JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S
RELATIVE MOTION

1. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when: (i) they are moving in the same direction, and (ii) in the aopposite direction is :

[*JEE Main-2019* (*January*)]

(*a*) $\frac{11}{5}$ (*b*) $\frac{5}{2}$

(*c*) $\frac{3}{2}$ (*d*) $\frac{25}{11}$

2. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in : **[*JEE Main - 2019* (*April*)]**

(*a*) 4.2 hrs. (*b*) 2.2 hrs.

(*c*) 3.2 hrs. (*d*) 2.6 hrs.

3. The stream of a river is flowing with a speed of 2km/h. A swimmer can swim at a speed of 4km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight ?

[*JEE Main-2019* (*April*)]

(*a*) 60° (*b*) 150°

(*c*) 90° (*d*) 120°

4. A particle is moving along the x-axis with its coordinate with time t given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is

[*JEE Main-2020* (*January*)]

(*a*) 580 (*b*) 500

(*c*) 540 (*d*) 560

5. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed ν, he sees that rain drops are coming at an angle 60º from the horizontal. On further increasing the speed of the car to $(1 + \beta)\nu$, this angle changes to 45º. The value of β is close to

[*JEE Main-2020* (*September*)]

(*a*) 0.37 (*b*) 0.41

(*c*) 0.73 (*d*) 0.50

PROJECTILE

6. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[*JEE Main - 2019* (*January*)]**

(*a*) $y = x^2 +$ constant (*b*) $y^2 = x +$ constant

(*c*) $y^2 = x^2 +$ constant (*d*) $xy =$ constant

7. Two guns A and B can fire bullets at speed 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the buyllets fired by the two guns, on the ground is:

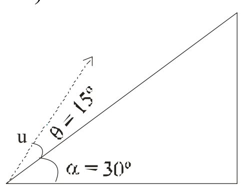
[*JEE Main-2019* (*January*)]

(*a*) 1 : 16 (*b*) 1 : 2

(*c*) 1 : 4 (*d*) 1 : 8

8. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$ from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to: **[*JEE Main-2019 (April)*]**

(Take $g = 10\ ms^{-2}$)



(*a*) 14 cm (*b*) 20 cm
(*c*) 18 cm (*d*) 26cm

9. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2 . Which of the following is correct ? **[*JEE Main-2019 (April)*]**

(*a*) $R^2 = 2\ h_1 h_2$ (*b*) $R^2 = 16\ h_1 h_2$
(*c*) $R^2 = 4\ h_1 h_2$ (*d*) $R^2 = h_1 h_2$

10. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$) **[*JEE Main-2019 (April)*]**

(*a*) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

(*b*) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

(*c*) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$

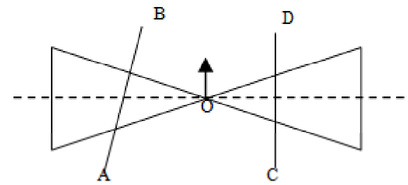
(*d*) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$

11. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product $t_1 t_2$ is: **[*JEE Main-2019 (April)*]**

(*a*) R/g (*b*) R/4g
(*c*) 2R/g (*d*) R/2g

CIRCULAR MOTION

12. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to: **[*JEE Mains-2016*]**



(*a*) turn left
(*b*) turn right.
(*c*) go straight.
(*d*) turn left and right alternately.

13. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then, **[*JEE Main-2018*]**

(*a*) $T \propto R^{\frac{n}{2}+1}$ (*b*) $T \propto R^{(n+1)/2}$
(*c*) $T \propto R^{n/2}$ (*d*) $T \propto R^{3/2}$ for any n

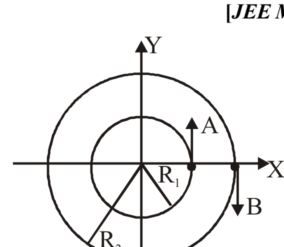
14. A body is projected at $t = 0$ with a velocity $10\ ms^{-1}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1$ s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\ ms^{-2}$ the radius of R is : **[*JEE Main-2019 (January)*]**

(*a*) 10.3 m (*b*) 2.8 m
(*c*) 2.5 m (*d*) 5.1 m

15. A particle is moving along a circular path with a constant speed of $10\ ms^{-1}$. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60^{o+} around the centre of the circle ? **[*JEE Main-2019 (January)*]**

