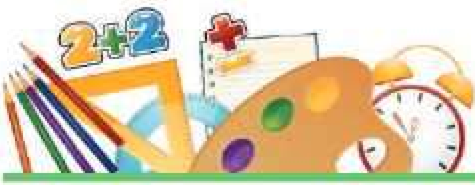


CHAPTER - 3

Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

Objectives :
1. To provide information about Multiples and Factors.
2. To give knowledge about the concept of HCF and LCM with the help of different activities.
3. To give knowledge about various methods of finding HCF and LCM.
4. To develop their ability to use HCF and LCM in real life situations.
5. To make them familiar with Even, Odd, Prime and Composite numbers with the help of activities.
6. To prepare them for competitive exams.

Introduction

3.1 Multiples

When two or more than two numbers are multiplied, we get the product. This product is the multiple of each multiplied number. Observe the following:

(1) $4 \times 7 = 28$　　(2) $8 \times 6 = 48$

(3) $2 \times 3 \times 4 = 24$　　(4) $9 \times 10 = 90$

From the above multiplications, we get the following

(1) 28 is the multiple of 4 and 7.

(2) 48 is the multiple of 6 and 8.

(3) 24 is the multiple of 2, 3 and 4.

(4) 90 is the multiple of 9 and 10.

So , the multiples of a number are obtained by multiplying the given number with natural numbers (1, 2, 3, 4, 5, ...)

Multiples of 3	**Multiples of 8**
$3 \times 1 = 3$	$8 \times 1 = 8$
$3 \times 2 = 6$	$8 \times 2 = 16$
$3 \times 3 = 9$	$8 \times 3 = 24$
$3 \times 4 = 12$	$8 \times 4 = 32$
$3 \times 5 = 15$	$8 \times 5 = 40$
.. ..	.. ..
.. ..	.. ..
.. ..	.. ..
.. ..	.. ..

So, the above numbers 3, 6, 9, ... and 8, 16, 24, ... are the multiples of 3 and 8 respectively.

Things to Remember

- Every number is a multiple of itself.
- Every number is a multiple of 1.
- Every multiple of a number is greater than or equal to the number.
- The smallest multiple of a number is the number itself.

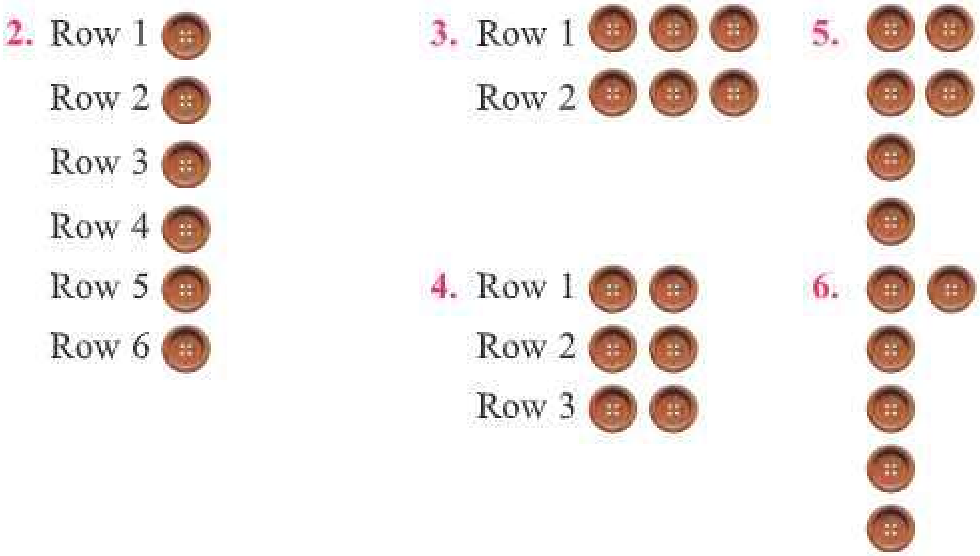
3.2 Factors :

Activity

In class distribute 6-6 buttons to each student and tell them to place them in the form of every possible horizontal line, vertical line,square or rectangular shape that will lead to the factors of the given number.

Students can make the following possible lines and shapes.

1. Row : 



In figure 1, buttons are in 1 row.

In figure 2, buttons are in 6 rows.

In figure 3, buttons are in 2 rows.

In figure 4, buttons are in 3 rows.

In figure 5 and 6, buttons are not in the shape of line or square /rectangle. So these do not form factors.

So, factors according to the number of rows from figure 1 to 4 are 1, 6, 2 and 3.

So factors of 6 = 1, 2, 3, 6

Methods of Finding Factors of a number

We can find all the factors of a number in two ways :

(*a*) By Multiplication	(*b*) By Division
$1 \times 6 = 6$	$6 \div 1 = 6$
$2 \times 3 = 6$	$6 \div 2 = 3$
	$6 \div 3 = 2$
	$6 \div 6 = 1$

So 1, 2, 3, 6 are all factors of 6.

