Chapter 5

Torsion of Shafts and Springs, Columns

CHAPTER HIGHLIGHTS

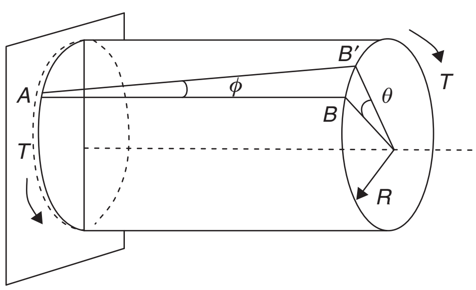
- *Torsion of Circular Shafts*
- *Torsional Equation*
- *Polar Modulus and Stiffness*
- *Comparison of stiffness*
- *Shaft Combinations*
- *Shafts in Series*
- *Torsion of a Tapering Shaft*
- *Helical Springs (Closed Coiled)*
- *Springs in Parallel*
- *Theory of Columns and Struts*
- *Analysis of the Critical Load for Long Column by Euler's Formula*
- *Effective Lengths for Different End Conditions of Column*
- *Rankine's Formula*

Torsion of Circular Shafts

When a moment is applied on a shaft about its axis, it is twisted about its axis. The shaft is then said to be in torsion. The applied moment is called twisting moment or torsional moment. Shafts transmitting power, springs, etc. are examples.

In actual practice a member/shaft may be subjected to combined effect of torsion, axial forces and bending moments. It is said to be under pure torsion if only torsional moments are acting.

Torsional Equation



When a torsional moment is applied on the shaft the effects are:

1. There is an angular displacement of a cross-section of one end with respect to the other end.
2. Shearing stresses are set up on any cross-section perpendicular to the axis.

From the previous figure, Line AB is twisted to a position AB'. The surface of the shaft is moved by angle ϕ. The cross section at B is twisted by an angle θ.

Here,

ϕ = Shear strain

θ = Angle of twist

It can be seen that

$R\theta = L\phi$

But $\phi = \dfrac{q_s}{G}$

Where q_s = Shear stress at surface

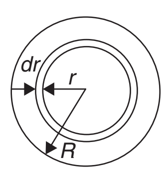
G = Modulus of rigidity

$$\therefore \quad R\theta = L\frac{q_s}{G}$$

or
$$\boxed{\frac{q_s}{R} = \frac{G\theta}{L}}$$

$\dfrac{G\theta}{L}$ being constant, it can be seen that shear stress is directly proportional to radius.

$$\therefore \quad \frac{q_s}{R} = \frac{q}{r}$$



Considering an elemental ring of thickness dr shearing resistance of the ring = $q2\pi r$dr

$\therefore$ Total resisting moment $T = \int_O^R \frac{q_s}{R} 2\pi r^3 dr$

Since $q = \frac{q_s}{R} r$

$$T = \frac{q_s \pi R^4}{R2} = q_s \frac{\pi R^3}{2}$$

$= \frac{q_s}{R} J$; where $J = \frac{\pi R^4}{2}$, the polar moment of inertia

or
$$\frac{q_s}{R} = \frac{T}{J}$$

$\therefore$
$$\boxed{\frac{q_s}{R} = \frac{T}{J} = \frac{G\theta}{L}}$$

This may be compared with the equation for bending moment

$$\frac{f}{y} = \frac{M}{I} = \frac{E}{R}$$

Polar Modulus and Stiffness

$$T = q_s \frac{J}{R} = q_s Z_p$$

Zp = Polar modulus

Also,

$$\frac{T}{J} = \frac{G\theta}{L}$$

or
$$GJ = \frac{TL}{\theta}$$

GJ is called the torsional rigidity or stiffness. It is the torque required for a twist of one radian per unit length of the shaft.

For hollow shafts,

$$J = \frac{\pi}{2}(R^4 - r^4)$$

$$= \frac{\pi}{32}(D^4 - d^4)$$

Power Transmitted

Power transmitted by a shaft $= \frac{2\pi NT}{60}$ NM/s or W

where $N = rpm$

Consider a solid shaft and hollow shaft of the same material, same length and same weight. Torque carrying capacity of both the shafts is to be compared.

Since mass remain the same

$\rho v_h = \rho v_s$

$\Rightarrow \quad v_h = v_s$

$\frac{\pi}{4}(d_o^2 - d_i^2) \times \ell = \frac{\pi}{4} d^2 \times \ell$, ℓ being the same

$$d_o^2 - d_i^2 = d^2 \quad (1)$$

But, $\frac{T}{J} = \frac{f_s}{R}$

Torque carrying capacity depends on $\frac{f_s J}{R}$. Since f_s is the same for both, it depends on $\frac{J}{R}$, the polar modulus.

$$T_h \propto \frac{J_h}{R} = \frac{\pi}{32} \frac{(d_o^4 - d_i^4)}{do} \times 2 = \frac{\pi}{16} \frac{d_o^4\left[1 - \left(\frac{d_i}{d_o}\right)^4\right]}{d_o}$$

$$T_h \propto \frac{\pi}{16} d_o^3 [1 - k^4] - (1); \text{ where } k = \frac{d_i}{d_o}$$

$$T_s \propto \frac{\pi}{16} d^3$$

$$\therefore \frac{T_h}{T_s} = \frac{do^3[1-k^4]}{d^3} = \frac{do^3(1-k^4)}{(do^2 - di^2)^{\frac{3}{2}}}$$

$$= \frac{do^3[1-k^4]}{do^3[1-k^2]^{\frac{3}{2}}}$$

$$\Rightarrow \frac{T_h}{T_s} = \frac{1-k^4}{(1-k^2)^{\frac{3}{2}}} = \frac{1-k^4}{(1-k^2)\sqrt{1-k^2}}$$

$$= \frac{(1+k^2)(1-k^2)}{(1-k^2)\sqrt{1-k^2}} = \frac{1+k^2}{\sqrt{1-k^2}}$$

$$\therefore \frac{T_h}{T_s} > 1$$

Therefore, torque carrying capacity of hollow shaft is more than that of solid shaft provided,

1. they are of the same material
2. they are of the same mass
3. they are of the same length

$$\boxed{\frac{T_h}{T_s} = \frac{1+k^2}{\sqrt{1-k^2}}}$$

Comparison of Stiffness

A solid shaft and a hollow shaft of same material, same mass and same length may be considered. It is required to compare their stiffness.

We have the equation $\frac{T}{J} = \frac{N\theta}{\ell}$

$\frac{T}{\theta}$ = Stiffness $= \frac{NJ}{\ell}$; 'N' and 'ℓ' being constant, Stiffness $\propto J$

(Stiffness)$_{\text{solid}} \propto J_s$

(Stiffness)$_{\text{hollow}} \propto J_H$

$$\therefore \quad \frac{(\text{stiffnesess})_{\text{hollow}}}{(\text{stiffness})_{\text{solid}}} = \frac{J_u}{J_s} = \frac{\frac{\pi}{64} d_o^4 (1-k^4)}{\frac{\pi}{64} d^4}$$

$$= \frac{d_o^4(1-k^4)}{d^4}$$

$\therefore$ But, $d^2 = d_o^2 - d_i^2$

$$\therefore \quad \frac{(\text{stiffness})_{\text{hollow}}}{(\text{stiffness})_{\text{solid}}} = \frac{d_o^4(1-k^4)}{(d_o^2 - d_i^2)^2}$$

$$= \frac{d_o^4(1-k^4)}{d_o^4[1-k^2]^2}$$

$$= \frac{(1-k^2)(1+k^2)}{(1-k^2)^2} = \frac{1+k^2}{1-k^2} > 1$$

Therefore, hollow shaft is more stiff when compared to solid shaft provided,

1. they are of same mass
2. they are of same length
3. they are of same material

Shaft Combinations

A shaft may consists of various small shafts of different cross-sectional areas or different materials.

