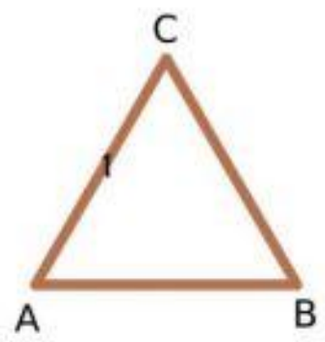
Chapter 2:

CIRCLES

Focus Points

- ⋆ Angle in the semicircle is 90°
- ⋆ Angle outside the semicircle is less than 90°
- ⋆ Angle inside the semicircle is greater than 90°
- ⋆ An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement
- ⋆ Angle formed by the arc in the complement is half the angle at the centre
- ⋆ Sum of the angles at the centre and in the complement is 180°
- ⋆ Angles on an arc are equal
- ⋆ If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.
- ⋆ The sum of the opposite angles of a cyclic quadrilateral is 180°.
- ⋆ The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is 180° it will be a cyclic quadrilateral.
- ⋆ Square, rectangle and isosceles trapezium are cyclic .
- ⋆ Two chords of a circle AB and CD intersect at the point P inside the circle .It can be proved that $PA \times PB = PC \times PD$
- ⋆ This relation can be used to construct a rectangle having equal area of another rectangle.
- ⋆ If the chords intersect outside the circle ,the same relation holds. $PA \times PB = PC \times PD$
- ⋆ In the case of the intersecting chords of a circle,if one chord AB is the diametre of the circle and other chordCD is perpendicular to the diametre ,then $PA \times PB = PC^2$
- ⋆ This relation is used to construct a square with same area of a rectangle.It can be used to draw the lines of irrational lengths.

1) $ABC$is a triangle in which $AB = AC = BC$



a) What are the measure of its angles?

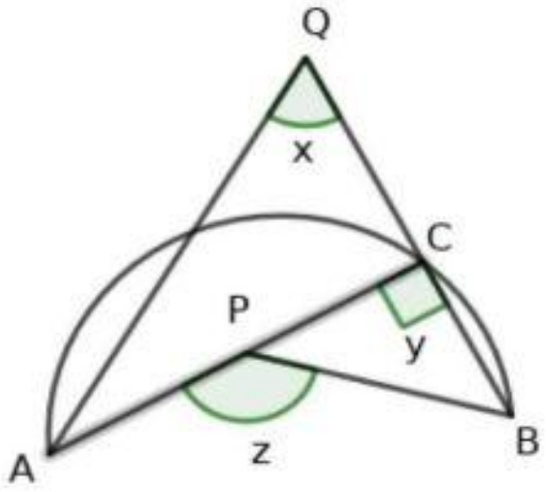
b) What is the position of the vertex C based on the circle with diametreAB?Is it inside ,outside or on the circle.

Answers

a) 60°

b) Outside the circle

2) In the figure AB is the diametre of a semicircle.Three angles x, y, z are marked outside, on the semicircle and inside the semicircle.



a) What is the value of y?

b) If x, y, z are in an arithmetic sequence,then what is $x + z$?

c) If the common difference of the sequence is 50 then find x and z

Answers

a) $y = 90^\circ$

b) $x + z = 2 \times 90 = 180^\circ$ (Refer the propery of arithmetic sequence)

c) $d = 50 \therefore x = 90 - 50 = 40^\circ, z = 90 + 50 = 140^\circ$

3) a) Draw a circle of radius 3cm. Construct a square with vertices are on the circle.

b) What is the length of its side?

c) Calculate the area of the square.

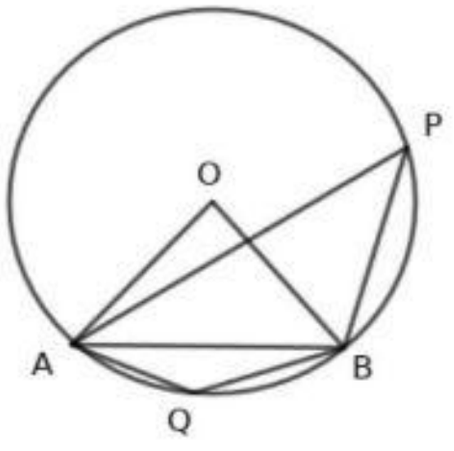
Answers

a) Draw a circle of radius 3cm and diametreAB.

b) Draw another diametre CDperpendicular to AB. Join the end points of the diametre . $ACBD$ will be a square

c) Side of the square $AC = \sqrt{3^2 + 3^2} = 3\sqrt{2}$cm.
Area $= 3\sqrt{2} \times 3\sqrt{2} = 18$sq.cm

4) In the figure $\angle BDE = 40°$



a) What is the measure of angleADB?

b) Wha is the measure of angleACB?

c) What is the measure of angle AOB?