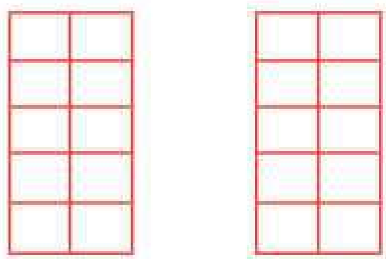
Things to Remember

- 1 is a factor of every number except zero.
- Every non-zero number is a factor of itself.
- The smallest factor of a number is 1.
- The largest factor of a number is the number itself.
- If 1st number is factor of 2nd number then 2nd number is the multiple of 1st number.

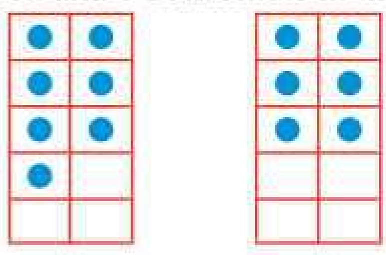
3.3 Even and Odd Numbers :

Activity

Ask the students to draw two tables with 10 boxes.



The teacher will write '7' and 6 on the blackboard and will ask the students whether it is even/odd. Teacher will tell the students to paste/stick seven dots/bindis in one box and 6 dots/bindis in the other box.

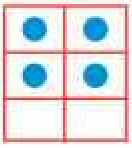


7 6

We observe that in the first three rows, dots/bindis form pairs but in the fourth row, no pair is formed. When the dots/bindis form pairs then the number is an even number i.e., '6' but when the dot/bindi does not form a pair, it is an odd number i.e., '7' as in the above figure.

- Teacher is advised to give some more examples which he finds suitable.

Now, we will take two digits number say 74. To see whether 74 is even or odd we need not to paste 74 dots/bindis. We will put its ones place digit i.e., 4 in the box.

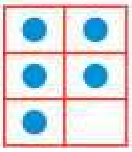


In number 74 ones digit 4 forms two pairs of dots/bindis.

So 74 is an even number.

Now consider 3 digits number 175 :

1 7 $\boxed{5}$ Here we see that ones digit is 5.



Digit 5 does not form complete pairs of dots/bindis. So 175 is an odd number.

In this way, we can tell whether the number is even or odd just by looking at unit digit of a number.

- Sum of two even numbers is always even as 2 + 4 = 6
- Sum of two odd numbers is always even as 1 + 3 = 4
- Sum of even and odd numbers is always odd as 2 + 3 = 5

Things to Remember

1. If a digit at ones place is 0, 2, 4, 6, 8 then it is an even number and if a digit at ones place is 1, 3, 5, 7, 9 then it is an odd number.
2. Even number is always divided by 2 and 2 is the factor of every even number.

Example 1 : Is 45 a multiple of 9 ?

Solution : Divide 45 by 9

$$
\begin{array}{r}
9\overline{)\,4\ 5\,}(05 \\
-0 \\
\hline
4\ 5 \\
-4\ 5 \\
\hline
0 \\
\hline
\end{array}
$$

45 is completely divisible by 9. So 45 is a multiple of 9.

Example 2 : Is 82 a multiple of 8 ?

Solution : Divide 82 by 8

$$
\begin{array}{r|l}
8 & \overline{8\;\;2}\,(10 \\
 & -8 \\
\hline
 & 0\;\;2 \\
 & -\;\;0 \\
\hline
 & \;\;\;\;2 \text{ Remainder}
\end{array}
$$

82 is not completely divisible by 8. So 82 is not a multiple of 8.

Example 3 : Write first four multiples of 9.

Solution : $9 \times 1 = 9$, $9 \times 2 = 18$, $9 \times 3 = 27$, $9 \times 4 = 36$

So, the first four multiples of 9 are 9, 18, 27, 36.

Example 4 : Write factors of 12.

Solution : **By Multiplication :**

$$1 \times 12 = 12$$
$$2 \times 6 = 12$$
$$3 \times 4 = 12$$
$$\left.\begin{array}{r}4 \times 3 = 12 \\ 6 \times 2 = 12 \\ 12 \times 1 = 12\end{array}\right\} \text{Here factors are repeating.}$$

So, 1, 2, 3, 4, 6 & 12 are the factors of 12

Division Method :

$$
\begin{array}{r|l}
1 & \overline{1\;\;2}\,(12 \\
 & -1 \\
\hline
 & 0\;\;2 \\
 & \;\;-2 \\
\hline
 & \;\;\;\;0
\end{array}
\qquad
\begin{array}{r|l}
2 & \overline{1\;\;2}\,(06 \\
 & -0 \\
\hline
 & 1\;\;2 \\
 & -1\;\;2 \\
\hline
 & 0\;\;0
\end{array}
\qquad
\begin{array}{r|l}
3 & \overline{1\;\;2}\,(04 \\
 & -0 \\
\hline
 & 1\;\;2 \\
 & -1\;\;2 \\
\hline
 & 0\;\;0
\end{array}
$$