(*a*) $10\sqrt{3}$ m/s (*b*) 0
(*c*) $10\sqrt{2}$ m/s (*d*) 10 m/s

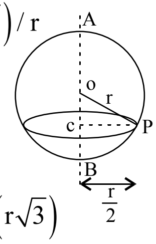
16. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure: **[*JEE Main - 2019 (January)*]**



The relative velocity $\overrightarrow{v_A} - \overrightarrow{v_B}$ at $t = \frac{\pi}{2\omega}$ is given by :

(*a*) $\omega(R_1 + R_2)\hat{i}$ (*b*) $-\omega(R_1 + R_2)\hat{i}$
(*c*) $\omega(R_2 - R_1)\hat{i}$ (*d*) $\omega(R_1 - R_2)\hat{i}$

17. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to: **[*JEE Main-2019 (April)*]**



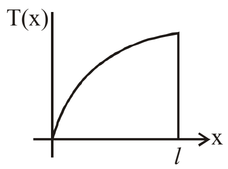
(*a*) $\left(g\sqrt{3}\right)/r$

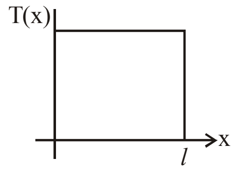
(*b*) $\frac{\sqrt{3}g}{2r}$

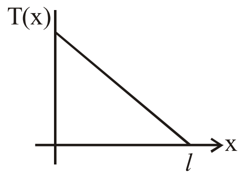
(*c*) $2g/r$

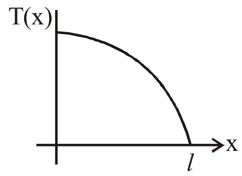
(*d*) $2g/\left(r\sqrt{3}\right)$

18. A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely? **[*JEE Main-2019 (April)*]**

(*a*) 

(*b*) 

(*c*) 

(*d*) 

19. A particle of mass m is fixed to one end of light spring having force constant k and unstretched length ℓ. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is : **[*JEE Main-2020 (January)*]**

(*a*) $\frac{m\ell\omega^2}{k - m\omega^2}$ (*b*) $\frac{m\ell\omega^2}{k - \omega m}$

(*c*) $\frac{m\ell\omega^2}{k + m\omega^2}$ (*d*) $\frac{m\ell\omega^2}{k + m\omega}$

20. A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ where ω is a constant and t is time, then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle : **[*JEE Main-2020 (January)*]**

(*a*) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(*b*) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$

(*c*) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

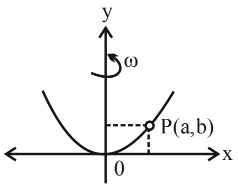
(*d*) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin.

21. A spring mass system (mass m, spring constant k and natural length ℓ) rests in equilibrium on a horizontal disc. The free and of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option **[*JEE Main-2020 (January)*]**

(*a*) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (*b*) $\frac{m\omega^2}{3k}$

(*c*) $\frac{2m\omega^2}{k}$ (*d*) $\frac{m\omega^2}{k}$

22. A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) **[*JEE Main-2020 (September)*]**



(*a*) $2\sqrt{2gC}$ (*b*) $2\sqrt{gC}$

(*c*) $\sqrt{\frac{2g}{C}}$ (*d*) $\sqrt{\frac{2gC}{ab}}$

23. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of **[*JEE Main-2020 (September)*]**

(*a*) 10^{-1} (*b*) 10^{-2}

(*c*) 10^{-4} (*d*) 10^{-3}

24. A mosquito is moving with a velocity $\vec{v} = 0.5t\hat{i}\hat{j} + 3t\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s? **[*JEE Main-2021* (*March*)]**

(*a*) $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from *y*-axis

(*b*) $\tan^{-1}\left(\frac{5}{2}\right)$ from *y*-axis

(*c*) $\tan^{-1}\left(\frac{2}{3}\right)$ from *x*-axis

(*d*) $\tan^{-1}\left(\frac{5}{2}\right)$ from *x*-axis

25. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection 8 and the maximum height attained H are respectively given by: **[*JEE Main-2021* (*March*)]**

