2 QUADRATIC EQUATION

QUADRATIC EQUATIONS WITH REAL COEFFICIENTS

The expression, $ax^2 + bx + c$, where $a, b, c \in R$ and $a \neq 0$ is called as a quadratic expression in x. The quadratic expression when equated to zero, $ax^2 + bx + c = 0$, is called as a quadratic equation in x. Where the numbers a, b, c are called the coefficients of the equation.

1. The values of x which satisfy the quadratic equation is called roots (also called solutions or zeros) of the quadratic equation.
2. This equation has two roots which are given by

   $$x = \frac{-b \pm \sqrt{D}}{2a},$$

   where D (or Δ) $= b^2 - 4ac$ is called as discriminant of the quadratic equation.
3. If α and β denote the roots of $ax^2 + bx + c = 0$, then,

   (*i*) $\alpha + \beta = -\frac{b}{a}$ (*ii*) $\alpha\beta = \frac{c}{a}$ (*iii*) $|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$

NATURE OF ROOTS

- Consider the quadratic equation $ax^2 + bx + c = 0$ where a, b, c $\in$ R & $a \neq 0$ then;
  - (*i*) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal)
  - (*ii*) $D = 0 \Leftrightarrow$ roots are real & coincident (equal)
  - (*iii*) $D < 0 \Leftrightarrow$ roots are imaginary
  - (*iv*) If $p + iq$ is one root of a quadratic equation, then the other must be it's conjugate $p - iq$ & vice versa. ($p, q \in R$ & $i = \sqrt{-1}$)
- Consider the quadratic equation $ax^2 + bx + c = 0$ where a, b, c $\in$ Q & $a \neq 0$ then ;
  - (*i*) If $D > 0$ & is a perfect square , then roots are rational & unequal.
  - (*ii*) If $\alpha = p + \sqrt{q}$ is one root in this case, (where p is rational & $\sqrt{q}$ is a surd) then the other root must be the conjugate of it i.e. $\beta = p - \sqrt{q}$ & vice versa.

TRAIN YOUR BRAIN

Example 1: Determine the values of m for which the equation $5x^2 - 4x + 2 + m(4x^2 - 2x - 1) = 0$ will have

(*a*) equal roots

(*b*) product of the roots as 2

Sol. $5x^2 - 4x + 2 + m(4x^2 - 2x - 1) = 0$

$$(5 + 4m)x^2 - 2(m + 2)x + (2 - m) = 0 \quad \ldots(1)$$

(*a*) For equal roots, the discriminant of (1) should be equal to zero.

$$\therefore\ 4(m+2)^2 - 4.(5+4m)(2-m) = 0$$

$$\Rightarrow m^2 + 4m + 4 - (10 + 8m - 5m - 4m^2) = 0$$

$$\Rightarrow 5m^2 + m - 6 = 0 \quad \Rightarrow (5m + 6)(m - 1) = 0$$

$$\Rightarrow m = -\frac{6}{5} \quad \text{or} \quad m = 1$$

(*b*) $\frac{2 - m}{5 + 4m} = 2$

$$\therefore\ 2 - m = 10 + 8m \Rightarrow m = -\frac{8}{9}$$

RELATION BETWEEN ROOTS AND CO-EFFICIENTS

(*i*) The solutions of quadratic equation, $ax^2 + bx + c = 0, (a \neq 0)$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(*ii*) If α, β are the roots of quadratic equation,

$ax^2 + bx + c = 0 \quad \ldots(i)$

then equation (*i*) can be written as

$a(x - \alpha)(x - \beta) = 0$

or $ax^2 - a(\alpha + \beta)x + a\alpha\beta = 0 \quad \ldots(ii)$

$$\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence we conclude that the quadratic equation whose roots are α & β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Now consider the following cases.

Case		Nature of roots
Case-I	$a > 0,\ b > 0, c > 0$ $\Rightarrow \alpha + \beta < 0, \alpha\beta > 0$	Both roots are negative.
Case-II	$a > 0,\ b > 0, c < 0$ $\Rightarrow \alpha + \beta < 0, \alpha\beta < 0$	Both roots are opposite in sign; Magnitude of negative root is more than the magnitude of positive root.
Case-III	$a > 0,\ b < 0, c > 0$ $\Rightarrow \alpha+\beta > 0,\ \alpha\beta > 0$	Both roots are positive.
Case-IV	$a > 0,\ b < 0, c < 0$ $\Rightarrow \alpha + \beta > 0, \alpha\beta < 0$	Roots are opposite in sign. Magnitude of positive root is more than magnitude of negative root.

Remember:

(*i*) Roots are rational $\Leftrightarrow$ D is a perfect square.

(*ii*) Roots are irrational $\Leftrightarrow$ D is positive but not a perfect square.

(*iii*) If $a + b + c = 0$, then 1 is a root of the equation $ax^2 + bx + c = 0$.

(*iv*) If a and c are of opposite sign, the roots must be of opposite sign.

(*v*) If the roots are equal in magnitude but opposite in sign, then $b = 0$, $ac < 0$.

(*vi*) If the roots are reciprocal of each other, then $c = a$.

(*vii*) The quadratic equation whose roots are reciprocals of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$ (*i.e.*, the coefficients are written in reverse order).

(*viii*) If $a = 1$, $b, c \in Z$ and the roots are rational numbers, then these roots must be integers.

(*ix*) If $ax^2 + bx + c = 0$ is satisfied by more than two values, it is an identity and $a = b = c = 0$ and vice-versa.

(*x*) If $P(x) = a_1x^2 + b_1x + c_1$ and $Q(x) = a_2x^2 + b_2x + c_2$, then P(x). Q(x) = 0 have atleast two real roots if $D_1 + D_2 > O$.

SYMMETRIC FUNCTION OF THE ROOTS

The following results may be useful.

(*i*) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

(*ii*) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$

(*iii*) $\alpha^4 + \beta^4 = (\alpha^3 + \beta^3)\,(\alpha + \beta) - \alpha\beta\,(\alpha^2 + \beta^2)$

(*iv*) $\alpha^5 + \beta^5 = (\alpha^3 + \beta^3)\,(\alpha^2 + \beta^2) - \alpha^2\beta^2(\alpha + \beta)$

(*v*) $|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

(*vi*) $\alpha^2 - \beta^2 = (\alpha + \beta)\,(\alpha - \beta)$

(*vii*) $\alpha^3 - \beta^3 = (\alpha - \beta)\,[(\alpha + \beta)^2 - \alpha\beta]$

(*viii*) $\alpha^4 - \beta^4 = (\alpha + \beta)\,(\alpha - \beta)\,(\alpha^2 + \beta^2)$

QUADRATIC EXPRESSION

Let $y = ax^2 + bx + c = a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right)$ …(1)

1. Represents a parabola with vertex $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$ and axis of the parabola is $x = \frac{-b}{2a}$

If $a > 0$, the parabola opens upward while if $a < 0$, the parabola opens downward. The parabola cuts the x-axis at points corresponding to roots of $ax^2 + bx + c = 0$. If this equation has

(*i*) $D > 0$, the parabola cuts x-axis at two real and distinct points.

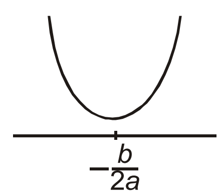
(*ii*) $D = 0$, the parabola touches x-axis at $x = \frac{-b}{2a}$.

(*iii*) $D < 0$, then;
if $a > 0$, parabola lies above x-axis.
if $a < 0$, parabola lies below x-axis.

GRAPHS OF QUADRATIC EXPRESSIONS

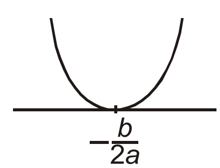
Let $f(x) = ax^2 + bx + c$ and $\alpha, \beta\ :\ \alpha < \beta$, be its roots

(*i*) $a > 0$ and $D < 0 \Leftrightarrow f(x) > 0 \quad \forall \quad x \in R$



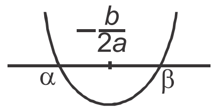
α and β are complex conjugates

(*ii*) $a > 0$ and $D = 0 \Leftrightarrow f(x) \geq 0 \quad \forall \quad x \in R$



$f(x) = 0$ at $x = \frac{-b}{2a}$

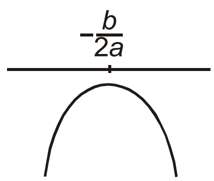
(*iii*) $a > 0$ and $D > 0$; then,



$f(x) > 0 \ \forall \ x \in (-\infty, \alpha) \cup (\beta, \infty)$
$f(x) < 0 \ \forall \ x \in (\alpha, \beta)$

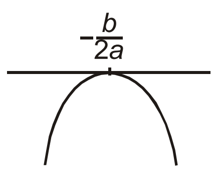
$$f(x)_{\min} = -\frac{D}{4a} \text{ at } x = -\frac{b}{2a}$$

(*iv*) $a < 0$ and $D < 0 \Leftrightarrow f(x) < 0 \quad \forall \quad x \in R$



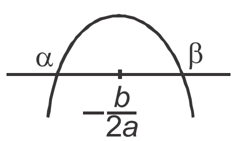
α and β are complex conjugates

(*v*) $a < 0$ and $D = 0 \Leftrightarrow f(x) \le 0 \quad \forall \quad x \in R$



$f(x) = 0$ at $x = \frac{-b}{2a}$

(*vi*) $a < 0$ and $D > 0$; then



$f(x) < 0 \quad \forall \quad x \in (-\infty, \alpha) \cup (\beta, \infty)$

$f(x) > 0 \quad \forall \quad x \in (\alpha, \beta)$

$$f(x)_{\max} = -\frac{D}{4a} \ at \ x = -\frac{b}{2a}$$

EQUATION V/S IDENTITY

A quadratic equation is satisfied by exactly two values of 'x' which may be real or imaginary. The equation, $a x^2 + b x + c = 0$ is:

A **quadratic equation** if $a \neq 0$ and An **identity** if $a = b = c = 0$ Infinite Roots

If a equation is satisfied by three distinct values of 'x', then it is an identity.

$(x + 1)^2 = x^2 + 2x + 1$ is an identity in x.

SOLUTION OF QUADRATIC INEQUALITIES

The values of 'x' satisfying the inequality, $ax^2 + bx + c > 0$ $(a \neq 0)$ are:

(*i*) If $D > 0$, i.e. the equation $ax^2 + bx + c = 0$ has two different roots $\alpha < \beta$.
$a > 0 \Rightarrow x \in (\infty, \alpha) \cup (\beta, \infty)$
$a < 0 \Rightarrow x \in (\alpha, \beta)$

(*ii*) If $D = 0$, i.e. roots are equal, i.e. $\alpha = \beta$.
$a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\alpha, \infty)$
$a < 0 \Rightarrow x \in \phi$

(*iii*) If $D < 0$, i.e. the equation $ax^2 + bx + c = 0$ has no real roots.
$a > 0 \Rightarrow x \in R$
$a < 0 \Rightarrow x \in \phi$

(*iv*) Inequalities of the form $\frac{P(x)}{Q(x)} \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0$ can be solved using the method of intervals.

Example 2: Solve inequatility, $\frac{x^2 + x + 1}{|x + 1|} > 0$.

Sol. $\frac{x^2 + x + 1}{|x + 1|} > 0$ ...(*i*)

$\therefore$ $|x + 1| > 0, \ \forall x \in R - \{-1\}$
$\therefore$ (i) becomes $x^2 + x + 1 > 0 \ x \neq -1$
$\because$ $x^2 + x + 1 > 0, \forall x \in R$ (as its $a > 0$ and $D < 0$)
$\therefore$ Solution of (i) is $x \in R - \{-1\}$

Example 3: Solve $\left| \frac{x^2 - 3x - 1}{x^2 + x + 1} \right| < 3$.

Sol. $\frac{|x^2 - 3x - 1|}{x^2 + x + 1} < 3$.

$\because$ $(x^2 + x + 1) > 0, \forall x \in R$
$\therefore$ $|x^2 - 3x - 1| < 3(x^2 + x + 1)$
$\Rightarrow$ $(x^2 - 3x - 1)^2 - \{3(x^2 + x + 1)\}^2 < 0$
$\Rightarrow$ $(4x^2 + 2)(-2x^2 - 6x - 4) < 0$
$\Rightarrow$ $(2x^2 + 1)(x + 2)(x + 1) > 0$
$\Rightarrow$ $x \in (-\infty, -2) \cup (-1, \infty)$

COMMON ROOTS

One Root Common:

If $\alpha \neq 0$ is a common root of the equation

$a_1x^2 + b_1x + c_1 = 0$ ...(*i*)

and $a_2x^2 + b_2x + c_2 = 0$ ...(*ii*)

then we have

$a_1\alpha^2 + b_1\alpha + c_1 = 0$ and
$a_2\alpha^2 + b_2\alpha + c_2 = 0$

These give $\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{c_1a_2 - c_2a_1}$

$$= \frac{1}{a_1b_2 - a_2b_1} (a_1b_2 - a_2b_1 \neq 0).$$

Thus, the required condition for one common root is $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (c_1a_2 - c_2a_1)^2$ and the value of the common root is

$$\alpha = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \text{ or } \frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}.$$

Both Roots Common

If the equations (i) and (ii) have both roots common, then these equations will be identical. Thus the required condition for both roots common is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \qquad \text{(If no root is equal to zero)}$$

Remember

(*i*) To find the common root of two equations, make the coefficient of second degree terms in two equations equal and subtract. The value of x so obtained is the required common root.

(*ii*) If two quadratic equations with real and rational coefficients have a common imaginary or irrational root, then both roots will be common and the two equations will be identical. The required condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

(*iii*) If α is a repeated root of the quadratic equation

$$f(x) = ax^2 + bx + c = 0,$$

then α is also a root of the equation $f'(x) = 0$.

Note:

If $f(x) = 0$ & $g(x) = 0$ are two polynomial equation having some common root(s) then those common root(s) is/are also the root(s) of $h(x) \equiv a\,f(x) + b\,g(x) = 0$.

TRAIN YOUR BRAIN

Example 4: Find m and n in order that the equations $mx^2 + 5x + 2 = 0$ and $3x^2 + 10x + n = 0$ may have both the roots common.

Sol. The equations are $mx^2 + 5x + 2 = 0$ and $3x^2 + 10x + n = 0$. Since they have both the roots common,

$$\frac{m}{3} = \frac{5}{10} = \frac{2}{n} \qquad \because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_1}$$

From the first-relation, $m = \frac{15}{10} = \frac{3}{2}$.

From the last-relation, $n = 4$.

RANGE OF QUADRATIC EXPRESSION

$$\mathbf{f(x) = ax^2 + bx + c}$$

(*i*) **Range when $x \in R$:** If $a > 0 \Rightarrow f(x) \in \left[-\frac{D}{4a}, \infty\right)$

$a < 0 \Rightarrow f(x) \in \left(-\infty, \frac{D}{4a}\right]$

Maximum & Minimum Value of $y = ax^2 + bx + c$ occurs at $x = -(b/2a)$ according as $a < 0$ or $a > 0$ respectively

(*ii*) **Range in restricted domain :** Given $x \in [x_1, x_2]$

(a) If $-\frac{b}{2a} \notin [x_1, x_2]$ then,

$$f(x) \in \left[\min\{f(x_1), f(x_2)\},\ \max\{f(x_1), f(x_2)\}\right]$$

(b) If $-\frac{b}{2a} \in [x_1, x_2]$ then,

$$f(x) \in \left[\min\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\},\ \max\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\}\right]$$

TRAIN YOUR BRAIN

Example 5: Find the minimum value of $f(x) = x^2 - 5x + 6$.

Sol. Minimum value of $f(x) = -\frac{D}{4a} = -\left(\frac{25-24}{4}\right)$

$= -\frac{1}{4}\left(\text{at } x = -\frac{b}{2a} = \frac{5}{2}\right)$. Hence range is $\left[-\frac{1}{4}, \infty\right)$.

Example 6: Find the range of $y = \frac{x+2}{2x^2+3x+6}$, if x is real.

Sol. $y = \frac{x+2}{2x^2+3x+6}$

$\Rightarrow 2yx^2 + 3yx + 6y = x + 2$

$\Rightarrow 2yx^2 + (3y-1)x + 6y - 2 = 0$ ...(*i*)

case-I:

if $y \neq 0$, then equation (*i*) is quadratic in x

$\because$ x is real

$\therefore D \geq 0$

$\Rightarrow (3y-1)^2 - 8y(6y-2) \geq 0$

$\Rightarrow (3y-1)(13y+1) \leq 0$

$y \in \left[-\frac{1}{13}, \frac{1}{3}\right] - \{0\}$

case-II : if $y = 0$, then equation becomes

$x = -2$

which is possible as x is real

$\therefore$ Range $y \in \left[-\frac{1}{13}, \frac{1}{3}\right]$

THEORY OF EQUATIONS

If $\alpha_1, \alpha_2, \alpha_3, \ldots..\alpha_n$ are the roots of the equation ; $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$ where $a_0, a_1, \ldots a_n$ are all real & $a_0 \neq 0$ then,

$$\Sigma\alpha_1 = -\frac{a_1}{a_0},\ \Sigma\alpha_1\alpha_2 = +\frac{a_2}{a_0},\ \Sigma\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0},\ \ldots.., \alpha_1\alpha_2\alpha_3\ldots\ldots\alpha_n = (-1)^n\frac{a_n}{a_0}$$

Note:

(*i*) If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $(x-\alpha)$ or $(x-\alpha)$ is a factor of $f(x)$ and conversely.

(*ii*) Every equation of nth degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(*iii*) If the coefficients of the equation $f(x) = 0$ are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. **imaginary roots occur in conjugate pairs**.

(*iv*) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not a perfect square.

(*v*) If there be any two real numbers 'a' & 'b' such that f(a) & f(b) are of opposite signs, then $f(x) = 0$ must have atleast one real root between 'a' and 'b'.

(*vi*) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term.

TRAIN YOUR BRAIN

Example 7: If $x = 1$ and $x = 2$ are solutions of the equation $x^3 + ax^2 + bx + c = 0$ and $a + b = 1$, then find the value of b.

Sol. $a + b + c = -1 \Rightarrow c = -2$ & $8 + 4a + 2b + c = 0$

$\Rightarrow 4a + 2b = -6 \Rightarrow 2a + b = -3$

$\Rightarrow a = -4,\ b = 5$

Example 8: A polynomial in x of degree greater than 3 leaves the remainder 2, 1 and –1 when divided by $(x-1)$; $(x+2)$ & $(x+1)$ respectively. Find the remainder, if the polynomial is divided by, $(x^2-1)(x+2)$.

Sol. $f(x) = Q_1(x-1) + 2 = Q_2(x+2) + 1 = Q_3(x+1) - 1$

$\Rightarrow f(1) = 2\ ;\ f(-2) = 1\ ;\ f(-1) = -1$

Let $f(x) = Q_r(x^2-1)(x+2) + ax^2 + bx + c$

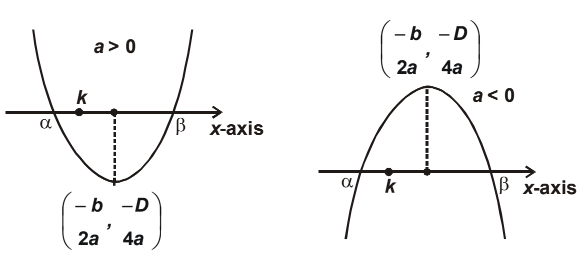
Hence $a + b + c = 2\ ;\ 4a - 2b + c = 1$

and $a - b + c = -1$

LOCATION OF ROOTS

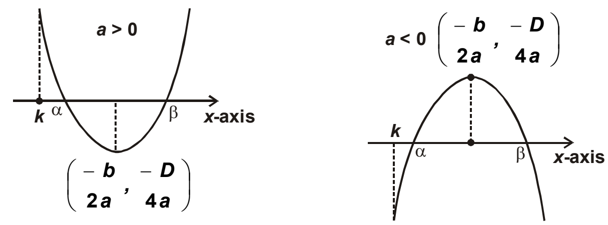
Let $f(x) = ax^2 + bx + c,\ a, b, c \in R,\ a \neq 0$ and α, β be roots of $f(x) = 0$

1. A real number k lies between the roots of $f(x) = 0$



(*i*) $D > 0$, (*ii*) $af(k) < 0$, where $\alpha < \beta$

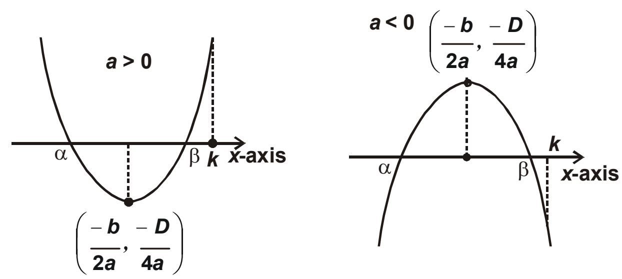
2. If both roots of quadratic equation $f(x) = 0$ are greater than k, then



(*i*) $D \geq 0$, (*ii*) $af(k) > 0$

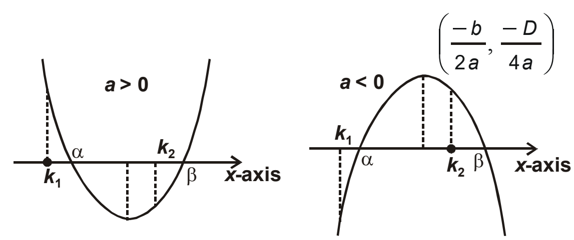
(*iii*) $k < \dfrac{-b}{2a}$, where $\alpha \leq \beta$

3. If both roots are less than real number k, then



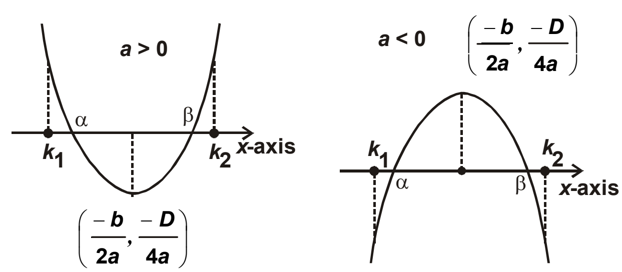
(*i*) $D \geq 0$, (*ii*) $af(k) > 0$ (*iii*) $k > \dfrac{-b}{2a}$, where $\alpha \leq \beta$

4. Exactly one root lies between real numbers k_1 and k_2, where $k_1 < k_2$.



(i) $D > 0$, (ii) $f(k_1).f(k_2) < 0$, where $\alpha < \beta$

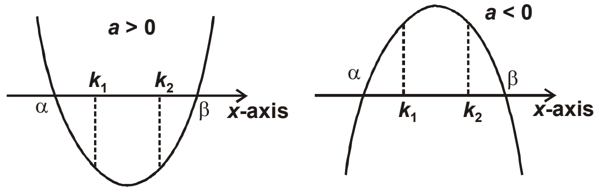
5. If both roots of $f(x) = 0$ are confined between real numbers k_1 and k_2, where $k_1 < k_2$.