4) 1 2 (03 − 0 1 2 − 1 2 0 0	5) 1 2 (02 − 0 1 2 − 1 0 0 2 Remainder	6) 1 2 (02 − 0 1 2 − 1 2 0 0
7) 1 2 (01 − 0 1 2 − 7 0 5 Remainder	8) 1 2 (01 − 0 1 2 − 8 0 4 Remainder	9) 1 2 (01 − 0 1 2 − 9 0 3 Remainder
10) 1 2 (01 − 0 1 2 − 1 0 0 2 Remainder	11) 1 2 (01 − 0 1 2 − 1 1 0 1 Remainder	12) 1 2 (01 − 0 1 2 − 1 2 0 0

Here 12 is divisible by 1, 2, 3, 4, 6 and 12.

So, factors of 12 are 1, 2, 3, 4, 6 and 12.

Example 5 : Is 8 a factor of 72 ?

Solution : Divide 72 by 8.

8) 7 2 (09
− 0
7 2
− 7 2
0

72 is completely divisible by 8. So 8 is factor of 72.

Exercise-3.1

1. Write the first five multiples of the following numbers :

(*a*) 5 (*b*) 9 (*c*) 10 (*d*) 12

(*e*) 16 (*f*) 17

2. Find the Factors of the given numbers :

(*a*) 5	1	2	3	4	5								
(*b*) 8	1	2	3	4	5	6	7	8					
(*c*) 14	1	2	3	4	6	7	8	9	10	11	12	13	14
(*d*) 12	1	2	3	4	5	6	7	8	9	10	11	12	
(*e*) 25	1	2	3	4	5	10	15	20	25	30	35	40	45
(*f*) 36	1	2	3	4	5	6	7	12	18	20	24	30	36

3. Write Factors of the following numbers :

(*a*) 18 (*b*) 24 (*c*) 35 (*d*) 36

(*e*) 45 (*f*) 21

4. Find out the Even Numbers from the following :

(*a*)	12	23	34	16	19	28
(*b*)	35	48	53	69	72	90
(*c*)	450	213	568	664	789	98
(*d*)	235	456	968	604	731	888
(*e*)	63	136	245	446	1278	2341
(*f*)	47	168	999	1729	5864	6859

5. Find out the Odd Numbers from the following :

(*a*)	11	23	54	16	19	35
(*b*)	36	45	58	69	76	97
(*c*)	451	215	508	614	789	983
(*d*)	237	416	948	654	739	666
(*e*)	631	135	249	746	1279	2851
(*f*)	49	178	765	1729	9261	6859

6. Fill in the blanks :

(*a*) If $4 \times 9 = 36$ then factors of 36 are .... and ....

(*b*) If $8 \times 7 = 56$ then factors of 56 are .... and ....

(*c*) If $3 \times 5 \times 6 = 90$ then factors of 90 are ...., ... and ....

(*d*) In $8 \times 10 = 80$, the multiple of 8 and 10 is ....

(*e*) In $2 \times 3 \times 5 = 30$ then 30, is the multiple of ...., ... and ....

7. Write True or False :

(*a*) 24 is a factor of 24. ☐

(*b*) 2 is a factor of every number. ☐

(*c*) 24 is an even number. ☐

(*d*) 136 is an odd number. ☐

(*e*) There are infinite multiples of a number. ☐

(*f*) 36 is a multiple of 5 and 7. ☐

(*g*) Sum of two even numbers is an odd number. ☐

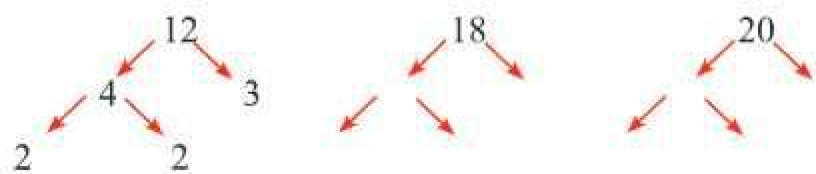
(*h*) Smallest even number is 0. ☐

(*i*) 152 is a odd number. ☐

(*j*) There are five single digit even numbers. ☐

8. Observe the Factorisation Pattern and complete:

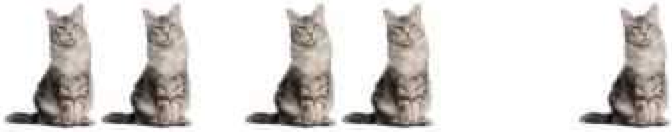
(*a*) (*b*) (*c*)

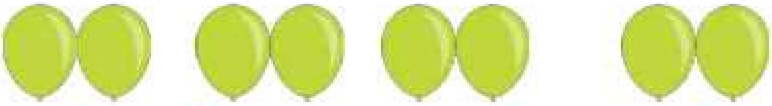


9. Look at the pattern and solve :

(*a*)	2	77	13
(*b*)	3	38	19
(*c*)	5	39	7
(*d*)	11	85	17

10. Understand and complete the following :

(a)  **Odd**

(b) 

(c) 

(d) 

(e) 

Even Odd

2, 4, 6, 8 and 0 if these are units of ones

No matter how long is the amount, these are even buns

1, 3, 5, 7 and 9, if these are units of ones

No matter how long is the amount, these are odd runs.

