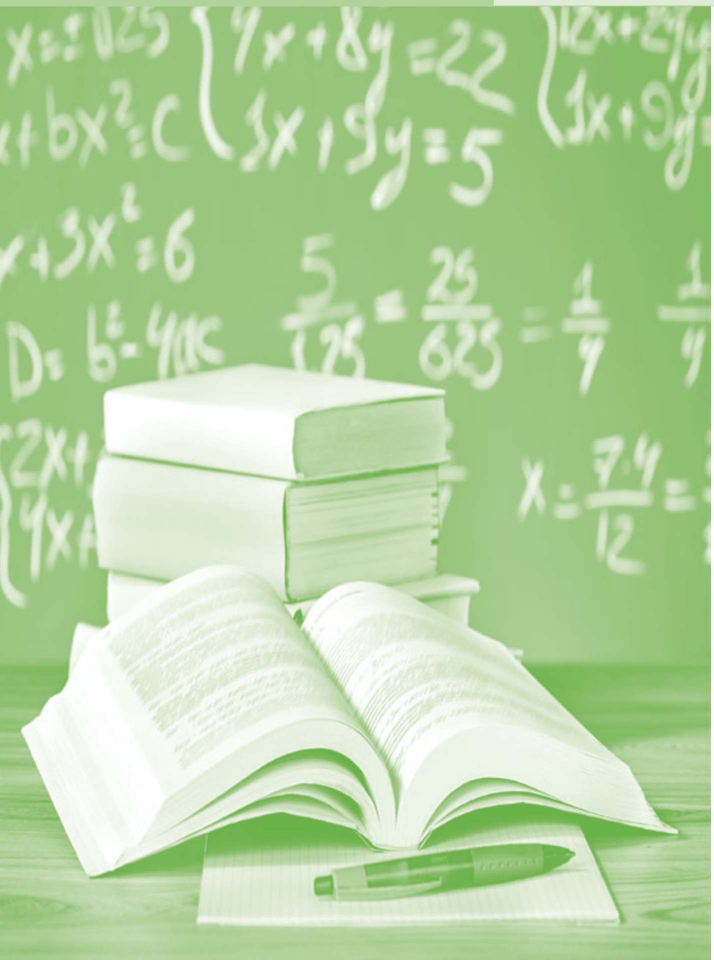
Chapter

2

Logarithms



REMEMBER

Before beginning this chapter, you should be able to:

- Explain laws of indices
- Solve exponential equations

KEY IDEAS

After completing this chapter, you should be able to:

- Understand the system of logarithms
- Learn properties and laws of logarithms
- Understand variation of $\log_a x$ with x and learn the signs of $\log_a x$ for different values of x and a
- Find the log of a number using log tables obtain the antilog from a given problem

INTRODUCTION

Please recall that you have earlier learnt about indices. One of the results we learnt is that if $2^x = 2^3$, $x = 3$ and if $4^x = 4^y$, then $x = y$, i.e., if two powers of the same base are equal and the base is not equal to −1, 0 or 1, then the indices are equal.

But when $3^x = 5^2$, just by using the knowledge of indices, we cannot find the numerical value of x. The necessity of the concept of logarithms arises here. Logarithms are useful in long calculations involving multiplication and division.

Definition

The logarithm of any positive number to a given base (a positive number not equal to 1) is the index of the power of the base which is equal to that number. If N and a ($\neq 1$) are any two positive real numbers and for some real number x, $a^x = N$, then x is said to be logarithm of N to the base a. It is written as $\log_a N = x$, i.e., if $a^x = N$, then $x = \log_a N$.

If in a particular relation, all the log expressions are to the same base, we normally do not specify the base.

Examples:

1. $2^3 = 8 \Rightarrow 3 = \log_2 8$
2. $5^4 = 625 \Rightarrow \log_5 625 = 4$

From the definition of logs, we get the following results:

When $a > 0$, $b > 0$ and $b \neq 1$,

1. $\log_a a^n = n$, e.g., $\log_4 4^3 = 3$
2. $a^{\log_a b} = b$, e.g., $2^{\log_2 16} = 16$.

SYSTEM OF LOGARITHMS

Though we can talk of the log of a number to any positive base not equal to 1, there are two systems of logarithms, natural logarithms and common logarithms, which are used most often.

1. **Natural logarithms:** These were discovered by Napier. They are calculated to the base e which is approximately equal to 2.71. These are used in higher mathematics.
2. **Common logarithms:** Logarithms to the base 10 are known as common logarithms. This system was introduced by Briggs, a contemporary of Napier.