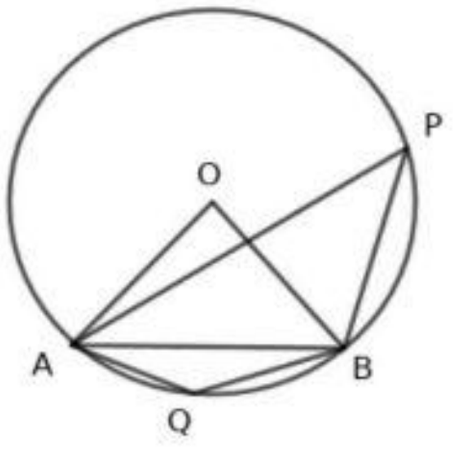
Answers

a) $\angle ADB = 180 - 40 = 140°$

b) $\angle ACB = 180 - 140 = 40°$

c) $\angle AOB = 2 \times 40 = 80°$

5) Triangle OAB is an equilateral triangle



a) What is the measure of angleAOB?

b) What is the measure of angleAPB?

c) What is the measure of angleAQB?

Answers

a) $\angle AOB = 60°$

b) $\angle APB = \frac{1}{2} \times 60° = 30°$

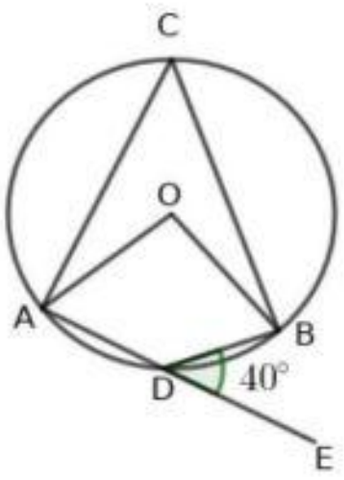
c) $\angle AQB = 180 - 30 = 150°$

6) Draw a circle of radus 3cm.Construct the angles $30°$ and $150°$ with vertices on the circle using compasses and scale only.

Answers

- ⋆ Draw a circle of radius 3cm.Mark the center of the circle as O
- ⋆ Mark a point A on the circle.Draw the radius OA.
- ⋆ With A as the centre and OA as radius , draw an arc which cut the circle at B. Join $OB, \angle AOB = 60°$
- ⋆ Mark a point Pon the complement of the arc AB,which makes $60°$ at the center.$\angle APB = \frac{1}{2} \times 60 = 30°$
- ⋆ Mark a point Q on the arc AB. $\angle AQB = 180 - 30 = 150°$

7) In the figure $BDE = 40°$



a) What is the measure of $\angle ADB$?

b) What is the measure of $\angle ACB$?

c) What is the measure of $\angle AOB$

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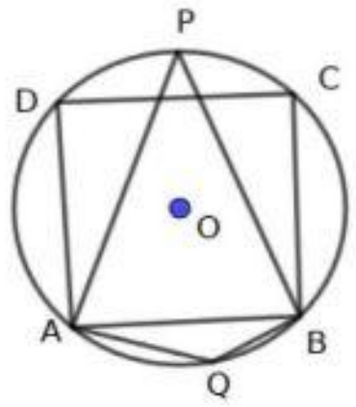
Answer

a) $\angle ADB = 180 - 40 = 140°$

b) $\angle ACB = 180 - 140 = 40°$

c) $\angle AOB = 2 \times 40 = 80°$

8) $ABCD$ is a square .The diagonals AC and BD intersect at O.



a) What is the measure of angle AOB?

b) What is the measure of angle APB?

c) What is the measure of angle AQB

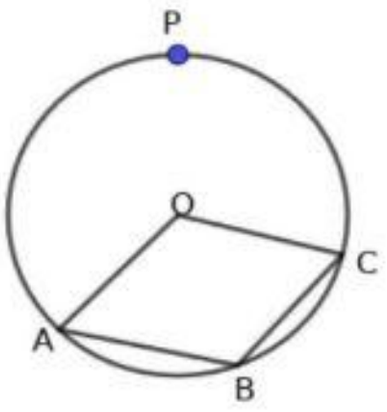
Answers

a) Diagonals of a square are perpendicular to eachother.$\angle AOD = 90°$

b) $\angle APD = 45°$

c) $\angle AQD = 180 - 45 = 135°$

9) $OABC$ is a parallelogram.Three vertices are on a circle and one at the centre.P is a point on the circle



a) Draw AP and CP, mark the angle$APC = x$. What is $\angle AOC$

b) What is angle ABC?