(*i*) $D \geq 0$, (*ii*) $af(k_1) > 0$

(*iii*) $af(k_2) > 0$, (*iv*) $k_1 < \dfrac{-b}{2a} < k_2$, where $\alpha \leq \beta$

6. If real numbers k_1 and k_2 lie between roots of $f(x) = 0$, where $k_1 < k_2$.



(*i*) $D > 0$, (*ii*) $af(k_1) < 0$

(*iii*) $af(k_2) < 0$, where $\alpha < \beta$

TRAIN YOUR BRAIN

Example 9: Let $x^2 - (m - 3)x + m = 0$ $(m \in R)$ be a quadratic equation, then find the values of 'm' for which

(*a*) both the roots are greater than 2.

(*b*) both roots are positive.

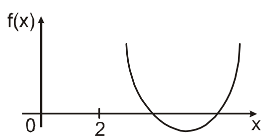
(*c*) one root is positive and other is negative.

(*d*) One root is greater than 2 and other smaller than 1

(*e*) Roots are equal in magnitude and opposite in sign.

(*f*) both roots lie in the interval (1, 2)

Sol. (*a*)



Condition - I: $D \geq 0 \Rightarrow (m-3)^2 - 4m \geq 0$

$\Rightarrow m^2 - 10m + 9 \geq 0$

$\Rightarrow (m-1)(m-9) \geq 0$

$\Rightarrow m \in (-\infty, 1] \cup [9, \infty)$ ...(*i*)

Condition - II: $f(2) > 0$

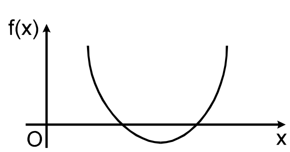
$\Rightarrow 4 - (m-3)2 + m > 0$

$\Rightarrow m < 10$ ...(*ii*)

Condition - III: $-\dfrac{b}{2a} > 2 \Rightarrow m > 7$ ...(*iii*)

Intersection of (*i*), (*ii*) and (*iii*) gives $m \in [9, 10)$

(*b*)



Condition - I $D \geq 0$

$\Rightarrow m \in (-\infty, 1] \cup [9, \infty)$

Condition - II $f(0) > 0$

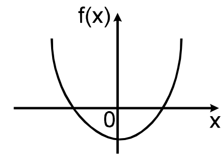
$\Rightarrow m > 0$

Condition - III $-\dfrac{b}{2a} > 0 \Rightarrow \dfrac{m-3}{2} > 0$

$\Rightarrow m > 3$

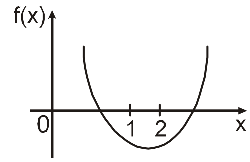
intersection gives $m \in [9, \infty)$ **Ans.**

(*c*)



Condition - I $f(0) < 0 \Rightarrow m < 0$ **Ans.**

(*d*)



Condition - I $f(1) < 0 \Rightarrow 4 < 0 \Rightarrow m \in \phi$

Condition - II $f(2) < 0 \Rightarrow m > 10$

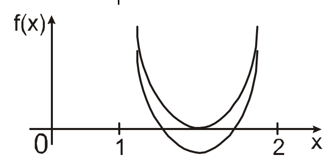
Intersection gives $m \in \phi$ **Ans.**

(*e*) sum of roots $= 0 \Rightarrow m = 3$

and $f(0) < 0 \Rightarrow m < 0$

$\therefore$ $m \in \phi$ **Ans.**

(*f*)



Condition - I $D \geq 0 \Rightarrow m \in (-\infty, 1] \cup [9, \infty)$

Condition - II $f(1) > 0 \Rightarrow 1 - (m-3) + m > 0 \Rightarrow 4 > 0$ which is true $\forall$ $m \in R$

Condition - III $f(2) > 0 \Rightarrow m < 10$

Condition - IV $1 < -\dfrac{b}{2a} < 2$

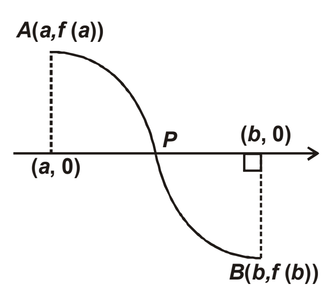
$\Rightarrow 1 < \dfrac{m-3}{2} < 2$

$\Rightarrow 5 < m < 7$

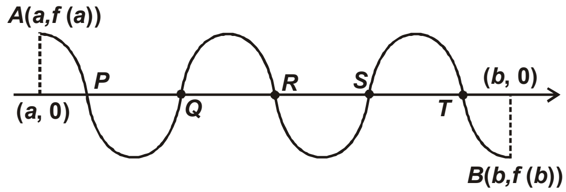
intersection gives $m \in \phi$ **Ans.**

NUMBER OF ROOTS OF A POLYNOMIAL EQUATION

1. If $f(x)$ is an increasing function in $[a, b]$, then $f(x) = 0$ will have atmost one root in $[a, b]$.
2. Let $f(x) = 0$ be a polynomial equation and a, b are two real numbers. Then $f(x) = 0$ will have at least one real root or odd number of real roots in (a, b) if $f(a)$ and $f(b)$ $(a < b)$ are of opposite signs.

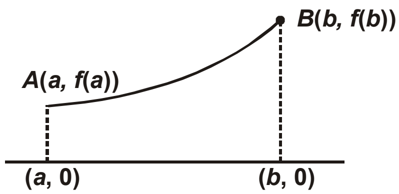


One real root

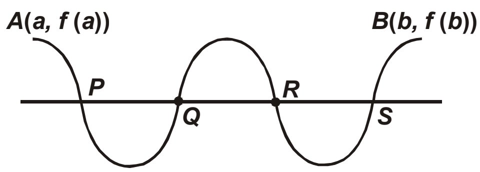


Odd number of real roots

But if $f(a)$ and $f(b)$ are of same signs, then either $f(x) = 0$ has no real root or even number of real roots in (a, b).



No real root



Even number of real roots

3. If the equation $f(x) = 0$ has two real roots a and b then $f'(x) = 0$ will have atleast one real root lying between a and b (using Rolle's Theorem).

EQUATIONS REDUCIBLE TO QUADRATIC EQUATIONS

1. **Reciprocal equation** of the standard form can be reduced to an equation of half its dimension.
2. **Equations of the form**
   (a) $(x-a)(x-b)(x-c)(x-d) = A$ where $a < b < c < d$ can be solved by change of variable.
   $$y = x - \left(\frac{a+b+c+d}{4}\right)$$
   (b) $(x-a)(x-b)(x-c)(x-d) = Ax^2$ where $ab = cd$ can be solved by assumption $y = x + \frac{ab}{x}$.
3. **For the equation of the type $(x-a)^4 + (x-b)^4 = A$**

   Substitute $y = \frac{(x-a)+(x-b)}{2}$

TRAIN YOUR BRAIN

Example 10: Solve $2x^4 + x^3 - 11x^2 + x + 2 = 0$

Sol. Since $x = 0$ is not a solution hence, divide by x^2

We get,

$$2x^2 + x - 11 + \frac{1}{x} + \frac{2}{x^2} = 0$$

or $2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 11 = 0$

Let $x + \frac{1}{x} = y \Rightarrow 2(y^2 - 2) + y - 11 = 0$

or $2y^2 + y - 15 = 0 \Rightarrow y = -3, \frac{5}{2}$

Corresponding values of x are $\frac{1}{2}, 2, \frac{-3 \pm \sqrt{5}}{2}$

Example 11: $(x+4)(x+6)(x+8)(x+12) = 1680x^2$

Sol. Here $(-4 \times -12) = 48 = (-6) \times (-8)$

$(x+4)(x+12)(x+6)(x+8) = 1680x^2$

$\Rightarrow (x^2 + 16x + 48)(x^2 + 14x + 48) = 1680x^2$

$\Rightarrow \left(x + 16 + \frac{48}{x}\right)\left(x + 14 + \frac{48}{x}\right) = 1680 \quad (\because x \neq 0)$

Let $y = x + \frac{48}{x} \qquad \Rightarrow (y+16)(y+14) = 1680$

$\Rightarrow y^2 + 30y - 1680 + 224 = 0$

$\Rightarrow y^2 + 30y - 1456 = 0 \Rightarrow y^2 + 56y - 26y - 1456 = 0$

$\Rightarrow (y+56)(y-26) = 0 \qquad \Rightarrow y = 26, -56$

$\therefore x + \frac{48}{x} = 26 \qquad$ and $x + \frac{48}{x} = -56$

$\Rightarrow x^2 - 26x + 48 = 0 \Rightarrow x^2 + 56x + 48 = 0$

$\Rightarrow x = 2, 24 \Rightarrow x = \frac{-56 \pm \sqrt{2944}}{2} = -28 \pm \sqrt{736}$

$= -28 \pm 4\sqrt{46}$

$\therefore x = \left\{2, 24, -28 - 4\sqrt{46}, -28 + 4\sqrt{46}\right\}$

IMPORTANT THEOREMS AND RESULTS

1. If α is a root of the equation $f(x) = 0$, then $f(x)$ is exactly divisible by $(x - \alpha)$ and conversely, if $f(x)$ is exactly divisible by $(x - \alpha)$ then α is a root of the equation $f(x) = 0$.
2. Every equation of an odd degree has atleast one real root, whose sign is opposite to that of its last term, provided that the coefficient of the first term is positive.
3. Every equation of an even degree whose last term is negative has at least two real roots, one positive and one negative, provided that the coefficient of the first term is positive.
4. If an equation has no odd powers of x, then all roots of the equation are complex provided all the coefficients of the equation are having positive sign.
5. If $x = \alpha$ is root repeated m times in $f(x) = 0$, ($f(x) = 0$ is an nth degree equation in x) then
   $$f(x) = (x - \alpha)^m g(x)$$
   when $g(x)$ is a polynomial of degree $(n - m)$ and the root $x = \alpha$ is repeated $(m - 1)$ times in $f'(x) = 0$, $(m - 2)$ times in $f''(x) = 0, \ldots (m - (m - 1))$ times in $f^{m-1}(x) = 0$.

6. The condition that a quadratic function $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ may be resolved into two linear factor if
$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$$\text{or} \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \text{ is 0.}$$

TRAIN YOUR BRAIN

Example 12: If $0 \le x \le \pi$, then the solution of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is given by x equal to

(*a*) $\frac{\pi}{6}, \frac{\pi}{3}$ (*b*) $\frac{\pi}{3}, \frac{\pi}{2}$

(*c*) $\frac{\pi}{6}, \frac{\pi}{2}$ (*d*) None of these

Sol. Let $16^{\sin^2 x} = y$, then $16^{\cos^2 x} = 16^{1-\sin^2 x} = \frac{16}{y}$

Hence, $y + \frac{16}{y} = 10$

$\Rightarrow y^2 - 10y + 16 = 0$ or $y = 2, 8$

Now $16^{\sin^2 x} = 2 \Rightarrow 4\sin^2 x = 1$

$\therefore \sin x = \pm\frac{1}{2} \Rightarrow x = \frac{\pi}{6}$

Also $16^{\sin^2 x} = 8 \Rightarrow 4\sin^2 x = 3$

$\therefore \sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$

Hence **(a)** is the correct answer

Topicwise Questions

Quadratic Equation & Nature of Roots

1. If $a + b + c = 0$, and $a, b, c \in R$, then the roots of the equation $4ax^2 + 3bx + 2c = 0$ are

(*a*) Equal (*b*) Imaginary

(*c*) Real (*d*) Both (*a*) and (*b*)

2. The roots of the given equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$ are
[Where $a \neq b$]

(*a*) Rational (*b*) Irrational

(*c*) Real (*d*) Imaginary

3. If $a, b, c \in Q$, then roots of the equation $(b + c - 2a)x^2 + (c + a - 2b)x + (a + b - 2c) = 0$ are

(*a*) Rational (*b*) Non-real

(*c*) Irrational (*d*) Equal

4. The value of k for which the quadratic equation, $kx^2 + 1 = kx + 3x - 11x^2$ has real and equal roots are

(*a*) $-11, -3$ (*b*) $5, 7$

(*c*) $5, -7$ (*d*) $-7, 25$

5. $x^2 + x + 1 + 2k(x^2 - x - 1) = 0$ is a perfect square for how many values of k

(*a*) 2 (*b*) 0

(*c*) 1 (*d*) 3

6. If $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q$, then $(pq) =$

(*a*) (3, 4) (*b*) (4, 5)

(*c*) (4, 3) (*d*) (5, 4)

7. The roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are

(*a*) $\frac{c-a}{b-c}, 1$ (*b*) $\frac{a-b}{b-c}, 1$

(*c*) $\frac{b-c}{a-b}, 1$ (*d*) $\frac{c-a}{a-b}, 1$

8. If a, b, c are integers and $b^2 = 4(ac + 5d^2)$, $d \in N$, then roots of the equation $ax^2 + bx + c = 0$ are

(*a*) Irrational (*b*) Rational & different

(*c*) Complex conjugate (*d*) Rational & equal

9. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has

(*a*) real roots (*b*) imaginary roots

(*c*) exactly one root (*d*) none of these

10. Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5, 100]$. Total number of different values of 'n' so that the given equation has integral roots, is

(*a*) 4 (*b*) 6

(*c*) 8 (*d*) 3

11. The entire graph of the expression $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if

(*a*) $k < 7$ (*b*) $-5 < k < 7$

(*c*) $k > -5$ (*d*) None

12. If $a, b \in R$, $a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $a + b + 1$ is:

(*a*) positive (*b*) negative

(*c*) zero (*d*) depends on the sign of b

13. If a and b are the non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is

(*a*) $\frac{3}{2}$ (*b*) $\frac{9}{4}$

(*c*) $-\frac{9}{4}$ (*d*) 1

14. If both roots of the quadratic equation $(2 - x)(x + 1) = p$ are distinct & positive, then p must lie in the interval

(*a*) $(2, \infty)$ (*b*) $(2, 9/4)$

(*c*) $(-\infty, -2)$ (*d*) $(-\infty, \infty)$

15. If $(1 - p)$ is root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are

(*a*) 0, 1 (*b*) –1, 1

(*c*) 0, –1 (*d*) –1, 2

Sum and Product of Roots

16. If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other, then k =

(*a*) 0 (*b*) 5

(*c*) 1/6 (*d*) 6

17. If α and β are the roots of the equation $4x^2 + 3x + 7 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

(*a*) $-\frac{3}{7}$ (*b*) $\frac{3}{7}$

(*c*) $-\frac{3}{5}$ (*d*) $\frac{3}{5}$

18. If the sum of the roots of the equation $ax^2 + bx + c = 0$ be equal to the sum of their squares, then

(*a*) $a(a + b) = 2bc$ (*b*) $c(a + c) = 2ab$

(*c*) $b(a + b) = 2ac$ (*d*) $b(a + b) = ac$

19. If α, β be the roots of the equation $x^2 - 2x + 3 = 0$, then the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ is

(*a*) $x^2 + 2x + 1 = 0$ (*b*) $9x^2 + 2x + 1 = 0$

(*c*) $9x^2 - 2x + 1 = 0$ (*d*) $9x^2 + 2x - 1 = 0$

20. If the product of roots of the equation, $mx^2 + 6x + (2m - 1) = 0$ is –1, then the value of m will be

(*a*) 1 (*b*) – 1

(*c*) $\frac{1}{3}$ (*d*) $-\frac{1}{3}$

21. If α and β are the roots of the equation $x^2 - 4x + 1 = 0$ the value of $\alpha^3 + \beta^3$ is

(*a*) 76 (*b*) 52

(*c*) –52 (*d*) –76

22. If α, β, γ are the roots of the equation $x^3 + x + 1 = 0$, then the value of $\alpha^3 \beta^3 \gamma^3$

(*a*) 0 (*b*) -3

(*c*) 3 (*d*) -1

23. If a, b are the roots of quadratic equation $x^2 + px + q = 0$ and g, d are the roots of $x^2 + px - r = 0$, then $(a - g) . (a - d)$ is equal to:

(*a*) $q + r$ (*b*) $q - r$

(*c*) $-(q + r)$ (*d*) $-(p + q + r)$

24. Two real numbers a & b are such that $a + b = 3$ & $|a - b| = 4$, then a & b are the roots of the quadratic equation:

(*a*) $4x^2 - 12x - 7 = 0$ (*b*) $4x^2 - 12x + 7 = 0$

(*c*) $4x^2 - 12x + 25 = 0$ (*d*) None of these

25. Let conditions C_1 and C_2 be defined as follows: $C_1 : b^2 - 4ac \geq 0$, C_2 : a, – b, c are of same sign. The roots of $ax^2 + bx + c = 0$ are real and positive, if

(*a*) both C_1 and C_2 are satisfied

(*b*) only C_2 is satisfied

(*c*) only C_1 is satisfied

(*d*) None of these

26. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the value of $\alpha^3 + \beta^3$ is

(*a*) $\dfrac{3abc + b^3}{a}$ (*b*) $\dfrac{a^3 + b^3}{3abc}$

(*c*) $\dfrac{3abc - b^3}{a^3}$ (*d*) $\dfrac{-(3abc + b^3)}{a^3}$

Common Roots

27. If both the roots of $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then $2r - p$ is equal to

(*a*) –1 (*b*) 0

(*c*) 1 (*d*) 2

28. If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$, have a common root,(Where $p \neq q$) then $p + q + 1 =$

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) -1

29. If a, b, p, q are non-zero real numbers, then the equations $2a^2x^2 - 2abx + b^2 = 0$ and $p^2x^2 + 2pqx + q^2 = 0$ have:

(*a*) no common root

(*b*) one common root if $2a^2 + b^2 = p^2 + q^2$

(*c*) two common roots if $3pq = 2ab$

(*d*) two common roots if $3qb = 2ap$

Theory of Equation and Identity, Inequalities

30. If x is real and satisfies $x + 2 > \sqrt{x + 4}$, then

(*a*) $x < -2$ (*b*) $x > 0$

(*c*) $-3 < x < 0$ (*d*) $-3 < x < 4$

31. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$ is

(*a*) $(-\infty, -2) \cup (2, \infty)$ (*b*) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

(*c*) $(-\infty, -1) \cup (1, \infty)$ (*d*) $(\sqrt{2}, \infty)$

32. Number of values of 'p' for which the equation $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ possess more than two roots, is:

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) None

33. The number of the integer solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) None of these

34. The complete set of values of 'x' which satisfy the inequations : $5x + 2 < 3x + 8$ and $\dfrac{x+2}{x-1} < 4$ is

(*a*) $(-\infty, 1)$ (*b*) $(2, 3)$

(*c*) $(-\infty, 3)$ (*d*) $(-\infty, 1) \cup (2, 3)$

35. The complete solution set of the inequality $\dfrac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} \geq 0$ is:

(*a*) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$

(*b*) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

(*c*) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$

(*d*) none of these

36. If the inequality $(m - 2)x^2 + 8x + m + 4 > 0$ is satisfied for all $x \in R$, then the least integral 'm' is:

(*a*) 4 (*b*) 5

(*c*) 6 (*d*) none

37. The complete set of real 'x' satisfying $||x - 1| - 1| \leq 1$ is:

(*a*) $[0, 2]$ (*b*) $[-1, 3]$

(*c*) $[-1, 1]$ (*d*) $[1, 3]$

38. If $\log_{1/3} \dfrac{3x - 1}{x + 2}$ is less than unity, then 'x' must lie in the interval:

(*a*) $(-\infty, -2) \cup (5/8, \infty)$ (*b*) $(-2, 5/8)$

(*c*) $(-\infty, -2) \cup (1/3, 5/8)$ (*d*) $(-2, 1/3)$

39. Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is:

(*a*) $[-4, 1]$ (*b*) $[-4, -3) \cup (0, 1]$

(*c*) $(-\infty, -3) \cup (1, \infty)$ (*d*) $(-\infty, -4) \cup [1, \infty)$

40. The set of all the solutions of the inequality $\log_{1-x}(x - 2) \geq 0$ is

(*a*) $(-\infty, 0)$ (*b*) $(2, \infty)$

(*c*) $(-\infty, 1)$ (*d*) ϕ

41. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval
(*a*) $(2, \infty)$ (*b*) $(1, 2)$
(*c*) $(-2, -1)$ (*d*) None of these

42. If $\log_{0.5} \log_5 (x^2-4) > \log_{0.5} 1$, then 'x' lies in the interval
(*a*) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$
(*b*) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3\sqrt{5})$
(*c*) $(\sqrt{5}, 3\sqrt{5})$
(*d*) ϕ

43. The set of all solutions of the inequality $(1/2)^{x^2-2x} < 1/4$ contains the set
(*a*) $(-\infty, 0)$ (*b*) $(-\infty, 1)$
(*c*) $(1, \infty)$ (*d*) $(3, \infty)$

44. If $\dfrac{6x^2-5x-3}{x^2-2x+6} \le 4$, then least and the highest values of $4x^2$ are
(*a*) 0 & 81 (*b*) 9 & 81
(*c*) 36 & 81 (*d*) None of these

45. If two roots of the equation $x^3 - px^2 + qx - r = 0$ are equal in magnitude but opposite in sign, then:
(*a*) $pr = q$ (*b*) $qr = p$
(*c*) $pq = r$ (*d*) None

46. If α, β & γ are the roots of the equation $x^3 - x - 1 = 0$ then,
$\dfrac{1+\alpha}{1-\alpha} + \dfrac{1+\beta}{1-\beta} + \dfrac{1+\gamma}{1-\gamma}$ has the value equal to:
(*a*) zero (*b*) -1
(*c*) -7 (*d*) 1

47. For what value of a and b the equation $x^4 - 4x^3 + ax^2 + bx + 1 = 0$ has four real positive roots?
(*a*) $(-6, -4)$ (*b*) $(-6, 5)$
(*c*) $(-6, 4)$ (*d*) $(6, -4)$

48. If α, β are roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$ is
(*a*) $ab\,x^2 - (a+b)\,cx + (a+b)^2 = 0$
(*b*) $ac\,x^2 - (a+c)\,bx + (a+c)^2 = 0$
(*c*) $ac\,x^2\,(a+c)\,bx - (a+c)^2 = 0$
(*d*) None of these

49. If S is the set of all real x such that $\dfrac{2x-1}{2x^3+3x^2+x}$ is positive, then S contains
(*a*) $(-\infty, -3/2)$ (*b*) $(-3/2, 1/4)$
(*c*) $(-1/4, 1/2)$ (*d*) $(-1/2, 3)$

Max and Min Value, Factorization

50. If x is real, then the maximum and minimum values of the expression $\dfrac{x^2-3x+4}{x^2+3x+4}$ will be
(*a*) 2, 1 (*b*) $5, \dfrac{1}{5}$
(*c*) $7, \dfrac{1}{7}$ (*d*) 2, 7

51. The smallest value of $x^2 - 3x + 3$ in the interval $(-3, 3/2)$ is
(*a*) 3/4 (*b*) 5·
(*c*) -15 (*d*) -20

52. If $y = -2x^2 - 6x + 9$, (for $x \in R$) then
(*a*) maximum value of y is -11 and it occurs at $x = 2$
(*b*) minimum value of y is -11 and it occurs at $x = 2$
(*c*) maximum value of y is 13.5 and it occurs at $x = -1.5$
(*d*) minimum value of y is 13.5 and it occurs at $x = -1.5$