For the rest of this chapter, we shall use the short form log rather than logarithm.

Properties

1. Logs are defined only for positive real numbers.
2. Logs are defined only for positive bases different from 1.
3. In $\log_b a$, neither a nor b is negative but the value of $\log_b a$ can be negative.

   Example: As $10^{-2} = 0.01$, $\log_{10} 0.01 = -2$

4. Logs of different numbers to the same base are different, i.e., if $a \neq b$, then $\log_m a \neq \log_m b$. In other words, if $\log_m a = \log_m b$, then $a = b$.

   Example: $\log_{10} 2 \neq \log_{10} 3$

   $\log_{10} 2 = \log_{10} y \Rightarrow y = 2$

5. Logs of the same number to different bases have different values, i.e., if $m \neq n$, then $\log_m a \neq \log_n a$. In other words, if $\log_m a = \log_n a$, then $m = n$.

   Example: $\log_2 16 \neq \log_4 16$

   $\log_2 16 = \log_n 16 \Rightarrow n = 2$

6. Log of 1 to any base is 0.

   Example: $\log_2 1 = 0$ ($\therefore\ 2^0 = 1$)

7. Log of a number to the same base is 1

   Example: $\log_4 4 = 1$.

8. Log of 0 is not defined.

Laws

1. $\log_m(ab) = \log_m a + \log_m b$

   Example: $\log 15 = \log(5 \times 3) = \log 5 + \log 3$

2. $\log_m\left(\dfrac{a}{b}\right) = \log_m a - \log_m b$

   Example: $\log\left(\dfrac{15}{20}\right) = \log 15 - \log 20$

3. $\log a^m = m \log a$

   Example: $\log 36 = \log 6^2 = 2\log 6$

4. $\log_b a \log_c b = \log_c a$ (chain rule)

   Example: $\log_2 4 \log_4 16 = \log_2 16$

5. $\log_b a = \dfrac{\log_c a}{\log_c b}$

   Example: $\log_4 16 = \dfrac{\log_2 16}{\log_2 4}$ (change of base rule)

   In this relation, if we take $a = c$, we get the following result:

$$\log_b a = \frac{1}{\log_a b}.$$

Variation of $\log_a x$ with x

For $1 < a$ and $0 < p < q$, $\log_a p < \log_a q$
For $0 < a < 1$ and $0 < p < q$, $\log_a p > \log_a q$

Example: $\log_{10} 2 < \log_{10} 3$ and $\log_{0.1} 2 > \log_{0.1} 3$

Bases which are greater than 1 are called strong bases and bases which are less than 1 are called weak bases. Therefore, for strong bases log increases with number and for weak bases log decreases with number.

Sign of $\log_a x$ for Different Values of x and a

Strong bases ($a > 1$)

1. If $x > 1$, $\log_a x$ is positive.

   Example: $\log_2 10$, $\log_5 25$ are positive.

2. If $0 < x < 1$, then $\log_a x$ is negative.

   Example: $\log_3 0.2$, $\log_{10} 0.25$ are negative.

   Consider $\log_3 0.2 = \dfrac{\log 0.2}{\log 3} = \dfrac{\log 2 - \log 10}{\log 3}$

   $\log 2 < \log 10$ and $0 < \log 3$ for strong bases.

   As $\dfrac{\log 2 - \log 10}{\log 3} < 0$, $\log_3 0.2 < 0$

Weak bases ($0 < a < 1$)

3. If $x > 1$ and, then $\log_a x$ is negative.
   Example: Consider $\log_{0.4} 2$

   $$= \frac{\log 2}{\log 0.4}$$

   $$= \frac{\log 2}{\log 4 - \log 10}$$

   $\log 4 < \log 10$ (for any base)
   $\log 4 - \log 10 < 0$
   $\dfrac{\log 2}{\log 4 - \log 10} < 0$. (for strong bases)

4. If $0 < x < 1$, then $\log_a x$ is positive. For example, $\log_{0.1} 0.2$, $\log_{0.4} 0.3$ are positive. To summarize, logs of big numbers (>1) to strong bases and small numbers (<1) to weak bases are positive.

EXAMPLE 2.1

If $x^2 + y^2 = 3xy$, then choose the correct answer of $2\log(x - y)$ from the following options:

(a) $\log x - \log y$ **(b)** $\log x + \log y$ **(c)** $\log(xy)$ **(d)** Both (b) and (c)

HINT

Find $(x - y)^2$.