c) Find x

d) Find the angles of the parallelogram

Answers

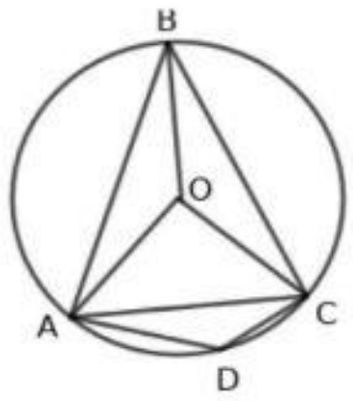
a) $\angle AOC = 2x$

b) $\angle ABC = 2x$ Opposite angles of a parallelogram are equal

c) $\angle APC + \angle ABC = 180°, x + 2x = 180, 3x = 180, x = 60$

d) Angles are $120°, 60°, 120°, 60°$

10) In the figure O is the centre of the circle,$\angle BAO = 20°, \angle BCO = 10°$



a) What is the measure of angleABC?

b) What is the measure of angleAOC?

c) What is the measure of angleADC?

d) Find the angles of triangle AOC

e) If the diametre of the circle is 10cm then find the length of the chord AB

Answers

a) In triangle OAB,$OA = OB$. Angles opposite to the equal sides are equal.Similarly in the case of triangle OBCalso.
$\angle ABC = 20 + 10 = 30°$

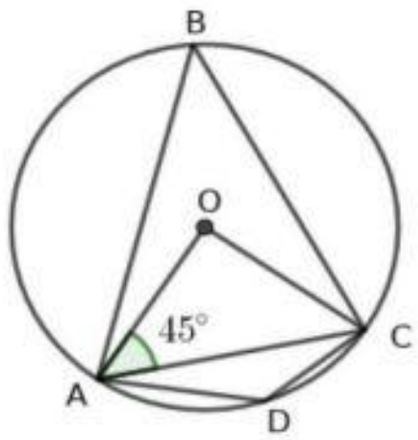
b) $\angle AOC = 2 \times 30 = 60°$

c) $\angle ADC = 180 - 30 = 150°$

d) Triangle AOC, $OA = OC$, $\angle OAC = \angle OCA = \frac{180-60}{2} = 60° \triangle OAC$ is an equilateral triangle. Angles are $60°$each.

e) $OA = AC = OC = 5$cm, radius 5cm.

11) In the figure $O$is the centre of the circle, $\angle OAC = 45°$then



a) What kind of triangle is OAC?

b) What is the measure of angleABC?

c) What is the measure of angle ADC?

d) If the radius of the circle is 6cm then what is the length of the chord AC.

Answers

a) $OA = OC, \angle OAC = \angle OCA = 45°, \angle AOC = 90°$. $\triangle OAC$ is an isosceles right triangle

b) $\angle ABC = \frac{1}{2} AOC = 45°$

c) $\angle ADC = 180 - 45 = 135°$

d) $AC = \sqrt{6^2 + 6^2} = 6\sqrt{2}$cm

12) Draw a circle of radius 3cm , construct an equilateral triangle with vertices on the circle. What is the length of the side?

Answers

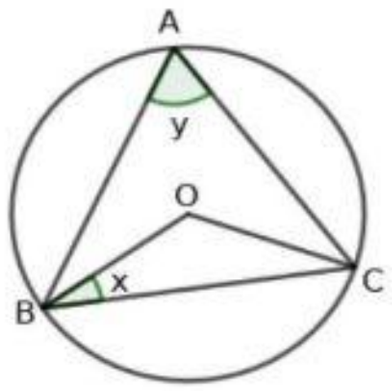
★ Draw a circle with centre O and radius 3cm. Mark a point A on the circle and radiusOA.

★ Divide the angle around the centre $120°$each and mark the points B, Con the circle

★ Draw triangle ABC .