53. If 'x' is real and $k = \dfrac{x^2-x+1}{x^2+x+1}$, then:
(*a*) $\dfrac{1}{3} \le k \le 3$ (*b*) $k \ge 5$
(*c*) $k \le 0$ (*d*) None of these

54. Consider $y = \dfrac{2x}{1+x^2}$, where x is real, then the range of expression $y^2 + y - 2$ is
(*a*) $[-1, 1]$ (*b*) $[0, 1]$
(*c*) $[-9/4, 0]$ (*d*) $[-9/4, 1]$

55. The values of x and y besides y can satisfy the equation (x, y $\in$ real numbers) $x^2 - xy + y^2 - 4x - 4y + 16 = 0$
(*a*) 2, 2 (*b*) 4, 4
(*c*) 3, 3 (*d*) None of these

56. If x is real, then $\dfrac{x^2-x+c}{x^2+x+2c}$ can take all real values if
(*a*) $c \in [0, 6]$ (*b*) $c \in [-6, 0]$
(*c*) $c \in (-\infty, -6) \cup (0, \infty)$ (*d*) $c \in (-6, 0)$

Location of Roots

57. The real values of 'a' for which the quadratic equation $2x^2 - (a^3 + 8a - 1)\,x + a^2 - 4a = 0$ possesses roots of opposite sign is given by:
(*a*) $a > 5$ (*b*) $0 < a < 4$
(*c*) $a > 0$ (*d*) $a > 7$

58. If a, b are the roots of the quadratic equation $x^2 - 2p\,(x-4) - 15 = 0$, then the set of values of 'p' for which one root is less than 1 & the other root is greater than 2 is:
(*a*) $(7/3, \infty)$ (*b*) $(-\infty, 7/3)$
(*c*) $x \in R$ (*d*) None of these

59. The values of k, for which the equation $x^2 + 2(k-1)\,x + k + 5 = 0$ possess atleast one positive root, are
(*a*) $[4, \infty)$ (*b*) $(-\infty, -1] \cup [4, \infty)$
(*c*) $[-1, 4]$ (*d*) $(-\infty, -1]$

Learning Plus

1. If a > 0, b > 0, c > 0 then both the roots of the equation $ax^2 + bx + c = 0$

(*a*) Are real and negative

(*b*) Have negative real parts

(*c*) Are rational numbers

(*d*) Both (*a*) and (*c*)

2. The value of k for which the equation $(k-2)x^2 + 8x + k + 4 = 0$ has both real, distinct and negative is -

(*a*) 0 (*b*) 2

(*c*) 3 (*d*) -4

3. If α, β are the roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is

(*a*) $acx^2 + (a+c)bx + (a+c)^2 = 0$

(*b*) $abx^2 + (a+c)bx + (a+c)^2 = 0$

(*c*) $acx^2 + (a+b)cx + (a+c)^2 = 0$

(*d*) $acx^3 + (a+c)bx + (a+c)^3$

4. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$

(*a*) $\frac{2}{a}$ (*b*) $\frac{2}{b}$

(*c*) $\frac{2}{c}$ (*d*) $-\frac{2}{a}$

5. If the roots of the equation $12x^2 - mx + 5 = 0$ are in the ratio 2 : 3, then m =

(*a*) $5\sqrt{10}$ (*b*) $3\sqrt{10}$

(*c*) $2\sqrt{10}$ (*d*) $10\sqrt{5}$

6. $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, if $a =$

(*a*) 24 (*b*) 0, 24·

(*c*) 3, 24 (*d*) 0, 3

7. If α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$, then $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}$

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

8. If the roots of $x^2 + x + a = 0$ exceed a, then

(*a*) $2 < a < 3$ (*b*) $a > 3$

(*c*) $-3 < a < 3$ (*d*) $a < -2$

9. The value of *p* for which both the roots of the equation $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2, lies in

(*a*) $(4/5, 2)$ (*b*) $(2, \infty)$

(*c*) $(-1, -4/5)$ (*d*) $(-\infty, -1)$

10. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has -

(*a*) No root (*b*) One root

(*c*) Two equal root (*d*) Infinitely many roots

11. The number of quardratic equations which are unchanged by squaring their roots

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) None of these

12. For what value of the curve $y = x^2 + ax + 25$ touches the x-axis

(*a*) 0 (*b*) ± 5

(*c*) ± 10 (*d*) None of these

13. If $b^2 < 2ac$, then equation $ax^3 + bx^2 + cx + d = 0$ has

(*a*) exactly one real roots

(*b*) Has three real roots

(*c*) at least two roots

(*d*) None of these

14. Let α and β are the roots of the equation $x^2 + x + 1 = 0$ then

(*a*) $\alpha^2 + \beta^2 = 4$ (*b*) $(\alpha - \beta)^2 = 3$

(*c*) $\alpha^3 + \beta^3 = 2$ (*d*) $\alpha^4 + \beta^4 = 1$

15. If $\sec\alpha, \tan\alpha$ are roots of $ax^2 + bx + c = 0$, then

(*a*) $a^4 - b^4 + 4ab^2c = 0$ (*b*) $a^4 + b^4 - 4ab^2c = 0$

(*c*) $a^2 - b^2 = 4ac$ (*d*) $a^2 + b^2 = ac$

16. Let α, β be the roots of $ax^2 + bx + c = 0$, γ, δ be the roots of $px^2 + qx + r = 0$ and D_1 and D_2 be their respective discriminant. If $\alpha, \beta, \gamma, \delta$, are in A.P., then the ratio $D_1 : D_2$ is equal to

(*a*) $\frac{a^2}{b^2}$ (*b*) $\frac{a^2}{p^2}$

(*c*) $\frac{b^2}{q^2}$ (*d*) $\frac{c^2}{r^2}$

17. If one root of the equation $(l - m)x^2 + lx + 1 = 0$ is double the other and if l is real, then the greatest value of m is

(*a*) $\frac{9}{8}$ (*b*) $\frac{7}{8}$

(*c*) $\frac{8}{9}$ (*d*) $\frac{8}{9}$

18. If p, q, r are real numbers satisfying the condition $p + q + r = 0$, then the roots of the quadratic equation $3px^2 + 5qx + 7r = 0$ are

(*a*) Positive (*b*) Negative
(*c*) Real and distinct (*d*) Imaginary

19. If a, b, c are in G.P., then the equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

(*a*) A.P. (*b*) G.P.
(*c*) H.P. (*d*) $ab = cd$

20. If $a < b < c < d$ and $K > 0$, then the quadratic equation $(x - a)(x - c) + k(x - b)(x - d) = 0$ has

(*a*) All roots real and distinct
(*b*) All roots real but not necessarily distinct
(*c*) All root real and negative
(*d*) May be imaginary

21. The number of integers satisfying the inequality $\frac{x}{x+6} \le \frac{1}{x}$ is:

(*a*) 7 (*b*) 8
(*c*) 9 (*d*) 3

22. Sum of values of x and y satisfying the equation $3^x - 4^y = 77$; $3^{x/2} - 2^y = 7$ is:

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 5

23. If the value of $m^4 + \frac{1}{m^4} = 119$, then the value of $\left|m^3 - \frac{1}{m^3}\right| =$

(*a*) 11 (*b*) 18
(*c*) 24 (*d*) 36

24. The number of integral roots of the equation $x^8 - 24x^7 - 18x^5 + 39x^2 + 1155 = 0$ is:

(*a*) 0 (*b*) 2
(*c*) 4 (*d*) 6

25. If the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the product of the roots is:

(*a*) $-2(p^2 + q^2)$ (*b*) $-(p^2 + q^2)$
(*c*) $-\frac{(p^2 + q^2)}{2}$ (*d*) $-pq$

26. If $\alpha \ne \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$, then the equation with roots $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ is:

(*a*) $3x^2 - 25x + 3 = 0$
(*b*) $x^2 + 5x - 3 = 0$
(*c*) $x^2 - 5x + 3 = 0$
(*d*) $3x^2 - 19x + 3 = 0$

27. Minimum possible number of positive root of the quadratic equation $x^2 - (1 + \lambda)x + \lambda - 2 = 0$, $\lambda \in R$:

(*a*) 2 (*b*) 0
(*c*) 1 (*d*) Can not be determined

28. If $\alpha, \beta, \gamma, \delta \in R$ satisty

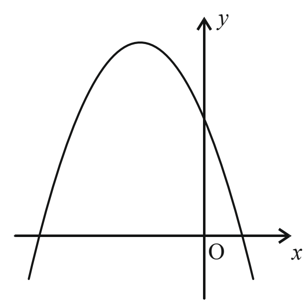
$$\frac{(\alpha+1)^2 + (\beta+1)^2 + (\gamma+1)^2 + (\delta+1)^2}{\alpha+\beta+\gamma+\delta} = 4$$

If biquadratic equation $a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$ has the roots $\left(\alpha + \frac{1}{\beta} - 1\right), \left(\beta + \frac{1}{\gamma} - 1\right), \left(\gamma + \frac{1}{\delta} - 1\right), \left(\delta + \frac{1}{\alpha} - 1\right)$.

Then the value of a_2/a_0 is:

(*a*) 4 (*b*) –4
(*c*) 6 (*d*) None the these

29. If graph of the quadratic $y = ax^2 + bx + c$ is given below:

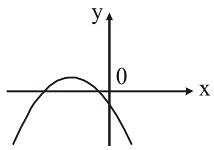


(*a*) $a > 0, b > 0, c > 0$
(*b*) $a < 0, b > 0, c < 0$
(*c*) $a < 0, b < 0, c > 0$
(*d*) $a < 0, b < 0, c < 0$

Advanced Level Multiconcept Questions

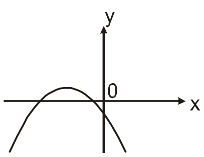
MCQ/COMPREHENSION/MATCHING/ NUMERICAL

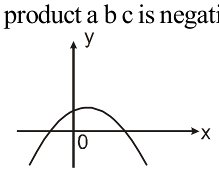
1. The graph of the quadratic polynomial $y = ax^2 + bx + c$ is as shown in the figure. Then:

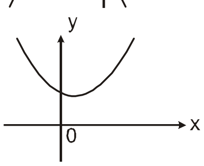


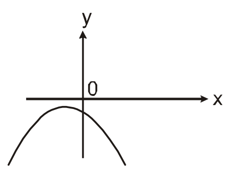
(*a*) $b^2 - 4ac > 0$ (*b*) $b < 0$
(*c*) $a > 0$ (*d*) $c < 0$

2. For which of the following graphs of the quadratic expression $y = ax^2 + bx + c$, the product abc is negative?

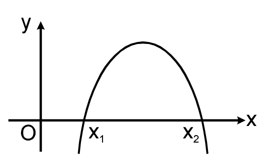
(*a*)


(*b*)


(*c*)


(*d*)


3. The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then



(*a*) $a < 0$
(*b*) $b^2 < 4ac$
(*c*) $c > 0$
(*d*) a and b are of opposite sign

4. For the equation $|x|^2 + |x| - 6 = 0$, the correct statement (s) is (are):
(*a*) sum of roots is 0 (*b*) product of roots is – 4
(*c*) there are 4 roots (*d*) there are only 2 roots

5. If a, b are the roots of $ax^2 + bx + c = 0$, and $a + h$, $b + h$ are the roots of $px^2 + qx + r = 0$, (where h ¹ 0), then

(*a*) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ (*b*) $h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

(*c*) $h = \frac{1}{2}\left(\frac{b}{a} + \frac{q}{p}\right)$ (*d*) $\frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

6. If a, b are non-zero real numbers and α, β the roots of $x^2 + ax + b = 0$, then
(*a*) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$
(*b*) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax + 1 = 0$
(*c*) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b - a^2)x + b = 0$
(*d*) $(\alpha - 1), (\beta - 1)$ are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

7. $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is
$(a, b, c, d \in R)$
(*a*) $-d/a$ (*b*) d/a
(*c*) $(b - a)/a$ (*d*) $(a - b)/a$

8. If $\frac{1}{2} \leq \log_{0.1} x \leq 2$, then
(*a*) maximum value of x is $\frac{1}{\sqrt{10}}$
(*b*) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$
(*c*) minimum value of x is $\frac{1}{\sqrt{10}}$
(*d*) minimum value of x is $\frac{1}{100}$

9. If the quadratic equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, then the equation containing their other roots is/are:
(*a*) $x^2 + a(b + c)x - a^2bc = 0$
(*b*) $x^2 - a(b + c)x + a^2bc = 0$
(*c*) $a(b + c)x^2 - (b + c)x + abc = 0$
(*d*) $a(b + c)x^2 + (b + c)x - abc = 0$

10. If the quadratic equations $ax^2 + bx + c = 0$ $(a, b, c \in R, a \neq 0)$ and $x^2 + 4x + 5 = 0$ have a common root, then a, b, c must satisfy the relations:
(*a*) $a > b > c$
(*b*) $a < b < c$
(*c*) $a = k; b = 4k; c = 5k$ $(k \in R, k \neq 0)$
(*d*) $b^2 - 4ac$ is negative.

11. If α, β are the real and distinct roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always
(*a*) two real roots
(*b*) two negative roots
(*c*) two positive roots
(*d*) one positive root and one negative root

12. If $(x^2+x+1)+(x^2+2x+3)+(x^2+3x+5)+......+(x^2+20x+39)=4500$, then x is equal to

(*a*) 10 (*b*) – 10
(*c*) 20.5 (*d*) –20.5

13. If roots of equation, $x^3+bx^2+cx-1=0$ forms an increasing G.P., then

(*a*) $b+c=0$
(*b*) $b \in (-\infty, -3)$
(*c*) one of the roots = 1
(*d*) one root is smaller than 1 & other > 1

14. Let $f(x)=\dfrac{3}{x-2}+\dfrac{4}{x-3}+\dfrac{5}{x-4}$, then $f(x)=0$ has

(*a*) exactly one real root in (2, 3)
(*b*) exactly one real root in (3, 4)
(*c*) at least one real root in (2, 3)
(*d*) None of these

Paragraph for question no. 15 & 16: A quadratic polynomial $f(x)=px^2+qx+r$ has two distinct roots x_1 & x_2. If its vertex (of parabola) is V and x_1, x_3, x_2 are in A.P., then answer the following

Comprehension – 1 (No. 15 to 17)

Consider the equation $x^4-\lambda x^2+9=0$. This can be solved by substituting $x^2=t$ such equations are called as pseudo quadratic equations.

15. If the equation has four real and distinct roots, then λ lies in the interval

(*a*) $(-\infty,-6)\cup(6,\infty)$ (*b*) $(0,\infty)$
(*c*) $(6,\infty)$ (*d*) $(-\infty,-6)$

16. If the equation has no real root, then λ lies in the interval

(*a*) $(-\infty,0)$ (*b*) $(-\infty,6)$
(*c*) $(6,\infty)$ (*d*) $(0,\infty)$

17. If the equation has only two real roots, then set of values of λ is

(*a*) $(-\infty,-6)$ (*b*) $(-6,6)$
(*c*) $\{6\}$ (*d*) ϕ

18. **Match the column.**

Column – I	**Column – II**
(*a*) If α, $\alpha+4$ are two roots of $x^2-8x+k=0$, then possible value of k is	(P) 2
(*b*) Number of real roots of equation $x^2-5\lvert x\rvert+6=0$ are 'n', then value of $\dfrac{n}{2}$ is	(Q) 3
(*c*) If $3-i$ is a root of $x^2+ax+b=0$ (a, b $\in$ R), then b is	(R) 12
(*d*) If both roots of $x^2-2kx+k^2+k-5=0$ are less than 5, then 'k' may be equal to	(S) 10

NUMERICAL BASED QUESTIONS

19. Find number of integer roots of equation $x(x+1)(x+2)(x+3)=120$.

20. Find product of all real values of x satisfying $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$

21. The least prime integral value of '2a' such that the roots α, β of the equation $2x^2+6x+a=0$ satisfy the inequality $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}<2$ is

22. If a, b are the roots of $x^2+px+1=0$ and c, d are the roots of $x^2+qx+1=0$. Then find the value of $(a-c)(b-c)(a+d)(b+d)/(q^2-p^2)$.

23. α, β are roots of the equation $\lambda(x^2-x)+x+5=0$. If λ_1 and λ_2 are the two values of λ for which the roots α, β are connected by the relation $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=4$, then the value of $\left(\dfrac{\dfrac{\lambda_1}{\lambda_2}+\dfrac{\lambda_2}{\lambda_1}}{14}\right)$ is

24. Let α, β be the roots of the equation $x^2+ax+b=0$ and γ, δ be the roots of $x^2-ax+b-2=0$. If $\alpha\beta\gamma\delta=24$ and $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}+\dfrac{1}{\delta}=\dfrac{5}{6}$, then find the value of a.

25. The least value of expression $x^2+2xy+2y^2+4y+7$ is:

26. If $a>b>0$ and $a^3+b^3+27ab=729$ then the quadratic equation $ax^2+bx-9=0$ has roots α, β ($\alpha<\beta$). Find the value of $4\beta-a\alpha$.

27. Let α and β be roots of $x^2-6(t^2-2t+2)x-2=0$ with $\alpha>\beta$. If $a_n=\alpha^n-\beta^n$ for $n\geq 1$, then find the minimum value of $\dfrac{a_{100}-2a_{98}}{a_{99}}$ (where $t \in R$)

28. If roots of the equation $x^2-10ax-11b=0$ are c and d and those of $x^2-10cx-11d=0$ are a and b, then find the value of $\dfrac{a+b+c+d}{110}$. (where a, b, c, d are all distinct numbers)

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEARS

1. The sum of all real values of x satisfying the equation **[*JEE Main-2016*]**

$(x^2-5x+5)^{x^2+4x-60}=1$ is:

(*a*) 3 (*b*) –4
(*c*) 6 (*d*) 5

2. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + \ldots\ldots + (x+\overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to: **[*JEE Main-2017*]**

(*a*) 11 (*b*) 12
(*c*) 9 (*d*) 10

3. If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to- **[*JEE Main-2018*]**

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) –1

4. Let α and β be two roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}$ is equal to:- **[*JEE Main-2019 (January)*]**

(*a*) –256 (*b*) 512
(*c*) –512 (*d*) 256

5. If both the roots of the quadratic equation $x^2-mx+4=0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval **[*JEE Main-2019 (January)*]**

(*a*) (4, 5) (*b*) (3, 4)
(*c*) (5, 6) (*d*) (–5, –4)

6. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha=0$ are rational numbers is: **[*JEE Main-2019 (January)*]**

(*a*) 2 (*b*) 5
(*c*) 3 (*d*) 4

7. Consider the quadratic equation $(c-5)x^2-2cx+(c-4)=0, c\neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is: **[*JEE Main-2019 (January)*]**

(*a*) 18 (*b*) 12
(*c*) 10 (*d*) 11

8. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is: **[*JEE Main-2019 (January)*]**

(*a*) –81 (*b*) 100
(*c*) 144 (*d*) –300

9. The number of integral values of m for which the quadratic expression, $(1+2m)x^2-2(1+3m)x+4(1+m)$, $x \in R$, is always positive, is:- **[*JEE Main-2019 (January)*]**

(*a*) 3 (*b*) 8
(*c*) 7 (*d*) 6

10. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2=0$, then the least value of m for which $\lambda+\frac{1}{\lambda}=1$, is **[*JEE Main-2019 (January)*]**

(*a*) $2-\sqrt{3}$ (*b*) $4-3\sqrt{2}$
(*c*) $-2+\sqrt{2}$ (*d*) $4-2\sqrt{3}$

11. If α and β be the roots of the equation $x^2-2x+2=0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n=1$ is: **[*JEE Main-2019 (April)*]**

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 5

12. If three distinct numbers a,b,c are in G.P. and the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root, then which one of the following statements is correct? **[*JEE Main-2019 (April)*]**

(*a*) d, e, f are in A.P. (*b*) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in G.P.
(*c*) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P. (*d*) d, e, f are in G.P.

13. The number of integral values of m for which the equation $(1+m^2)x^2-2(1+3m)x+(1+8m)=0$ has no real root is: **[*JEE Main-2019 (April)*]**

(*a*) infinitely many (*b*) 2
(*c*) 3 (*d*) 1

14. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2+px+q=0$, then: **[*JEE Main-2019 (April)*]**

(*a*) $q^2+4p+14=0$ (*b*) $p^2-4q-12=0$
(*c*) $q^2-4p-16=0$ (*d*) $p^2-4q+12=0$

15. If m is chosen in the quadratic equation $(m^2+1)x^2-3x+(m^2+1)^2=0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:- **[*JEE Main-2019 (April)*]**

(*a*) $8\sqrt{3}$ (*b*) $4\sqrt{3}$
(*c*) $10\sqrt{5}$ (*d*) $8\sqrt{5}$

16. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)(\alpha-\beta)^{24}}$ is equal to:

[*JEE Main-2019* (*April*)]

(*a*) $\frac{2^6}{(\sin\theta+8)^{12}}$ (*b*) $\frac{2^{12}}{(\sin\theta-8)^6}$

(*c*) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ (*d*) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

17. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is : **[*JEE Main-2019* (*April*)]**

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 1

18. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r + \lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to:

[*JEE Main-2019* (*April*)]

(*a*) $\frac{21}{346}$ (*b*) $\frac{29}{358}$

(*c*) $\frac{1}{12}$ (*d*) $\frac{7}{116}$

19. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to:

[*JEE Main-2019* (*April*)]

(*a*) $\beta\gamma$ (*b*) 0

(*c*) $\alpha\gamma$ (*d*) $\alpha\beta$

20. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which of the following statements is not true? **[*JEE Main-2020* (*January*)]**

(*a*) $p_5 = p_2 \cdot p_3$ (*b*) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(*c*) $p_3 = p_5 - p_4$ (*d*) $p_5 = 11$

21. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is ________.

