \cos^2\theta + \sin^2\theta(1 - \cos^2\theta)$

$$= 1 - \frac{1}{4}\sin^2 2\theta \Rightarrow \frac{3}{4} \le A \le 1$$

**(B)** $\tan A < 0 \Rightarrow A > \dfrac{\pi}{2} \Rightarrow 0 < B + C < \dfrac{\pi}{2}$

$$\Rightarrow \tan(B+C) > 0 \Rightarrow \frac{\tan B + \tan C}{1 - \tan B\tan C} > 0$$

$$\Rightarrow 0 < \tan B\tan C < 1$$

**(C)** Let $y = \dfrac{\cos^2\theta - 1}{\cos^2 + \cos\theta}$

$$\Rightarrow (y-1)\cos^2\theta + y\cos\theta + 1 = 0$$

$$\Rightarrow \cos\theta = -1 \text{ or } \cos\theta = \frac{1}{1-y}$$

$$-1 < \frac{1}{1-y} < 1$$

$$\Rightarrow y < 0 \text{ or } y > 2$$

**(D)** $y = \tan A\tan B = \tan A\tan\left(\dfrac{\pi}{3} - A\right)$

$$= x\left(\frac{\sqrt{3} - x}{1 + \sqrt{3}x}\right), \text{ where } x = \tan A$$

$$\Rightarrow x^2 + \sqrt{3}x(y-1) + y = 0$$

$$\because x \in R \Rightarrow 3(y-1)^2 - 4y \ge 0 \Rightarrow y \le \frac{1}{3} \text{ or } y \ge 3$$

Also, $0 < A, B < \dfrac{\pi}{3} \Rightarrow 0 < \tan A, \tan B < \sqrt{3}$

$$\Rightarrow 0 < \tan A\tan B < 3$$

$$\therefore 0 < y \le \frac{1}{3}$$