The shaft combination may be

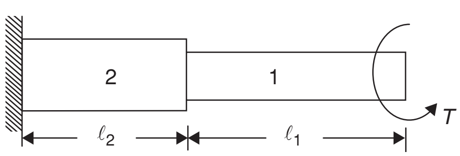
1. Shafts in series or stepped shafts
2. Shafts in parallel or composite shafts
3. Indeterminate shafts

In analyzing these shafts some points to be noted are:

1. At fixed end a torque is developed to keep the shaft in equilibrium
2. At the ends of any portion the torque developed are equal and opposite
3. At common point between two portions angle of twist remain same

Shafts in Series

One end fixed and torque applied at the free end.



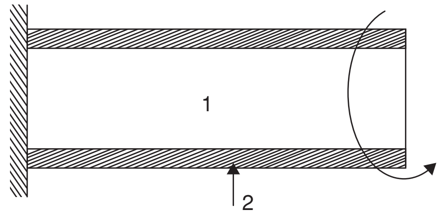
Here torque transmitted by each shaft is same i.e., $T_1 = T_2 = T$

Angle of twist

$$\theta = \theta_1 + \theta_2$$

$$= \frac{T}{J}\left(\frac{\ell_1}{G_1} + \frac{\ell_2}{G_2}\right)$$

Shafts in Parallel

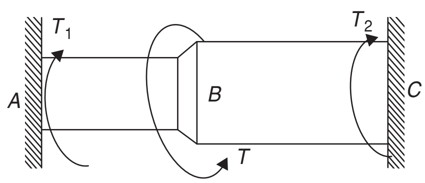


In this case, angle of twist is same for each shaft.

That is, $\theta = \theta_1 + \theta_2$ and torque, $T = T_1 + T_2$

Indeterminate Shafts

The shaft is fixed at both ends and torque is applied at a common point.



Torque T is applied at the point B. Torque T_1 and T_2 are developed at the ends.

Here, $T_1 + T_2 = T$

and $\theta_{1B} = \theta_{2B}$

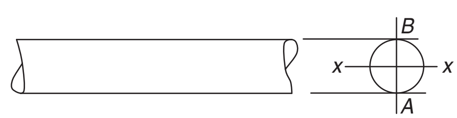
Torsion of a Tapering Shaft

It can be shown that the angle of twist in a tapering shaft of length L and end radii r_1 and r_2 when a constant torque, T is acting, is

$$\theta = \frac{2TL}{3\pi G} \frac{r_1^2 + r_1 r_2 + r_2^2}{r_1^3 r_2^3}$$

Combined Bending and Torsion

A shaft is generally subjected to torsional shear stresses. But due to self-weight, eccentric thrust, etc., there may be bending moments also.



Bending stresses and torsional shear stresses are maximum at the extreme fibres A and B.

Bending stress, $f = \frac{M}{I} y = \frac{32\text{ M}}{\pi d^3}$

Shear stress, $q = \frac{T}{J}R = \frac{16T}{\pi d^3}$

Maximum principal stress

$$p_1 = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$$

$$= \frac{1}{2}\left(\frac{32\text{ M}}{\pi d^3}\right) + \sqrt{\frac{1}{4}\left(\frac{32\text{ M}}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

Let M_e be the equivalent bending moment

then $p_{\max} = \frac{32M_e}{\pi d^3} = p_1$

$\therefore \quad M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$

Maximum shear stress

$$q_{\max} = \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$$

$$= \sqrt{\frac{1}{4}\left(\frac{32\text{ M}}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

If T_e is the equivalent twisting moment, then

$$q_{\max} = \frac{16T_e}{\pi d^3}$$

$\therefore \quad T_e = \sqrt{M^2 + T^2}.$

Strain Energy in Torsion

Strain energy, $U = \frac{1}{2}T\theta$

It can be shown that $U = \frac{q_s^{\,2}}{4G} \times \text{volume}$

Solved Examples

Example 1: *A* 2 m long steel drive shaft with 6 cm outer and 4 cm inner diameter transmits 150 kw at 1,500 rpm.

Taking modulus of rigidity

$$G = 8 \times 10^6 \text{ N/cm}^2$$

determine maximum shear stress.

Solution: Power transmitted $= \frac{2\pi NT}{60}$ W

That is, $150 \times 10^3 = \frac{2\pi \times 1500 \times T}{60}$

$\Rightarrow \quad T = 954.93$ Nm

$= 95{,}493$ Ncm

$$\frac{T}{J} = \frac{q_s}{R}$$

$$J = \frac{\pi}{32}(D^4 - d^4)$$

$$= \frac{\pi}{32}(6^4 - 4^4)$$

$$= 102.1 \text{ cm}^4$$

$\therefore \quad \frac{T}{J} = \frac{95493}{102.1} = 935.29$

Maximum shear stress =

$$q = \frac{TR}{J} = 935.29 \times 3 \text{ N/cm}^2$$

$$= 2805.87 \text{ N/cm}^2.$$

Example 2: In the above problem find angle of twist of the shaft.

Solution:

$$\frac{T}{J} = \frac{G\theta}{L}$$

Angle of twist $\theta = \frac{TL}{JG} = 935.29 \times \frac{200}{8 \times 10^6}$

$= 0.0234$ radian

$= 1.34°\left(1 \text{ rad} = \frac{180°}{\pi}\right)$

Example 3: A hollow shaft is to transmit a torque 3500 Nm.The diametral ratio of the hollow shaft is 0.5. The permissible shear stress of the material is 80 MPa. The outside diameter of the shaft is

(A) 28 m (B) 31 mm
(C) 25.5 mm (D) 35 mm

Solution:

$$T = \frac{f_s \pi d_o^{\,3}(1 - k^4)}{16}$$

$$700 \times 1{,}000 = \frac{80 \times \pi d_o^{\,3}(1 - 0.5^4)}{16}$$

$\Rightarrow \quad 700{,}000 = \pi d_o^{\,3}\,[1 - 0.0625]$

$\Rightarrow \quad d_o = 61.94 = 62$ mm

$\therefore \quad d_i = 31$ mm

NOTES

1. Torque carrying capacity of a shaft is represented by the polar modulus $\left(\frac{J}{R}\right)$.

   Stiffness of the shaft is represented by the polar moment of inertia (J).
2. Torque carried by solid shaft of diameter 'd',

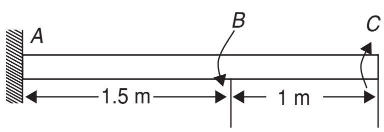
$$T = f_s \frac{\pi d^3}{16}$$

   Torque carried by hollow shaft

$$T = f_s \frac{\pi do^3}{16}(1-k^4)$$

   where k – diameter ratio $\frac{di}{do}$ for the hollow shaft.

Example 4: A 2.5 m long steel shaft of circular cross-section is subjected to torques as shown in the figure.