★ Length of side $= 3\sqrt{3}$cm

13) $O$is the centre of the circumcircle of triangle ABC.
If $\angle BAC = y, \angle OBC = x$then



a) What is the measure of $\angle BCO$?

b) What is the measure of $\angle BOC$?

c) Prove that $x + y = 90°$

Answers

a) Since $OB = OC$ opposite angles of these sides in triangle OBC are equal.
$\angle BCO = x$.

b) $\angle BOC = 180 - 2x$

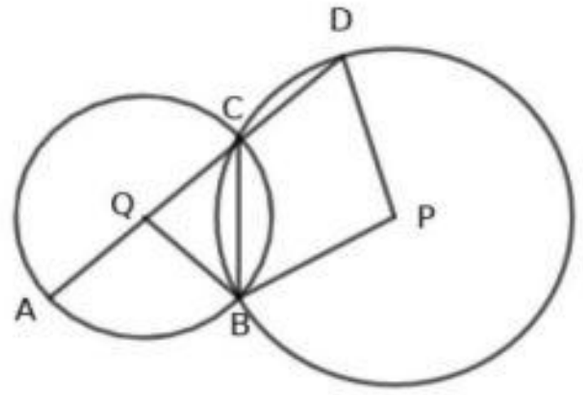
c) We know that $\angle BOC = 2 \times \angle BAC$
$180 - 2x = 2y, 2x + 2y = 180, x + y = 90°$

14) In triangle ABC, $\angle A = 70°, \angle B = 80°$. The vertices of the triangle are on the circumcircle of the triangle .Radius of the circumcircle is 3cm.Construct the triangle.

Answers

- ⋆ Draw a circle of radius 3 , mark its centre as$O$ and a point A on the circle. Draw the radius OA
- ⋆ Mark a point B on the circle such that $\angle AOB = 2 \times 70 = 140°$
- ⋆ Mark a point C on the circle such that $\angle BOC = 2 \times 80 = 160°$
- ⋆ Draw triangle ABC .

15) P and Q are the centre of the circles shown in the figure. Circles intersect at B and C.If $\angle AQB = 130°$ then



a) What is the measure of $\angle ACB$?

b) What is the measure of $\angle BCD$?

c) What is the measure of $\angle BPD$

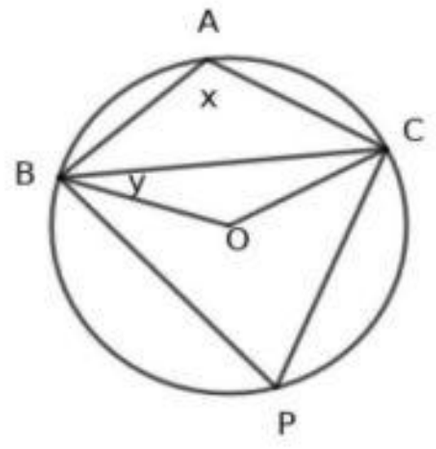
Answers

a) $\angle ACB = \frac{1}{2} \times 130 = 65°$

b) $\angle BCD = 180 - 65 = 115°$

c) The central angle of the complement of the arcBCD is $2 \times 115 = 230°$.Therefore $\angle BPD = 360 - 230 = 130°$

16) In the figure $\angle BAC = x, \angle CBO = y, O$ is the centre of the circle.



a) What is the measure of $\angle BCO$?

b) What is the measure of $\angle BOC$?

c) What is the measure of $\angle BPC$?

d) Prove that $x - y = 90°$?

Answers

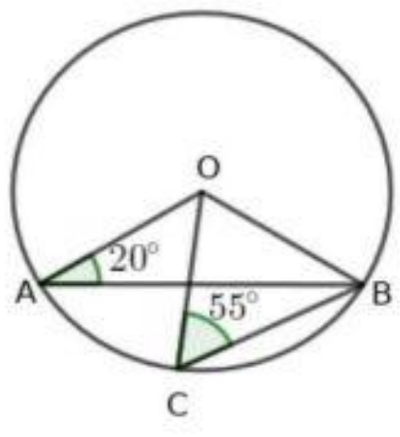
a) $\angle BCO = y$
(In triangle $BCO, OB = OC$. Angles opposite to equal sides are equal.)

b) $\angle BOC = 180 - 2y$

c) $\angle BPC = \frac{1}{2} \times \angle BOC = \frac{1}{2} \times (180 - 2y) = 90 - y$

d) $\angle BAC + \angle BPC = 180, x + 90 - y = 180, x - y = 180 - 90 = 90°$

17) In the figure O is the centre of the circle .If $\angle BCO = 55°, \angle BAO = 20°$ then



a) What is the measure of $\angle OBC$?

b) What is the measure of $\angle BOC$?

c) What is the measure of $\angle AOC$?

d) What is the measure of $\angle ABC$?