EXAMPLE 2.2

Choose the correct answer from the following option for: $\log_2[\log_4\{\log_3(\log_3 27)\}] =$

(a) 0 **(b)** 1 **(c)** $\log_2 3$ **(d)** undefined

SOLUTION

$\log_2[\log_4\{\log_3(\log_3 27)\}]$

$= \log_2[\log_4\{\log_3(\log_3 3^3)\}]$

$= \log_2[\log_4\{\log_3 3(\log_3 3)\}]$

$= \log_2[\log_4\{\log_3 3(1)\}]$

$= \log_2[\log_4\{\log_3 3\}]$

$= \log_2[\log_4 1] = \log_2 0$, which is not defined.

EXAMPLE 2.3

If $\log 2 = 0.301$, then find the number of digits in 2^{1024} from the following options:

(a) 307 **(b)** 308 **(c)** 309 **(d)** 310

SOLUTION

Let $x = 2^{1024}$

$\Rightarrow \log x = \log 2^{1024}$

$= 1024 \log 2 = 1024(0.301)$

$\Rightarrow \log x = 308.22$

$\Rightarrow$ The characteristic is 308

$\therefore$ The number of digits in 2^{1024} is 309.

To Find the log of a Number to Base 10

Consider the following numbers:
2, 20, 200, 0.2 and 0.02.

We see that $20 = 10(2)$ and $200 = 100(2)$

$\therefore \log 20 = 1 + \log 2$ and $\log 200 = 2 + \log 2$

Similarly, $\log 0.2 = -1 + \log 2$ and $\log 0.02 = -2 + \log 2$

From the tables, we see that $\log 2 = 0.3010$. (Using the tables, this is explained in more detail in later examples.)

$\therefore \log 20 = 1.3010$, $\log 200 = 2.3010$, $\log 0.2 = -1 + 0.3010$ and $\log 0.02 = -2 + 0.3010$.

We note two points:

1. Multiplying or dividing by a power of 10 changes only the integral part of the log, not the fractional part.
2. For numbers less than 1, (for example 0.2) it is more convenient to leave the log value as $-1 + 0.3010$ instead of changing it to -0.6090. We refer to the first form (in which the fraction is positive) as the standard form and the second form as the normal form. Both the forms represent the same number.

   For numbers less than 1, it is more convenient to express the log in the standard form. As the negative sign refers only to the integral part, it is written above the integral part, rather than in front, i.e., $\log 0.2 = \bar{1}.3010$ and not -1.3010.

   The convenience of the standard form will be clear when we learn how to take the antilog, which is explained in more detail later.

   antilog (-0.6090) = antilog $(-1 + 0.3010)$ = antilog $\bar{1}.3010 = 0.2$.

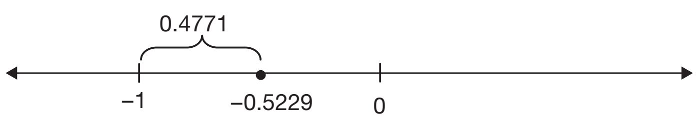
When the logs of numbers are expressed in the standard form, (for numbers greater than 1, the standard form of the log is the same as the normal form), the integral part is called the characteristic and the fractional part (which is always positive) is called the *mantissa*.

EXAMPLE 2.4

Express -0.5229 in the standard form and locate it on the number line.

SOLUTION

$$-0.5229 = -1 + 1 - 0.5229 = \bar{1}.4771$$



The Rule to Obtain the Characteristic of log *x*

1. If $x > 1$ and there are n digits in x, the characteristic is $n - 1$.
2. If $x < 1$ and there are m zeroes between the decimal point and the first non-zero digit of x, the characteristic is $-m$, more commonly written as $\bar{m}$.

Note $-4 = \bar{4}$ but $-4.01 \neq \bar{4}.01$

To Find the log of a Number from the log Tables

EXAMPLE 2.5

Find the value of $\log 25$, $\log 250$ and $\log 0.025$.

SOLUTION

In the log table, we find the number 25 in the left-hand column. In this row, in the next column (under 0), we find 0.3979. (The decimal point is dropped in other columns)

For the log of all numbers whose significant digits are 25 and this number 0.3979, is the *mantissa*. Prefixing the characteristics we have,

$$\log 25 = 1.3979$$
$$\log 250 = 2.3979$$
$$\log 0.025 = \bar{2}.3979.$$

EXAMPLE 2.6

Find the value of $\log 2.54$, $\log 0.254$ and $\log 25400$.