(*a*) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$ (*b*) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$

(*c*) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$ (*d*) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

26. A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a: **[*JEE Main-2021* (*March*)]**

(*a*) hyperbola

(*b*) parabola in the direction of motion of plane

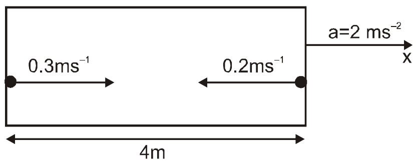
(*c*) straight line vertically down the plane

(*d*) parabola in a direction opposite to the motion of plane

JEE-ADVANCED PREVIOUS YEAR'S

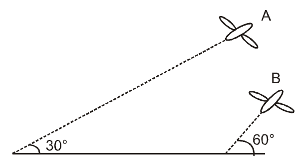
RELATIVE MOTION

1. A rocket is moving in a gravity free space with a constant acceleration of 2ms^{-2} along $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of 0.3ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is: **[*JEE Advanced-2014*]**



PROJECTILE

2. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure . The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0s, an observer in A finds B at a distance of 500m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is: **[*JEE Advanced-2014*]**



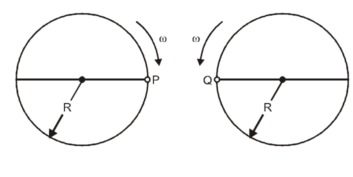
3. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is .... **[*JEE Advanced-2018*]**

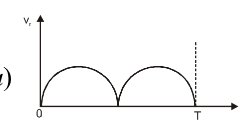
4. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1

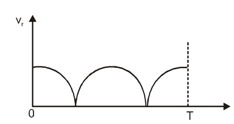
After hitting the ground, ball rebounds at the same angle q but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is______ **[*JEE Advanced-2019*]**

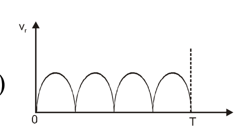
CIRCULAR MOTION

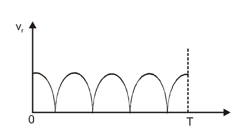
5. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane.At time t = 0 , the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. as function of times best represented by **[*IIT JEE-2012*]**



(*a*) 

(*b*) 

(*c*) 

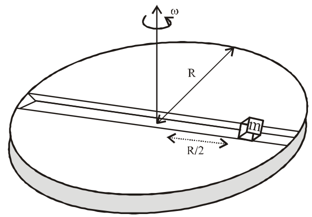
(*d*) 

Comprehension– 01 (Q.6 and 7)

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass *m* moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in inertial frame of reference is $\vec{F}_{rot} = \vec{F}_{in} + 3m((\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$,

Where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter –clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r}$ = (R/2) $\hat{i}$ at t = 0 and is constrained to move only along the slot. **[*JEE Advanced-2016*]**



6. The distance *r* of the block at time *t* is

(*a*) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (*b*) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

(*c*) $\frac{R}{2}\cos 2\,\omega t$ (*d*) $\frac{R}{2}\cos \omega t$

7. The net reaction of the disc on the block is

(*a*) $-m\omega^2 R\cos\omega t\,\hat{j} - mg\hat{k}$

(*b*) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

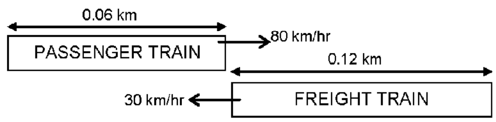
(*c*) $m\omega^2 R\sin\omega t\,\hat{j} - mg\hat{k}$

(*d*) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEAR'S

1. (*a*) -



Time taken if both moving in same direction

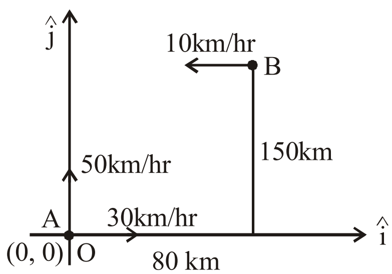
$$t_1 = \frac{\text{distance}}{\text{speed}} = \frac{0.12+0.06}{80-30} = \frac{0.18}{50}$$

Time taken if moving in opposite direction.