[*JEE Main-2020* (*January*)]

22. Let S be the set of all real roots of the equation,

[*JEE Main-2020* (*January*)]

$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S:

(*a*) contains exactly two elements

(*b*) is a singleton

(*c*) contains at least four elements

(*d*) is an empty set

23. The number of real roots of the equation,

[*JEE Main-2020* (*January*)]

$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$

(*a*) 3 (*b*) 1

(*c*) 4 (*d*) 2

24. Let a, b $\in$ R, a $\neq$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, $\alpha^2 + \beta^2$ is equal to:

[*JEE Main-2020* (*January*)]

(*a*) 25 (*b*) 26

(*c*) 24 (*d*) 28

25. Let f(x) be a quadratic polynomial such that f(–1) + f(*b*) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in: **[*JEE Main-2020* (*September*)]**

(*a*) (–1, 0) (*b*) (–3, –1)

(*c*) (0, 1) (*d*) (1, 3)

26. Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, , n = 1, 2, 3, ..., then:

[*JEE Main-2020* (*September*)]

(*a*) $5S_6 + 6S_5 = 2S_4$

(*b*) $6S_6 + 5S_5 + 2S_4 = 0$

(*c*) $6S_6 + 5S_5 = 2S_4$

(*d*) $5S_6 + 6S_5 + 2S_4 = 0$

27. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0,1) is:

[*JEE Main-2020* (*September*)]

(*a*) (–3, –1) (*b*) (2, 4]

(*c*) (0, 2) (*d*) (1, 3]

28. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to:

[*JEE Main-2020* (*September*)]

(*a*) $\frac{9}{4}(9 - q^2)$ (*b*) $\frac{9}{4}(9 + p^2)$

(*c*) $\frac{9}{4}(9 + q^2)$ (*d*) $\frac{9}{4}(9 - p^2)$

29. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to:

[*JEE Main-2020* (*September*)]

(*a*) 18 (*b*) 9

(*c*) 27 (*d*) 36

30. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio (2q + p): (2q – p) is:

[*JEE Main-2020* (*September*)]

(*a*) 3 : 1 (*b*) 5 : 3

(*c*) 9 : 7 (*d*) 33 : 31

31. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to:

[*JEE Main-2020* (*September*)]

(*a*) $\frac{1}{24}$ (*b*) $\frac{27}{32}$

(*c*) $\frac{3}{8}$ (*d*) $\frac{27}{16}$

32. The product of the roots of the equation $9x^2 - 18\,|\,x\,| + 5 = 0$, is: **[*JEE Main-2020* (*September*)]**

(*a*) $\frac{25}{9}$ (*b*) $\frac{25}{81}$

(*c*) $\frac{5}{9}$ (*d*) $\frac{5}{27}$

33. If α and β are the roots of the equation $2x(2x+1) = 1$, then β is equal to: **[*JEE Main-2020* (*September*)]**

(*a*) $2\alpha^2$ (*b*) $-2\alpha(\alpha+1)$

(*c*) $2\alpha(\alpha-1)$ (*d*) $2\alpha(\alpha+1)$

34. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is:

[*JEE Main-2020* (*September*)]

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 1

35. Let p and q be two positive number such that $p + q = 2$ and $p^4 + p^4 = 272$. Then p and q are roots of the equation: **[*JEE Main-2021* (*February*)]**

(*a*) $x^2 - 2x + 2 = 0$ (*b*) $x^2 - 2x + 8 = 0$

(*c*) $x^2 - 2x + 136 = 0$ (*d*) $x^2 - 2x + 16 = 0$

36. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), $(2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is:

[*JEE Main-2021* (*February*)]

(*a*) $\frac{71}{256}$ (*b*) $-\frac{69}{256}$

(*c*) $\frac{69}{256}$ (*d*) $-\frac{71}{256}$

37. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is **[*JEE Main-2021* (*February*)]**

38. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is

[*JEE Main-2021* (*February*)]

(*a*) $\frac{1}{54}$ (*b*) $\frac{1}{72}$

(*c*) $\frac{1}{36}$ (*d*) $\frac{5}{216}$

39. The integer 'k'. for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R is:

[*JEE Main-2021* (*February*)]

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 0

40. If α, $\beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + az + \beta = 0$, then $(\alpha - \beta)$ is equal to:

[*JEE Main-2021* (*February*)]

(*a*) 7 (*b*) –3

(*c*) 3 (*d*) –7

41. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = a^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is:

[*JEE Main-2021* (*February*)]

(*a*) 4 (*b*) 1

(*c*) 2 (*d*) 3

42. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is **[*JEE Main-2021* (*February*)]**

43. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $P_n = (\alpha)^n + (\beta)^n$, $P_{n-1} = 11$ and $P_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of P_n^2 is:

[*JEE Main-2021* (*February*)]

44. The number of elements in the set $\{x \in R: (|x| - 3)|x + 4| = 6\}$ is equal to

[*JEE Main-2021* (*March*)]

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 1

45. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and $P(x)$ leaves remainder 5 when it is divided by $(x-2)$. Then the value of $9(b+c)$ is equal to: **[*JEE Main-2021* (*March*)]**

(*a*) 9 (*b*) 15

(*c*) 7 (*d*) 11

46. The value of $4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + \ldots\ldots\infty}}}}$

[*JEE Main-2021* (*March*)]

(*a*) $2 + \frac{2}{5}\sqrt{30}$ (*b*) $2 + \frac{4}{\sqrt{5}}\sqrt{30}$

(*c*) $4 + \frac{4}{\sqrt{5}}\sqrt{30}$ (*d*) $5 + \frac{2}{5}\sqrt{30}$

47. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to— **[*JEE Main-2021* (*March*)]**

JEE-ADVANCED
PREVIOUS YEARS

1. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has **[*JEE Advanced-2014*]**

(*a*) only purely imaginary roots
(*b*) all real roots
(*c*) two real and two purely imaginary roots
(*d*) neither real nor purely imaginary roots

2. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S? **[*JEE Advanced-2015*]**

(*a*) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$

(*b*) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(*c*) $\left(0, \frac{1}{\sqrt{5}}\right)$

(*d*) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Comprehension–1 (3 and 4)

Let p, q_n be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$ where $\alpha \neq \beta$. For n = 0, 1, 2,........let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$. **[*JEE Advanced-2017*]**

3. $a_{12} =$

(*a*) $a_{11} + 2a_{10}$
(*b*) $2a_{11} + a_{10}$
(*c*) $a_{11} - a_{10}$
(*d*) $a_{11} + a_{10}$

4. If $a_4 = 28$, then $p + 2q =$

(*a*) 14 (*b*) 7
(*c*) 21 (*d*) 12

5. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define

$$a^n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1$$ **[*JEE Advanced-2019*]**

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$.

Then which of the following options is/are correct?

(*a*) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \geq 1$

(*b*) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(*c*) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(*d*) $b_n = \gamma^n + \beta^n$ for all $n \geq 1$

6. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define

$a^n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 1$, $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$.

Then which of the following options is/are correct? **[*JEE Advanced-2019*]**

(*a*) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \geq 1$

(*b*) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(*c*) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(*d*) $b_n = \gamma^n + \beta^n$ for all $n \geq 1$

7. Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c,d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of

$ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$ is **[*JEE(Advanced)-2020*]**

(*a*) 0 (*b*) 8000
(*c*) 8080 (*d*) 16000

ANSWER KEY

Topicwise Questions

1. (*c*) **2.** (*d*) **3.** (*a*) **4.** (*c*) **5.** (*a*) **6.** (*d*) **7.** (*b*) **8.** (*a*) **9.** (*a*) **10.** (*c*) **11.** (*b*) **12.** (*a*)
13. (*c*) **14.** (*b*) **15.** (*c*) **16.** (*b*) **17.** (*a*) **18.** (*c*) **19.** (*b*) **20.** (*c*) **21.** (*b*) **22.**(*d*) **23.** (*c*) **24.** (*a*)
25. (*a*) **26.** (*c*) **27.** (*b*) **28.** (*a*) **29.** (*a*) **30.** (*b*) **31.** (*b*) **32.** (*b*) **33.** (*d*) **34.** (*d*) **35.** (*b*) **36.** (*b*)
37. (*b*) **38.** (*a*) **39.** (*b*) **40.** (*d*) **41.** (*a*) **42.** (*a*) **43.** (*d*) **44.** (*a*) **45.** (*c*) **46.** (*c*) **47.** (*d*) **48.** (*d*)
49. (*a*) **50.** (*c*) **51.** (*a*) **52.** (*c*) **53.** (*a*) **54.** (*c*) **55.** (*b*) **56.** (*d*) **57.** (*b*) **58.** (*b*) **59.** (*d*)

Learning Plus

1. (*b*) **2.** (*c*) **3.** (*a*) **4.** (*d*) **5.** (*a*) **6.** (*b*) **7.** (*c*) **8.** (*d*) **9.** (*d*) **10.** (*a*) **11.** (*c*) **12.** (*c*)
13. (*a*) **14.** (*c*) **15.** (*a*) **16.** (*b*) **17.** (*a*) **18.** (*c*) **19.** (*a*) **20.** (*a*) **21.** (*a*) **22.** (*d*) **23.** (*d*) **24.** (*a*)
25. (*c*) **26.** (*d*) **27.** (*c*) **28.** (*c*) **29.** (*c*)

Advanced Level Multiconcept Questions

MCQ/COMPREHENSION/MATCHING/NUMERICAL

1. (*a*,*b*,*d*) **2.** (*a*,*b*,*c*,*d*) **3.** (*a*,*d*) **4.** (*a*,*b*,*d*) **5.** (*b*,*d*) **6.** (*b*,*c*,*d*) **7.** (*a*,*d*) **8.** (*a*,*b*,*d*) **9.** (*b*,*d*) **10.** (*c*,*d*) **11.** (*a*,*d*)
12. (*a*,*d*) **13.** (*a*,*b*,*c*,*d*) **14.** (*a*,*b*) **15.** (*c*) **16.** (*b*) **17.** (*d*) **18.** $(a) \to$ (r),$(b) \to$ (p),$(c) \to$ (s), $(d) \to$ (p, q)
19. (2) **20.** (8) **21.** (11) **22.** (1) **23.** (73) **24.** (10) **25.** (3) **26.** (13) **27.** (6) **28.** (11)

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEARS

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*a*) **5.** (Bonus) **6.** (*c*) **7.** (*d*) **8.** (*d*) **9.** (*c*) **10.** (*b*) **11.** (*c*) **12.** (*c*)
13. (*a*) **14.** (*b*) **15.** (*d*) **16.** (*d*) **17.** (*d*) **18.** (*c*) **19.** (*a*) **20.** (*a*) **21.** 8 **22.** (*b*) **23.** (*b*) **24.** (*a*)
25. (*a*) **26.** (*a*) **27.** (*d*) **28.** (*d*) **29.** (*a*) **30 .** (*c*) **31.** (*d*) **32.** (*b*) **33.** (*b*) **34.** (*b*) **35.** (*d*) **36.** (*d*)
37. (2) **38.** (*d*) **39.** (*a*) **40.** (*d*) **41.** (*c*) **42.** (1) **43.** (324) **44.** (*b*) **45.** (*c*) **46.** (*a*) **47.** (0)

JEE-ADVANCED PREVIOUS YEARS

1. (*d*) **2.** (*a*, *d*) **3.** (*d*) **4.** (*d*) **5.** (*a*,*b*,*c*) **6.** (*a*,*b*,*c*) **7.** (*d*)

2 QUADRATIC EQUATION

Topicwise Questions

1. (*c*) We have $4ax^2 + 3bx + 2c = 0$ Let roots are α and β
Let $D = B^2 - 4AC = 9b^2 - 4(4a)(2c) = 9b^2 - 32\,ac$
Given that, $(a + b + c) = 0 \Rightarrow b = -(a + c)$
Putting this value, we get
$= 9(a + c)^2 - 32\,ac = 9(a - c)^2 + 4ac$
Hence roots are real.

2. (*d*) Given equation
$2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$
Let $A = 2(a^2 + b^2)$, $B = 2(a + b)$ and $C = 1$
$B^2 - 4AC = 4(a^2 + b^2 + 2ab) - 4.2\,(a^2 + b^2)1$
$\Rightarrow B^2 - 4AC = -4(a - b)^2 < 0$
Thus given equation has imaginary roots.

3. (*a*) Here $(b + c - 2a) + (c + a - 2b) + (a + b - 2c) = 0$
Therefore the roots are rational.

4. (*c*) The quadratic is $(k + 11)x^2 - (k + 3)x + 1 = 0$
Accordingly, $(k + 3)^2 - 4(k + 11)(1) = 0 \Rightarrow k = -7, 5$

5. (*a*) Given equation $(1 + 2k)x^2 + (1 - 2k)x + (1 - 2k) = 0$
If equation is a perfect square then root are equal
i.e., $(1 - 2k)^2 - 4(1 + 2k)(1 - 2k) = 0$
i.e., $k = \frac{1}{2}, \frac{-3}{10}$. Hence total number of values $= 2$.

6. (*d*) $x^2 - 3x + 2$ be factor of $x^4 - px^2 + q = 0$
Hence $(x^2 - 3x + 2) = 0 \Rightarrow (x - 2)(x - 1) = 0$
$\Rightarrow x = 2, 1$ putting these values in given equation
so $4p - q - 16 = 0$ .....(i)
and $p - q - 1 = 0$ .....(ii)
Solving (i) and (ii), we get $(p, q) = (5, 4)$

7. (*b*) check by options
$x = 1$ is root
Let other root = a
$\therefore$ Product of the roots $= (1)(1) = \frac{a - b}{b - c}$
$\Rightarrow$ roots are $1, \frac{a - b}{b - c}$

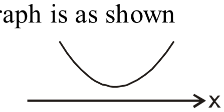
8. (*a*) $D = b^2 - 4ac = 20d^2$
$\sqrt{D} = 2\sqrt{5}d$ here $\sqrt{5}$ is irrational
So roots are irrational.

9. (*a*) $D = b^2 - 4ac = b^2 - 4a(-4a - 2b) = b^2 + 16a^2 + 8ab$
Since $ab > 0$
$\therefore D > 0$
So equation has real roots.

10. (*c*) For integral roots, D of equation should be perfect sq.
$\because D = 4(1 + n)$
By observation, for $n \in N$, D should be perfect sq. of even integer.
So $D = 4(1 + n) = 6^2, 8^2, 10^2, 12^2, 14^2, 16^2, 18^2, 20^2$
No. of values of $n = 8$.

11. (*b*) Here for $D < 0$, entire graph will be above x-axis
$(\because a > 0)$
$\Rightarrow (k - 1)^2 - 36 < 0$
$\Rightarrow (k - 7)(k + 5) < 0$
$\Rightarrow -5 < k < 7$

12. (*a*) Let $f(x) = ax^2 - bx + 1$
Given $D < 0$ & $f(0) = 1 > 0$
$\therefore$ possible graph is as shown



i.e. $f(x) > 0 \;\forall\; x \in R$
or $f(-1) > 0$
$f(-1) = a + b + 1 > 0$

13. (*c*) $x^2 + ax + b = 0 \begin{cases} a \\ b \end{cases}$
$a + b = -a$
$\Rightarrow 2a + b = 0$
and $ab = b$
$ab - b = 0$
$b(a - 1) = 0$
$\Rightarrow$ Either $b = 0$ or $a = 1$
But $b \neq 0$(given)
$\therefore\; a = 1$

$\therefore \quad b = -2$

$\therefore \quad f(x) = x^2 + x - 2$

Least value occurs at $x = -\dfrac{1}{2}$

Least value $= \dfrac{1}{4} - \dfrac{1}{2} - 2 = -\dfrac{9}{4}$

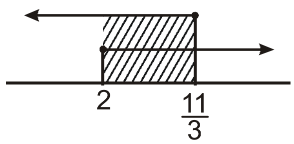
14. (*b*) $(2 - x)(x + 1) = p$

$(x - 2)(x + 1) + p = 0$

$\Rightarrow x^2 - x - 2 + p = 0$

$\dfrac{c}{a} > 0 \Rightarrow \; p - 2 > 0$

& $D > 0 \Rightarrow 1 - 4(p - 2) > 0 \Rightarrow p < \dfrac{9}{4}$



$\dfrac{-b}{2a} > 0, \; \dfrac{-1}{2(2-p)} > 0, P \in (2, \infty)$

Taking intersection of all $p \in \left(2, \dfrac{9}{4}\right)$

15. (*c*) $x^2 + px + (1 - p) = 0$

$(1 - p)^2 + p(1 - p) + (1 - p) = 0$

$(1 - p)[1 - p + p + 1] = 0 \Rightarrow p = 1$

Q.E. will be $\Rightarrow x^2 + x = 0 \Rightarrow x(x + 1) = 0$

$\Rightarrow x = 0, -1$

Aliter

$\alpha + 1 - p = -p \Rightarrow \alpha = -1$

Satisfies

$1 - p + 1 - p = 0 \Rightarrow p = 1$

$\beta = 1 - p = 0 \Rightarrow \beta = 0$

16. (*b*) Let first root $= \alpha$ and second root $= \dfrac{1}{\alpha}$

Then $\alpha, \dfrac{1}{\alpha} = \dfrac{k}{5} \Rightarrow k = 5$

17. (*a*) Given equation $4x^2 + 3x + 7 = 0$, therefore

$\alpha + \beta = -\dfrac{3}{4}$ and $\alpha\beta = \dfrac{7}{4}$

Now $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha + \beta}{\alpha\beta} = \dfrac{-3/7}{7/4} = \dfrac{-3}{4} \times \dfrac{4}{7} = -\dfrac{3}{7}$

18. (*c*) Let α and β be two roots of $ax^2 + bx + c = 0$

Then $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \dfrac{b^2}{a^2} - 2\dfrac{c}{a}$

So under condition $\alpha + \beta = \alpha^2 + \beta^2 \; \alpha + \beta = a^2 + \beta^2$

$\Rightarrow \; -\dfrac{b}{a} = \dfrac{b^2 - 2ac}{a^2} \Rightarrow b(a + b) = 2ac$

19. (*b*) α, β be the roots of $x^2 - 2x + 3 = 0$, then $\alpha + \beta = 2$ and $\alpha\beta = 3$.

Now required equation whose roots are $\dfrac{1}{\alpha^2}, \dfrac{1}{\beta^2}$ is

$$x^2 - \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)x + \frac{1}{\alpha^2\beta^2} = 0$$

$$\Rightarrow x^2 - \left(-\frac{2}{9}\right)x + \frac{1}{9} = 0 \Rightarrow 9x^2 + 2x + 1 = 0$$

20. (*c*) According to condition

$\dfrac{2m - 1}{m} = -1 \Rightarrow 3m = 1 \Rightarrow m = \dfrac{1}{3}$

21. (*b*) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (4)^3 - 3 \times 1(4) = 52$

22. (*d*) We know that the roots of the equation $ax^3 + bx^2 + cx + d = 0$ follows $\alpha\beta\gamma = -d/a$

Comparing above equation with given equation we get $d = 1, a = 1$

So, $\quad \alpha\beta\gamma = -1$ or $\alpha^3\beta^3\gamma^3 = -1$.

23. (*c*) $a + b = -p$

$ab = q$

$g + d = -p$

$gd = -r$

$(a - g)(a - d) = a^2 - a(g + d) + gd$

$= a^2 + pa - r = a(a + p) - r = -ab - r$

$= -q - r = -(q + r)$

24. (*a*) $|a - b| = 4 \Rightarrow (a - b)^2 = 16$

$\Rightarrow (a + b)^2 - 4ab = 16$

$\Rightarrow 9 - 4ab = 16 \Rightarrow ab = -\dfrac{7}{4}$

$\Rightarrow$ equation is $x^2 - 3x - \dfrac{7}{4} = 0$

25. (*a*) $C_1 : b^2 - 4ac \geqslant 0$,

$ax^2 + bx + c = 0$ real roots C_1 satisfied

C_2 : a, –b, c are same sign

$\alpha + \beta > 0 \Rightarrow \dfrac{-b}{a} > 0$

$\alpha\beta > 0 \Rightarrow \dfrac{c}{a} > 0$

C_2 satisfied C_1 & C_2 are satisfied

26. (*c*) $ax^2 + bx + c = 0, \alpha + \beta = \dfrac{-b}{a}, \alpha\beta = \dfrac{c}{a}$

$\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$$\alpha^3 + \beta^3 = \left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 3\frac{c}{a}\right]$$

$$= \frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right] = \frac{-b}{a}\frac{(b^2 - 3ac)}{a^2} = \frac{3abc - b^3}{a^3}$$

27. (*b*) Given equation can be written as

$(6k+2)x^2 + rx + 3k - 1 = 0$ .....(i)

and $2(6k+2)x^2 + px + 2(3k-1) = 0$ .....(ii)

Condition for common roots is

$$\frac{12k+4}{6k+2} = \frac{p}{r} = \frac{6k-2}{3k-1} = 2 \text{ or } 2r - p = 0$$

28. (a) Let α is the common root,

so $\alpha^2 + p\alpha + q = 0$ .....(i)

and $\alpha^2 + q\alpha + p = 0$ ....(ii)

from (i) – (ii),

$\Rightarrow (p-q)\alpha + (q-p) = 0 \Rightarrow \alpha = 1$

Put the value of α in (i), $p + q + 1 = 0$.

29. (*a*) $D_1 = 4a^2b^2 - 8a^2b^2 = -4a^2b^2 < 0$ img. root

$D_2 = 4p^2q^2 - 4p^2q^2 = 0$ equal, real roots

So no common roots.

30. (*b*) Given, $x + 2 > \sqrt{x+4} \Rightarrow (x+2)^2 > (x+4)$

$\Rightarrow x + 4x + 4 > x + 4 \Rightarrow x^2 + 3x > 0$

$\Rightarrow x(x+3) > 0 \Rightarrow x < -3 \text{ or } x > 0 \Rightarrow x > 0$

31. (*b*) **Case I:** When $x + 2 \geq 0$ i.e. $x \geq -2$

Then given inequality becomes

$x^2 - (x+2) + x > 0 \Rightarrow x^2 - 2 > 0 \Rightarrow |x| > \sqrt{2}$

$\Rightarrow x < -\sqrt{2}$ or $x > \sqrt{2}$

As $x \geq -2$, therefore, in this case the part of the solution set is $[-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

Case II: When $x + 2 \leq 0$ *i.e.* $x \leq -2$,

Then given inequality becomes $x^2 + (x+2) + x > 0$

$\Rightarrow x^2 + 2x + 2 > 0 \Rightarrow (x+1)^2 + 1 > 0$, which is true for all real x

Hence, the part of the solution set in this case is $(-\infty, -2]$. Combining the two cases, the solution set is

$(-\infty, -2) \cup ([-2, -\sqrt{2}] \cup (\sqrt{2}, \infty) = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

32. (*b*) For $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ to be an identity

$p^2 - 3p + 2 = 0 \Rightarrow p = 1, 2$ ...(1)

$p^2 - 5p + 4 = 0 \Rightarrow p = 1, 4$ ...(2)

$p - p^2 = 0 \Rightarrow p = 0, 1$ ...(3)

For (1), (2) & (3) to hold simultaneously

$p = 1$.

33. (*d*) $x^2 + 9 < (x+3)^2 < 8x + 25$

$x^2 + 9 < x^2 + 6x + 9 \Rightarrow x > 0$

$\& (x+3)^2 < 8x + 25$

$x^2 + 6x + 9 - 8x - 25 < 0$

$x^2 - 2x - 16 < 0$

$1 - \sqrt{17} < x < 1 + \sqrt{17}$ & $x > 0$

$\Rightarrow x \in (0, 1 + \sqrt{17})$

Integer x = 1, 2, 3, 4, 5

No. of integer are = 5

34. (*d*) $5x + 2 < 3x + 8 \Rightarrow 2x < 6 \Rightarrow x < 3$ ...(i)

$$\frac{x+2}{x-1} < 4 \Rightarrow \frac{x+2}{x-1} - 4 < 0 \Rightarrow \frac{-3x+6}{x-1} < 0$$

$\Rightarrow \dfrac{x-2}{x-1} > 0 \Rightarrow x \in (-\infty, 1) \cup (2, \infty)$ ...(ii)

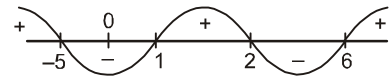
Taking intersection of (i) and (ii) $x \in (-\infty, 1) \cup (2, 3)$

35. (*b*) $\dfrac{x^2(x^2 - 3x + 2)}{x^2 - x - 30} \geq 0$

$\Rightarrow \dfrac{x^2(x-1)(x-2)}{(x+5)(x-6)} \geq 0$

$x \neq -5, 6$

$x \in (-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$



36. (*b* $\because$ $(m-2)x^2 + 8x + m + 4 > 0 \ \forall x \in R$

$\Rightarrow m > 2 \ \& \ D < 0$

$64 - 4(m-2)(m+4) < 0$

$16 - [m^2 + 2m - 8] < 0$

$\Rightarrow m^2 + 2m - 24 > 0$

$\Rightarrow (m+6)(m-4) > 0$

$m \in (-\infty, -6) \cup (4, \infty)$

But $m > 2$

$\Rightarrow m \in (4, \infty)$

Then least integral m is m = 5.