Torque at B = 500 Nm (anticlockwise)
Torque at C = 1,000 Nm (clockwise)
Determine the diameter of the shaft if permissible shear stress is 6000 kN/m^2

Modulus of rigidity = 80 GN /m^2

Solution:

$$\frac{T}{J} = \frac{q_s}{R}$$

or $q_s = \dfrac{16T}{\pi D^3}$

Maximum value of torque is to be considered for selecting the diameter.
For section BC, torque is 1,000 Nm
For section AB, torque is 1,000 – 500 = 500 Nm

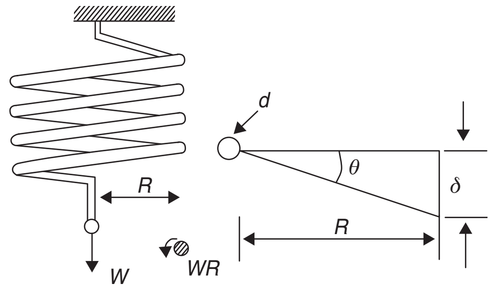
$$D^3 = \frac{16T}{\pi \times q_s}$$

$$= \frac{16 \times 1000}{\pi \times 6000 \times 10^3} = 0.849 \times 10^{-3}$$

$\therefore D = 0.0947$ m

$= 9.47$ cm

Helical Springs (Closed Coiled)



When a load W is attached at the end of the spring the torque, T on every section of the rod of the spring will be WR.

But, $T = q_s \dfrac{\pi r^3}{2}$

$$= q_s \frac{\pi d^3}{16}$$

That is, $WR = q_s \times \dfrac{\pi d^3}{16}$

$\therefore$ Shear stress,

$$\boxed{q_s = 16\frac{WR}{\pi d^3}}$$

Length of the spring = $n 2\pi R$

Strain Energy in Springs

$$= \frac{q_s^{\,2}}{4G} \times \text{volume}$$

$$= \frac{32W^2R^3n}{Gd^4}$$

If vertical displacement due to the load is δ,

Work done $= \dfrac{1}{2}W\delta$

Equating with strain energy

$$\frac{1}{2}W\delta = \frac{32W^2R^3n}{Gd^4}$$

$$\boxed{\delta = \frac{64WR^n n}{Gd^4}} = R\theta$$

Stiffness of the spring or spring constant

$$= \frac{W}{\delta} = \frac{Gd^4}{64R^3n}$$

Wahl stress factor (K) is given by the formula

$$K = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

Where c = spring index $= \dfrac{D}{d}$

Shear stress after considering the stress factor becomes, $q_s = \dfrac{16WRk}{\pi d^3}$.

Springs in Series

In series, total extension is equal to sum of individual extension of the springs.

That is, $\delta = \delta_1 + \delta_2$

$$\therefore \quad \frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2}$$

where k = stiffness of the springs

From the above, $k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$

Springs in Parallel

Here,

$$F = F_1 + F_2$$

$\therefore k_{eq} \cdot \delta = k_1\delta + k_2\delta$

$[\because \delta = \delta_1 = \delta_2]$

$\therefore k_{eq} = k_1 + k_2$

Example 5: A close coiled helical spring of 10 cm mean diameter carries an axial load of 80 N. The spring is having 20 turns of 8 mm diameter wire. Find shear stress developed and deflection. (Modulus of rigidity $= 8.4 \times 10^6$ N/cm^2)

Solution:

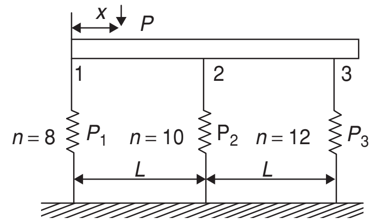
$$q_s = \frac{16T}{\pi d^3} = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 80 \times 5}{\pi (0.8)^3}$$

$$= 3981 \text{ N/cm}^2$$

$$\delta = \frac{64WR^3 n}{8.4 \times 10^6 \times (0.8)^4} = 3.72 \text{ cm.}$$

Example 6: 3 springs of same wire diameter are vertically arranged in a line over which a stiff bar of negligible weight is placed. A load P is acting in between the first two springs. Find the distance of the load from the first spring if the springs are equally spaced and number of turns are and 8, 10 and 12, respectively. Mean radii are in the proportion 10 : 12 : 15.



Solution: Let loads on springs are P_1, P_2 and P_3.
All the springs have same deflection δ.

$$\delta = \frac{64P_1R^3 \times 8}{Gd^4} = \frac{64P_2(1.2R)^3 \times 10}{Gd^4}$$

$$= \frac{64P_3(1.5R)^3 \times 12}{Gd^4}$$

That is, $8P_1 = 1.2^3 \times 10P_2 = 1.5^3 \times 12P_3$

$\therefore P_1 = 5.06P_3$

$P_2 = 2.34P_3$

Taking algebraic sum of the moments about the point where P acts.

$$P_1x = P_2(L - x) + P_3(2L - x)$$

$$5.06\, P_3x = 2.34\, P_3(L - x) + P_3(2L - x)$$

i.e.,

$$5.06x = 2.34\,(L - x) + (2L - x)$$

$$5.06x = (2.34 + 2)L - (2.34 + 1)x$$

$$8.4x = 4.34L$$

$$x = 0.517L$$

Theory of Columns and Struts

Euler's theory of columns: A column is a compressive member that under gradually increasing loads fails by buckling at loads considerably less than those required to cause failure by crushing. Long column fails by buckling, intermediate by a combination of crushing and buckling, short compression blocks by crushing.

An ideal column is homogenous that is initially straight and subjected to axial compressive loads. However, actual columns have small imperfections of material and fabrication as well as unavoidable accidental eccentricities of load. The initial crookedness of the column, together with the placement of the load, causes an intermediate eccentricity 'e' with respect to the centroid of a typical section.

Compression member of a truss is called strut. Both columns and struts are subjected mainly to compressive forces and their analysis can be treated together.

Mainly there are three types of columns:

1. Centrally loaded short columns
2. Eccentrically loaded masonry columns
3. Elastic long columns

Eccentrically Loaded Columns

When a column is subjected to an eccentric load, apart from the main compressive stress a bending moment also is resulted causing tension on one side and compression on the other side.

For a rectangular column loaded eccentrically with respect to only one axis,

$$f_{\min} = \frac{P}{A} - \frac{P_e \frac{b}{2}}{\mathrm{I}_y}$$

$$= \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$f_{\max} = \frac{P}{A}\left(1 + \frac{6_e}{b}\right)$$

If eccentricity is with respect to both x and y axis,

$$f_{\min} = \frac{P}{A}\left(1 - \frac{6e_1}{b} - \frac{6e_2}{d}\right)$$

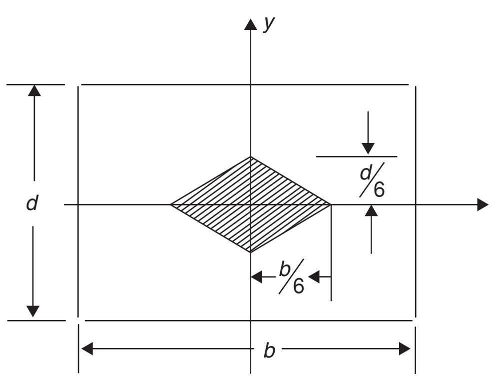
$$f_{\max} = \frac{P}{A}\left(1 + \frac{6e_1}{b} - \frac{6e_2}{d}\right)$$

For no tension,
$F_{\min}$ should not be negative.

$$\therefore \quad f_{\min} = 0 = 1 - \frac{6e_1}{b} - \frac{6e_2}{d}$$

when $e_1 = 0,\ e_2 = \dfrac{d}{6}$

when $e_2 = 0,\ e_1 = \dfrac{b}{6}$



If load acts in the shaded area, tension will not be developed. This area is known as kern of the section.