SOLUTION

In the log table, we locate 25 in the first column. In this row, in the column under 4 we find 0.4048. As done in the earlier example, the same line as before gives the *mantissa* of logarithms of all numbers which begin with 25. From this line we pick out the *mantissa* which is located in the column numbered 4. This gives 4048 as the *mantissa* for all numbers whose significant digits are 254.

$$\log 2.54 = 0.4048$$
$$\log 0.254 = \bar{1}.4048$$
$$\log 25400 = 4.4048.$$

EXAMPLE 2.7

Find the value of $\log 2.546$ and $\log 25460$.

SOLUTION

As found in the above example, we can find the mantissa for the sequence of digits 4048.

Since there are four significant digits in 2546, in the same row where we have found 4048, under column 6 in the mean difference column, we find the number 10, the mantissa of the logarithm of 2546 will be 4048 + 10 = 4058.

Thus,
$$\log 2.546 = 0.4058$$
$$\log 0.2546 = \bar{1}.4058$$
$$\log 25460 = 4.4058.$$

ANTILOG

As $\log_2 8 = 3$, 8 is the antilogarithm of 3 to the base 2. Antilog b to base m is m^b.

In example 3 above, we saw that, $\log 2.546 = 0.4058$. Therefore, Antilog 0.4058 = 2.546

To Find the Antilog

EXAMPLE 2.8

To find the antilog of 1.301.

SOLUTION

Step 1: In the antilog table, we find the number 30 in the left hand column. In that row in the column under 1, we find 2000.

Step 2: As the characteristic is 1, we place the decimal after two digits from the left. That is, antilog 1.301 = 20.00.

If the characteristic was 2, we would place the decimal after three digits from the left. That is, antilog 2.301 = 200.0.

If the characteristic was 3, we would place the decimal after four digits from the left. That is, antilog 3.301 = 2000.

EXAMPLE 2.9

To find the antilog of 2.3246.

SOLUTION

We have to locate 0.32 in the left hand column and slide along the horizontal line and pick out the number in the vertical column headed by 4. We see that the number is 2109. The mean difference for 6 in the same line is 3.

The significant digits in the required number are = 2109 + 3 = 2112. As the characteristic is 2, the required antilog is 211.2.

EXAMPLE 2.10

Find the value of $\dfrac{5.431 \times 0.061}{12.38 \times 0.041}$ to four significant digits.

SOLUTION

log of a fraction = (log of numerator) − (log of denominator)

log of numerator = $\log 5.431 + \log 0.061 = 0.7349 + \bar{2}.7853 = \bar{1}.5202$

log of denominator = $\log 12.38 + \log 0.041 = 1.0927 + \bar{2}.6128 = \bar{1}.7055$

log of given expression = $\bar{1}.5202 - \bar{1}.7055 = \bar{1}.8147$

antilog of 8147 is 6516 + 11 = 6527

Since the characteristic is $\bar{1}$, the decimal should be kept before the four digits.

$\therefore$ The answer is 0.6527.

EXAMPLE 2.11

If $\log_{10}3 = 0.4771$, and $\log_{10}2 = 0.3010$, Find the value of $\log_{10}48$.