Answers

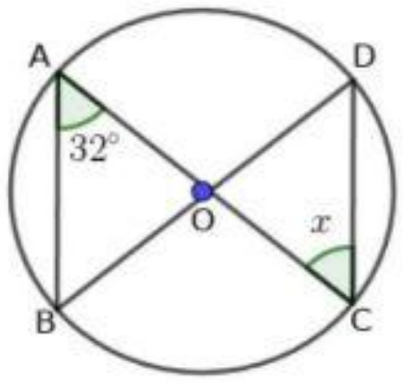
a) $\angle OBC = 55°$
(In triangle $OBC, OB = OC$. Angles opposite to equal sides are equal.)

b) $\angle BOC = 180 - (55 + 55) = 180 - 110 = 70°$

c) In triangle $AOB, \angle B = 20°, \angle ACB = 180 - 40 = 140°$
$\angle AOC = 140 - 70 = 70°$

d) $\angle ABC = \frac{1}{2} \times 70 = 35°$

18) In the figure O is the centre of the circle.If $\angle BAC = 32°$ then



a) Find the angles of triangle OAB

b) What is the measure of $\angle DOC$?

c) Find x

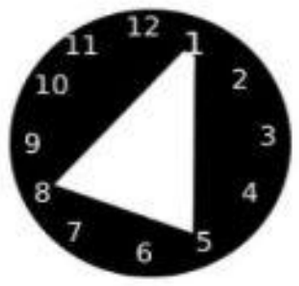
Answers

a) In triangle OAB,$OA = OB$. Therefore $\angle B = 32°$, $\angle AOB = 180 - 64 = 116°$

b) $\angle DOC = 116°$
(Opposite angles are equal)

c) In triangle OCD,$\angle D = x$
$x + x + 116 = 180, 2x = 64, x = 32$

19) This is the picture of a clock face.$1, 8, 5$ are joined to make a triangle.Find the angles of this triangle.



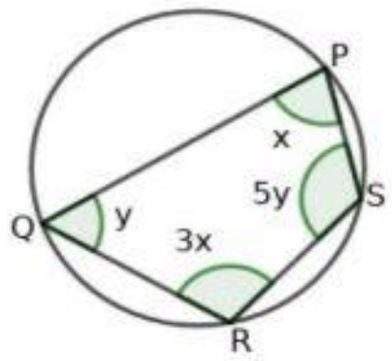
Answers

★ The central angle of the arc in between two numbers(say $1, 2$) is $\frac{1}{12} \times 360 = 30°$.
The central angle of the arc between 1 and 5 is $4 \times 30 = 120°$.
The angle of the triangle with vertex at 8 is $\frac{1}{2} \times 120 = 60°$

★ The central angle of the arc between 8 and 5 is $3 \times 30 = 90°$.
The angle of the triangle with vertex at 1 is $\frac{1}{2} \times 90 = 45°$

★ The central angle of the arc between 8 and 1 is $5 \times 30 = 150°$.
The angle of the triangle with vertex at 5 is $\frac{1}{2} \times 150 = 75°$

20) In the figure $PQRS$ is a cyclic quadrilateral.$\angle P = x, \angle Q = y, \angle R = 3x, \angle S = 5y$.



a) Find x and y

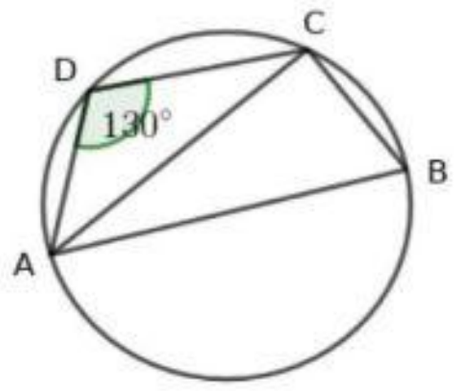
b) Find the angles of the quadrilateral.

Answers

a) $\angle P + \angle R = 180°$
$x + 3x = 180, 4x = 180, x = 45$
$\angle Q + \angle S = 180°, y + 5y = 180, 6y = 180, y = 30$

b) $\angle P = 45°, \angle R = 3 \times 45 = 135° \ \angle Q = 30°, \angle S = 5 \times 30 = 150°$

21) $ABCD$ is a cyclic quadrilateral.AB is the diametre of the circle , $AD = CD$ and $\angle ADC = 130°$.



a) What is the measure of $\angle ACB$?

b) What is the measure of $\angle ABC$?

c) Find $\angle DCB$.

d) What is the measure of $\angle BAD$?