Product Multiple and Factor

When we multiply some number, to know how they conduct Answer of the multiplication is called their product.

Now those number of multiplication are factors of the product.

Product is also the multiple of those number, you can instruct

3.4 Prime and Composite Numbers :

We can tell whether a given number is a prime or a composite number by counting the number of factors. Factors of first 10 natural numbers are as follows :

Factors of 1 = 1

Factors of 2 = 1, 2

Factors of 3 = 1, 3

Factors of 4 = 1, 2, 4

Factors of 5 = 1, 5

Factors of 6 = 1, 2, 3, 6

Factors of 7 = 1, 7

Factors of 8 = 1, 2, 4, 8

Factors of 9 = 1, 3, 9

Factors of 10 = 1, 2, 5, 10

In the above table, numbers 2, 3, 5 and 7 have two factors : 1 and the number itself. These numbers are called Prime Numbers. Numbers 4, 6, 8, 9, 10 have more than two factors, these numbers are called Composite Numbers.

Prime Numbers : The numbers which have exactly two factors are called Prime Numbers. Example 2, 3, 5, 7 - etc.

Composite Numbers : The numbers which have more than two factors are called Composite Numbers. Example 4, 6, 8, 9, 10, etc.

Now the Question arises : Is 1 prime or composite ? The number 1 has only one factor, so it is neither a prime nor composite.

Prime Numbers between 1 and 100.

The steps to find prime numbers between 1 to 100 are given below :

$\boxed{1}$	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Step 1 : Write numbers from 1 to 100.

Step 2 : Encircle 2 and cross all the numbers which are multiples of 2.

Step 3 : Encircle 3 and cross all the numbers which are multiples of 3.

Step 4 : Encircle 5 and cross all the numbers which are multiples of 5.

Step 5 : Encircle 7 and cross all the numbers which are multiples of 7.

Step 6 : Encircle 11 and cross all the numbers which are multiples of 11.

Step 7 : Continue this process till all the numbers are either crossed or encircled.

Step 8 : Make a box around number 1, because it is a unique number.

All the encircled numbers are prime numbers and the crossed-out are Composite Numbers. Prime Numbers between 1 and 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

This is called the sieve of Eratosthenes.

Things to Remember

- 1 is neither a prime nor a composite number.
- Only 2 is an even prime number.
- The smallest prime number is 2.
- The smallest composite number is 4.

3.5 Prime Factorisation :

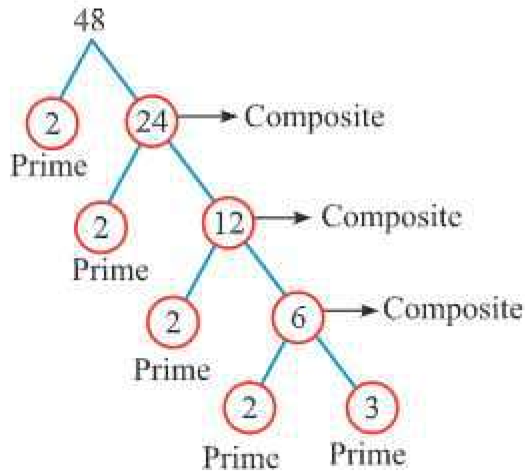
A composite number can be written as the product of prime factors. This is called prime factorisation. There are two methods of prime factorisation.

(*a*) Factor Tree Method (*b*) Division Method

(*a*) Factor Tree Method :

In this method, we factorise a composite number till we get all prime factors.

Let us factorise 48 using this method.



The Prime factorisation of 48 is $= 2 \times 2 \times 2 \times 2 \times 3$.

(*b*) Division Method :

In this method, we start dividing the given number by the smallest prime number and continue division by prime numbers till we reach 1.

2	48	[Divide by the smallest prime number]
2	24	
2	12	
3	6	
2	2	
	1	[Continue on dividing with the prime numbers till we get 1].

The prime factorisation of $48 = 2 \times 2 \times 2 \times 2 \times 3$

3.5 Highest Common Factor (H.C.F.)

Activity :

Teacher - How many students are there in 4th class of our school.

Students - Sir, 18 students.

Teacher - Now tell, how many students are in 5th class ?

Students - Sir, 27 students.

Teacher - We will play a game by forming teams from both classes. In each team, we will take equal number of students. Now tell, how many students can participate in one team so that no student is left ?

Students - Sir, there can be 7 students in each team.

Teacher - No students, we can take 9 students in each team.