SOLUTION

$\log_{10}48 = \log_{10}16 + \log_{10}3$

$= \log_{10}2^4 + \log_{10}3 = 4\log_{10}2 + \log_{10}3$

$= 4(0.3010) + 0.4771 = 1.6811.$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $\log_x A^n =$ _____.
2. Expand $\log_3\left(\dfrac{xy^2}{z^3}\right)$.
3. Can we write $\log_x \dfrac{a}{b}$ as $\dfrac{\log_x a}{\log_x b}$?
4. Express $0.001 = (0.1)^3$ in the logarithmic form.
5. $(5)^{2\log_5 2} =$ _______.
6. $\log_5 2 + \log_5 20 - \log_5 8 =$ _______.
7. The value of $\dfrac{3+\log_{10}(10)^2}{\log_5 5}$ is _______.
8. $\log_x ab = (\log_x a) \times (\log_x b)$. State True or False.
9. If $\log_{10} 2 = 0.3010$, then $\log_{10} 2000 =$ _______.
10. Evaluate $3 - \log_{10} 100$.
11. Given $3 = \log_2 x + 4\log_2 8$. Then the value of $x =$ _______.
12. If $\log_{10} 2 = 0.3010$, then $\log_{10} 5 =$ _______.
13. If $x = \log_5 3$ and $y = \log_5 8$, then $\log_5 24$ in terms of x, y is equal to _______.
14. If $\log_{16} 25 = k \log_2 5$, then $k =$ _______.
15. If $5\log 3 + \log x = 5\log 6$, then $x =$ _______.
16. If $\log x = \left(\dfrac{\log_a y}{\log_a x}\right)^k$, then $k =$ _______.
17. If $a > 1$ and $m > n$, then which is greater, $\log_a m$ (or) $\log_a n$?
18. If $2\log x + 2\log y = k$ and $xy = 1$, then $k =$ _______.
19. If $\log 198.9 = 2.2987$, then the characteristic of $\log 198.9 =$ and mantissa of $\log 198.9 =$ _______.
20. When $0 < a < 1$ and $m < n$, then which is greater, $\log_a m$ (or) $\log_a n$?
21. Given $\log_{10} x = y$. If the characteristic of y is 10, then the number of digits to the left the decimal point in x is _____.
22. Find the value of $\log_{\sqrt{8}} 16$.
23. $\log_y x \times \log_z y \times \log_x z =$ _____.
24. If the characteristics of the logarithm of two numbers $abcd \cdot abef$ and $a \cdot bcdabef$ are x and y respectively, then $x - y =$ _____.
25. If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, then find the values of $\log 210$.
26. Given, antilog$(2.375) = x$. Characteristic of $\log x$ is _____.
27. If $\log(21.73) = 1.3371$, then find the values of $\log(2.173)$.
28. If antilog $(0.2156) = 1.643$, then find the values of antilog (1.2156).
29. Without using the logarithm tables find the value of $3\log_3 27$.
30. Find the value of $\log_{0.6}\left(\dfrac{9}{25}\right)$.

Short Answer Type Questions

31. Prove that $\log 5040 = 4\log 2 + 2\log 3 + \log 5 + \log 7$.
32. Find the value of $\log_{2^{-1}}(0.0625)$.
33. Express the following as a single logarithm. $\dfrac{1}{3}\log x - \dfrac{8}{5}\log y + \dfrac{7}{2}\log z$.
34. If $x^2 + y^2 = 25xy$, then prove that $2\log(x + y) = 3\log 3 + \log x + \log y$.
35. If $x^2 + y^2 = z^2$, then prove that $\log_y(z + x) + \log_y(z - x) = 2$.
36. Prove that $\log_2[\log_4\{\log_5(625)^4\}] = 1$.
37. If $\log_{10} 2 = 0.3010$, then find the number of digits in $(16)^{10}$.
38. If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, then find the value of $\log 75$.
39. If $x^4 + y^4 = 83x^2y^2$, then prove that $\log\left(\dfrac{x^2 - y^2}{9}\right) = \log x + \log y$.
40. Prove that
$$2\log\frac{35}{192} + 2\log\frac{114}{91} + \log 48 + 2\log\left(\frac{13}{19}\right) = \log\left(\frac{75}{64}\right).$$

PRACTICE QUESTIONS

41. Solve for x:

$\log x + \log 5 = 2 + \log 64$

42. If $x^6 - y^6 = z^6$, then prove that

$\log_z(x^2 - y^2) + \log_z(x^2 + y^2 - xy) + \log_z(x^2 + y^2 + xy) = 6$.

43. Using the tables find the value of $\sqrt[4]{(32)^3}$.

44. Using the tables find the value of $\sqrt{(0.12)^3}$.

45. Given $\log 3 = 0.4771$, then the number of digits in 3^{1000} is ______.

Essay Type Questions

46. If $\log_{x+1} 2x - 1 + \log_{2x-1} x + 1 = 2$, find x.

47. If $a = b^{1/3} = c^{1/5} = d^{1/7} = e^{1/9}$, find $\log_a abcde$.

48. Arrange the following in ascending order.

$A = \log_9 6561$ $\quad$ $B = \log_{\sqrt{5}} 625$

$C = \log_{\sqrt{3}} 243$ $\quad$ $D = \log_{\sqrt{2}} 256$

49. If $\log_y x - \log_{y^3} x^2 = 9(\log_x y)^2$ and $x = 9y$, find y.

50. $\log\left(\frac{a^2}{b}\right) + \log\left(\frac{a^4}{b^3}\right) + \log\left(\frac{a^6}{b^3}\right) + \cdots + \log\left(\frac{a^{2n}}{b^n}\right) = ?$

CONCEPT APPLICATION

Level 1

1. $\log_y a \times \log_x y =$ ______.