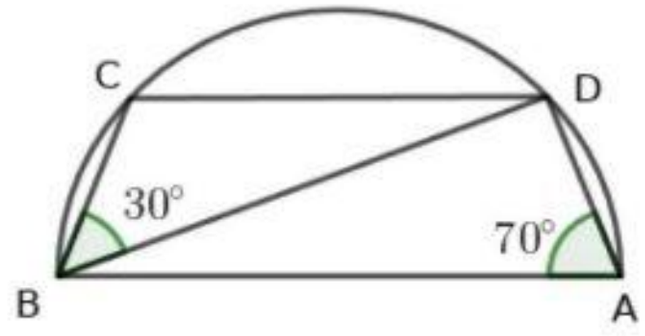
Answers

a) $\angle ACB = 90°$(Angle in the semicircle)

b) $\angle ABC = 180 - 130 = 50°$

c) Since $CD = AD$,the angles opposite to the equal sides of triangle $ADC$are equal.
$\angle DCA = 25°, \angle DCB = 90 + 25 = 115°$

d) $\angle BAD = 180 - 115 = 65°$



a) What is the measure of $\angle BCD$?

b) What is the measure of $\angle CDB$?

c) What is the measure of $\angle ADC$?

d) What is the measure of $\angle ABD$?

Answers

a) $\angle BCD = 180 - 70 = 110°$

b) $\angle CDB = 180 - (30 + 110) = 180 - 140 = 40°$

c) $\angle ADC = \angle ADB + \angle BDC = 90 + 40 = 130°$

d) $\angle ABD = 180 - (90 + 70) = 180 - 160 = 20°$

25) The parallelogram which is not a rectangle is not cyclic. Justify this statement

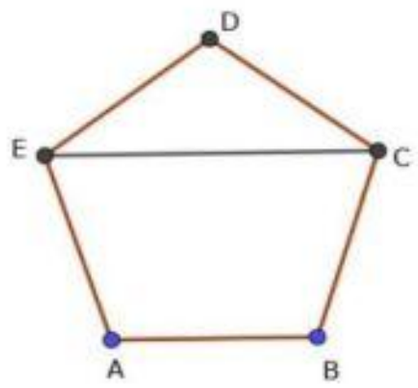
Answers

⋆ $ABCD$ is a parallelogram(Draw a parallelogram and name it as $ABCD$ in an order.
$\angle A = \angle C$

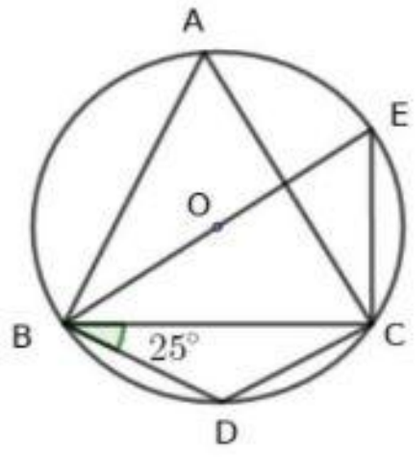
⋆ Since $ABCD$ is not a rectangle $\angle A \neq 90°, \angle C \neq 90°$

⋆ $\angle A + \angle C \neq 180°$.
$\therefore ABCD$ is not cyclic

26 In the figure $ABCDE$is a regular pentagon.Prove that $ABCE$is a cyclic quadrilateral.



22 In the figure $BD = CD, \angle DBC = 25°$



a) What is the measure of $\angle BDC$?

b) What is the measure of $\angle BAC$?

c) What is the measure of $\angle EBC$?

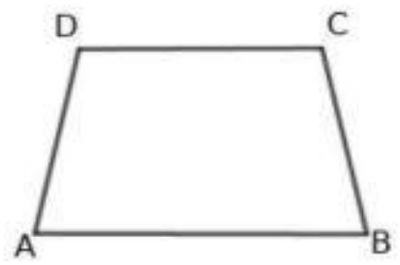
Answers

a) In triangle $BDC, BD = CD$. Angle opposite to these sides are equal .$\angle BCD = 25°$
$\angle BDC = 180 - (25 + 25) = 130°$

b) $\angle BAC = 180 - 130 = 50°$

c) $\angle BEC = \angle BAC = 180 - 130 = 50°, \angle EBC = 180 - (90 + 50) = 180 - 140 = 40°$

23) In the figure $ABCD$ is a quadrilateral in which AB is parallel to CD and $AD = BC$



Prove that $ABCD$ is a cyclic quadrilateral.

Answers

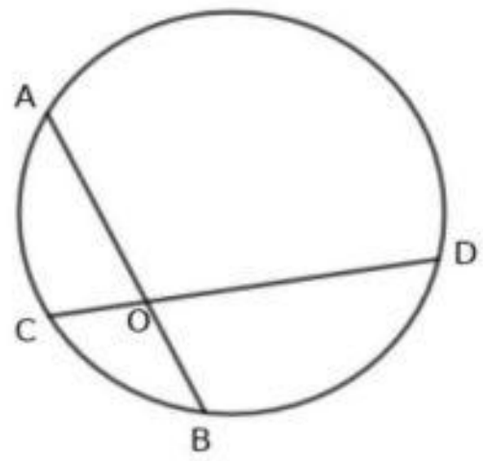
- ⋆ The line AB is parallel to CD. Therefore $\angle A + \angle D = 180°$
- ⋆ Since $AD = BC$then $ABCD$ is an isosceles trapezium $\angle A = \angle B$
- ⋆ Therefore $\angle B + \angle D = 180°$. $ABCD$is a cyclic quadrilateral .