Let us learn how we can divide them :

Factors of 18 = 1, 2, 3, 6, 9, 18

Factors of 27 = 1, 3, 9, 27

Highest common factor of 18 and 27 is 9. So, we shall make teams of 9 students each so that no child is left out of the team. Therefore, 9 is the H.C.F. of 18 and 27.

For Example : 35 is divisible by 5.

So 5 is a factor of 35 and 35 is multiple of 5 i.e, $5 \times 7 = 35$.

5 and 7 are factors of 35 or 35 is multiple of 5 and 7.

Similarly : $2 \times 3 \times 5 = 30$; 2, 3 and 5 are factors of 30.

In above example, observe the factors of 30 and 35. In both, 5 is the common factor.

So, 5 is the HCF of 30 and 35.

Things to Remember

- The greatest common factor of two or more than two numbers is their H.C.F.
- If HCF of two numbers is 1 then that numbers are called Co-Prime numbers.

Hints For Teacher Teacher will take examples from daily life with the help of a measuring tape to measure exact length and breadth of the floor, measuring weight etc

H.C.F.

We can find HCF by two methods :

(*a*) Prime factorisation method   (*b*) Division method

3.5.1 Prime Factorisation Method

First of all, we will find all the prime factors of the given numbers and list the prime factors which are common to all.

Now, the product of these common factors is the required H.C.F.

Example 1 : Find HCF of 20 and 30 using prime factorisation method.

Solution :

$$\begin{array}{c|c} 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Prime factorisation of 20 $= 2 \times 2 \times 5$

Prime factorisation of 30 $= 2 \times 3 \times 5$

Common prime factors = 2 and 5

So HCF is $2 \times 5 = 10$.

Example 2 : Find HCF of 45, 90 and 105 using prime factorisation method.

Solution :

$$\begin{array}{c|c} 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 3 & 105 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Prime factorisation of 45 $= 3 \times 3 \times 5$

Prime factorisation of 90 $= 2 \times 3 \times 3 \times 5$

Prime factorisation of 105 $= 3 \times 5 \times 7$

Common prime factors = 3 and 5

So HCF $= 3 \times 5 = 15$

3.5(2) To find HCF using Division method :

To find HCF of two numbers, we follow the steps given below:

- Make the smaller number as divisor and the larger number as dividend and divide.
- The remainder (if not zero) becomes the new divisor and the last divisor becomes the new dividend.
- Continue the process till we get zero as remainder.
- The last divisor is the required HCF.

The following example will show how we get HCF using division method

Example 1 : Find the HCF of 75 and 105 by using the division method.

Solution : We divide 105 by 75 the remainder is 30. Now divide 75 by 30 and continue this process till we get zero as remainder.

```
75)1 0 5(1
   -7 5
   ------
     3 0)7 5(2
        - 6 0
        ------
          1 5)3 0(2
             - 3 0
             -----
                 0
```

So, H.C.F. = 15

Example 2 : Find the HCF of 60, 90 and 130 by using division method.

Solution : First find HCF of any two numbers.

```
60) 9 0 (1
   -6 0
   ------
    30)6 0(2
      - 6 0
      -----
          0
```

HCF of 60 and 90 is 30.
Now find HCF of 30 and 130.

```
30)1 3 0(4
  - 1 2 0
  -------
     1 0)3 0(3
        - 3 0
        -----
            0
```

So, HCF of 60, 90 and 130 is 10.

Example 3 : Three containers contain 18 *l*, 27 *l* and 36 *l* oil respectively. What capacity of measuring vessel can measure oil of all the three containers exactly ?

Solution : We need such a measuring vessel which can measure oil of all three containers exactly. For that, we will find the HCF of all three containers.

$$
\begin{array}{l}
18\overline{)\,27\,}(1 \\
\quad -18 \\
\quad \overline{\quad 9)\,18\,(2} \\
\qquad -18 \\
\qquad \overline{\quad 0 \quad}
\end{array}
\qquad\qquad
\begin{array}{l}
9\overline{)\,36\,}(4 \\
\; -36 \\
\; \overline{\quad 0 \quad}
\end{array}
$$

A vessel of 9*l* can measure oil of all three containers exactly.

Exercise-3.2

1. Write the prime numbers from the following :

(*a*)	12	8	5	7	6	3
(*b*)	2	9	11	13	16	21
(*c*)	10	5	25	35	42	33
(*d*)	18	41	23	17	19	27
(*e*)	27	29	37	47	49	39

2. Write the composite numbers from the following :

(*a*)	14	7	9	6	5
(*b*)	21	12	18	17	11
(*c*)	23	32	37	41	15
(*d*)	10	25	5	7	9
(*e*)	43	24	47	49	50

3. Find HCF of following numbers using Prime factorisation.

(a) 18, 27 (b) 21, 63 (c) 80, 100 (d) 42, 98

4. Find HCF of following numbers using Prime factorisation.

(a) 30, 50, 70 (b) 24, 32, 40

(c) 36, 60, 72 (d) 25, 30, 35

5. **Find HCF of following number using Prime factorisation.**

(a) 42, 84 (b) 45, 90

(c) 16, 64, 80 (d) 45, 90, 105

6. **Find HCF of following numbers using Division method.**

(a) 48, 60 (b) 120, 140

(c) 12, 18, 64 (d) 60, 96, 128

7. Find the largest number which divides 60,75 and 90 without any remainder

8. There are three drums that contain 36 *l*, 45 *l* and 72 *l* milk respectively. Find the largest vessel which can measure milk of all three drums completely.