For circular sections,

$$f_{\max} = \frac{P}{A}\left(1 + \frac{8s}{d}\right)$$

$$f_{\min} = \frac{P}{A}\left(1 - \frac{8s}{d}\right)$$

where s = distance from centre

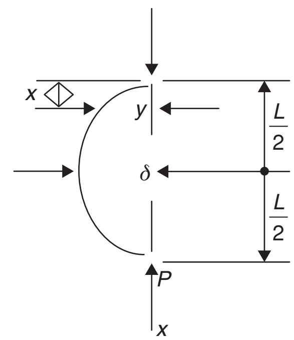
For no tension, $S = \dfrac{d}{8}$

$\therefore$ Kern of a circular section is a circle of radius $\dfrac{d}{8}$.

Analysis of the Critical Load for Long Column by Euler's Formula

The analysis is based on the differential equations of the elastic curve.

$$EI\frac{d^2 y}{dx^2} = M = P(-y) = -Py$$



$$M\frac{d^2 x}{dt^2} = -kx$$

for which the general equations are

$$X = C_1 \sin\left(t\sqrt{\frac{k}{m}}\right) + C_2 \cos\left(t\sqrt{\frac{k}{m}}\right)$$

$$Y = C_1 \sin\left(x\sqrt{\frac{p}{EI}} + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right)\right)$$

Putting $y = 0$ at $x = 0$
we get $C_2 = 0$

Again putting $y = 0$ at $x = L$ we get $0 = C_1 \sin\left(L\sqrt{\dfrac{P}{EI}}\right)$
$C_1 = n\pi$

$$P = n^2 \frac{EIx^2}{L^2}$$

Special cases

1. For fixed end columns

$$P_{Cr} = \frac{4\pi^2 EI}{L^2}$$

2. One end fixed and the other hinged

$$P_{cr} = \frac{2\pi^2 EI}{L^2}$$

3. Both ends hinged $P_{Cr} = \dfrac{\pi^2 EI}{L^2}$

4. One end fixed and the other end free $P_{Cr} = \dfrac{\pi^2 EI}{4L^2}$

Limitations

1. The value of I in the column formulas is always with the least moment of inertia of the cross-section. Any tendency to buckle, therefore occurs about the least axis of inertia of the cross-section.
2. Euler's formula also shows that the critical load that causes buckling depends not only on the elastic modulus of the material, but also with dimensions and modulus of elasticity.
3. In order for Euler's formula to be applicable, the stress accompanying the bending that occurs during buckling must not exceed the proportional limit.
4. Euler's formula determines critical loads but not working loads.

All the above cases may be represented by a common expression

$$P_{Cr} = \frac{\pi^2 EI}{L_e^2}$$

where L_e = effective length.

So effective length for fixed end column $= \dfrac{L}{2}$

For one end fixed and other end free it is $= 2L$ etc.

Effective Lengths for Different End Conditions of Column

1. Fixed end columns

$$L_e = \frac{L}{2}$$

2. One end fixed and other end hinged

$$L_e = \frac{L}{\sqrt{2}}$$

3. Both ends hinged

$$L_e = L$$

4. One end fixed and other end free

$$L_e = 2L$$

Rankine's Formula

Euler's formula holds good only for long columns with higher values of slenderness ratios $\left(\dfrac{L}{K}\right)$ where K = radius of gyration.

Rankine's formula is one of the empirical formulae which take care of entire range of slenderness ratios.

It is based on the relationship between actual crippling load (P_{Cr}), crushing load (P_c) and Euler's buckling load (P_E). The relationship established is

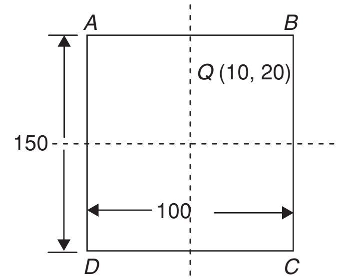
$$\frac{1}{P_{Cr}} = \frac{1}{P_c} + \frac{1}{P_E}$$

Rankine's formula is derived from the above. According to this,

$$P_{Cr} = \frac{f_c A}{1 + a\left(\dfrac{L}{K}\right)^2}$$

where $a = \dfrac{f_c}{\pi^2 E}$, the Rankine's constant and f_c = crushing strength

Example 7:



(Dimensions in mm)

The above figure shows section of a pillar. A load of 100 kN was applied at point Q. Find the stress developed at point D.

Solution:

$$f = \frac{P}{A} + \frac{Pe_2}{I_x} y + \frac{Pe_1}{I_y} x,$$

where P = load

A = sectional area

$$= \frac{100{,}000}{150 \times 100} + \frac{100{,}000 \times 20y}{\frac{1}{12} \times 100 \times 150^3} + \frac{100{,}000 \times 10x}{\frac{1}{12} \times 150 \times 100^3}$$

$$= \frac{100{,}000}{150 \times 100}\left[1 + \frac{12 \times 20}{150^2} y + \frac{12 \times 10}{100^2} x\right]$$

$= 6.667\ [1 - 0.8 - 0.6]$

as $y = -75$ and $x = -50$

$= -2.67\ \text{N/mm}^2 = 2.67\ \text{N/mm}^2$ (Tensile).

Example 8: A hollow cylindrical column carries an axial load of 1,000 kN. Length of the column is 3 m and ends are fixed. The internal diameter is half of outside diameter. Find the diameter of the column using the following data.

$$f_c = 550 \text{ N/mm}^2$$

Factor of safety = 4

Rankine's constant for both ends hinged $= \dfrac{1}{1600}$.

Solution:

Effective Length $= \dfrac{3}{2} = 1.5$ m

$$D = 2d$$

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$K^2 = \frac{I}{A}$$

$$= \frac{\dfrac{(D^4 - d^4)}{64}}{\dfrac{(D^2 - d^2)}{4}}$$

$$= \frac{1}{16}(D^2 + d^2)$$

$$= \frac{1}{16}D^2\left[1 + \left(\frac{1}{2}\right)^2\right] = 0.078125D^2$$

$$K = 0.2795\text{D mm}$$

$$\frac{L}{K} = \frac{1500}{0.2795D} = \frac{5366.56}{D}$$

Working load = 1000 kN

Critical load = 1000 × FS

$$(P_{Cr}) = 4000 \text{ kN}$$

using Rankine's formula,

$$P_{Cr} = \frac{F_c A}{1 + a\left(\dfrac{L}{K}\right)^2}$$

That is, $4000 \times 10^3 = \dfrac{550 \times \dfrac{\pi}{4}(D^2 - d^2)}{1 + \dfrac{1}{1600} \times \left(\dfrac{5366.56}{D}\right)^2}$

$$= \frac{550 \times \dfrac{\pi}{4} D^2\left[1 - \left(\dfrac{1}{2}\right)^2\right]}{1 + \dfrac{18000}{D^2}}$$

$$= \frac{323.98D^2 \times D^2}{D^2 + 18,000}$$

That is, $12.35 \times 10^3 = \dfrac{D^4}{D^2 + 18,000}$

$\Rightarrow$ $12.35 \times 10^3\, D^2 + 222,300 \times 10^3 = D^4$

$\Rightarrow$ $D^4 - 12350\, D^2 - 2.223 \times 10^8 = 0$

$\Rightarrow$ $D^2 = \dfrac{12350 + \sqrt{12350^2 + 4 \times 2.223 \times 10^8}}{2}$

$\Rightarrow$ $D^2 = 22312.86$

$\Rightarrow$ $D = 149.37$ mm

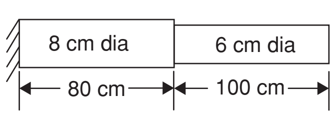
$\therefore$ $d = 74.69$ mm

Exercises

Practice Problems I

Direction for questions 1 to 15: Select the correct alternative from the given choices.

Direction for questions 1 and 2: A stepped shaft is made of brass and steel as shown in the figure. The brass end is fixed and the steel end is free.