(a) $\log_a y$ $\quad$ (b) $\log_x a$

(c) $\log_y a$ $\quad$ (d) $\log_a x$

2. If $\log x = 123.242$, then the characteristic of $\log x$ is

(a) 0.242 $\quad$ (b) 122

(c) 123 $\quad$ (d) 124

3. Pick up the false statement.

(A) Logarithms are defined only for positive real numbers.

(B) $\log_a N$ is always unique.

(C) The log form of $2^3 = 8$ is $3 = \log_8 2$.

(D) $\log 1 = 0$

(a) B $\quad$ (b) C

(c) D $\quad$ (d) A

4. $\log\left(\frac{169}{9}\right) - 2\log 13 + 2\log 3 = ?$

(a) 1 $\quad$ (b) 0

(c) $\log\left(\frac{13}{3}\right)$ $\quad$ (d) $\log\left(\frac{x}{yz}\right)$

5. $\log_{z^2} x^2y^2 = ?$

(a) $2(\log x + \log y - \log z)$

(b) $\log x + \log y - \log z$

(c) $\dfrac{\log x^2 + \log y}{\log z}$

(d) $\dfrac{\log x + \log y}{\log z}$

6. If $x^3 - y^3 = 3xy(x - y)$, then $\log(x - y)^3 =$ ______.

(a) 0 $\quad$ (b) 1

(c) Undefined $\quad$ (d) None of these

7. $\log(a^3 + b^3) - \log(a + b) - \log(a^2 - ab + b^2) =$ ______.

(a) $a^3 - b^3$

(b) 0

(c) $\log 1$

(d) Both (b) and (c)

8. Which is greatest among the following:

(a) $\log_2 20$ $\quad$ (b) $\log_7 35$

(c) $\log_5 70$ $\quad$ (d) $\log_3 68$

9. $\log(a + b) + \log(a - b) - \log(a^2 - b^2) =$ ______.

(a) 0 $\quad$ (b) 1

(c) $(a^2 - b^2)$ $\quad$ (d) $a^2 + b^2$

10. If $x^3 + y^3 = 4xy(x + y)$. Then $\log(x + y)^3 =$ ______.

(a) $\log x + \log y + \log(x + y) - \log 7$

(b) $\log(x) - \log y + \log(x + y) + \log 7$

(c) $\log x + \log y + \log(x - y) + \log 7$

(d) $\log x + \log y + \log(x + y) + \log 7$

11. If $\frac{\log x}{\log y} = \frac{\log 49}{\log 7}$, then the relation between x and y.

(a) $x = \sqrt{y}$

(b) $x = y^3$

(c) $y = x^2$

(d) $x = y^2$

12. $\log(x) - \log(2x - 3) = 1$, then $x =$?

(a) $\frac{30}{19}$ (b) $\frac{20}{19}$

(c) $\frac{19}{30}$ (d) $\frac{19}{20}$

13. If $2\log(x + 4) = \log 16$, then $x =$?

(a) 0, −8 (b) −8

(c) −2 (d) 0

14. The value of x when $\log_x 343 = 3$, is

(a) 7 (b) 8

(c) 3 (d) 27

15. $\log_{16}3 \cdot \log_{17}4 \cdot \log_9 17 =$ ______.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{2}{3}$

16. $\log_2 [\log_2 \{\log_2 (\log_3 81)\}] =$

(a) 1

(b) 0

(c) $\log 3$

(d) Undefined

17. $\log_{11}3 \cdot \log_3 1331 =$ ______.

(a) 3 (b) 11

(c) 121 (d) 9

18. $\log_{121}\left(\frac{\sqrt{14641}}{121}\right) =$ ______.

(a) 11 (b) 121

(c) 0 (d) 1

19. $\frac{\log_3 729 + \log_6 216}{4 + \log_2 16 - 2\log_4 64} =$ ______.

(a) 9 (b) 4

(c) $\frac{9}{2}$ (d) $\frac{1}{2}$

20. If $\log_{x^n} y^m = k\log_x y$, then the value of k is

(a) $\frac{m}{n}$ (b) mn

(c) m^n (d) n^m

21. If $\log_y x = 2$, then ${}_a\log_a{}^{(\log_x y)} =$ ______.

(a) −2 (b) 4

(c) $\frac{1}{2}$ (d) $\frac{-1}{4}$

22. If $p = \log_6 216$ and $q = \log_3 25$ then $p^q =$ ______.

(a) 3 (b) 25

(c) 15 (d) Cannot be determined

23. $2^{(16 - \log_2 1024)} =$ ______.