3.6 Lowest Common Multiple (L.C.M.)

Activity

Teacher : In our 5th class, find out the least number of students that can form teams of 3 students each and 4 students each in a way that no student is left out.

Teacher - Teacher will call the first team of 3 students and ask them if it is possible to make a team of 4 students out of 3 ?

Students - No, Sir.

Now, teacher will call another team of 3 students and ask them if it is possible to make a team of 4 students out of these. If possible, is any child left out of the team ?

Students - Yes Sir, two students will be left out after forming of team of 4 students.

Teacher - (Calls one more team of 3 students). Is it possible to make team of 4 students out of all students, standing near to me ? If so then how many students will be left out ?

Students - Yes, one more team can be formed but one student will be left out.

Teacher - (Calls one more team of 3 students.) Now can we make another team of 4 students ? If possible, is any student left out of the team.

Students - Yes, one more team will be formed. No student will be left out.

The Teacher will explain that first, we made four teams of 3 students each, then with these students, we formed teams of 4 students. We find that we need minimum 12 students to form teams of 3 student and 4 students each. This activity is based on Lowest Common Multiple (LCM).

So, the smallest multiple of 3 and 4 is 12. This smallest multiple is called LCM.

Activity

To find LCM using a game activitiy i.e., write numbers 1 to 100 in a grid of 10 × 10.

To find LCM of 3, 4 and 6, call three students.

1. Tell the first student to put the blue dart on the multiples of 3.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. Tell the second student to put the yellow dart on the multiples of 4.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3. Tell the third student to put the green dart on the multiples of 6.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Teacher will tell the students that in the grid when three darts of different colours are together in the same box the first number with three coloured darts will be the L.C.M.

So, LCM of 3, 4 and 6 is 12.

Note Teachers are advised to use button, bindi etc. as per their convenience.

The teacher will give some more examples using different numbers.

Now write the multiples of 8 and 12

First Child - Multiples of 8 = 8, 16, 24, 32, 40, 48, 56, 64, 72 .....

Second Child - Multiples of 12 = 12, 24, 36, 48, 60, 72 .....

Look at these multiples carefully and write the common multiples.

So the common multiples are : 24, 48, 72 ....

The multiples of two different numbers will be infinite.

But out of these multiples, the smallest common multiples is known as L.C.M.

So, the LCM of 8 and 12 is 24

Things to Remember

- The LCM is the smallest common multiple among the multiples of two or more than two numbers.
- If one number is also the multiple of another number then the multiple itself is the L.C.M. of the two numbers

We can find LCM by using the following methods :

(a) LCM through multiples

(b) LCM using Prime factorisation

(c) LCM using Division Method.

Hint for the Teacher The teacher will do on activity. He will choose two students, one of whom will be told to jump to a distance of 2 feet and the other one to a distance of 3 feet. They will keep convering distance with jumps till they reach the same distance. Similarly LCM method can be used in making teams and in other day to day activites.

3.6(1) LCM through Multiples :

In this method, first of all we will find the multiples of given numbers. Then we will list the common multiples of given numbers. Now the lowest common multiple is the required LCM.

Let us consider one example.

Example 1 : Find LCM of 3, 6 and 9.

Solution : Multiples of 3 = 3, 6, 9, 12, 15, 18, 21, 24...., ..., ...

Multiples of 6 = 6, 12, 18, 24, 30, 36, 42 ...., ..., ...

Multiples of 9 = 9, 18, 27, 36, 45...., ..., ...

Common multiples of 3, 6 and 9 = 18, 36 ...., ..., ...

Lowest common Multiple = 18

So, LCM of 3, 6 and 9 is 18.

3.6(2) LCM using Prime Factorisation :

In this method, we will first list the prime factors of the numbers and then multiply the common factors and the remaining prime factors.

Let us consider one example.

Example 2 : Find LCM of 12 and 48 using Prime factorisation.

Solution : $12 = 2 \times 2 \times 3$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

Common Factors $= 2 \times 2 \times 3$

Remaining factors $= 2 \times 2$

L.C.M. $= 2 \times 2 \times 3 \times 2 \times 2$

$= 48$

2	12
2	6
3	3
	1

2	48
2	24
2	12
2	6
3	3
	1

Or

In both, prime factorisation , 2 has occurred maximum four times and 3 has occurred maximum one time.