$$t_2 = \frac{\text{distance}}{\text{speed}} = \frac{0.12+0.06}{80+30} = \frac{0.18}{110}$$

$$\frac{t_1}{t_2} = \frac{11}{5}$$

2. (*d*) If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as :



$\vec{v}_A = (30\hat{i} + 50\hat{j})$ km/hr

$\vec{v}_B = -10\hat{i}$ km/hr

$\vec{r}_A = 0\hat{i} + 0\hat{j}$

$\vec{r}_B = (80\hat{i} + 150\hat{j})$ km

Time after which distance between them will be mini mum

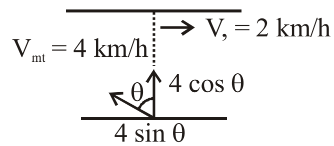
$t = -\dfrac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{|\vec{v}_{BA}|^2}$; where $\vec{r}_{BA} = (80\hat{i} + 150\hat{j})$ km

$\vec{v}_{BA} = -10\hat{i} - (30\hat{i} + 50\hat{j}) = (-40\hat{i} - 50\hat{j})$ km/hr

$$\therefore \quad t = -\frac{(80\hat{i} + 150\hat{j}) \cdot (-40\hat{i} - 50\hat{j})}{|-40\hat{i} + 50\hat{j}|^2}$$

$$= \frac{3200 + 7500}{4100} \text{ hr} = \frac{10700}{4100} \text{ hr} = 2.6 \text{ hrs}$$

3. (*d*)



For swimmer to cross the river straight

$\Rightarrow 4 \sin\theta = 2$

$\Rightarrow \sin\theta = \dfrac{1}{2} \Rightarrow \theta = 30^\circ$

So, angle with direction of river flow = 90º + θ = 120º

Option (*d*)

4. [580] $X_1 = -3t^2 + 8t + 10$

$\vec{v}_1 = (-6t + 8)\hat{i}$

$= 2\hat{i}$

$Y_2 = 5 - 8t^3$

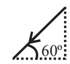
$\sqrt{v} = |\vec{v}_2 - \vec{v}_1| = |-24\hat{j} - 2\hat{i}|$

$\sqrt{v} = \sqrt{24^2 + 2^2}$

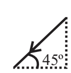
v = 580

5. (*c*) Given

$\dfrac{v^1}{v} = \tan 60^\circ$ ...(i) $\longrightarrow v$



$\dfrac{v^1}{v(1+\beta)} = \tan 45^\circ$ ...(ii) $\longrightarrow v(1+\beta)$



From (i) and (ii)

β = 0.73

6. (*c*) $\frac{dx}{dt} = y; \frac{dy}{dt} = x$

$\frac{dx}{dy} = \frac{y}{x}$

$\Rightarrow y^2 = x^2 + c$

7. (*a*) Area $\propto \pi(\text{Range})^2 \propto V^4$

$\therefore \frac{A_1}{A_2} = \left(\frac{1}{2}\right)^4$

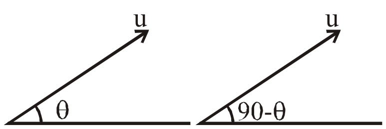
8. (*b*) $t = \frac{2 \times 2\sin 15^\circ}{g\cos 30^\circ}$

$S = 2\cos 15^\circ \times t - \frac{1}{2} g \sin 30^\circ t^2$

Put values and solve

S = 20 cm

9. (*b*)



For same range angle of projection will be θ & 90 – θ

$R = \frac{u^2 2\sin\theta\cos\theta}{g}$

$h_1 = \frac{u^2 \sin^2\theta}{g}$

$h_2 = \frac{u^2 \sin^2(90-\theta)}{g}$

$\frac{R^2}{h_1 h_2} = 16$

10. (*a*) Equation of trajectory is given as

$y = 2x - 9x^2$ ....(i)

Comparing with equation :

$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}.x^2$ ....(ii)

We get ;

$\tan\theta = 2$

$\therefore \cos\theta = \frac{1}{\sqrt{5}}$

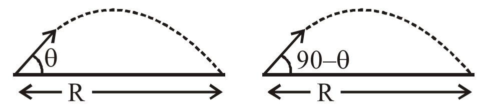
Also, $\frac{g}{2u^2\cos^2\theta} = 9$

$\Rightarrow \frac{10}{2 \times 9 \times \left(\frac{1}{\sqrt{5}}\right)^2} \times u^2$