24) C, D are two points in a semicircle of diametre AB.
If $\angle BAD = 70°, \angle DBC = 30°$then

Answers

- ⋆ One angle of the regular pentagon is $= \frac{(5-2)\times 180}{5} = 108°$
- ⋆ In triangle $EDC, ED = CD, \angle DEC = \angle DCE = \frac{180-108}{2} = 36°$
- ⋆ $\angle ECB = 108 - 36 = 72°$. In the quadrilateral $ABCE, \angle A + \angle C = 108 + 72 = 180°$.
  $ABCE$ is cyclic.

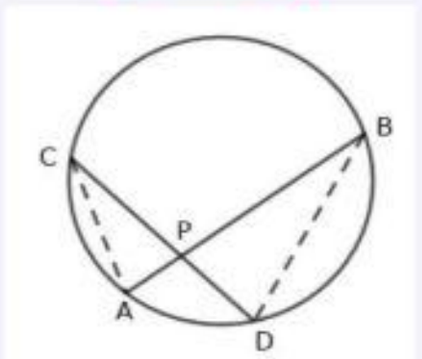
27) In the figure the chord AB has length 8cm and $OA = 5$cm.



a) What is the length of OB?

b) If$OC = 2.5$cm, what is the length of OD?

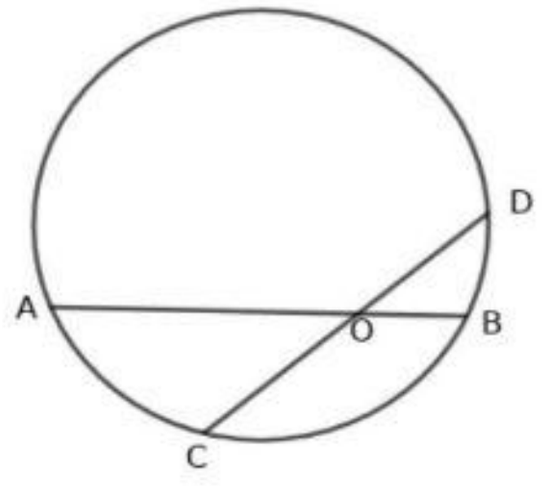
Answers



a) $OB = 8 - 5 = 3$cm

b) $OA \times OB = OC \times OD$
$5 \times 3 = 2.5 \times OD, OD = \frac{15}{2.5} = 6$cm

28) The chords AB and CD intersect at O .This point divide each chord into two segments



a) What is the relation between these segments?

b) If $CD = 10$cm and $OD = 4$cm then what is the length OC?

c) If $OA = 8$cm , $OC = 6$cm and $OD = 4$cm then what is the length OB?

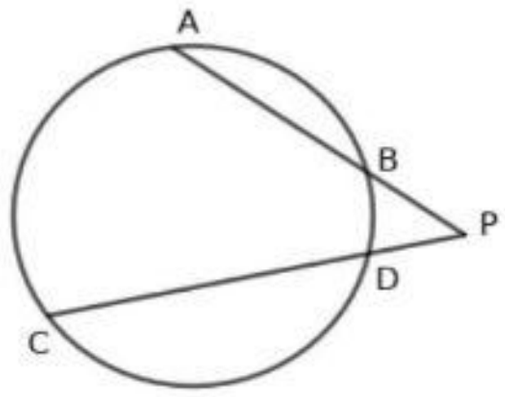
Answers

a) $OA \times OB = OC \times OD$

b) $OC = CD - OD = 10 - 4 = 6$ cm

c) $8 \times OB = 6 \times 4, OB = 3$cm

29) The chords AB and CD intersect at P outside the circle.



a) What is the relation between PA, PB, PC and PD?

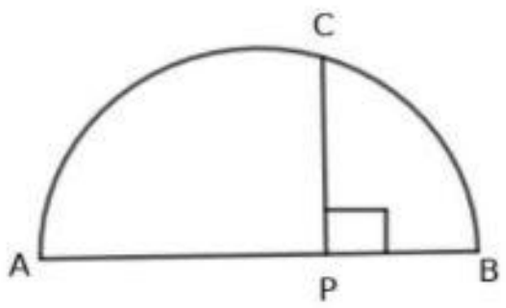
b) If $AB = 5$cm , $PB = 3$cm , $PD = 2$cm then what is the length CD?