The following values may be taken for steel and brass.

	Steel	Brass
J	127 cm^4	402 cm^4
q_s	100 N/mm^2	80 N/mm^2
G	80k N/mm^2	40k N/mm^2

1. Find the maximum torque that can be applied at the free end.
   (A) 3244 Nm (B) 2962 Nm
   (C) 4233 Nm (D) 3050 Nm
2. Find the angle of twist at the free end.
   (A) 3.05° (B) 3.59°
   (C) 2.56° (D) 3.18°
3. A solid circular shaft transmits 150 kW at 200 rpm. The twist in the shaft length of 2 m is limited to 1°. Maximum shear stress is 60 N/mm^2 and shear modulus G is 1×10^5 N/mm^2.
   The diameter of the shaft is
   (A) 85.77 mm (B) 84.7 mm
   (C) 95.62 mm (D) 90.62 mm
4. A solid shaft transmits a power of 6,000 watts at 1,500 rpm. To transmit the same power a hollow shaft of same material with diametral ratio 0.6 is chosen. The

percentage of material saving is (permissible shear stress of material is 80 MPa)

(A) 20.83% (B) 26.25%
(C) 29.88% (D) 30.25%

5. A closed-coiled spring to have a stiffness of 1 N/cm under a maximum load of 5 N and a maximum shearing stress of 12,500 N/cm^2. The length of the spring when coils are touching is to be 4.5 cm. Modulus of rigidity $G = 42 \times 10^5$ N/cm^2.

Wire diameter of the spring is

(A) 2.2 mm (B) 1.1 mm
(C) 1.8 mm (D) 1.6 mm

6. A closed-coiled helical spring made of 1 cm diameter steel wire has 15 coils of 10 cm mean diameter. Modulus of rigidity is $G = 8.16 \times 10^6$ N/cm^2. When the spring is subjected to an axial load of 100 N, the stiffness of the spring is

(A) 68 N/cm (B) 72 N/cm
(C) 76 N/cm (D) 66 N/cm

7. A solid shaft of diameter D carries a twisting moment that develops maximum shear stress f. If the shaft is replaced by a hollow shaft of outside diameter D and inside diameter $\frac{D}{2}$, then maximum shear stress will be

(A) $1.143f$ (B) $1.330f$
(C) $2f$ (D) $1.067f$

8. A hollow cylindrical shaft used as a column 4.5 m long with both ends fixed has internal diameter 0.6 times the external diameter. The column is to carry an axial load of 250 kN.

Take $f_c = 550$ N/mm^2 and Rankine's constant $a = \frac{1}{1600}$

The relation between radius of gyration and outer diameter of column is

(A) 0.392 D (B) 0.2915 D
(C) 0.1763 D (D) 0.3535 D

9. A strut is made of a bar of circular section and 5 m long which pin jointed at both ends. When the bar is used as a simple supported beam gives a midspan deflection of 10 mm with a load of 10 N at centre. Critical load of the strut is

(A) 1,136 N (B) 1,226 N
(C) 1,029 N (D) 1,185 N

10. An m. s column is built up using an I-section and m. s plates, the cross-section of which is shown in the figure. It is 4 m long and both ends are hinged. Given that $f_c = 315$ N/mm^2.

Rankine's constant $= \frac{1}{7500}$

For I-section,

c.s. Area = 3,671 mm^2

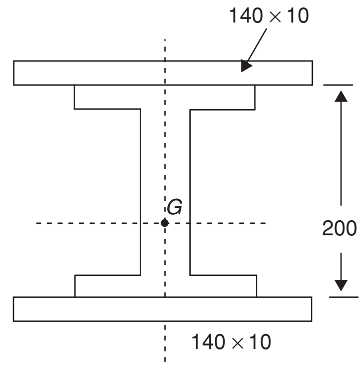
$I_{xx} = 26.245 \times 10^6$ mm^4

$I_{yy} = 3.288 \times 10^6$ mm^4

$K_x = 84.6$ mm

$K_y = 29.9$ mm

The safe axial load for the column, assuming a factor of safety of 4 and using Rankine's formula, is



(A) 184.86 kN (B) 196.84 kN
(C) 172.92 kN (D) 176.76 kN

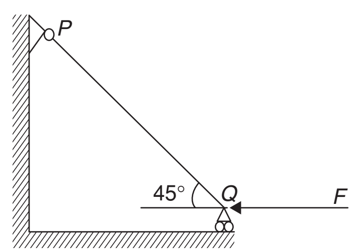
11. At a certain cross-section, a shaft of 100 mm diameter is subjected to a bending moment of 4 kNm and a twisting moment of 8 kNm. Maximum principal stress induced in the section is

(A) 72.8 N/mm^2 (B) 6.17 N/mm^2
(C) 65.9 N/mm^2 (D) 68.6 N/mm^2

12. A solid shaft of 150 mm diameter is transmitting a torque of 20 kNm. At the same time it is subjected to a bending moment of 10 kNm and an axial thrust of 150 kN. Maximum shear stress developed will be

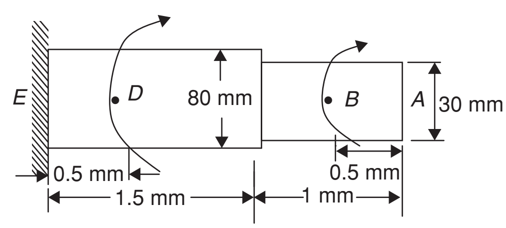
(A) 35.71 N/mm^2 (B) 37.62 N/mm^2
(C) 34.83 N/mm^2 (D) 38.13 N/mm^2

13. A rod PQ of 60 mm diameter and 2.5 m long is fitted both ends hinged as shown in the figure. Young's modulus $= 2 \times 10^5$ N/mm^2. The minimum force F under which the rod will buckle is



(A) 277 kN (B) 285 kN
(C) 281 kN (D) 296 kN

14. The stepped shaft shown in the figure is subjected torques of 150 Nm and 300 Nm at points B and D, respectively. Modulus of rigidity is 80 kN/mm^2. The rotation of the free end will be



(A) 0.742° (B) 0.951°
(C) 0.633° (D) 0.872°

15. A solid shaft of 210 mm diameter is to be replaced by a hollow shaft of external diameter D and internal diameter $\frac{D}{2}$. If same power is to be transmitted at the same speed and at same level of shear stress, the external diameter of hollow shaft is
(A) 218.6 mm
(B) 204.3 mm
(C) 216.4 mm
(D) 214.6 mm

Practice Problems 2

Direction for questions 1 to 10: Select the correct alternative from the given choices.

1. In a shaft of 5 m length a stress of 75 MPa was developed. Find the diameter of the shaft if the angle of twist was 3°. Take G = 83 GPa
(A) 16.24 cm (B) 15.89 cm
(C) 17.26 cm (D) 18.22 cm

2. In the above problem find the torque developed.
(A) 65.8×10^6 Nmm (B) 79.5×10^6 Nmm
(C) 62.3×10^6 Nmm (D) 75.7×10^6 Nmm

Direction for questions 3 and 4: A closely coiled spring having mean diameter of 200 mm is made of 30 mm diameter rod and has 30 turns. A weight of 1.5 kN is dropped from a height on the spring such that the spring compresses by 120 mm ($G = 8 \times 10^4$ N/mm^2).