$\Rightarrow u^2 = \frac{25}{9}$

$\Rightarrow u = \frac{5}{3}$ m/s

11. (*c*) Range will be same for time t_1 & t_2, so angles of projection will be 'θ' & '90° – θ'



$t_1 = \frac{2u\sin\theta}{g}$ $t_2 = \frac{2u\sin(90^\circ - \theta)}{g}$

and $R = \frac{u^2 \sin 2\theta}{g}$

$t_1 t_2 = \frac{4u^2 \sin\theta\cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right]$

$\frac{2R}{g}$

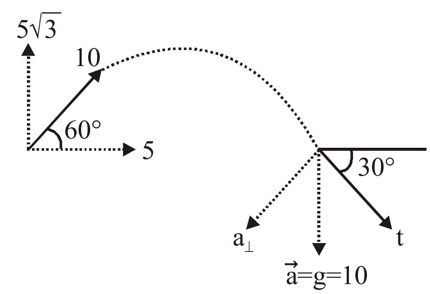
12. (*a*)

13. (*b*) $m\omega^2 R = \text{Force} \propto \frac{1}{R^n}$ $\Rightarrow \omega^2 \propto \frac{1}{R^{n+1}}$

$\Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$

time period $T = \frac{2\pi}{\omega} \propto R^{\frac{n+1}{2}}$

14. (*b*)



at t = 1

$u_x = 5, u_y = 5\sqrt{3}$

$v_y = 5\sqrt{3} - 10$; $v_x = 5$

$\tan\theta = -(2 - \sqrt{3}) \Rightarrow \theta = -30^\circ$

$R = \frac{v^2}{a_\perp} = \frac{10^2}{(10\cos 30^\circ)}$

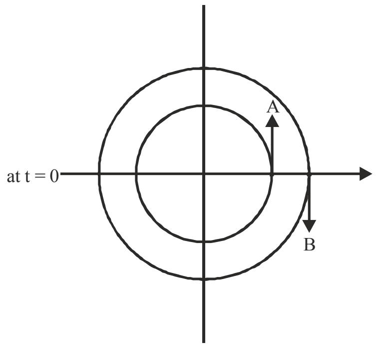
$= \frac{10}{\sqrt{3}} \times 2 = \frac{20}{\sqrt{3}}$ m

$\frac{5^2 + (10 - 5\sqrt{3})^2}{10\cos\theta} = \frac{200 - 100\sqrt{3}}{10 \times 0.965} = 2.8$ m

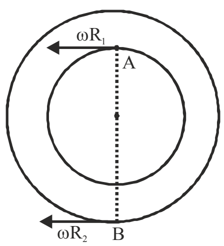
15. (*d*) $\Delta\vec{v} = 2v\sin\left(\frac{\theta}{2}\right)$

$= 2 \times 10 \times \sin(30°) \quad = 10$ m/s

16. (*c*)



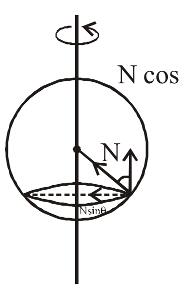
at $t = \frac{\pi}{2\omega}$



$\vec{v}_A - \vec{v}_B = -\omega R_1\hat{i} + \omega.R_2\hat{i}$

$= \omega(R_2 - R_1)\hat{i}$

17. (*d*)



$N\sin\theta = m\frac{r}{2}\omega^2$ .....(*a*)

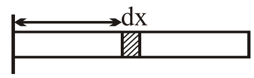
$N\cos\theta = mg$ .....(*b*)

$\tan\theta = \frac{r\omega^2}{2g}$

$$\frac{r}{2\frac{\sqrt{3}r}{2}} = \frac{r\omega^2}{2g}$$

$$\omega^2 = \frac{2g}{\sqrt{3}r}$$

18. (*d*)

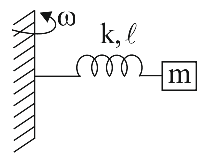


$$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell} \frac{m}{\ell} dx\omega^2 x$$

$$= \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$$

$$T = \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$$

19. (*a*)