(a) 16 (b) 32

(d) 64 (d) 8

24. $2^{3\log_2 2} + 3^{2\log_3 2} =$ ______.

(a) 8 (b) 4

(c) 9 (d) 2

25. $\log_{a+b}(a^3 + b^3) - \log_{a+b}(a^2 - ab + b^2) =$ ______.

(a) $\log_{a+b}(a - b)$ (b) 2

(c) $a + b$ (d) 1

26. $4^{\log_{16}25} =$ ______.

(a) 25 (b) 5

(c) 16 (d) 4

27. $\frac{1}{\log_{xy} x} + \frac{1}{\log_{xy} y} =$ ______?

(a) 1 (b) 2

(c) 0 (d) $\frac{1}{\log_{xy}x \times \log_{xy}y}$

28. If $y = \log_{x-3}(x^2 - 6x + 9)$, then find y.

(a) 4 (b) 8

(c) 2 (d) 32

PRACTICE QUESTIONS

Level 2

29. If $\log_{10}2 = 0.3010$, then the number of digits in 16^{12} is

(a) 14 (b) 15

(c) 13 (d) 16

30. If $\log_{64}p^2 = 1\frac{2}{3}$, then $\log_2 \frac{p}{16} =$ ______.

(a) 16 (b) 2

(c) 32 (d) 1

31. $\log_2\log_2\log_5 125 =$ ______.

(a) 4 (b) 8

(c) −1 (d) 1

32. If $\log_x y = \frac{\log_a y}{p}$, then the value of p is

(a) $\log_y x$ (b) $\log_x a$

(c) $\log_a x$ (d) $\log_a y$

33. $\log\left[\frac{\sqrt[3]{x^2} \times y}{\sqrt[5]{z^2}}\right] =$ ______

(a) $\log x^{2/3} - \log z^{2/5} + \log y$

(b) $\log x^{3/2} + \log y - \log z^{5/2}$

(c) $\log x^{2/3} - \log y + \log z^{2/5}$

(d) None of these

34. If $\log[4 - 5\log_{32}(x + 3)] = 0$, find x.

(a) 32 (b) 8

(c) 3 (d) 5

35. If $x = \log_3 \log_2 \log_2 256$, then $2^{\log_4 2^{2x}} =$ ______.

(a) 4 (b) 8

(c) 2 (d) 1

36. If $\log_a\left(\frac{13^2}{\sqrt{2^3} \times 5}\right) = 2\log_a 13 - \log_a 5 - x$ then

(a) $a^x = 2^{3/2}$ (b) $x^a = 2^{3/2}$

(c) $a^x = 2^{2/3}$ (d) $x^a = 2^{2/3}$

37. If $\log 81 - \log 3 = \log a$, then $4_9^{\log a} =$ ______.

(1) 4 (2) 16

(3) 2 (4) 8

38. If $a^{\log_a n} = 3$ then $a^{2\log b} - b^{\log a} =$ ______.

(a) 6 (b) 9

(c) 3 (d) Cannot be determined

39. If $\log_3(x - 5) + \log_3(x + 2) = \log_3 8$, then $x =$ ______.

(a) −3 (b) 6

(c) 6, −3 (d) 3, −6

40. If $\log(x + y) = \log x + \log y$, then $x =$ ______.

(a) $\frac{-y}{1+y}$ (b) $\frac{-y}{1-y}$

(c) 1 (d) $\frac{y}{1+y}$

41. If $2^{\log 5} \cdot 5^{\log 2} = 2^{\log x}$, then $\log_5 \sqrt[3]{x^2} =$ ______.

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $\frac{1}{3}$ (d) 3

42. If $3^{\log x} + x^{\log 3} = 54$, find $\log x$.

(a) 3 (b) 2

(c) 4 (d) Cannot be determined

43. If $\log_{10}x - \log_{10}y = 1$ and $x + y = 11$, then $x =$ ______.

(a) 10 (b) 1

(c) 11 (d) 2

44. If $\log_{49}3 \times \log_9 7 \times \log_2 8 = x$, then find the value of $\frac{4x}{3}$.

(a) 3 (b) 7

(c) 8 (d) 1

45. The value of $\log_{a-b}(a^3 - b^3) - \log_{a-b}(a^2 + ab + b^2)$ is ______. $(a > b)$

(a) 0 (b) 1

(3) 3 (c) Undefined

46. If $\log_3 2 = x$, then the value of $\frac{\log_{10} 72}{\log_{10} 24}$ is

(a) $\frac{1+x}{1-x}$ (b) $\frac{2+3x}{1-3x}$

(c) $\frac{2-3x}{2+3x}$ (d) $\frac{3x+1}{3x+2}$

47. $\log\left(\frac{1}{2}\right)+\log\left(\frac{2}{3}\right)+\log\left(\frac{3}{4}\right)+\cdots+\log\left(\frac{99}{100}\right)$ = ______.