So, LCM $= 2 \times 2 \times 2 \times 2 \times 3 = 48$.

3.6(3) LCM using Common Division Method :

In this method, we follow the steps given below:

- Divide with the smallest prime number, which can divide at least one of the given numbers. Bring down the numbers that cannot be divided further.
- Continue division with the smallest possible prime numbers till the last point till we get 1.
- In this way, the prime factors will be multiplied resulting in LCM.

LCM can be further understood with the following example.

Example 3 : Find LCM of 6 and 12 using division method.

Solution :

2	6,	12
2	3,	6
3	3,	3
	1,	1

LCM of 6 and 12 = $2 \times 2 \times 3 = 12$

Example 4 : Find LCM of 8, 12 and 24.

Solution :

2	8,	12,	24
2	4,	6,	12
2	2,	3,	6
3	1,	3,	3
	1,	1,	1

LCM of 8, 12 and 24 = $2 \times 2 \times 2 \times 3 = 24$

Exercise-3.3

1. Find LCM of the following :

(a) 5, 10 (b) 6, 18 (c) 25, 50 (d) 6, 24

2. Find LCM of the following :

(a) 4, 8 and 12 (b) 6, 12 and 24

(c) 15, 18 and 27 (d) 24, 36 and 40

3. Find LCM of following using Prime factorisation.

(a) 32, 40
(b) 24, 36
(c) 15, 30 and 45
(d) 40, 44 and 48

4. Find LCM of following using Division method:

(a) 15, 20
(b) 12, 38
(c) 30, 45 and 50
(d) 40, 68 and 60

5. Find the smallest number which is divisible by 12, 15 and 20 completely.

6. One child jumps 3 feet high and another child jumps 4 feet high. If both the children continue jumping together in same direction then after how many feet they will be together again ?

7. How many minimum number of students are required from a class to make groups of 4 each and 5 each so that no student is left ?

8. Three bells ring with a time gap of 10 min, 20 min and 30 min respectively in a school. If all bells are rung together at 8 : 00 am then after how long the bells would ring together again ?

Multiple Choice Questions (MCQs)

1. Which number is smallest even Prime number ? ....

(a) 0 (b) 1 (c) 2 (d) 4

2. Which number is neither Prime nor Composite ?

(a) 1 (b) 2 (c) 3 (d) 4

3. Which numbers are Prime numbers between 70 and 80 ?

(a) 71, 72, 73 (b) 71, 75, 79 (c) 71, 80 (d) 71, 73 79

4. HCF of 75 and 90 = ...

(a) 5 (b) 10 (c) 15 (d) 20

5. LCM of 12, 18 and 24 = ...

(a) 72 (b) 36 (c) 48 (d) 24

6. If HCF of any two numbers is 8 then out of following which can not be LCM of that numbers ?

(a) 48 (b) 60 (c) 24 (d) 56

7. What is length of largest tape which can measure the lengths of 24 m and 30 m ?

(a) 4 m (b) 5 m (c) 6 m (d) 7 m

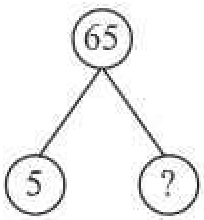
8. What is the smallest number which is divisible by 8 and 12 ?

(a) 16 (b) 48 (c) 72 (d) 24

9. LCM of 26 and 39 = ...

(a) 13 (b) 78 (c) 39 (d) 26

10.



(a) 5 (b) 65 (c) 12 (d) 13

11. Which number is composite number in the following ?

(a) 43 (b) 23 (c) 21 (d) 37

12. Out of the following, which number is multiple of 19 ?

(a) 171 (b) 172 (c) 173 (d) 174

13. HCF of 15, 45 and 105 = ...

(a) 15 (b) 5 (c) 30 (d) 45

14. What is the HCF of two prime numbers ?

(a) 1 (b) 2 (c) 3 (d) 4

15. Three bells ring with the time gap of 10 min, 15min and 20min respectively in a school. If all bells are rung together at 9 : 00 am then after how long the bells would ring together again ?

(a) 11:00 o'clock (b) 08:00 o'clock

(c) 10:00 o'clock (d) 12:00 o'clock

Read this pattern carefully and answer questions (16 – 20)

First odd number	1	$1 =$	1	$= 1 \times 1$
First 2 odd numbers	1, 3	$1 + 3 =$	4	$= 2 \times 2$
First 3 odd numbers	1, 3, 5	$1 + 3 + 5 =$	9	$= 3 \times 3$
First 4 odd numbers	1, 3, 5, 7	$1 + 3 + 5 + 7 =$	16	$= 4 \times 4$

First Even number	2	$2 =$	2	$= 1 \times 2$
First 2 even numbers	2, 4	$2 + 4 =$	6	$= 2 \times 3$
First 3 even numbers	2, 4, 6	$2 + 4 + 6 =$	12	$= 3 \times 4$
First 4 even numbers	2, 4, 6, 8	$2 + 4 + 6 + 8 =$	20	$= 4 \times 5$