$m\omega^2(\ell + x) = kx$

$$\left(\frac{\ell}{x} + 1\right) = \frac{k}{m\omega^2}$$

$$x = \frac{\ell m\omega^2}{k - m\omega^2}$$

20. (*d*) $\vec{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \omega\left(-\sin\omega t\hat{i} + \cos\omega t\hat{j}\right)$$

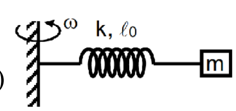
$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\hat{i} + \sin\omega t\hat{j})$$

$\vec{a} = -\omega^2\vec{r}$ $\therefore$ $\vec{a}$ is antiparallel to $\vec{r}$

$\vec{v}.\vec{r} = \omega\left(-\sin\omega t \ \cos\omega t + \cos\omega t \ \sin\omega t\right) = 0$

so $\vec{v} \perp \vec{r}$

21. (*d*)



$m\omega^2(\ell_0 + x) = kx$

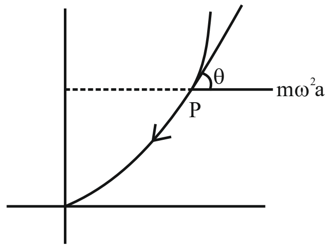
$$\left(\frac{\ell_0}{x} + 1\right) = \frac{k}{m\omega^2}$$

$$x = \frac{\ell_0 m\omega^2}{k - m\omega^2}$$

$k >> m\omega^2$

So, $\frac{x}{\ell_0}$ is equal to $\frac{m\omega^2}{k}$.

22. (*a*)



y = 4Cx²

$\Rightarrow \frac{dy}{dx} = \tan\theta = 8Cx$

At P, tanθ = 8Ca

For steady circular motion

mgsinθ = mw²acosθ

$\Rightarrow \tan\theta = \frac{\omega^2 a}{g}$

$\Rightarrow 8Ca \times g = \omega^2 \times a$

$\Rightarrow \omega = \sqrt{8gC}$

$\Rightarrow \omega = 2\sqrt{2gC}$

23. (*d*) $a = \omega^2 \times \ell$

$= \left(\frac{2\pi}{T}\right)^2 \times \ell$

$= \left(\frac{2\pi}{60}\right)^2 \times 0.1$

$= 1.1 \times 10^{-3}$ m/s²

24. (*a*) $\vec{v} = \left(0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}\right)$m / s

At $t = 2$s

$\vec{v} = (2\hat{i} + \hat{j}\hat{j} + g\hat{k})$

Direction cosine along y-axis,

$\cos\theta = \frac{(v\hat{j})}{\sqrt{g^2 + \epsilon^2 + 2^2}} = \frac{6}{\sqrt{121}} = \frac{6}{11}$

$\therefore \sin\theta = \frac{\sqrt{85}}{11}$

and $\tan\theta = \frac{\sqrt{85}}{6}$

$\therefore$ Mosquito make angle $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y -axis.

25. (*a*) $y = \alpha x - \beta x^2$

comparing with trajectory equation

$y = x\tan\theta - \frac{1}{2}\frac{\xi y^2}{x^2\cos^2\phi}$

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}\alpha$

$\Rightarrow \beta = \frac{1}{2}\frac{\theta}{2\cos^2\theta}$

$\Rightarrow u^2 = \frac{g}{2\beta 00 < 9}$

Maximum height H:

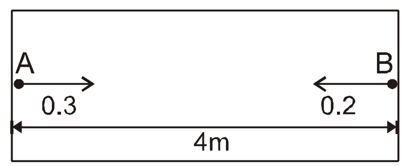
$H = \frac{2^2\sin^2\theta}{2\theta} = \frac{\theta}{2\beta\,\mathrm{coc}\,\alpha\beta}\frac{\sin^2\beta}{2\theta}$

$\Rightarrow H = \frac{\tan^2\beta}{4\beta} = \frac{\alpha^2}{4\beta}$

26. (*c*) Relative velocity of bomb w.r.t. observer in plane = 0. Bomb will fall down vertically So, it will move in straight line w.r.t. Observer.