37. (*b*) $-1 \leq |x-1| - 1 \leq 1$

$\Rightarrow 0 \leq |x-1| \leq 2$

$\Rightarrow 0 \leq |x-1|$

$\Rightarrow x \in R$ ...(1)

and $|x-1| \leq 2$

$\Rightarrow -2 \leq x - 1 \leq 2$

$\Rightarrow -1 \leq x \leq 3$ ...(2)

$(1) \cap (2)$

$\Rightarrow x \in [-1, 3]$.

38. (*a*) $\log_{1/3} \dfrac{3x-1}{x+2} < 1$

$\Rightarrow \dfrac{3x-1}{x+2} > 0 \Rightarrow x \in (-\infty, -2) \cup \left(\dfrac{1}{3}, \infty\right)$ ....(i)

and $\dfrac{3x-1}{x+2} > \dfrac{1}{3}$

$\Rightarrow \dfrac{8x-5}{x+2} > 0$

$\Rightarrow x \in (-\infty, -2) \cup \left(\dfrac{5}{8}, \infty\right)$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in (-\infty, -2) \cup \left(\dfrac{5}{8}, \infty\right)$

39. (*b*) $2-\log_2(x^2+3x)\ge 0$

$\Rightarrow \quad \log_2(x^2+3x)\le 2$

$x^2+3x>0$

$\Rightarrow \quad x\in(-\infty,-3)\cup(0,\infty)$ .....(i)

and $x^2+3x\le 4$

$\Rightarrow \quad (x-1)(x+4)\le 0$

$\Rightarrow \quad x\in[-4,1]$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow x\in[-4,-3)\cup(0,1]$

40. (*d*) $\log_{1-x}(x-2)\ge 0$

$x>2$ ....(1)

(i) When $0<1-x<1 \Rightarrow 0<x<1$

So no common range comes out.

(ii) When $1-x>1\Rightarrow x<0$ but $x>2$

here, also no common range comes out. , hence no solution.

Finally, no solution

41. (*a*) $\log_{0.3}(x-1)<\log_{0.09}(x-1)$

$\log_{0.3}(x-1)<\dfrac{\log_{0.3}(x-1)}{2}$

$\Rightarrow \log_{0.3}(x-1)<0 \quad\Rightarrow\quad x-1>1 \quad\Rightarrow\quad x>2$

42. (*a*) $\log_{0.5}\log_5(x^2-4)>\log_{0.5}1$

$\log_{0.5}\log_5(x^2-4)>0$

$\Rightarrow x^2-4>0$

$\Rightarrow x\in(-\infty,-2)\cup(2,\infty)$ ....(i)

$\log_5(x^2-4)>0\Rightarrow x^2-5>0$

$\Rightarrow x\in(-\infty,-\sqrt{5})\cup(\sqrt{5},\infty)$ ....(ii)

$\log_5(x^2-4)<1$

$\Rightarrow x^2-9<0\Rightarrow x\in(-3,\ 3)$ ....(iii)

(i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow x\in(-3,\sqrt{5})\cup(\sqrt{5},3)$

43. (*d*) $\left(\frac{1}{2}\right)^{x^2-2x}<\left(\frac{1}{2}\right)^2$ here base is less than zero so inequality change

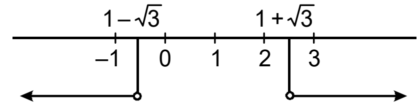
$\Rightarrow x^2-2x>2\Rightarrow x^2-2x-2>0$

$$\alpha,\beta=\frac{2\pm\sqrt{4+8}}{2}=\frac{2\pm 2\sqrt{3}}{2}$$

$a=1-\sqrt{3}$, $b=1+\sqrt{3}$

$(x-a)(x-b)>0$

$x\in\left(-\infty,\ 1-\sqrt{3}\right)\cup\left(1+\sqrt{3},\ \infty\right)$, x can be in $(3,\infty)$



44. (*a*) $\dfrac{6x^2-5x-3}{x^2-2x+6}\le 4$

D^r is always >0

$6x^2-5x-3-4x^2+8x-24\le 0$

$\Rightarrow 2x^2+3x-27\le 0$

$\Rightarrow (2x+9)(x-3)\le 0\Rightarrow x\in\left[\frac{-9}{2},3\right]$

least value of $4x^2=4.0^2=0$

Highest value of $4x^2$ is $=\max\left(4\cdot\left(-\frac{9}{2}\right)^2,\ 4\cdot 3^2\right)$

$=\max(81,36)=81$

45. (*c*) Let the roots be a, b, –b

then $\alpha+\beta-\beta=p$

$\Rightarrow \alpha=p$ ...(1)

and $\alpha\beta-\alpha\beta-\beta^2=q$

$\Rightarrow \beta^2=-q$ ...(2)

also $-\alpha\beta^2=r$

$\Rightarrow pq=r$ [using (1)].

46. (*c*) $x^3-x-1=0$ α, β, γ

then $\alpha^3-\alpha-1=0$.............(1)

Let $\dfrac{1+\alpha}{1-\alpha}=y \Rightarrow \quad \alpha=\dfrac{y-1}{y+1}$

from equation (1) $\left(\dfrac{y-1}{y+1}\right)^3-\left(\dfrac{y-1}{y+1}\right)-1=0$

$\Rightarrow \quad y^3+7y^2-y+1=0$ $\frac{1+\alpha}{1-\alpha}$, $\frac{1+\beta}{1-\beta}$, $\frac{1+\gamma}{1-\gamma}$

then $\dfrac{1+\alpha}{1-\alpha}+\dfrac{1+\beta}{1-\beta}+\dfrac{1+\gamma}{1-\gamma}=-7$ Ans.

47. (*d*) $x^4-4x^3+ax^2+bx+1=0$

real & positive roots

$\alpha+\beta+r+\delta=4$ & $\alpha\beta r\delta=1$

$\Rightarrow \alpha=\beta=r=\delta=1$

$\Sigma\alpha\beta=a\Rightarrow a=6$

$\Sigma\alpha\beta r=-b\Rightarrow b=-4$

or $(x-1)^4=x^4-4x^3+6x^2-4x+1$

48. (*d*) $ax^2 + bx + c = 0 \Big< {\alpha \atop \beta}$

sum of roots $= (2\alpha + 3\beta) + (3\alpha + 2\beta)$

$= 5(\alpha + \beta) = 5\left(-\dfrac{b}{a}\right)$

Product of roots $= 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta$

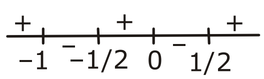
$= 6\left(\dfrac{-b}{a}\right)^2 + \dfrac{c}{a} \quad = \dfrac{6b^2}{a^2} + \dfrac{c}{a}$

Q. E. $\quad x^2 + \dfrac{5b}{a}x + \dfrac{6b^2}{a^2} + \dfrac{c}{a} = 0$

$a^2x^2 + 5abx + 6b^2 + ac = 0$

49. (*a*) $\Rightarrow \dfrac{(2x-1)}{x(2x^2+3x+1)} > 0$

$\Rightarrow \dfrac{(2x-1)}{x(x+1)(2x+1)} > 0$



consontains $\left(-\infty, \dfrac{-3}{2}\right)$

50. (*c*) Let $y = \dfrac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$\Rightarrow (y-1)x^2 + 3(y+1)x + 4(y-1) = 0$

For x is real $D \ge 0$

$\Rightarrow\ 9(y+1)^2 - 16(y-1)^2 \ge 0\ 9(y+1)^2 - 16(y-1)^2 \ge 0$

$\Rightarrow\ -7y^2 + 50y - 7 \ge 0 \Rightarrow 7y^2 - 50y + 7 \le 0$

$\Rightarrow\ (y-7)\,(7y-1) \le 0$

Now, the product of two factors is negative if one in –ve and one in +ve.

Case I : $(y-7) \ge 0$ and $(7y-1) \le 0$

$\Rightarrow\ y \ge 7$ and $y \ge \dfrac{1}{7}$. But it is impossible

Case II : $(y-7) \le 0$ and $(7y-1) \ge 0$

$\Rightarrow y \le 7$ and $y \ge \dfrac{1}{7} \Rightarrow \dfrac{1}{7} \le y \le 7$

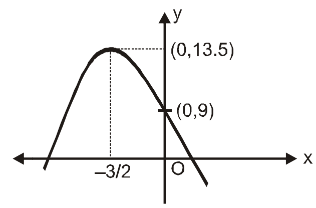
Hence maximum value is 7 and minimum value is $\dfrac{1}{7}$

51. (*a*) $x^2 - 3x + 3 = \left(x - \dfrac{3}{2}\right)^2 + \dfrac{3}{4}$

Therefore, smallest value is $\dfrac{3}{4}$, which lie in $\left(-3, \dfrac{3}{2}\right)$

52. (*c*) $y = -2x^2 - 6x + 9$

$\therefore\ \dfrac{-b}{2a} = \dfrac{6}{2(-2)} = -\dfrac{3}{2} = -1.5$



& $D = 36 - 4(-2)(9) = 36 + 72 = 108$

$\therefore\ -\dfrac{D}{4a} = -\dfrac{108}{4(-2)} = +\dfrac{108}{8} = 13.5$

$\Rightarrow\ y \in (-\infty, 13.5]$

53. (*a*) $k = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow\ (k-1)x^2 + (k+1)x + (k-1) = 0$

Q $\quad$ x is real

$\therefore\ \ D \ge 0$

$\Rightarrow (k+1)^2 - 4\,(k-1)^2 \ge 0$

$\Rightarrow (3k-1)\,(k-3) \le 0$

$\Rightarrow\ \ k \in \left[\dfrac{1}{3}, 3\right]$

54. (*c*) $y = \dfrac{2x}{1+x^2}, x \in R$

$\Rightarrow yx^2 - 2x + y = 0$

$\Rightarrow D \ge 0 \Rightarrow 4 - 4y^2 \ge 0$

$\Rightarrow (y^2 - 1) \le 0 \Rightarrow y \in [-1, 1]$

$\therefore$ Range of $f(y) = y^2 + y - 2$

Min value $= \dfrac{-D}{4a} = \dfrac{-9}{4}$ at $y = \dfrac{-b}{2a} = \dfrac{-1}{2}$

$y = \dfrac{-1}{2} \in [-1, 1]$

$f(-1) = 1 - 1 - 2 = -2$

$f(1) = 1 + 1 - 2 = 0$

max value is $= 0$

Range $\left[\dfrac{-9}{4},\ 0\right]$

55. (*b*) $x^2 - xy + y^2 - 4x - 4y + 16 = 0,\ x, y \in R$

$x^2 - x(y+4) + (y^2 - 4y + 16) = 0 \qquad \ldots(1)$

$x \in R \Rightarrow D \ge 0$

$(y+4)^2 - 4(y^2 - 4y + 16) \ge 0$

$\Rightarrow y^2 + 8y + 16 - 4y^2 + 16y - 64 \ge 0$

$\Rightarrow\ y^2 - 8y + 16 \le 0$

$\Rightarrow\ (y-4)^2 \le 0 \Rightarrow y = 4$

Put is given equation (i)

$x^2 - 8x + 16 = 0$

$\Rightarrow (x-4)^2 = 0 \Rightarrow x = 4$

56. (*d*) $(y-1)x^2 + (y+1)x + (2cy - c) = 0$

$D \ge 0 \therefore\ x \in R$

$\Rightarrow (y+1)^2 - 4(y-1)(2cy - c) \ge 0$

$y^2 + 2y + 1 - 8cy^2 + 12cy - 4c \ge 0$

$(1-8c)y^2 + (2+12c)y + (1-4c) \ge 0$

$\forall\, y \in R, D \le 0$

$(2+12c)^2 - 4(1-8c)(1-4c) \le 0$

$(1+6c)^2 - (1-8c)(1-4c) \le 0$

$4c^2 + 24c \le 0 \Rightarrow c \in [-6, 0]$

& N^r & D^r have no any common root

(i) both common factor (root) (not possible)

$\dfrac{1}{1} = \dfrac{-1}{+1} = \dfrac{c}{2c}$

(ii) If one common root is α

$(\alpha^2 - \alpha + c = 0) \times 2$

& $\alpha^2 + \alpha + 2c = 0$

$\alpha^2 - 3\alpha = 0$

$\alpha = 0 \Rightarrow c = 0$

or $\quad \alpha = 3 \Rightarrow c = -6$

$\therefore \quad c \ne 0\ \&\ c \ne -6$

$\therefore \quad c \in (-6, 0)$

57. (*b*) $2x^2 - (a^3 + 8a - 1)x + (a^2 - 4a) = 0$

since the roots are of opposite sign, $f(0) < 0$

$\Rightarrow a^2 - 4a < 0$

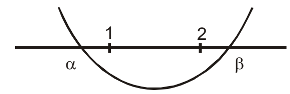
$\Rightarrow a(a-4) < 0$

$\Rightarrow a \in (0, 4)$

58. (*b*) $x^2 - 2px + (8p - 15) = 0$

$f(1) < 0$ and $f(2) < 0$

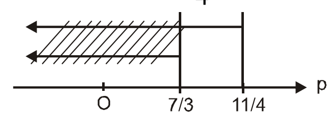
$\Rightarrow f(1) = 1 - 2p + 8p - 15 < 0$



$\Rightarrow p < 7/3$

and $f(2) = 4 - 4p + 8p - 15 < 0$

$\Rightarrow 4p - 11 < 0 \Rightarrow\ p < \dfrac{11}{4}$



Hence $p \in (-\infty, 7/3)$ Ans.

59. (*d*) $x^2 + 2(k-1)x + k + 5 = 0$

Case - I $\quad$ (i) $D \ldots 0$

$\Rightarrow 4(k-1)^2 - 4(k+5) \ldots 0$

$\Rightarrow k^2 - 3k - 4 \ldots 0 \Rightarrow (k+1)(k-4) \ldots 0$

$\Rightarrow k \in (-\infty, -1] \cup [4, \infty)$

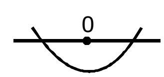
& (ii) $f(0) > 0 \Rightarrow k + 5 > 0 \Rightarrow k \in (-5, \infty)$

& (iii) $\dfrac{-b}{2a} > 0 \Rightarrow \dfrac{-2(k-1)}{2} > 0$

$\Rightarrow k \in (-\infty, 1) \therefore k \in [-5, -1]$

Case - II $\quad f(0) \le 0 \Rightarrow k + 5 \le 0$

$\Rightarrow k \in (-\infty, -5]$



Finally $k \in$ (Case - I) $\cup$ (Case - II)

$k \in (-\infty, -1]$

Learning Plus

1. (*b*) The roots of the equations are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) Let $b^2 - 4ac > 0, b > 0$

Now if $a > 0, c > 0$, $b^2 - 4ac < b^2$

$\Rightarrow$ the roots are negative.

(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{(4ac - b^2)}}{2a},\ (i = \sqrt{-1})$$

Which are imaginary and have negative real part

$(\because b > 0)$

$\therefore$ In each case, the roots have negative real part.

2. (*c*) From options put k = 3 $\Rightarrow x^2 + 8x + 7 = 0$

$\Rightarrow (x+1)(x+7) = 0 \Rightarrow x = -1, -7$

means for k = 3 roots are negative.

3. (*a*) Here $\alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

If roots are $\alpha+\frac{1}{\beta}$, $\beta+\frac{1}{\alpha}$ then sum of roots are

$$= \left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right) = (\alpha+\beta)+\frac{(\alpha+\beta)}{\alpha\beta} = \frac{b}{ac}(a+c)$$

and product $= \left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$$= \alpha\beta+1+1+\frac{1}{\alpha\beta} = 2+\frac{c}{a}+\frac{a}{c}$$

$$= \frac{2ac+c^2+a^2}{ac} = \frac{(a+c)^2}{ac}$$

Hence required equation is given by

$$x^2+\frac{b}{ac}(a+c)x+\frac{(a+c)^2}{ac} = 0$$

$\Rightarrow acx^2 + (a+c)bx + (a+c)^2 = 0$

Trick : Let a = 1 , b = – 3, c = 2, then α = 1, β = 2b =–3, c = 2, then α= 1, β = 2

$\therefore\ \alpha+\frac{1}{\beta} = \frac{3}{2}$ and $\beta+\frac{1}{\alpha} = 3$

Therefore, required equation must be

$(x-3)(2x-3) = 0$ i.e. $2x^2 - 9x + 9 = 0$

Here (1) gives this equation on putting

a = 1, b = – 3, c = 2

4. (*d*) $\alpha+\beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$

and $\alpha^2+\beta^2 = \frac{(b^2-2ac)}{a^2}$

Now $\frac{\alpha}{a\beta+b}+\frac{\beta}{a\alpha+b} = \frac{\alpha(a\alpha+b)+\beta(a\beta+b)}{(a\beta+b)(a\alpha+b)}$

$$= \frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{\alpha\beta a^2+ab(\alpha+\beta)+b^2} = \frac{a\frac{(b^2-2ac)}{a^2}+b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2+ab\left(-\frac{b}{a}\right)+b^2}$$

$$= \frac{b^2-ac-b^2}{a^2c-ab^2+ab^2} = \frac{-ac}{a^2c} = -\frac{2}{a}$$

5. (*a*) Let roots are α, β so, $\frac{\alpha}{\beta} = \frac{2}{3} \Rightarrow \alpha = \frac{2\beta}{3}$

$\therefore\ \alpha+\beta = \frac{m}{12}$

$\Rightarrow \frac{2\beta}{3}+\beta = \frac{m}{12} \Rightarrow \frac{5\beta}{3} = \frac{m}{12}$ …..(i)

and $\alpha\beta = \frac{5}{12} \Rightarrow \frac{2\beta}{3}.\beta = \frac{5}{12} \Rightarrow \beta^2 = \frac{5}{8}$

$\Rightarrow \beta = \sqrt{5/8}$

Put the value of β in (i), $\frac{5}{3}.\sqrt{\frac{5}{8}} = \frac{m}{12} \Rightarrow m = 5\sqrt{10}$.

6. (*b*) Expressions are $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, then

$$\Rightarrow \frac{x^2}{-22a+14a} = \frac{x}{a-2a} = \frac{1}{-14+11}$$

$$\Rightarrow \frac{x^2}{-8a} = \frac{x}{-a} = \frac{1}{-3} \Rightarrow x^2 = \frac{8a}{3} \text{ and } x = \frac{a}{3}$$

$$\Rightarrow \left(\frac{a}{3}\right)^2 = \frac{8a}{3} \Rightarrow \frac{a^2}{9} = \frac{8a}{3} \Rightarrow a = 0, 24.$$

Trick : We can check by putting the values of *a* from the options.

7. (*c*) If α, β, γ are the roots of the equation.

$x^3 - px^2 + qx - r = 0$

$\therefore\ (\alpha+\beta)^{-1}+(\beta+\gamma)^{-1}+(\gamma+\alpha)^{-1} = \frac{p^2+q}{pq-r}$

Given, p = 0, q = 4 , r = – 1

$\Rightarrow \frac{p^2+q}{pq-r} = \frac{0+4}{0+1} = 4$

8. (*d*) If the roots of the quadratic equation $ax^2+bx+c=0$ exceed a number k , then $ak^2+bk+c>0$ if $a>0$, $b^2-4ac \ge 0$ and sum of the roots > 2k. Therefore, if the roots of $x^2+x+a=0$ exceed a number a , then

$a^2+a+a>0$, $1-4a \ge 0$ and $-1 > 2a$

$\Rightarrow a(a+2) > 0$, $a \le \frac{1}{4}$ and $a < -\frac{1}{2}$ $\Rightarrow a > 0$ or

$a < -2, a < \frac{1}{4}$ and $a < -\frac{1}{2}$

Hence a < – 2.

9. (*d*) Let

$f(x) = 4x^2 - 20px + (25p^2 + 15p - 66) = 0$ .....(i)

The roots of (i) are real if $b^2 - 4ac = 400p^2 - 16(25p^2 + 15p - 66) = 16(66 - 15p) \ge 0$

$\Rightarrow p \le 22/5$ .....(ii)

Both roots of (i) are less than 2. Therefore f(2) > 0 and sum of roots < 4.

$\Rightarrow 4.2^2 - 20p.2 + (24p^2 + 15p - 66) > 0$ and $\frac{20p}{4} < 4$

$\Rightarrow p^2 - p - 2 > 0$ and $p < \frac{4}{5}$

$\Rightarrow (p+1)(p-2) > 0$ and $p < \frac{4}{5}$

$\Rightarrow p < -1$ or $p > 2$ and $p < \frac{4}{5} \Rightarrow p < -1$ .....(iii)

From (ii) and (iii), we get $p < -1$ i.e. $p \in (-\infty, -1)$.

10. (*a*) $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

Since, $x - 1$ is in denominator

$x - 1 \neq 0$

$x \neq 1$

$x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

$\Rightarrow x = 1$ But x can't be 1. So, No roots.

11. (*c*) If roots are α and β.

Then, $\alpha + \beta = \alpha^2 + \beta^2$

$\alpha\beta = \alpha^2\beta^2$

$\Rightarrow \alpha^2\beta^2 - \alpha\beta = 1$

$\alpha\beta(\alpha\beta - 1) = 0$

$\alpha\beta = 0$ or $\alpha\beta = 1$

$\Rightarrow \alpha = 0$ or $\beta = 0$ or $\alpha\beta = 1$

C1: if $\alpha = 0$

$0 + \beta = 0 + \beta^2 \Rightarrow \beta^2 - \beta = 0$

$\beta(\beta - 1) = 0 \Rightarrow \beta = 0$ or 1.

Roots are 0,0 and 0,1

C2: Similarly, for $\beta = 0 \Rightarrow \alpha = 0$ or 1

Roots will be 0,0 and 1,0

C3: $\alpha\beta = 1$

$\alpha^2 + \beta^2 = \alpha + \beta$

$(\alpha + \beta)^2 - 2\alpha\beta = \alpha + \beta$

$(\alpha + \beta)^2 - (\alpha + \beta) - 2 = 0$

Let $\alpha + \beta = t$

$t^2 - t - 2 = 0$

$(t - 2)(t + 1) = 0$

$t = 2$ or $t = -1$

$\alpha + \beta = 2$ or $\alpha + \beta = -1$

for $\alpha + \beta = 2, \alpha\beta = 1$

Roots are 1, 1

for $\alpha + \beta = -1, \alpha\beta = 1$

Roots are w, w^2.