16. Read the above pattern and find the sum of first 6 Odd numbers.

(a) 30 (b) 12 (c) 25 (d) 36

17. Read the above pattern, find the sum of first 10 Odd numbrs

(a) 20 (b) 50 (c) 100 (d) 40

18. Read the above pattern, Find the sum of first 8 Even numbers

(a) 16 (b) 24 (c) 72 (d) 64

19. Read the above pattern, find sum of first 9 Even number.

(a) 19 (b) 18 (c) 45 (d) 90

20. On a given road, the poles are erected at a distance of 24 m each and pile of stones are lying at a distance of 30 m each. If the Ist pile of stones is lying adjacent to the fist pole, at what distance will the pole and the pile of stones be together again ?

(a) 100 m (b) 110 m

(c) 150 m (d) 120 m

Facts :

- A number with unit digit 0, 2, 4, 6, 8 is divisible by 2, then 2 is the factor of that number.
- A number with unit digit 0 and 5 is divisible by 5, then 5 is the factor of that number.
- A number with unit digit 0 is divisible by 10.
- If sum of digits of any number is divisible by 3 then that number is divisible by 3.

Learning Outcomes

Students will have learnt the following :

- Finding HCF and LCM of numbers.
- Use of different methods such as prime factorisation, division method of HCF and LCM.
- To solve the problems of HCF and LCM in daily life.
- Prepare for competitive exam.

Answers

Exercise 3.1

1. (a) 5, 10, 15, 20, 25 (b) 9, 18, 27, 36, 45
(c) 10, 20, 30, 40, 50 (d) 12, 24, 36, 48, 60
(e) 16, 32, 48, 64, 80 (f) 17, 34, 51, 68, 85

2. (a) 1, 5 (b) 1, 2, 4, 8
(c) 1, 2, 7, 14 (d) 1, 2, 3, 4, 6, 12
(e) 1, 5, 25 (f) 1, 2, 3, 4, 6, 9, 12, 18, 36

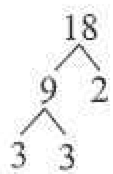
3. (a) 1, 2, 3, 6, 9, 18 (b) 1, 2, 3, 4, 6, 8, 12, 24
(c) 1, 5, 7, 35 (d) 1, 2, 3, 4, 6, 9, 12, 18, 36
(e) 1, 3, 5, 9, 15, 45 (f) 1, 3, 7, 21

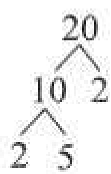
4. (a) 12, 34, 16, 28 (b) 48, 72, 90
(c) 450, 568, 664, 98 (d) 456, 968, 604, 888
(e) 136, 446, 1278 (f) 168, 5864

5. (a) 11, 23, 19, 35 (b) 45, 69, 97
(c) 451, 215, 789, 983 (d) 237, 739
(e) 631, 135, 249, 1279, 2851 (f) 49, 765, 1729, 9261, 6859

6. (a) 4 and 9 (b) 8 and 7
(c) 3, 5 and 6 (d) 80
(e) 2, 3 and 5

7. (a) True (b) False (c) True (d) False
(e) True (f) False (g) False (h) False
(i) False (j) False

8. (b) 

(c) 

9. $\boxed{2} - \boxed{38} - \boxed{19}$ $\boxed{3} - \boxed{39} - \boxed{13}$
$\boxed{5} - \boxed{85} - \boxed{17}$ $\boxed{11} - \boxed{77} - \boxed{7}$

10. (a) odd (b) even (c) odd (d) odd
(e) even

Exercise 3.2

1. (a) 5, 7, 3 (b) 2, 11, 13
(c) 5 (d) 41, 23, 17, 19
(e) 29, 37, 47

2. (a) 14, 9, 6 (b) 21, 12, 18 (c) 32, 15 (d) 24, 49, 50

3. (a) 9 (b) 21 (c) 20 (d) 14

4. (a) 10 (b) 8 (c) 12 (d) 5

5. (a) 42 (b) 45 (c) 16 (d) 15

6. (a) 12 (b) 20 (c) 2 (d) 4

7. 15

8. 9 *l*

Exercise 3.3

1. (a) 10	(b) 18	(c) 50	(d) 72
2. (a) 24	(b) 24	(c) 270	(d) 360
3. (a) 160	(b) 72	(c) 90	(d) 2640
4. (a) 60	(b) 228	(c) 450	(d) 2040
5. 60	**6.** 12 Feet	**7.** 20 Children	
8. 9.00 am			

Multi Choice Questions (MCQ)

1. c	**2.** a	**3.** d	**4.** c
5. a	**6.** b	**7.** c	**8.** d
9. b	**10.** d	**11.** c	**12.** a
13. a	**14.** a	**15.** c	**16.** d
17. c	**18.** c	**19.** d	**20.** d