3. The gradually applied load which produces spring deflection of 120 mm is
(A) 5,170 N (B) 4,050 N
(C) 6,269 N (D) 3,850 N

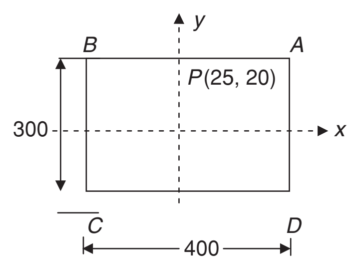
4. The drop height is
(A) 40 mm (B) 38 mm
(C) 42 mm (D) 36 mm

5. A hollow shaft of diameter ratio $\frac{3}{5}$ is required to transmit 600 kW at 110 rpm, the maximum torque being 20% greater than mean. The shear stress is not to exceed 6,300 N/cm^2. The maximum external diameter of the shaft is
(A) 16.86 cm (B) 17.98 cm
(C) 14.97 cm (D) 18.52 cm

6. If the twist in a length of 3 m is not to exceed 1.4° the maximum external diameter is
(Take $G = 84 \times 10^5$ N/cm^2)
(A) 16.88 cm (B) 17.56 cm
(C) 18.44 cm (D) 19.77 cm

7. A closed-coiled helical spring of 15 coils having a mean radius 9 cm is free to rotate at its ends. It is subjected to axial compressive load. If the spring absorbs 58 Nm of energy consistent with a maximum deflection of 5 cm, the diameter of the rod forming the spring is ($G = 8.5 \times 10^6$ N/cm^2)
(A) 2.5 cm (B) 2.8 cm
(C) 3.2 cm (D) 3.6 cm

8.



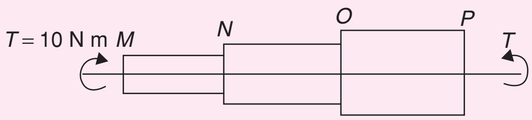
A load of 500 kN is applied at point P on a masonry pillar as shown in the figure. The stress developed at the corner D is
(A) 3.836 N/mm^2 (B) 4.087 N/mm^2
(C) 4.427 N/mm^2 (D) 3.576 N/mm^2

9. A wooden column of length 2 m and square cross-section is to be made. Taking E = 12 GPa and allowable stress 12 MPa, the size of the column to support a load of 100 kN is (use Euler's Crippling load with factor of safety 3)
(A) 110 mm (B) 105 mm
(C) 100 mm (D) 95 mm

10. A hollow cast iron how column of length 4.5 m is having an outside diameter of 200 mm and thickness of 20 mm. The safe load using Rankine's formula is (Take factor of safety 3, $E = 1 \times 10^5$ N/mm^2 , Rankine's constant = $\frac{1}{1600}$, f_c = 550 N/mm^2 and assume both ends fixed)
(A) 1,220 kN (B) 1,250 kN
(C) 1,010 kN (D) 1,170 kN

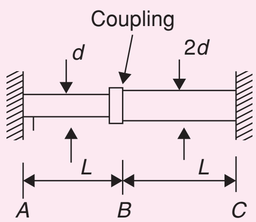
PREVIOUS YEARS' QUESTIONS

1. A torque of 10 Nm is transmitted through a stepped shaft as shown in the figure. The torsional stiffnesses of individual sections of lengths *MN*, *NO* and *OP* are 20 Nm/rad, 30 Nm/rad and 60 Nm/rad, respectively. The angular deflection between the ends *M* and *P* of the shaft is **[2004]**



(A) 0.5 rad (B) 1.0 rad
(C) 5.0 rad (D) 10.0 rad

2. The two shafts *AB* and *BC*, of equal length and diameters *d* and 2d, are made of the same material. They are joined at *B* through a shaft coupling, while the ends *A* and *C* are built-in (cantilevered). A twisting moment *T* is applied to the coupling. If T_A and T_C represent the twisting moments at the ends *A* and C, respectively, then **[2005]**



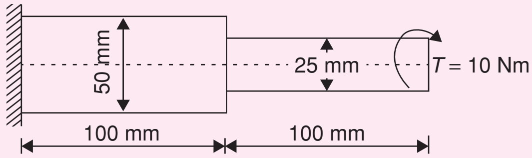
(A) $T_C = T_A$ (B) $T_C = 8T_A$
(C) $T_C = 16T_A$ (D) $T_A = 16T_C$

3. A pin-ended column of length *L*, modulus of elasticity *E* and second moment of the cross-sectional area *I* is loaded centrically by a compressive load *P*. The critical bucking load (P_{Cr}) is given by **[2006]**

(A) $P_{Cr} = \dfrac{EI}{\pi^2 L^2}$ (B) $P_{Cr} = \dfrac{\pi^2 EI}{3L^2}$

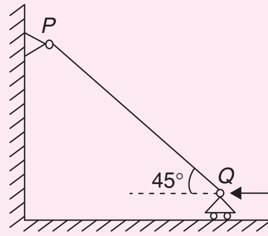
(C) $P_{Cr} = \dfrac{\pi EI}{L^2}$ (D) $P_{Cr} = \dfrac{\pi^2 EI}{L^2}$

4. A stepped steel shaft shown below is subjected to 10 Nm torque. If the modulus of rigidity is 80 GPa, the strain energy in the shaft in *N* mm is **[2007]**



(A) 4.12 (B) 3.46
(C) 1.73 (D) 0.86

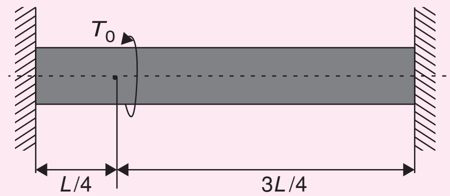
5. The rod *PQ* of length *L* and with flexural rigidity *EI* is hinged at both ends. For what minimum force *F* is it expected to buckle? **[2008]**



(A) $\dfrac{\pi^2 EI}{L^2}$ (B) $\dfrac{\sqrt{2}\,\pi^2 EI}{L^2}$

(C) $\dfrac{\pi^2 EI}{\sqrt{2}L^2}$ (D) $\dfrac{\pi^2 EI}{2L^2}$

6. A solid shaft of diameter, *d* and length *L* is fixed at both the ends. A torque, T_o is applied at a distance, *L*/4 from the left end as shown in the figure given below: **[2009]**



The maximum shear stress in the shaft is

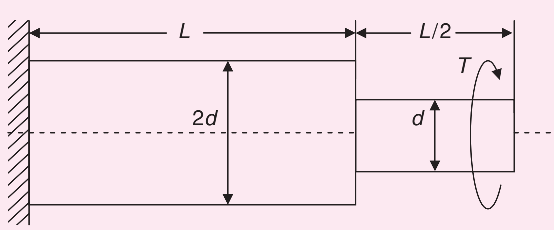
(A) $\dfrac{16T_o}{\pi d^3}$ (B) $\dfrac{12T_o}{\pi d^3}$

(C) $\dfrac{8T_o}{\pi d^3}$ (D) $\dfrac{4T_o}{\pi d^3}$

7. A column has a rectangular cross-section of 10 mm × 20 mm and a length of 1 m. The slenderness ratio of the column is close to **[2011]**

(A) 200 (B) 346
(C) 477 (D) 1000

8. A torque *T* is applied at the free end of a stepped rod of circular cross-sections as shown in the figure. The shear modulus of the material of the rod is *G*. The expression for *d* to produce an angular twist θ at the free end is **[2011]**



(A) $\left(\dfrac{32TL}{\pi\theta G}\right)^{\frac{1}{4}}$ (B) $\left(\dfrac{18TL}{\pi\theta G}\right)^{\frac{1}{4}}$

(C) $\left(\dfrac{16TL}{\pi\theta G}\right)^{\frac{1}{4}}$ (D) $\left(\dfrac{2TL}{\pi\theta G}\right)^{\frac{1}{4}}$