JEE-ADVANCED PREVIOUS YEAR'S

1. [2]



consider motion of two balls with respect to rocket

Maximum distance of ball A from left wall =

$\frac{u^2}{2a} = \frac{0.3 \times 0.3}{2 \times 2} = \frac{0.09}{4} \approx 0.02$ m

so collision of two balls will take place very near to left wall

For B

$S = ut + \frac{1}{2}at^2$

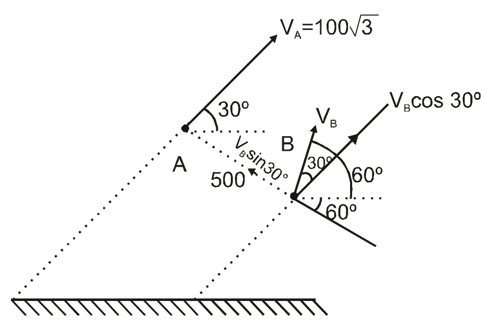
$-4 = -0.2t - \left(\frac{1}{2}\right)2t^2$

$\Rightarrow t^2 + 0.2t - 4 = 0$

$\Rightarrow t = \frac{-0.2 \pm \sqrt{0.04 + 16}}{2} = 1.9$

nearest integer = 2s

2. [**5**]



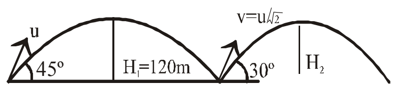
For relative motion perpendicular to line of motion of A

$V_A = 100\sqrt{3} = V_B \cos 30^\circ$

$\Rightarrow V_B = 100$ m/s

$$t_0 = \frac{50}{V_B \sin 30^\circ} = \frac{500}{200\times\frac{1}{2}} = 5\,\text{sec}$$

3. [30.00]



$$H_1 = \frac{u^2 \sin^2 45}{2g} = 120$$

$$\Rightarrow \frac{u^2}{4g} = 120 \qquad \text{......(i)}$$

when half of kinetic energy is lost $v = \frac{u}{\sqrt{2}}$

$$H_2 = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30}{2g} = \frac{u^2}{16g} \qquad \text{......(ii)}$$

from (i) & (ii)

$$H_2 = \frac{H_1}{4} = 30\text{m} \text{ on } 30.00$$

4. [4.00] Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

Total time taken $= t_1 + t_2 + t_3 + ...$

$$= t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + ...$$

$$\text{Total time} = \frac{t_1}{1-\frac{1}{\alpha}}$$

Total displacement $= v_1t_1 + v_2t_2 + ...l$

$$= v_1t_1 + \frac{v_1}{\alpha}.\frac{t_1}{\alpha} + ...$$

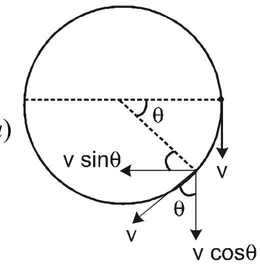
$$= \frac{v_1t_1}{1-\frac{1}{\alpha^2}}$$

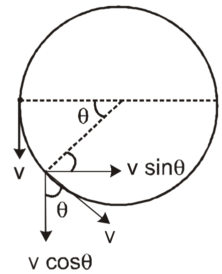
On solving

$$< v > = \frac{v_1\alpha}{\alpha+1} = 0.8v_1$$

$\alpha = 4.00$

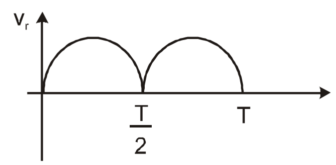
5. (*a*)





$v_r = |2 v \sin \theta)|$

$= |2v \sin \omega t)|$



6. (*b*) $a = \omega^2 r$

$$\Rightarrow \int_0^v v\,dv = \omega^2 \int_{R/2}^r r\,dr \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}}$$

$$\& \int_{R/2}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \omega\int_0^t dt$$

$$\Rightarrow r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

Hence, (*b*)

7. (*d*) $\vec{F}_{rot} = -m\omega^2 r\hat{i} + 2mv_{rot}\omega(-\hat{j}) + m\omega^2 r\hat{i}$

$$= 2mv_{rot}\omega(-\hat{j})$$

$$= 2m\frac{\omega R}{4}\left(e^{\omega t} - e^{-\omega t}\right)\omega(-\hat{j})$$

$$\vec{F}_{net} = -\vec{F}_{rot} + mg\hat{k} = \frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$

Hence, (*d*)