Hences total roots are possible.

0, 0

0, 1

1, 1

ω, ω^2

12. (*c*) for $y = x^2 + ax + 25$ to touch the x - axis, it should have equal & real roots, i.e D = 0

$a^2 - 4.1.25 = 0$

$a^2 - 100 = 0$

$(a - 10)(a + 10) = 0$

$a = 10$ or -10

13. (*a*) $ax^3 + bx^2 + cx + d = 0$

Let roots be α, β, γ.

$\alpha + \beta + \gamma = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\alpha\beta\gamma = -\frac{d}{a}$

Now, $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$$= \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$$

As $b^2 < 2ac$

$b^2 - 2ac < 0 \quad \Rightarrow \quad \alpha^2 + \beta^2 + \gamma^2 < 0$

$\downarrow$

Not possible if all α,β,γ are real.

And complex root occurs in pair, so two out of α,β,γ are complex & one is real.

14. (*c*) $x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \text{w, w}^2$$

$\alpha = \omega, \beta = \omega^2$, ω is cube toot of unity.

(*a*) $\alpha^2 + \beta^2 = \omega^2 + \omega^4 = \omega^2 + \omega = -1$ $(1 + \omega + \omega^2 = 0)$

(*b*) $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= -1 - 2.\omega^3$

$= -1 - 2 = -3$ $\quad (\omega^3 = 1)$

(*c*) $\alpha^3 + \beta^3 = \omega^3 + (\omega^2)^3 = 1 + 1 = 2$

15. (*a*) $ax^2 + bx + c = 0$

$\sec\alpha$ and $\tan\alpha$ are roots.

$\sec\alpha + \tan\alpha = -\frac{b}{a}$; $\sec\alpha . \tan\alpha = \frac{c}{a}$

$(\sec\alpha - \tan\alpha)^2 = (\sec\alpha + \tan\alpha)^2 - 4 \sec\alpha \tan\alpha$

$$= \frac{b^2}{a^2} - \frac{4c}{a} = \frac{b^2 - 4ac}{a^2}$$

$$\sec\alpha - \tan\alpha = \sqrt{\frac{b^2 - 4ac}{a}}$$

Now, we know that,

$\sec^2\alpha - \tan^2\alpha = 1 \Rightarrow (\sec\alpha - \tan\alpha)(\sec\alpha + \tan\alpha) = 1$

$$\Rightarrow \left(\sqrt{\frac{b^2 - 4ac}{a}}\right)\left(\frac{-b}{a}\right) = 1$$

$$\Rightarrow \frac{(b^2 - 4ac)b^2}{a^4} = 1 \text{ (Squaring both side)}$$

$a^4 - b^4 + 4ab^2c = 0$

16. (*b*) $ax^2 + bx + c = 0$

$$\alpha + \beta = \frac{-b}{a};\ \ \alpha\beta = \frac{c}{a};\ \ \text{D1} = b^2 - 4ac$$

$px^2 + qx + r = 0$

$$\gamma + \delta = \frac{-q}{p};\ \gamma\delta = \frac{r}{p};\ \text{D}_2 = q^2 - 4pr$$

$\alpha, \beta, \gamma, \delta$ are in AP.

$\alpha = \beta - \alpha = \delta - \gamma$

Common difference

$$\Rightarrow \frac{\sqrt{\text{D}_1}}{a} = \frac{\sqrt{\text{D}_2}}{p} \qquad \text{(Difference between roots).}$$

$$\sqrt{\frac{\text{D}_1}{\text{D}_2}} = \frac{a}{p}$$

$$\frac{\text{D}_1}{\text{D}_2} = \frac{a^2}{p^2}$$

17. (*a*) $(l - m)x^2 + lx + 1 = 0$

Roots are α and 2α.

$$\alpha + 2\alpha = \frac{-l}{l-m} \Rightarrow \alpha = \frac{-l}{3(l-3)} \qquad \text{...(i)}$$

$$\alpha.2\alpha = \frac{l}{l-m} \Rightarrow \alpha^2 = \frac{-l}{2(l-3)} \qquad \text{...(ii)}$$

from (i) & (ii),

$$\frac{l^2}{9(l-m)^2} = \frac{1}{2(l-3)}$$

$\Rightarrow 2(l - m)\, l^2 = 9(l - m)^2$

$\Rightarrow 2l^2 - 9l + 9m = 0$

for real l, $\text{D} \geq 0$

$81 - 4.2.9m \geq 0 \quad \Rightarrow 9 - 8m \geq 0$

$$m \leq \frac{9}{8}$$

18. (*c*) $3px^2 + 5qx + 7r = 0$

$\text{D} = (5q)^2 - 4.3p.7r$

$= 25q^2 - 84pr$

$= 25\,(p + r)2 - 84pr \qquad (p + q + r = 0)$

$= 25\,(p^2 + r^2 + 2pr) - 84pr$

$= 25p^2 + 25r^2 + 50pr - 84pr$

$= 25p^2 - 34pr + 25r^2$

$$= (5p)^2 - 2.(5p)\,\frac{(17r)}{5} + \frac{(17r)^2}{5} - \frac{(17r)^2}{5} + 25r^2$$

As $\text{D} > 0$

Roots are real & distinct.

19. (*a*) $ax^2 + 2bx + c = 0$

$\text{D} = (2b)^2 - 4ac = 4(b^2 - ac)$

As a,b,c are in G.P. So, $b^2 = ac$

$\text{D} = b^2 - ac = 0$

Hence, above eq^n has equal roots

$$x = \frac{-2b}{2a} = \frac{-b}{a}$$

This root will satisfy $dx^2 + 2ex + f = 0$

$$\text{So, } d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + f = 0$$

$db^2 - 2aeb + a^2f = 0$

$dac - 2aeb + a^2f = 0$

$dac + a^2f = 2aeb \Rightarrow dc + af = 2eb$

Dividing by ac, both side

$$\frac{d}{a} + \frac{f}{c} = \frac{2e}{b} \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

20. (*a*) $a < b < c < d$ and $k > 0$

$f(x) = (x - a)\,(x - c) + k(x - b)\,(x - d)$

$f(a) = k(a - b)\,(a - b) \Rightarrow f(a) > 0$

$f(b) = (b - a)\,(b - c) \Rightarrow f(b) < 0$

$f(c) = (c - b)\,(c - d) \Rightarrow f(c) < 0$

$f(d) = (d - a)\,(d - c) \Rightarrow f(d) < 0$

$a \quad b \quad c \quad d$

$f(x)$ has one root in (a,b) & another root in (c,d).

Hence, roots are real & distinct.

21. (*a*) $\dfrac{x}{x+6} - \dfrac{1}{x} \leq 0 \Rightarrow \dfrac{x^2 - x - 6}{x(x+6)} \leq 0$

$$\frac{(x+2)(x-3)}{x(x+6)} \leq 0$$

$+ \quad - \quad + \quad - \quad +$

$-6 \quad -2 \quad 0 \quad 3$

No. of integral values of $x = 7$

22. (*d*) Given, $3x - 4y = 77$

$3^{x/2} - 2^y = 7$

Let $3^{x/2} = p,\ 2^y = q$

$p - q = 7$

$$p^2 - q^2 = 77 \Rightarrow (p - q)\,(p + q) = 77 \Rightarrow p + q = \frac{77}{7} = 11$$

$$\left.\begin{array}{l} p - q = 7 \\ p + q = 11 \end{array}\right\} \Rightarrow p = 9,\ q = 2$$

$$3^{x/2} = 9 \Rightarrow \frac{x}{2} = 2 \Rightarrow x = 4$$

$2^y = 2 \Rightarrow y = 1$

$x + y = 4 + 1 = 5$

23. (*d*) $\left(m^2+\frac{1}{m^2}\right)^2-2=119 \Rightarrow \left(m^2+\frac{1}{m^2}\right)^2=121$

$m^2+\frac{1}{m^2}=11 \Rightarrow \left(m-\frac{1}{m}\right)^2+2=11$

$\left(m-\frac{1}{m}\right)^2=9 \Rightarrow m-\frac{1}{m}=3$

$\left|m^3-\frac{1}{m^3}\right|=\left|\left(m-\frac{1}{m}\right)\left(m^2+\frac{1}{m^2}+1\right)\right|$

$=|3.12|=36$

24. (*a*) $x^8-24x^7-18x^5+39x^2+1155=0$

$x^2(x^6-24x^5-18x^3+39)=-1155$

$=-3\times5\times7\times11$

As $x\in Z$.

LHS contains x^2

But in RHS, there is no perfect square.

So, there is no integral value of x.

25. (*c*) $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$

$\frac{(x+q)+(x+p)}{(x+p)(x+q)}=\frac{1}{r}$

$(2x+p+q)r=x^2+(p+q)x+pq$

$x^2+(p+q-2r)x+pq-(p+q)r=0$

Roots are $\alpha_1-\alpha$.

$(\alpha)+(-d)=-(p+q-2r)$

$\Rightarrow 2r=p+q \Rightarrow r=\frac{p+q}{2}$

Product of roots $=\alpha.(-\alpha)=pq-(p+q)r$

$=pq-(p+q)\frac{(p+q)}{2}=-\frac{(p^2+q^2)}{2}$

26. (*d*) $\alpha^2=5\alpha-3;\ \beta^2=5\beta-3$

$\left.\begin{array}{l}\alpha^2-5\alpha+3=0\\ \beta^2-5\beta+3=0\end{array}\right\}\alpha\ \&\ \beta$ are roots of $x^2-5x+3=0$

$\alpha+\beta=5$

$\alpha\beta=3$

Required Eq^n:

$x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+\left(\frac{\alpha}{\beta}.\frac{\beta}{\alpha}\right)=0$

$x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+1=0$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{25-2.3}{3}=\frac{19}{3}$

$x^2-\frac{19}{3}x+1=0$

$3x^2-19x+1=0$

27. (*c*) $x^2-(1+\gamma)x+\gamma-2=0$

α,β are roots.

$\alpha+\beta=(1+\gamma)$

$\alpha\beta=\gamma-2$

$\alpha+\beta-\alpha\beta=3$

$\alpha\beta-\gamma-\beta+3=0$

$\alpha\beta-\alpha-\beta+1+2=0$

$\alpha(\beta-1)-(\beta-1)=-2$

$(\alpha-1)(\beta-1)=-2$

Since, product is $(-ve)$

At least one root is $+ve$.

28. (*c*) $\frac{(\alpha+1)^2+(\beta+1)^2+(\gamma+1)^2(\delta+1)^2}{\alpha+\beta+\gamma+\delta}=4$

$(\alpha+1)^2+(\beta+1)^2+(\gamma+1)^2+(\delta+1)^2$

$=4\alpha+4\beta+4\gamma+4\delta$

$(\alpha+1)^2-4\alpha+(\beta+1)^2-4\beta+(\gamma+1)^2$

$-4\gamma+(\delta+1)^2-4\delta=0$

$(\alpha-1)^2+(\beta-1)^2+(\gamma-1)^2+(\delta-1)^2=0$

$\alpha=\beta=\gamma=\delta=1$

$a_0x^4+a_1x^3+a_2x^2+a_3x+a_4=0$

Roots are 1, 1, 1, 1

$\frac{a_2}{a_0}=S_2=$ Sum of product of roots taken two at a time.

$\frac{a_2}{a_0}=6$

29. (*c*) $ax^2+bx+c=0$

(i) Since downward parabola, $a<0$

(ii) As graph cuts $+ve$ y-axis, $c>0$

(iii) Vertex lies in 2nd Quardrant,

$\frac{-b}{2a}<0\Rightarrow\frac{b}{2a}>0$

$\Rightarrow b\ \&\ a$ must have same sign.

$b<0$

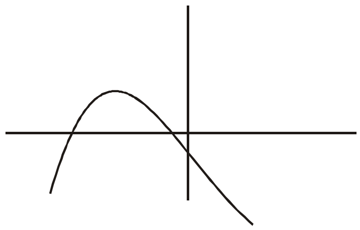
Hence, $a<0, b<0, c>0$

Advanced Level Multiconcept Questions

1. (*a*, *b*, *d*)

$y = ax^2 + bx + c$

Clearly $a < 0$



and $\frac{-b}{2a} < 0$

$\Rightarrow b < 0$

also $f(0) < 0 \Rightarrow c < 0$

and $D > 0$

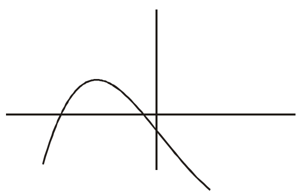
$\therefore$ (A), (B) and (D).

2. (*a*,*b*,*c*,*d*)

(*a*) $a < 0$,

$-\frac{-b}{2a} < 0 \Rightarrow b < 0$

& $f(0) < 0 \Rightarrow c < 0$



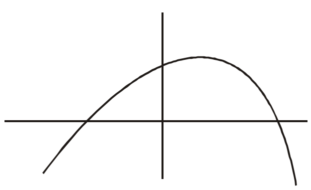
$\therefore$ $abc < 0$

(*b*) $a < 0$,

$\frac{-b}{2a} > 0 \Rightarrow b > 0$

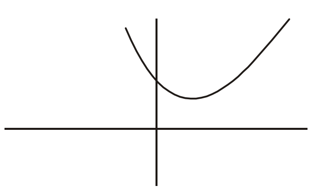
$f(0) > 0 \Rightarrow c > 0$

$\Rightarrow abc < 0$



(*c*) $a > 0$

$\frac{-b}{2a} > 0 \Rightarrow b < 0$

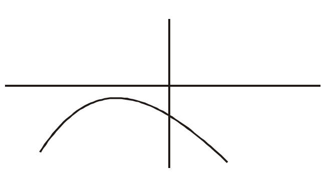


$f(0) > 0$ $c > 0$

$\Rightarrow abc < 0$

(*d*) $a < 0$

$\frac{-b}{2a} < 0 \Rightarrow b < 0$



$f(0) < 0 \Rightarrow c < 0$

$\therefore$ (*a*), (*b*), (*c*), (*d*)

3. (*a*,*d*)

Clearly $a < 0$

$\frac{-b}{2a} > 0 \Rightarrow b > 0$

$\therefore$ (*a*), (*d*)

4. (*a*, *b*, *d*)

$|x|^2 + |x| - 6 = 0 \Rightarrow |x| = -3, 2 \Rightarrow |x| = 2$

$\Rightarrow x = \pm 2$

5. (*b*, *d*)

$ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

$a + b = -b/a$, $ab = c/a$

$px^2 + qx + r = 0 \begin{matrix} \nearrow \alpha + h \\ \searrow \beta + h \end{matrix}$

$$(a + b) + 2h = \frac{-q}{p}$$

$$h = \frac{\frac{-q}{p} + \frac{b}{a}}{2} = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right) \text{ Ans.}$$

$$|\alpha - \beta| = |(\alpha + h) - (\beta + h)|$$

$$= \sqrt{[(\alpha + h) + (\beta + h)]^2 - 4(\alpha + h)(\beta + h)}$$

$$= \frac{b^2}{a^2} - \frac{4c}{a} = \frac{q^2}{p^2} - \frac{4r}{p} \Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2} \text{ Ans.}$$

6. (*b*,*c*,*d*)

(*a*) $S = a^2 + b^2 = a^2 - 2b$

$P = a^2 b^2 = b^2$

$\therefore$ equation is $x^2 - (a^2 - 2b)x + b^2 = 0$

(*b*) $S = \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{a}{b}$, $P = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{b}$

$\therefore x^2 + \frac{a}{b}x + \frac{1}{b} = 0$

$\Rightarrow bx^2 + ax + 1 = 0$

(*c*) $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{a^2 - 2b}{b}$

$P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$x^2 - \frac{a^2 - 2b}{b}x + 1 = 0 \Rightarrow bx^2 - (a^2 - 2b)x + b = 0$

(*d*) $S = a + b - 2 = -a - 2$

$P = (a-1)(b-1)$

$= ab - (a+b) + 1$

$= b + a + 1$

$\therefore$ equation is

$x^2 + (a+2)x + (a+b+1) = 0.$

7. (*a*,*d*)

$ax^3 + bx^2 + cx + d = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

Let $ax^3 + bx^2 + cx + d \equiv (x^2 + x + 1)(Ax + B)$

Roots of $x^2 + x + 1 = 0$ are imaginary, Let these are α, β

So the third root 'γ' will be real.

$\alpha + \beta + \gamma = \frac{-b}{a}$

$-1 + \gamma = \frac{-b}{a} \quad \Rightarrow \quad \gamma = \frac{a-b}{a}$

Also $\alpha\beta\gamma = \frac{-d}{a}$

But $\alpha\beta = 1$

$\therefore \gamma = \frac{-d}{a}$

$\therefore$ Ans are (*a*) & (*d*).

8. (*a*, *b*, *d*)

$\frac{1}{2} \le \log_{1/10} x \le 2$

$\Rightarrow \frac{1}{100} \le x \le \frac{1}{\sqrt{10}}$

9. (*b*, *d*)

$x^2 + abx + c = 0 \begin{cases} \alpha \\ \beta \end{cases}$ ...(1)

$\alpha + \beta = -ab, \alpha\beta = c$

$x^2 + acx + b = 0 \begin{cases} \alpha \\ \delta \end{cases}$ ...(2)

$\alpha + \delta = -ac, \alpha\delta = b$

$\alpha^2 + ab\alpha + c = 0$

$\alpha^2 + ac\alpha + b = 0$

$\frac{\alpha^2}{ab^2 - ac^2} = \frac{\alpha}{c-b} = \frac{1}{ac - ab}$

$\Rightarrow \alpha^2 = \frac{a(b^2 - c^2)}{a(c-b)} = -(b+c)$

& $\alpha = \frac{c-b}{a(c-b)} = \frac{1}{a}$ $\therefore$ common root, $\alpha = \frac{1}{a}$

$\therefore -(b+c) = \frac{1}{a^2} \Rightarrow a^2(b+c) = -1$

Product of the roots of equation (1) & (2) gives

$\beta \times \frac{1}{a} = c \Rightarrow \beta = ac$

& $\delta \times \frac{1}{a} = b \Rightarrow \delta = ab.$

$\therefore$ equation having roots β, δ is

$x^2 - a(b+c)x + a^2bc = 0$

$a(b+c)x^2 - a^2(b+c)^2 x + a.(b+c)a^2bc = 0$

$a(b+c)x^2 + (b+c)x - abc = 0.$

10. (*c*, *d*)

$\because$ D of $x^2 + 4x + 5 = 0$ is less than zero

$\Rightarrow$ both the roots are imaginary

$\Rightarrow$ both the roots of quadratic are same

$\Rightarrow b^2 - 4ac < 0$ & $\frac{a}{1} = \frac{b}{4} = \frac{c}{5} = k$

$\Rightarrow a = k, b = 4k, c = 5k.$

11. (*a*, *d*)

$x^2 + px + q = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha + \beta = -p$, $\alpha\beta = q$ and $p^2 - 4q > 0$

$x^2 - rx + s = 0 \begin{cases} \alpha^4 \\ \beta^4 \end{cases}$ .....(1)

Now $\alpha^4 + \beta^4 = r$

$\Rightarrow \quad \alpha^4 + \beta^4 = r, (\alpha\beta)^4 = s = q^4$

$\therefore (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = r$

$\Rightarrow [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2 = r$

$\Rightarrow (p^2 - 2q)^2 - 2q^2 = r$

$\Rightarrow (p^2 - 2q)^2 = 2q^2 + r > 0$ .....(2)

Now, for $x^2 - 4qx + 2q^2 - r = 0$

$D = 16q^2 - 4(2q^2 - r)$ by equation (2)

$= 8q^2 + 4r = 4(2q^2 + r) > 0$

$\Rightarrow$ $D > 0$ two real and distinct roots

Product of roots $= 2q^2 - r$

$= 2q^2 - [(p^2 - 2q)^2 - 2q^2]$

$= 4q^2 - (p^2 - 2q)^2$

$= -p^2(p^2 - 4q) < 0$ from (1)

So product of roots is – ve

hence roots are opposite in sign

12. (*a*, *d*)

$20x^2 + 210x + 400 = 4500 \Rightarrow 2x^2 + 21x - 410 = 0$

$\Rightarrow (2x + 41)(x - 10) = 0$

$\Rightarrow x = \frac{-41}{2}, x = 10 \quad \Rightarrow x = -20.5, \; x = 10$

13. (*a*,*b*,*c*,*d*)

$$x^3+bx^2+cx-1=0 \begin{cases} \alpha = \frac{a}{r} \\ \beta = a \\ r = ar \end{cases}$$

$$\frac{a}{r}+a+ar=-b \;\Rightarrow\; a\left(\frac{1}{r}+1+r\right)=-b$$

$$\& \quad \frac{a}{r}\times a\times ar = 1$$

$$a^3=1 \Rightarrow a=1$$

$$\& \quad \frac{a}{r}a+a\cdot ar+\frac{a}{r}\cdot ar = c$$

$$a^2\left(\frac{1}{r}+r+1\right)=c$$

$$\frac{1}{r}+r+1=-b \;\&\; \frac{1}{r}+r+1=c \Rightarrow b+c=0$$

$$\text{we know } \frac{1}{r}+r>2 \Rightarrow \left(\frac{1}{r}+r+1\right)>3$$

$$-b>3 \Rightarrow b<-3 \Rightarrow b\in(-\infty,-3)$$

$$\& \text{ other two roots are } \frac{1}{r}\;\&\; r$$

$$\text{if } \frac{1}{r}>1 \Rightarrow r<1 \text{ if } r>1 \Rightarrow r<1$$

14. (*a*,*b*)

$$f(x)=\frac{3}{(x-2)}+\frac{4}{(x-3)}+\frac{5}{(x-4)}=0$$

$$6x^2-14x-21x+49=0$$

$$(3x-7)(2x-7)=0$$

$$x=\frac{7}{2},x=\frac{7}{2}$$

$$2<\frac{7}{2}<33<\frac{7}{2}<4$$

2nd Method

$$g(x)=3(x-3)(x-4)+4(x-2)(x-4)+5(x-2)(x-3)=0$$

$$g(2)>0;g(3)<0,g(4)>0$$

one root lie b/w (2, 3) & other root lie b/w (3, 4)

15. (*c*)

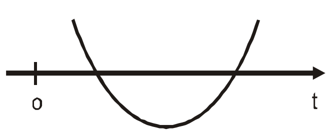
16. (*b*)

17. (*d*)

Sol. (15 to 16)

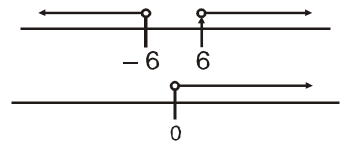
$$x^4-\lambda x^2+9=0 \Rightarrow \quad x^2=t \ge 0 \Rightarrow f(t)=t^2-\lambda t+9=0$$

15. given equation has four real & distinct roots



$D>0$

$\Rightarrow \lambda^2-36>0$



$\frac{-b}{2a}>0$

$\Rightarrow \frac{\lambda}{2}>0$

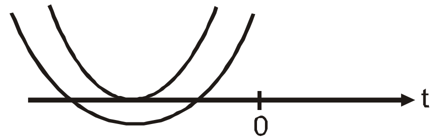
$\Rightarrow \quad \lambda>0$

$f(0)>0$

$\Rightarrow 9>0$

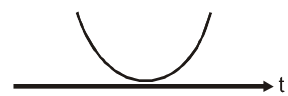
$\therefore \quad \lambda\in(6,\infty)$

16. Equation has no real roots.



case-I $\qquad D\ge 0 \Rightarrow \lambda^2-36\ge 0$

$\frac{-b}{2a}<0 \qquad \Rightarrow \lambda<0$



$f(0)>0 \qquad \Rightarrow \quad 9>0.$

$\therefore \quad \lambda\in(-\infty,-6]$

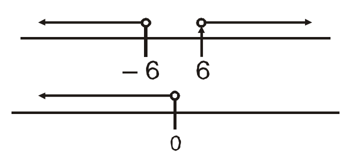
case-II $\qquad D<0$

$\Rightarrow \quad \lambda^2-36<0$

$\Rightarrow \quad \lambda\in(-6,6)$

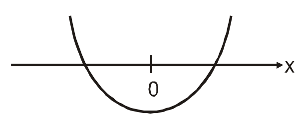
union of both cases gives

$\lambda\in(-\infty,6)$



17. Equation has only two real roots

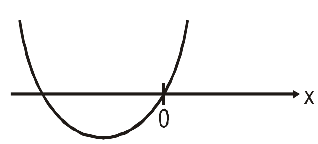
case-I $\qquad f(0)<0 \qquad 9<0$



which is false

case-II $f(0)=0$

and $\frac{-b}{2a}<0$



$\therefore$ No solution

$\therefore$ Final answer is ϕ

18. $(a) \to$ (r), $(b) \to$ (p), $(c) \to$ (s), $(d) \to$ (p, q)

(a) $x^2 - 8x + k = 0 \left\langle \begin{array}{l} \alpha \\ \alpha + 4 = \beta \end{array} \right.$

$\because (\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow 16 = 64 - 4k \Rightarrow 4k = 48 \Rightarrow k = 12$

(b) $\because$ $(|x| - 2)(|x| - 3) = 0$

$\Rightarrow x = \pm 2$; $x = \pm 3$

$\therefore$ $n = 4 \therefore \dfrac{n}{2} = 2$

(c) $\because$ $b = (3 - i)(3 + i)$

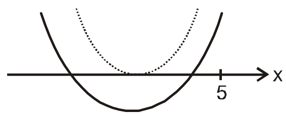
$b = 10$

(d) $x^2 - 2kx + (k^2 + k - 5) = 0$

(i) $D \ge 0$

$\Rightarrow 4k^2 - 4(k^2 + k - 5) \ge 0$

$\Rightarrow k - 5 \le 0$



(ii) $f(5) > 0$

$\Rightarrow$ $25 - 10k + k^2 + k - 5 > 0$

$\Rightarrow$ $k^2 - 9k + 20 > 0 \Rightarrow (k - 5)(k - 4) > 0$

(iii) $-\dfrac{b}{2a} < 5 \Rightarrow k < 5$

$\Rightarrow$ $k \in (-\infty, 4)$

So k may be 2, 3.