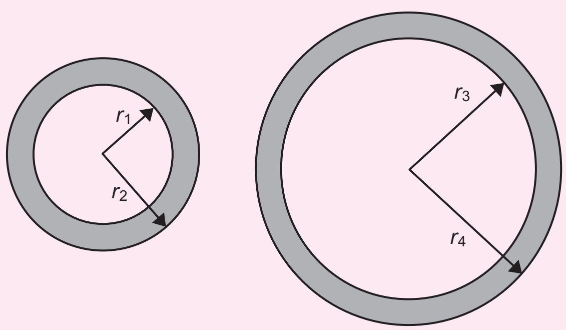
9. For a long slender column of uniform cross section, the ratio of critical buckling load for the case with both ends clamped to the case with both ends hinged is **[2012]**

(A) 1 (B) 2
(C) 4 (D) 8

10. Two solid circular shafts of radii R_1 and R_2 are subjected to same torque. The maximum shear stresses developed in the two shafts are τ_1 and τ_2. If $R_1/R_2 = 2$, then $\dfrac{\tau_2}{\tau_1}$ is ______. **[2014]**

11. A hollow shaft of 1 m length is designed to transmit a power of 30 kW at 700 RPM. The maximum permissible angle of twist in the shaft is 1°. The inner diameter of the shaft is 0.7 times the outer diameter. The modulus of rigidity is 80 GPa. The outside diameter (in mm) of the shaft is ______. **[2015]**

12. The cross-sections of two hollow bars made of the same material are concentric circles as shown in the figure. It is given that $r_3 > r_1$ and $r_4 > r_2$, and that the areas of the cross-sections are the same. J_1 and J_2 are the torsional rigidities of the bars on the left and right, respectively. The ratio J_2/J_1 is: **[2016]**

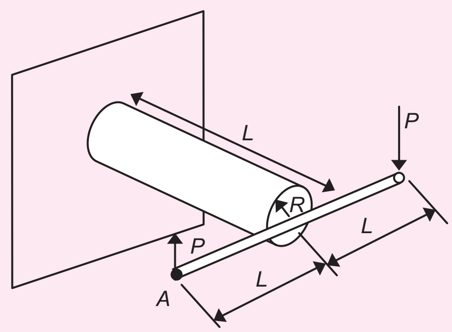


(A) > 1 (B) < 0.5
(C) = 1 (D) between 0.5 and 1

13. The spring constant of a helical compression spring DOES NOT depend on: **[2016]**

(A) coil diameter
(B) material strength
(C) number of active turns
(D) wire diameter

14. A rigid horizontal rod of length $2L$ is fixed to a circular cylinder of radius R as shown in the figure. Vertical forces of magnitude P are applied at the two ends as shown in the figure. The shear modulus for the cylinder is G and the Young's modulus is E. **[2016]**



The vertical deflection at point A is

(A) $PL^3/(\pi R^4 G)$ (B) $PL^3/(\pi R^4 E)$
(C) $2PL^3/(\pi R^4 E)$ (D) $4PL^3/(\pi R^4 G)$

15. Two circular shafts made of same material, one solid (S) and one hollow (H), have the same length and polar moment of inertia. Both are subjected to same torque. Here, θ_S is the twist and τ_S is the maximum shear stress in the solid shaft, whereas θ_H is the twist and τ_H is the maximum shear stress in the hollow shaft. Which one of the following is TRUE? **[2016]**

(A) $\theta_S = \theta_H$ and $\tau_S = \tau_H$
(B) $\theta_S > \theta_H$ and $\tau_S > \tau_H$
(C) $\theta_S < \theta_H$ and $\tau_S < \tau_H$
(D) $\theta_S = \theta_H$ and $\tau_S < \tau_H$

Answer Keys

Exercises

Practice Problems 1

1. C **2.** B **3.** C **4.** C **5.** B **6.** A **7.** D **8.** B **9.** C **10.** A
11. C **12.** A **13.** B **14.** A **15.** D

Practice Problems 2

1. C **2.** D **3.** B **4.** C **5.** B **6.** C **7.** A **8.** B **9.** B **10.** D

Previous Years' Questions

1. B **2.** C **3.** D **4.** C **5.** C **6.** B **7.** B **8.** B **9.** C
10. 7.9 to 8.1 **11.** 43 to 45 **12.** A **13.** B **14.** D **15.** D

Test

Strength of Materials

Time: 60 Minutes

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. The ratio of strain in the direction perpendicular to the direction of application of force to the strain along the direction of application of force is called
(A) Young's modulus (B) Bulk modulus
(C) Poisson's ratio (D) Modulus of rigidity

2. The maximum stress at which even a billion reversal of stress cannot cause failure of the material is called
(A) Safe stress (B) Proof stress
(C) Endurance limit (D) Fatigue stress

3. The ratio of load applied to the actual cross-section area of the specimen is known as
(A) Nominal stress (B) True stress
(C) Ultimate stress (D) Yield stress

4. The maximum strain energy which can be stored by a body without undergoing permanent deformation is known as
(A) Safe resilience (B) Modulus of rigidity
(C) Modulus of resilience (D) Proof resilience

5. In a uni-dimensional stress system, principal plain is defined as the one on which
(A) Shear stress is minimum
(B) Normal stress is zero and shear stress is maximum
(C) Shear stress is maximum
(D) Normal stress is maximum and shear stress is zero

6. If p_x and p_y are normal stresses on two mutually perpendicular sections and p_1 and p_2 are the principal stresses then radius of Mohr's cycle is
(A) $\frac{p_x + p_y}{2}$ (B) $\frac{p_1 + p_2}{2}$
(C) $\frac{p_1 - p_2}{2}$ (D) $\frac{p_x - p_y}{2}$

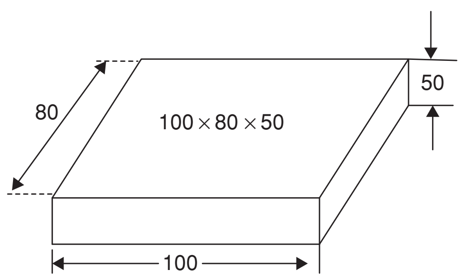
7. A body subjected to uni-axial tension will fail in a plane at 45° due to shear, if its shear strength is less than
(A) Tensile strength
(B) Compressive strength
(C) Half the tensile strength
(D) Difference between tensile and compressive strength.

8. At point of contraflexure
(A) +ve bending moment is maximum
(B) Bending moment have change in sign
(C) –ve bending moment is maximum
(D) Shear force is zero

9. A cantilever of span L subjected to a uniformly varying load, w/unit length at fixed end to zero at free end, undergoes a maximum bending moment of
(A) $\frac{wL^2}{6}$ (B) $\frac{wL^2}{8}$
(C) $\frac{wL^3}{6}$ (D) $\frac{wL^2}{12}$

10. Maximum shear stress in a beam of circular cross-section, when subjected to a shearing force is
(A) $\frac{5}{3}$ times the average shear stress
(B) $\frac{3}{2}$ times the average shear stress
(C) $\frac{4}{3}$ times the average shear stress
(D) Equal to average shear stress

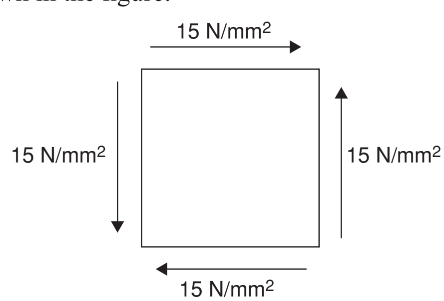
Direction for questions 11 and 12: The steel block shown in the figure is subjected to a uniform pressure of 150 MPa on all its faces. Young's modulus is 200 GPa and Poisson's ratio is 0.4.