Answers

a) $PA \times PB = PC \times PD$

b) $(5+3) \times 3 = (PD + CD) \times PD$
$(5+3) \times 3 = (2 + CD) \times 2$
$2 + CD = 12, CD = 10$cm

30) AB is the diametre of a semicircle, P is a point on AB and PC is perpendicular to AB



a) Prove that $PA \times PB = PC^2$

b) If$PA = 9$cm , $PB = 4$ cm then what is the length PC?

c) What is the area of the square with side PC?

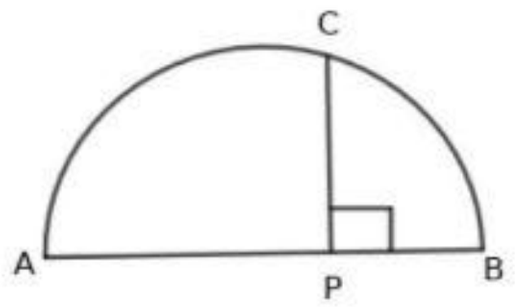
Answers

a) Draw AC, BC.Consider triangle APC and triangle BPC.If$\angle PAC = x$then $\angle PCA = 90 - x, \angle PCB = 90 - (90 - x) = x, \angle PBC = 90 - x$
These are similar triangles .Sides opposite to the equal angles are proportional .
$\frac{PC}{PB} = \frac{PA}{PC}$
$PA \times PB = PC^2$.

b) $PC^2 = 9 \times 4 = 36, PC = 6$cm

c) Area $PC^2 = 36$sq.cm

31)]AB is the diametre of a semicircle, P is a point on AB and PC is perpendicular to AB



a) If $PC = 6$cm,and $PB = 3$cm then what is the length of PA

b) What is the radius of the circle ?

c) What is the area of the square drawn with side PC?

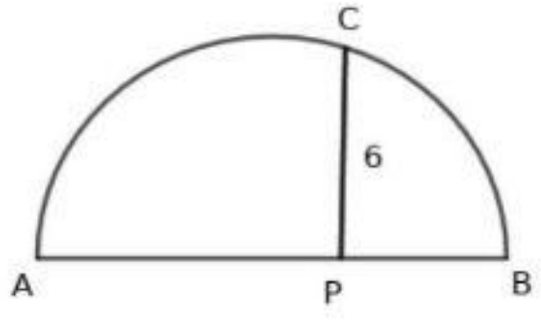
Answers

a) $PA \times PB = PC^2$
$PA \times 3 = 6^2, PA = 12$cm

b) $AB = 12 + 3 = 15$cm ,Radius 7.5cm.

c) Area $PC^2 = 36$sq.cm

32) In the figure AB is the diametre of the circle and PC is perpendicular to the diametre. $PA : PB = 2 : 1$ and $PC = 6$cm.



a) Write the relation between PA, PB and PC?

b) Find the lengths PA and PB

c) What is the radius of the circle?

Answers

a) $PA \times PB = PC^2$

b) If $PB = x$,$2x \times x = 6^2$,$2x^2 = 36, x^2 = 18, x = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$
$PA = 6\sqrt{2}, PB = 3\sqrt{2}$

c) $AB = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$
Radius $= \frac{9\sqrt{2}}{2}$ cm

33) Draw a rectangle of sides 5cm and 3cm .Construct a square whose area is same as the area of the rectangle

Answers

- ⋆ Draw the quadrilateral $ABCD$. $AB = 5$cm , $BC = 3$cm.
- ⋆ Produce AB and mark the point E such that $BC = BE$
- ⋆ Draw a semicircle of diametre AE.Produce BC, meet the semixircle at F.
- ⋆ $BA \times BE = BF^2$ can be written as $AB \times BC = BF^2$. $AB \times BC$ is the area of the rectangle
- ⋆ Draw a square of side BF.Area of rectangle is equal to the area of the square as per the relation $AB \times BC = BF^2$

34) Draw an equilateral triangle of one side $\sqrt{18}$cm

- ⋆ $18 = 6 \times 3, 6 + 3 = 9$
  Draw a line $AB$of length 9cm
- ⋆ Draw a semicircle with diameter AB. Mark a point P at the diatance 6cm from A. Draw a perpendicular from P to AB. This line cut the circle at C. $PC = \sqrt{18}$.
- ⋆ Draw an equilateral triangle with PC as side