NUMERICAL VALUE BASED

19. [2] $\left(x^2 + 3x + 2\right)\left(x^2 + 3x\right) = 120$

Let $x^2 + 3x = y$

$\Rightarrow y^2 + 2y - 120 = 0$

$\Rightarrow (y + 12)(y - 10) = 0$

$\Rightarrow y = -12 \Rightarrow x^2 + 3x + 12 = 0$

$\Rightarrow x \in \phi$

$y = 10$ $\Rightarrow$ $x^2 + 3x - 10 = 0$

$\Rightarrow (x + 5)(x - 2) = 0$ $\Rightarrow$ $x = \{-5, 2\}$

x = 2, – 5 are only two integer roots.

20. [8] $\left(5 + 2\sqrt{6}\right)^{x^2 - 3} + \dfrac{1}{\left(5 + 2\sqrt{6}\right)^{x^2 - 3}} = 10$

$\Rightarrow$ $t + \dfrac{1}{t} = 10$

$\Rightarrow$ $t^2 - 10t + 1 = 0$ $t = \dfrac{10 \pm \sqrt{96}}{2} = 5 \pm 2\sqrt{6}$

$\Rightarrow$ $(5 + 2\sqrt{6})^{x^2 - 3} = (5 + 2\sqrt{6})$ or $\dfrac{1}{5 + 2\sqrt{6}}$

$\Rightarrow$ $x^2 - 3 = 1$ or $x^2 - 3 = -1$

$\Rightarrow$ $x = 2$ or -2 or $-\sqrt{2}$ or $\sqrt{2}$

Product 8

21. [11] $2x^2 + 6x + a = 0$

Its roots are α, β

$\Rightarrow$ $\alpha + \beta = -3$ & $\alpha\beta = \dfrac{a}{2}$ $\Rightarrow$ $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} < 2$

$\Rightarrow$ $\dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} < 2$

$\Rightarrow$ $\dfrac{9 - a}{a} < 1$

$\Rightarrow$ $\dfrac{2a - 9}{a} > 0$

$\Rightarrow$ $a \in (-\infty, 0) \cup \left(\dfrac{9}{2}, \infty\right)$

$\Rightarrow$ $2a = 11$ is least prime.

22. [1]

$x^2 + px + 1 = 0 \left\langle \begin{array}{l} a \\ b \end{array} \right.$ $a + b = -p, ab = 1$; $x^2 + qx + 1 = 0 \left\langle \begin{array}{l} c \\ d \end{array} \right.$ $c + d = -q, cd = 1$

$a + b = -p$, $ab = 1 \Rightarrow c + d = -q$, $cd = 1$

RHS $= (a - c)(b - c)(a + d)(b + d)$

$= (ab - ac - bc + c^2)(ab + ad + bd + d^2)$

$= (1 - ac - bc + c^2)(1 + ad + bd + d^2)$

$= 1 + ad + bd + d^2 - ac - a^2cd - abcd - acd^2 - bc - abcd - b^2cd - bcd^2 + c^2 + adc^2 + bdc^2 + c^2d^2$

$= 1 + ad + bd + d^2 - ac - a^2 - 1 - ad - bc - 1 - b^2 - bd + c^2 + ac + bc + 1$

[$\because$ $ab = cd = 1$]

$= c^2 + d^2 - a^2 - b^2 = (c + d)^2 - 2cd - (a + b)^2 + 2ab$

$= q^2 - 2 - p^2 + 2 = q^2 - p^2 =$ LHS. **Proved.**

2nd Method :

RHS $= (ab - c(a + b) + c^2)(ab + d(ab + d(a + b) + d^2)$

$= (c^2 + pc + 1)(1 - pd + d^2)$ ...(1)

Since c & d are the roots of the equation $x^2 + qx + 1 = 0$

$\therefore$ $c^2 + qc + 1 = 0 \Rightarrow c^2 + 1 = -qc$ & $d^2 + qd + 1 = 0$

$\Rightarrow d^2 + 1 = -qd$.

$\therefore$ (i) Becomes $= (pc - qc)(-pd - qd) = c(p - q)(-d)(p + q) = -cd(p^2 - q^2)$

$= cd(q^2 - p^2) = q^2 - p^2 =$ LHS. **Proved.**

23. [73] $\because$ α, β are roots of $\lambda x^2 - (\lambda - 1)x + 5 = 0$

$\therefore$ $\alpha + \beta = \dfrac{\lambda - 1}{\lambda}$ and $\alpha\beta = \dfrac{5}{\lambda}$

$\because$ $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = 4$ $\Rightarrow$ $\dfrac{\alpha^2 + \beta^2}{\alpha\beta} = 4$

$\Rightarrow (\alpha+\beta)^2 = 6\alpha\beta \Rightarrow \dfrac{(\lambda-1)^2}{\lambda^2} = \dfrac{30}{\lambda}$

$\Rightarrow \lambda^2 - 32\lambda + 1 = 0$ ...(1)

$\because \lambda_1, \lambda_2$ are roots of (1)

$\therefore \lambda_1 + \lambda_2 = 32$ and $\lambda_1\lambda_2 = 1$

$\therefore \dfrac{\lambda_1}{\lambda_2} + \dfrac{\lambda_2}{\lambda_1} = \dfrac{(\lambda_1+\lambda_2)^2 - 2\lambda_1\lambda_2}{\lambda_1\lambda_2} = \dfrac{(32)^2-2}{1}$

$= 1022 \Rightarrow \left(\dfrac{\dfrac{\lambda_1}{\lambda_2}+\dfrac{\lambda_2}{\lambda_1}}{14}\right) = 73$

24. [10] $\alpha.\beta = b$; $\gamma\delta = b-2$

$\Rightarrow \alpha\beta\gamma\delta = b(b-2) = 24$

$\therefore bx^2 + ax + 1 = 0$ has roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

$\Rightarrow \dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{-a}{b}$

$(b-2)x^2 - ax + 1 = 0$ has root $\dfrac{1}{\gamma}, \dfrac{1}{\delta} \Rightarrow \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{a}{b-2}$

$\dfrac{1}{\gamma} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{-a}{b} + \dfrac{a}{b-2} = \dfrac{5}{6}$; $\dfrac{+2a}{b(b-2)} = \dfrac{5}{6}$;

$\dfrac{+2a}{24} = \dfrac{5}{6}$; $a = 10$.

25. [3] $x^2 + px + 1 = 0$

Roots = a,b

$a + b = -p$; $ab = 1$ ...(i)

$x^2 + qx + 1 = 0 \Rightarrow c + d = -q$; $cd = 1$ ...(ii)

Roots = c,d

Also, $c^2 + qc + 1 = 0$; $d^2 + qd + 1 = 0$ ...(iii)

$(a-c)(b-c)(a+b)(b+d) = (ab - (a+b)c + c^2)(ab + (a+b)d + d^2)$

$= (1 + p.c + c^2)(1 - pd + d^2)$ from ...(i)

$= (c^2 + 1 + pc)(d^2 + 1 - pd)$

$= (-qc + pc)(-qd - pd)$ from ...(iii)

$= -c(p-q)(p+q)d$

$= -cd(p^2 - q^2)$

$= -1(p^2 - q^2) = q^2 - p^2$

Required value $= \dfrac{(a-c)(b-c)(a+d)(b+d)}{q^2 - p^2}$

$= \dfrac{q^2 - p^2}{q^2 - p^2} = 1$

26. [13] $a^3 + b^3 + (-9)^3 = 3 \cdot a \cdot b\,(-9)$

$\Rightarrow a + b - 9 = 0$ or

$a = b = -9$. Which is rejected.

As $a > b > -9$

$\Rightarrow a + b - 9 = 0 \Rightarrow x = 1$ is a root

other root $= \dfrac{-9}{a}$. $\therefore \alpha = \dfrac{-9}{a}, \beta = 1$

$\Rightarrow 4\beta - a\alpha = 4 - a\left(\dfrac{-9}{a}\right) = 4 + 9 = 13.$

27. [6] Let $t^2 - 2t + 2 = k$

$\Rightarrow \alpha^2 - 6k\alpha - 2 = 0$

$\Rightarrow \alpha^2 - 2 = 6k\alpha$

$a_{100} - 2a_{98} = \alpha^{100} - 2.\alpha^{98} - \beta^{100} + 2.\beta^{98}$

$= \alpha^{98}(\alpha^2 - 2) - \beta^{98}(\beta^2 - 2) = 6k(\alpha^{99} - \beta^{99})$

$a_{100} - 2a_{98} = 6k.a_{99}$

$\dfrac{a_{100} - 2a_{98}}{a_{99}} = 6k = 6(t^2 - 2t + 2) = 6[(t-1)^2 + 1]$

$\therefore$ min. value of $\dfrac{a_{100} - 2a_{98}}{a_{99}}$ is 6.

28. [11] Given that, roots of equation $x^2 - 10ax - 11b = 0$ are c, d

So $c + d = 10a$ and $cd = -11b$ and a, b are the roots of equation $x^2 - 10cx - 11d = 0$

$\therefore a + b = 10c$, $ab = -11d$

So $a + b + c + d = 10(a + c)$ and $(c + d) - (a + b) = 10(a - c)$

$(c - a) - (b - d) + 10(c - a) = 0$

$\Rightarrow b + d = 9(a + c)$ ....(i)

$abcd = 121\,bd$

$\Rightarrow ac = 121$ ....(ii)

$b - d = 11(c - a)$ ....(iii)

c & a satisfies the equation $x^2 - 10ax - 11b = 0$ and $x^2 - 10cx - 11d = 0$ respectively

$\therefore c^2 - 10ac - 11b = 0$

$a^2 - 10ca - 11d = 0$

$(c^2 - a^2) - 11(b - d) = 0$

$(c - a)(c + a) = 11(b - d) = 11.\,11\,(c - a)$

(by equation (iii))

$c + a = 121$

$\Rightarrow a + b + c + d = 10(c + a)$

$\Rightarrow 10.121 \Rightarrow \dfrac{a + b + c + d}{110} = 11.$

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEARS

1. (*a*) $(x^2-5x+5)^{x^2+4x-60}=1$

$x^2-5x+5=1 \Rightarrow x=1,4$

or, $x^2+4x-60=0$ $x=-10, 6$

or , $x^2-5x+5=-1 \Rightarrow x=2,3$

But for $x=3$, $x^2+4x-60$ is odd

So Required values of x are $1,4,-10,6,2$ Sum = 3

2. (*a*) We have $\sum_{r=1}^{n}(x+r-1)(x+r)=10n$

$$\Rightarrow \sum_{r=1}^{n}\left(x^2+(2r-1)x+(r^2-r)\right)=10n$$

$\therefore$ On solving, we get

$$x^2+nx+\left(\frac{n^2-31}{3}\right)=0 \qquad \therefore (2\alpha+1)=-n$$

/ \

$\alpha \quad \alpha+1$

$$\Rightarrow \alpha=\frac{-(n+1)}{2} \qquad .....(1)$$

$$\text{and } \alpha(\alpha+1)=\frac{n^2-31}{3} \qquad .....(2)$$

$\Rightarrow$ $n^2=121$ (using (1) in (2))

or $n=11$

3. (*b*) α, β are roots of $x^2-x+1=0$

$\therefore$ $\alpha=-\omega$ and $\beta=-\omega^2$

where ω is non-real cube root of unity

so, $a^{101}+\alpha^{107}$

$\Rightarrow (-\omega)^{101}+(-\omega^2)^{107}$

$\Rightarrow -[\omega^2+\omega]$

$\Rightarrow -[-1]=1$

(As $1+\omega+\omega^2=0$ & $\omega^3=1$)

4. (*a*) $x^2+2x+2=0 \Rightarrow (x+1)^2=-1$

$$x=-1\pm i=\sqrt{2}e^{i\left(\pm\frac{3\pi}{4}\right)}$$

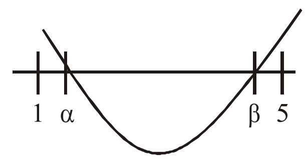
$$\therefore \alpha^{15}, \beta^{15}=\left(\sqrt{2}\right)^{15}\times 2\cos\left(15.\frac{3\pi}{4}\right)$$

$$=2^8\sqrt{2}\times\left(-\frac{1}{\sqrt{2}}\right)=-256$$

5. (Bonus)

$x^2-mx+4=0$

$\alpha, \beta \in [1, 5]$



(*a*) $D>0 \Rightarrow m^2-16>0$

$\Rightarrow m \in (-\infty, -4)\cup(4, \infty)$

(*b*) $f(1)\geq 0 \Rightarrow 5-m\geq 0 \Rightarrow m \in (-\infty, 5)$

(*c*) $f(5)\geq 0 \Rightarrow 29-5m\geq 0 \Rightarrow m \in \left(-\infty, \frac{29}{5}\right]$

(*d*) $1<\frac{-b}{2a}<5 \Rightarrow 1<\frac{m}{2}<5 \Rightarrow m \in (2, 10)$

$\Rightarrow m \in (4, 5)$

No option correct : Bonus

* If we consider $\alpha, \beta \in (1, 5)$ then option (1) is correct.

6. (*c*) D must be perfect square

$\Rightarrow 121-24\alpha=\lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha=1 \Rightarrow \lambda \notin 1$

$\alpha=2 \Rightarrow \lambda \notin 1$

$\alpha=3 \Rightarrow \lambda \in 1$

$\Rightarrow$ 3 integral values

$\alpha=4 \Rightarrow \lambda \in 1$

$\alpha=5 \Rightarrow \lambda \in 1$

7. (*d*) Case - 1

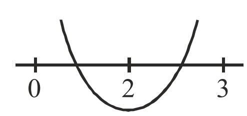
$c-5>0$ ....(i)

$f(0)>0$

$c-4>0$ ...(ii)

$f(2)<0$

$4(c-5)-4c+c-4<0$



$c<24$ ...(iii)

$f(2)>0$

$9(c-5)-6c+c-4>0$

$4c-49>0 \Rightarrow c>\frac{49}{4}$ ...(iv)

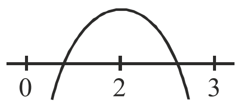
Here (i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv)

$c \in \left(\frac{49}{4}, 24\right)$

Case - II

$c - 5 < 0$ ...(i)

$f(0) < 0$



$c < 4$ ...(ii)

$f(2) > 0 \Rightarrow c > 24$ ...(iii)

$f(3) < 0 \Rightarrow c > 49$ ...(iv)

$4 \Rightarrow c \in \phi$

$c \in \left(\frac{49}{4}, 24\right)$.

8. (*d*) $\alpha + \alpha^3 = -\frac{K}{81}$ ....(1)

$\alpha^4 = \frac{256}{81}$

$\alpha = \pm\frac{4}{3}$ ....(2)

From (1) and (2)

$\frac{4}{3} + \frac{64}{27} = \frac{-K}{81}$

$K = -300$

9. (*c*) Expression is always positive if $2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$

and $D < 0 \Rightarrow m^2 - 6m - 3 < 0$

$3\sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ common interval is $3 - \sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ Integral value of m $\{0,1,2,3,4,5,6\}$.

10. (*b*) Let roots are α and β now

$\lambda + \frac{1}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1 \Rightarrow \alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 2\alpha\beta$

$\left(\frac{-m(m-4)}{3m^2}\right)^2 = 3\cdot\frac{2}{3m^2}$

$m^2 - 8m - 2 = 0$

$m = 4 \pm 3\sqrt{2}$

So least value of $m = 4 - 3\sqrt{2}$

11. (*c*) $(x-1)^2 + 1 = 0 \Rightarrow x = 1 + i, 1 - i$

$\therefore \left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm 1)^n = 1$

$\therefore$ n (least natural number) = 4

12. (*c*) a, b, c, in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

$x = -r$ is the common root, satisfies second equation

$d(-r)^2 + 2e(-r) + f = 0$

$\Rightarrow d.\frac{c}{a} - \frac{2ce}{b} + f = 0 \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

13. (*a*) $D < 0$

$4(1 + 3m)^2 - 4(Hm^2)(1 + 8m) < 0$

$\Rightarrow m(2m - 1)^2 > 0 \Rightarrow m > 0$

14. (*b*) In given question p, q $\in$ R. If we take other root as any real number α,

then quadratic equation will be

$x^2 - (\alpha + 2 - \sqrt{3})\,x + \alpha.\,(2 - \sqrt{3}) = 0$

Now, we can have none or any of the options can be correct depending upon 'α' Instead of p, q $\in$ R it should be p, q $\in$ Q then other root will be $2 + \sqrt{3}$

$\Rightarrow p = -(2 + \sqrt{3} - 2 - \sqrt{3}) = -4$

and $q = (2 + \sqrt{3})(2 - \sqrt{3}) = 1$

$\Rightarrow p^2 - 4q - 12 = (-4)^2 - 4 - 12$

$= 16 - 16 = 0$

Option (*b*) is correct

15. (*d*) $SOR = \frac{3}{m^2 + 1} \Rightarrow (S.O.R)_{max} = 3$

when $m = 0$

$x^2 - 3x + 1 = 0$ $\nearrow \alpha$ $\searrow \beta$

$\alpha + \beta = 3$

$\alpha\beta = 1$

$|\alpha^3 - \beta^2| = ||\alpha - \beta|(\alpha^2 + \beta^2 + \alpha\beta)|$

$= \left|\sqrt{(\alpha-\beta)^2 - \alpha\beta}\,((\alpha+\beta)^2 - \alpha\beta\right|$

$= \left|\sqrt{9-4}\,(9-1)\right|$

$= \sqrt{5} \times 8$

16. (*d*) $\frac{\alpha^{12} + \beta^{12}}{\left(\frac{1}{\alpha^{12}} + \frac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$

$= \frac{(\alpha\beta)^{12}}{\left[(\alpha+\beta)^2 - 4\alpha\beta\right]^{12}} = \left[\frac{\alpha\beta}{(\alpha+\beta)^2 - 4\alpha\beta}\right]^{12}$

$= \left(\frac{-2\sin\theta}{\sin^2\theta + 8\sin\theta}\right)^{12} = \frac{2^{12}}{(\sin\theta + 8)^{12}}$

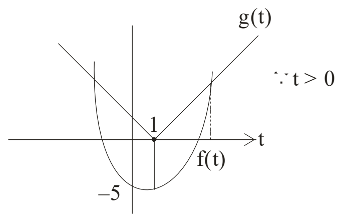
17. (*d*) Let $2^x = t$

$5 + |t-1| = t^2 - 2t$

$\Rightarrow |t-1| = (t^2 - 2t - 5)$

g(t) f(t)

From the graph



So, number of real root is 1.

18. (*c*) $375x^2 - 25x - 2 = 0$

$\alpha + \beta = \dfrac{25}{375}, \alpha\beta = \dfrac{-2}{375}$

$\Rightarrow (\alpha + \alpha^2 + ...$ upto infinite terms$) + (\beta + \beta^2 + ...$ upto

infinite terms$) = \dfrac{\alpha}{1-\alpha} + \dfrac{\beta}{1-\beta} = \dfrac{1}{12}$

19. (*a*) $\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t, \gamma = \alpha t^2$

$\therefore \alpha x^2 + 2\alpha t x + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore t^2 - t - 1 = 0$ ....(1)

Now

$\alpha(\beta + \gamma) = \alpha^2 (t + t^2)$

Option 1 $\beta\gamma = \alpha t . \alpha t^2 = \alpha^2 t^3 = a^2 (t^2 + t)$

from equation 1

20. (*a*) $\alpha^5 = 5\alpha + 3$

$\beta^5 = 5\beta + 3$

$p_5 = 5(\alpha + \beta) + 6$

$= 5(1) + 6$

$P_5 = 11$ and $p_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$

$P_2 = 3$ and $p_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1 = 2(1) + 2 = 4$

$P_2 \times P_3 = 12$ and $P_5 = 11 \Rightarrow P_5 \neq P_2 \times P_3$

21. 8 $D \geq 0$

$(a-10)^2 - 4(2)\left(\dfrac{33}{2} - 2a\right) \geq 0$

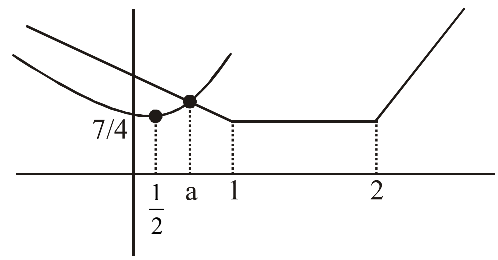
$(a-10)^2 - 4(33 - 4a) \geq 0$

$a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, -4] \cup [8, \infty).$

22. (*b*) Let $3^x = t$

$t(t-1) + 2 = |t-1| + |t-2|$

$t^2 - t + 2 = |t-1| + |t-2|$



are positive solution

$t = a$

$3^x = a$

$x = \log_3$ is singleton set.