11. The thickness of the block will
(A) Decrease by 0.01875 mm
(B) Decrease by 0.015 mm
(C) Decrease by .0075 mm
(D) Data insufficient

12. The volume of the steel block will
(A) Decrease by 180 mm^3 (B) Decrease by 300 mm^3
(C) Decrease by 250 mm^3 (D) Decrease by 281 mm^3

13. The state of stress at a point in a stressed element is shown in the figure.



The maximum tensile stress in the element will be

(A) 20 N/mm^2 (B) $15\sqrt{2}$ N/mm^2

(C) 15 N/mm^2 (D) Zero

14. In an element if the stresses are given by

p_x = 110 MPa

p_y = 30 MPa

q_{xy} = 30 MPa

the principal stresses in MPa are,

(A) 20, 120 (B) 110, 30

(C) 0, 140 (D) 20, 140

15. A round bar of diameter 40 mm and length 2.5 m is stretched 2.5 mm. Young's modulus of the material is 110 GN/m^2 and shear modulus is 42 GN/m^2. Then Lateral contraction is

(A) 0.111 mm (B) 0.0112 mm

(C) 0.0124 mm (D) 0.01 mm

Direction for questions 16 and 17: A steel rod of 4 m length is heated through a temperature of 50°C. The coefficient of linear expansion is 6.5×10^{-6}/°C and Young's modulus is 2×10^7 N/cm^2.

16. The increase in length of the rod is

(A) 1.3 mm (B) 1 mm

(C) 1.2 mm (D) 1.4 mm

17. Stress induced in the rod if the expansion due to heating is restricted, is

(A) 5,000 N/cm^2 (B) 6,000 N/cm^2

(C) 6,500 N/cm^2 (D) 7,000 N/cm^2

18. A cylindrical bar of 30 mm diameter and 1 m length is subjected to a tensile load. If the longitudinal strain is 2 times that of lateral strain and Young's modulus is 2×10^5 N/mm^2 the modulus of rigidity is

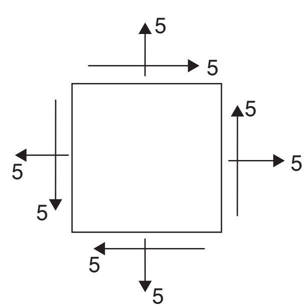
(A) 0.52×10^5 N/mm^2

(B) 0.67×10^5 N/mm^2

(C) 0.82×10^5 N/mm^2

(D) 0.77×10^5 N/mm^2

19. The state of stress in a material is given in the figure. Maximum principal stress will be



(A) 5 (B) 10

(C) 15 (D) 12.5

20. A simply supported beam of span 5 m carries a uniformly varying load from zero at one support to 2 kN/m at the other support. Maximum bending moment will be

(A) 3,000 Nm (B) 3,200 Nm

(C) 3,100 Nm (D) 3,300 Nm

21. A steel rod of 50 mm diameter and 6 m length is subjected to a tensile load of 100 kN. Poisson's ratio is 0.25 and Young's modulus is 2×10^5 N/mm^2.

Change in diameter of the rod is

(A) 2.92×10^{-3} mm (B) 3.62×10^{-3} mm

(C) 3.18×10^{-3} mm (D) 3.48×10^{-3} mm

22. A 100 N weight falls from a height of 100 mm on a collar attached to a bar of 20 mm diameter and 300 mm long. Young's modulus = 2×10^5 N/mm^2. The instantaneous stress produced is

(A) 198 N/mm^2 (B) 192 N/mm^2

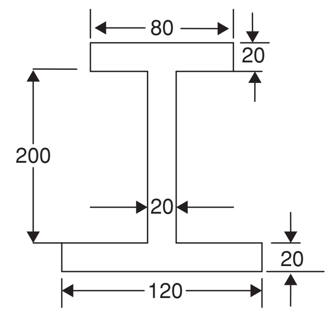
(C) 216 N/mm^2 (D) 206 N/mm^2

23. A simply supported beam of span 6 m carries a uniformly distributed load of 30 kN/m and a central point load of 50 kN. Moment of inertia of the cross-section about neutral axis is 1.34×10^9 mm^4. Bottom of the section is at a distance of 290 mm from neutral axis. Maximum bending stress at the bottom is

(A) 54.3 N/mm^2 (B) 45.4 N/mm^2

(C) 49.2 N/mm^2 (D) 47.6 N/mm^2

24. The I section shown in figure is subjected to a shear force of 40 kN.



(dimensions in mm)

Shear stress at the top of the web will be (Take $I = 65 \times 10^6$ mm$^{4)}$)

(A) 4.86 N/mm^2 (B) 6.32 N/mm^2

(C) 5.72 N/mm^2 (D) 5.96 N/mm^2

25. A stepped shaft is made of 2 materials 1 and 2. First part is having a length of 400 mm and diameter 40 mm. Second part is 800 mm long with 30 mm diameter. Young's modulus for 1 and 2 are 2×10^5 and 1×10^5 N/mm^2, respectively. The extension produced under an axial pull of 30 kN is

(A) 0.364 mm (B) 0.394 mm

(C) 0.387 mm (D) 0.412 mm

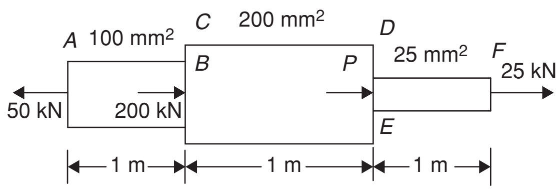
Direction for questions 26 and 27: Two wires of equal length made of steel and copper carry a common load of 8 kN at their end. Steel wire has an area of cross-section 1 cm^2 and for copper wire it is 2 cm^2. Young's modulus for steel and copper are 2×10^7 N/cm^2 and 1.2×10^7N/cm^2, respectively.

26. Load shared by steel wire is
(A) 4,354 N (B) 3,646 N
(C) 3,636 N (D) 4,364 N

27. Load shared by the copper wire is
(A) 4,354 N (B) 3,646 N
(C) 3,636 N (D) 4,364 N

Direction for questions 28 and 29: A compound steel bar is subjected to loads as shown in the figure.

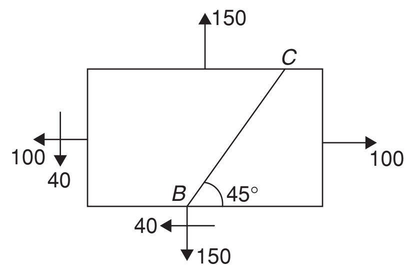
(Take $E = 200 \times 10^3$ N/mm^2)



28. For equilibrium, the value of force P is
(A) 170 kN (B) −170 kN
(C) 200 kN (D) −200 kN

29. Total increase in length is
(A) 8.25 mm (B) 4.25 mm
(C) 2.75 mm (D) 3.75 mm

30. The state of stress in a strained material is shown in the figure. The magnitude and direction of the resultant stress on plane BC is



(A) 166.88 and 8.62
(B) 196.47 and 20.14
(C) 190 and 14.74
(D) 166.88 and 14.74

Answer Keys

1. C	**2.** C	**3.** B	**4.** D	**5.** D	**6.** C	**7.** C	**8.** B	**9.** A	**10.** C
11. C	**12.** A	**13.** C	**14.** A	**15.** C	**16.** A	**17.** C	**18.** B	**19.** B	**20.** B
21. C	**22.** D	**23.** B	**24.** D	**25.** C	**26.** C	**27.** D	**28.** B	**29.** C	**30.** A