23. (*b*) Let $e^x = t \in (0, \infty)$

Given equation

$t^4 + t^3 - 4t^2 + t + 1 = 0$

$t^2 + t - 4 + \dfrac{1}{t} + \dfrac{1}{t^2} = 0$

$\left(t^2 + \dfrac{1}{t^2}\right) + \left(t + \dfrac{1}{t}\right) - 4 = 0$

Let $t + \dfrac{1}{t} = \alpha$

$(\alpha^2 - 2) + \alpha - 4 = 0$

$\alpha^2 + \alpha - 6 = 0$

$\alpha^2 + \alpha - 6 = 0$

$\alpha = -3, 2 \quad \Rightarrow \alpha = 2 \quad \Rightarrow e^x + e^{-x} = 2$

$x = 0$ only solution

24. (*a*) $2\alpha = \dfrac{2b}{a} \Rightarrow \alpha = \dfrac{b}{a}$ and $\alpha^2 = \dfrac{5}{a} \Rightarrow \dfrac{b^2}{a^2} = \dfrac{5}{a}$

$\Rightarrow b^2 = 5a$ .....(i) $(a \neq 0)$

$\alpha + \beta = 2b$ ....(ii)

$\alpha\beta = -10$ ....(iii)

$\alpha = \dfrac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

by (i) $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow 20a^2 = 5a$

$\Rightarrow a = \dfrac{1}{4}$ and $b^2 = \dfrac{5}{4}$

$\alpha^2 = 20$ and $\beta^2 = 5$

Now $\alpha^2 + \beta^2$

$= 5 + 20$

$= 25$

25. (*a*) Let $f(x) = ax^2 + bx + c$

Let roots are 3 and α

and $f(-1) + f(2) = 0$

$4a + 2b + c + a - b + c = 0$

$5a + b + 2c = 0$ ...(i)

$\because f(3) = 0 \Rightarrow 9a + 3b + c = 0$ ...(ii)

From equation (i) and (ii)

$$\frac{a}{1-6} = \frac{b}{18-5} = \frac{c}{15-9} \Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$$

$\therefore f(x) = k(-5x^2 + 13x + 6)$

$= -k(5x+2)(x-3)$

$=$ Root are 3 and $-\frac{2}{5}$

$\therefore -\frac{2}{5}$ lies in interval $(-1, 0)$

26. (*a*) $\because \alpha$ is a root of given equation, then

$5\alpha^2 + 6\alpha = 2$

$\Rightarrow 5\alpha^6 + 6\alpha^5 = 2\alpha^4$ ...(1)

Similarly $5\beta^6 + 6\beta^5 = 2\beta^4$ ...(2)

Adding (1) and (2), we get

$5S_6 + 6\,S_5 = 2S_4$

27. (*d*) $\because$ Equation is : $(\lambda^2 + 1)\,x^2 - 4\lambda x + 2 = 0$

$\because$ One root in interval (0,1)

$\therefore f(0).\,f(1) < 0$

$2.\,(\lambda^2 + 1 - 4\lambda + 2) < 0$

$(\lambda - 3)\,(\lambda - 1) < 0$

$\therefore \lambda \in (1, 3)$

If $\lambda = 3$, then roots are 1 and $\frac{1}{5}$

$\therefore \lambda \in (1, 3]$

28. (*d*) $\alpha.\beta = 2$ and $\alpha + \beta = -p$ also $\frac{1}{\alpha} + \frac{1}{\beta} = -q$

$\Rightarrow p = 2q$

Now $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$$= \left[\alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right]\left[\alpha\beta + \frac{1}{\alpha\beta} + 1 + 1\right]$$

$$= \frac{9}{2}\left[\frac{5}{2} - \frac{\alpha^2 + \beta^2}{2}\right] = \frac{9}{4}\left[5 - (p^2 - 4)\right]$$

$$= \frac{9}{4}(9 - p^2)$$

29. (*a*) Roots of $x^2 - x + 2\lambda = 0$ are α and β

and roots of $3x^2 - 10x + 27\lambda = 0$ and α and γ

Here,

$3\alpha^2 - 10\alpha + 27\lambda = 0$ ....(i)

$3\alpha^2 - 3\alpha + 6\lambda = 0$ ....(ii)

$\therefore \alpha = 3\lambda$

Now,

$3\lambda + \beta = 1$ and $3\lambda \cdot \beta = 2\lambda$

and, $3\lambda + \gamma = \frac{10}{3}$ and $3\lambda.\gamma = 9\lambda$

$\therefore \gamma = 3, \alpha = \frac{1}{3}$ and $\beta = \frac{2}{3}, \lambda = \frac{1}{9}$

$\frac{\beta\gamma}{\lambda} = 18$

30. (*c*) $\because \alpha, \beta, \gamma, \delta$ are in G.P, so $\alpha\delta = \beta\gamma$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \left|\frac{\alpha - \beta}{\alpha + \beta}\right| = \left|\frac{\gamma - \delta}{\gamma + \delta}\right|$$

$$\Rightarrow \sqrt{\frac{9 - 4p}{3}} = \sqrt{\frac{36 - 4p}{6}}$$

$\Rightarrow 36 - 16\,p = 36 - 4q$

$\Rightarrow q = 4p$

So, $\frac{2q + p}{2q - p} = \frac{9p}{7p} = \frac{9}{7}$

31. (*d*) $7x^2 - 3x - 2 = 0 \Rightarrow \alpha + \beta = \frac{3}{7}, \alpha\beta = \frac{-2}{7}$

Now $\frac{\alpha}{1 - \alpha^2} + \frac{\beta}{1 - \beta^2}$

$$= \frac{\alpha - \alpha\beta(\alpha + \beta) + \beta}{1 - (\alpha^2 + \beta^2) + (\alpha\beta)^2}$$

$$= \frac{(\alpha + \beta) - \alpha\beta(\alpha + \beta)}{1 - (\alpha^2 + \beta^2) + 2\alpha\beta + (\alpha\beta)^2}$$

$$= \frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \frac{9}{49} + 2 \times \frac{-2}{7} + \frac{4}{49}} = \frac{21 + 6}{49 - 9 - 28 + 4} = \frac{27}{16}$$

32. (*b*) Let $|x| = t$ we have

$9t^2 - 18t + 5 = 0$

$9t^2 - 15t - 3t + 5 = 0$

$(3t - 1)(3t - 5) = 0$

$\Rightarrow t = \frac{1}{3}$ or $\frac{5}{3} \Rightarrow |x| = \frac{1}{3}$ or $\frac{5}{3}$

Roots are $\pm\frac{1}{3}$ and $\pm\frac{5}{3}$

Product $= \frac{25}{81}$

33. (*b*) $\alpha+\beta=-\frac{1}{2}\Rightarrow -1=2\alpha+2\beta$

and $4\alpha^2+2\alpha-1=0$

$\Rightarrow 4\alpha^2+2\alpha+2\alpha+2\beta=0$

$\Rightarrow \beta=-2\alpha(\alpha+1)$

34. (*b*) $\frac{\alpha^{3/8}}{\beta^{5/8}}+\frac{\beta^{3/8}}{\alpha^{5/8}}=\frac{\alpha+\beta}{(\alpha\beta)^{5/8}}$

For $x^2-64x+256=0$

$\alpha+\beta=64$

$\alpha\beta=256$

$\therefore\ \frac{\alpha+\beta}{(\alpha\beta)^{5/8}}=\frac{64}{\left(2^8\right)^{5/8}}=\frac{64}{32}=2$

35. (*d*) Given, $p+q=2, p^4+q^4=272$

$\Rightarrow (p^2+q^2)-2p^2q^2=272$

$\Rightarrow ((p+q)^2-2pq)^2=2p^2q^2=272$

$\Rightarrow (4+2pq)^2-2(pq)^2=272$

$16+4(pq)^2-16pq-2(pq)^2=272$

$2(pq)^2-16(pq)-256=0$

$(pq)^2-8(pq)-128=0$

$pq=\frac{8\pm\sqrt{64+4.128}}{2}=\frac{8\pm 24}{2}=16$

As, p,q are $+ve_1$ So, $pq\neq -8$

$pq=16$

Quadrant equation is

$x^2-(p+q)x+pq=0$

$x^2-2x+16=0$

36. (*d*) $a,b,c\rightarrow \text{AP}\Rightarrow 2b=a+c$ ...(i)

Centroid is $\left(\frac{10}{3},\frac{7}{3}\right)$

$\frac{a+2+a}{3}=\frac{10}{3}$ & $\frac{c+b+b}{3}=\frac{7}{3}$

$2a+2=10$ & $c+2b=7$

$2a=8$

$a=4;$ ...(ii)

from (i) & (ii)

$2b=c+4$

$c+2b=7$

$c+c+4=7$

$2c=3$

$c=\frac{3}{2}$

$2b=c+4=\frac{11}{2}$

$b=\frac{11}{4}$

$ax^2+bx+4=0\Rightarrow 4x^2+\frac{11x}{4}+1=0$

$\alpha+\beta=\frac{-11}{4.4}=\frac{-11}{16};\ \alpha\beta=\frac{1}{4}$x

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\alpha^2+\beta^2-\alpha\beta=(\alpha+\beta)^2-3\alpha\beta=\frac{121}{256}-\frac{3}{4}=\frac{-71}{256}$

37. (2) $(x+1)^2+|x-5|=\frac{27}{4}$

Case 1: if $x\geq 5$

$(x+1)^2+x-5=\frac{27}{4}$

$x^2+3x-4=\frac{27}{4}\Rightarrow x^2\,3x-\frac{43}{4}=0$

$4x^2+12x-43=0$

$x=\frac{-12\pm\sqrt{144-4.4.(-43)}}{2.4}=\frac{-12\pm\sqrt{832}}{8}$

$=\frac{-12\pm 28.8}{8}$

$=\frac{-3\pm 7.2}{2}=\frac{-10.2}{2},\frac{+4.2}{2}$

But $x\geq 5$, So, no real root.

Case 2: if $x<5$

$(x+1)^2+(-(x-5))=\frac{27}{4}\Rightarrow x^2+1+2x-x+5=\frac{27}{4}$

$x^2+x+6-\frac{27}{4}=0\Rightarrow 4x^2+4x-3=0$

$x=\frac{-4\pm\sqrt{16+4.4.3}}{2.4}=\frac{-4\pm 8}{8}=-\frac{12}{8},\frac{4}{8}$

Two real roots.

38. (*d*) $ax^2+bx+c=0$

$a,b,c\in\{1,2,3,4,5,6,\}$

$n(s)=6\times 6\times 6=216$

for equal roots, $D=0\Rightarrow b^2-4ac=0$

$\Rightarrow ac=\frac{b^2}{4}\Rightarrow b^2$ must be divisible by 4.

(*a*) $b^2=4\Rightarrow b=2$ & $ac=\frac{4}{4}=1\Rightarrow a=1, c=1$

(*b*) $b^2=16\Rightarrow b=4$ & $ac=\frac{16}{4}=4\Rightarrow a=4, c=1$

$a=2, c=2$

or

$a=1, c=4$

(*d*) $b^2=36\Rightarrow b=6$ & $oc=\frac{36}{4}=9$

$\Rightarrow a=3, c=3$

Total favourable case = 5

Req. Probability $=\frac{5}{216}$

39. (*a*) for $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$

$D < 0$

$(-2(3k-1))^2 - 4.1.(8k^2-7) < 0$

$4(9k^2 + 1 - 6k) - 4(8k^2 - 7) < 0$

$k^2 - 6k + 8 < 0$

$(k-2)(k-4) < 0$

$k \in (2, 4)$

Integer value of $k = 3$

40. (*d*) $z^2 + \alpha z + \beta = 0$

$z = 1 - 2i$ is root

So, $(1-2i)^2 + \alpha(1-2i) + \beta = 0$

$1 + 4i^2 - 4i + \alpha - 2\alpha i + \beta = 0$

$(\alpha + \beta - 3) + i(4 + 2\alpha) = 0$

$\alpha + \beta - 3 = 0$ & $4 + 2\alpha = 0$

$2\alpha = -4$

$\alpha = -2$

$\alpha + \beta - 3 = 0$

$-2 + \beta - 3 = 0$

$\beta = 5$

$\alpha - \beta = -2 - 5 = -7$

41. (*c*) $x^2 - 6x - 2 = 0$

α, β roots

$\alpha + \beta = 6; \alpha\beta = -2$ ...(i)

$$\left.\begin{aligned}\alpha^2 - 6\alpha - 2 = 0\\ \beta^2 - 6\beta - 2 = 0\end{aligned}\right\} \quad ...(ii)$$

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)} = \frac{\alpha^8.6\alpha - \beta^8.6\beta}{3(\alpha^9 - \beta^9)} \text{ (from-(ii))}$$

$$= \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)} = 2$$

42. (1) $\log_4(x-1) = \log_2(x-3)$

$x > 1$ & $x > 3 \Rightarrow x > 3$ (according to definition of log)

$\Rightarrow \log_2(x-1) = \log_2(x-3)$

$\frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\log_2(x-1) = \log_2(x-3)^2$

$(x-1) = (x-3)^2$

$x^2 + 9 - 6x = x - 1$

$x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$x = 2$ or 5

But $x > B$

So, $x = 5$

Only one solution.

43. (324) $\alpha + \beta = 1; \alpha\beta = -1$

Quad. Eq^n: $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - x - 1 = 0$

α, β roots

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^2 = \alpha + 1$

Multiplying both side by, α^{n-1}

$\alpha^{n-1}, \alpha^2 = \alpha^{n-1}(\alpha + 1)$

$\alpha^{n+1} = \alpha^n + \alpha^{n-1}$ ...(i)

Similarly, $\beta^{n+1} = \beta^n + \beta^{n-1}$ ...(ii)

Adding (i) & (ii)

$\alpha^{n+1} + \beta^{n+1} = (\alpha^n + \beta^n) + (\alpha^{n-1} + \beta^{n-1})$

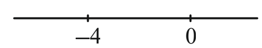
$P_{n+1} = P_n + P_{n-1}$

$29 = P_n + 11$

$P_n = 18$

$P_n^2 = 18^2 = 324$

44. (*b*) $(|x| - 3)(|x + 4|) = 6$



Case I:	**Case II:**	**Case III**
$x \geq 0$	$-4 \leq x < 0$	$x < -4$
$(x-3)(x+4) = 6$	$(-(x) - 3)(x+4) = 6$	$(-x-3)(-x-3)(-(x+4)) = 6$
$x^2 + x - 18 = 0$	$(x+3)(x+4) = -6$	$(x+3)(x+4) = 6$
$x = \frac{-1 \pm \sqrt{1 + 4.18}}{2}$	$x^2 + 7x + 18 = 0$	$x^2 + 7x + 6 = 0$
$x = \frac{-1 \pm \sqrt{73}}{2}$	$x = \frac{-7 \pm \sqrt{49 - 72}}{2}$	$(x+1)(x+6) = 0$
But $x \geq 0$	No real root	$x = -1$ or -6
$x = \frac{-1 \pm \sqrt{73}}{2}$		But $x < -4$
Only 1 solution		$x = -6$
		Only 1 solution

In total, there are 2 real solution.

45. (*c*) $\int_0^1 P(x)dx = 1 \Rightarrow \int_0^1 (x^2 + bx + c)dx = 1$

$$\left[\frac{x^3}{3} + \frac{bx^2}{2} + c\right]_0^1 = 1 \Rightarrow \frac{1}{3} + \frac{b}{2} + c = 1$$

$3b + 6c = 4$ ...(i)

Also, $P(2) = 5$

$(2)^2 + b(2) + c = 5$

$12b + 6c = 6$ ...(ii)

from (i) & (ii)

$b = \frac{2}{9}$ & $c = \frac{5}{9}$

$a(b + c) = \frac{9(2+5)}{9} = 7$

46. (*a*) Let $x = 4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + \ldots\ldots\ldots\infty}}}}$

Then,

$$x = 4 + \frac{1}{5 + \frac{1}{x}} \Rightarrow x - 4 = \frac{x}{5x+1}$$

$(x-4)(5x+1) = x$

$5x^2 - 20x - 4 = 0$

$$x = \frac{20 \pm \sqrt{400 + 4.5.4}}{2.5} = \frac{20 \pm \sqrt{480}}{10}$$

But x is $+ve$.

So, $x = \dfrac{20 + \sqrt{480}}{10} = \dfrac{2 + 4\sqrt{30}}{10}$

$$2 + \frac{2\sqrt{30}}{5}$$

47. (0) $P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(i)

$x^2 + x + 1 = 0$

$x = \omega, \omega^2$ (cube root of unity)

$P(x)$ is divisible by $x^2 + x + 1$

$P(\omega) = 0$; $P(\omega^2) = 0$

$P(\omega) = 0 \Rightarrow f(\omega^3) + \omega g(\omega^3) = 0$

$f(1) + \omega g(1) = 0$ ...(ii)

$P(\omega^2) = 0 \Rightarrow f(\omega^6) + \omega^2(g(\omega^6)) = 0$

$f(1) + \omega^2(g(1)) = 0$ ...(iii)

(ii) – (iii)

$(\omega - \omega^2)\, g(1) = 0$

As $(\omega - \omega^2) \neq 0$, So, only possibility is $g(1) = 0$

Putting value of $g(1)$ in –(*ii*),

$f(1) + 0 = 0 \Rightarrow f(1) = 0$

$P(1) = f(1) + g(1) = 0 + 0 = 0$

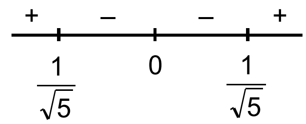
JEE-ADVANCED PREVIOUS YEARS

1. (*d*) p(x) will be of the form $ax^2 + c$. Since it has purely imaginary roots only.
Since p(x) is zero at imaginary values while $ax^2 + c$ takes real value only at real 'x', no root is real.
Also $p(p(x)) = 0 \Rightarrow p(x)$ is purely imaginary
$\Rightarrow$ $ax^2 + c$ = purely imaginary
Hence x can not be purely imaginary since x^2 will be negative in that case and $ax^2 + c$ will be real.
Thus. (*d*) is correct.

2. (*a*, *d*)

$(x_1 + x_2)^2 - 4x_1x_2 < 1$

$$\frac{1}{\alpha^2} - 4 < 1 \quad \Rightarrow 5 - \frac{1}{\alpha^2} > 0 \Rightarrow \frac{5\alpha^2 - 1}{\alpha^2} > 0$$



$$\alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right) \quad ...(1)$$

$D > 0$

$1 - 4\alpha^2 > 0$

$$\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right) \quad ...(2)$$

(a) & (2)

$$\alpha \in \left(-\frac{1}{2}, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right)$$

3. (*d*) As α and β are roots of equation $x^2 - x - 1 = 0$, we get:

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^2 = \alpha + 1$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^2 = \beta + 1$

$\therefore a_{11} + a_{10} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10}$

$= p\alpha^{10}(\alpha + 1) + q\beta^{10}(\beta + 1)$

$= p\alpha^{10} \times \alpha^2 + q\beta^{10} \times \beta^2$

$= p\alpha^{12} + q\beta^{12} = \alpha_{12}$

4. (*d*) $a_{n+2} = a_{n+1} + a_n$

$a_4 = a_3 + a_2 = 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p + q)$

As $\alpha = \dfrac{1+\sqrt{5}}{2}, \beta = \dfrac{1-\sqrt{5}}{2}$, we get

$$a_4 = 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2p + 2q = 28$$

$$\Rightarrow \left(\frac{3p}{2} + \frac{3q}{2} + 2p + 2q - 28\right) = 0 \quad ...(i)$$

$$\text{and} \Rightarrow \frac{3p}{2} - \frac{3q}{2} = 0. \quad ....(ii)$$

$\Rightarrow$ $p = q$ (from (ii))

$\Rightarrow$ $7p = 28$ (from (i) and (ii))

$\Rightarrow$ $p = 4$

$\Rightarrow$ $q = 4$

$\Rightarrow$ $p + 2q = 12$

5. (*a*,*b*,*c*)

α, β are roots of $x^2 - x - 1$

$$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta}$$

$$= \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$$

$$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta}$$

$$= \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} = a_{r+1}$$

$$\Rightarrow a_{r+2} - a_{r+1} = a_r$$

$$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta}$$

$$= a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$$

$$\text{Now } \sum_{r=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{r=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{r=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$\frac{\dfrac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1 - \frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10 - \alpha} - \frac{\beta}{10 - \beta}}{(\alpha - \beta)} = \frac{10}{(10 - \alpha)(10 - \beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n-1} + a_{n+1}$

$$= \frac{(\alpha^{n-1} - \beta^{n-1}) + (\alpha^{n+1} - \beta^{n+1})}{\alpha - \beta}$$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ & $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha - \beta) + (\alpha - \beta)\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$$

6. (*a*, *b*, *c*)

α, β are roots of $x^2 - x - 1$

$$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta}$$

$$= \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$$

$$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta} = \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} = a_{r+1}$$

$$\Rightarrow a_{r+2} - a_{r+1} = a_r$$

$$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta}$$

$$= a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$$

$$\text{Now } \sum_{r=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{r=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{r=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$\frac{\dfrac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1 - \frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10 - \alpha} - \frac{\beta}{10 - \beta}}{(\alpha - \beta)} = \frac{10}{(10 - \alpha)(10 - \beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n-1} + a_{n+1}$

$$= \frac{(\alpha^{n-1} - \beta^{n-1}) + (\alpha^{n+1} - \beta^{n+1})}{\alpha - \beta}$$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ & $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha - \beta) + (\alpha - \beta)\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$$

7. (*d*) $x^2 + 20x - 2020 = 0$ has two roots $a, b \in R$

$x^2 - 20x + 2020 = 0$ has two roots $c, d \in$ complex

$ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$

$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2(c + d) + b^2(c + d) - c^2(a + b) - d^2(a + b)$

$= (c + d)(a^2 + b^2) - (a + b)(c^2 + d^2)$

$= (c + d)((a + b)^2 - 2ab) - (a + b)((c + d)^2 - 2cd)$

$= 20[(20)^2 + 4040] + 20[(20)^2 - 4040]$

$= 20[(20)^2 + 4040 + (20)^2 - 4040]$

$= 20 \times 800 = 16000$