(a) −2 (b) −1

(c) 0 (d) 2

48. If $x^2 - y^2 = 1$, $(x > y)$, then find the value of $\log_{(x-y)}(x+y)$.

(a) −2 (b) 2

(c) −1 (d) 1

49. If $3^{\log_3 5} + 5^{\log_x 3} = 8$, then find the value of x.

(a) 3 (b) 5

(c) 4 (d) 8

50. $\log_2 1 \cdot \log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot \log_6 5 \ldots \log_{200} 199$ = ______.

(a) ∞ (b) 0

(c) 1 (d) Cannot be determined

51. If $x^2 + y^2 - 3xy = 0$ and $x > y$, then find the value of $\log_{xy}(x - y)$.

(a) $\frac{1}{4}$ (b) 4

(c) $\frac{1}{2}$ (d) 2

52. If $\log 3 = 0.4771$, then find the number of digits in 3^{100}.

(a) 47 (b) 48

(c) 49 (d) 50

Level 3

53. If $\log_5 x - \log_5 y = \log_5 4 + \log_5 2$ and $x - y = 7$, then x = ______.

(a) 1 (b) 8

(c) 7 (d) 6

54. If $\log_2\left[-1+\sqrt{x^2-14x+49}\right]=4$, then x = ______.

(a) 24 (b) −10

(c) 24, −10 (d) 10

55. If $\frac{\log p}{2}=\frac{\log q}{4}=\frac{\log r}{8}=k$ and $pqr = 100$, then k = ______.

(a) 14 (b) $\frac{1}{6}$

(c) $\frac{1}{7}$ (d) 2

56. If $\log 2 = 0.3010$, and $\log 3 = 0.4771$ then $\log 150$ = ______.

(a) 2.1761 (b) 2.8751

(c) 2.5762 (d) 2.6126

57. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{10}\left(\frac{2^3 \times 3^2}{5^2}\right)$ is

(a) 0.4592. (b) 0.5492.

(c) 0.4529. (d) 0.5429.

58. The value of $\frac{1}{1+\log_{ab} c}+\frac{1}{1+\log_{ac} b}+\frac{1}{1+\log_{bc} a}$ equals

(a) 2 (b) 0

(c) 1 (d) $\log abc$

59. If $x^2 + y^2 = z^2$, then $\frac{1}{\log_{(z+x)} y}+\frac{1}{\log_{(z-x)} y}$ = ______.

(a) 4 (b) 3

(c) 2 (d) 1

60. If $\log 2 = 0.301$, then find the number of digits in 2^{1024}.

(a) 307 (b) 308

(c) 309 (d) 310

61. If $x^2 - y^2 = 1$, $(x > y)$, then find the value of $\log_{(x-y)}(x + y)$.

(a) −2 (b) 2

(c) −1 (d) 1

62. If $x^2 + y^2 - 3xy = 0$ and $x > y$, then find the value of $\log_{xy}(x - y)$.

(a) $\frac{1}{4}$ (b) 4

(c) $\frac{1}{2}$ (d) 2

63. If $2^{\log_3 9} + 25^{\log_9 3} = 8^{\log_x 9}$, then $x =$ ______.

(a) 9 (b) 8

(c) 3 (d) 2

64. If $\log_a x = m$ and $\log_b x = n$, then $\log_{\left(\frac{a}{b}\right)} x =$ ______.

(a) $\frac{m}{m-n}$ (b) $\frac{mn}{m-n}$

(c) $\frac{n}{m-n}$ (d) $\frac{mn}{n-m}$

65. $\frac{\log_5 6}{\log_5 2 + 1} =$

(a) $\log_2 6$ (b) $\log_2 5$

(c) $\log_{10} 6$ (d) $\log_{10} 30$

66. If $x = \log_3 27$ and $y = \log_9 27$, then $\frac{1}{x} + \frac{1}{y} =$ ______.

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$

(c) 3 (d) 1

67. If $\log_6 x + 2\log_{36} x + 3\log_{216} x = 9$, then $x =$ ______.

(a) 6 (b) 36

(c) 216 (d) None of these

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $n\log_x A$
2. $\log_3 x + 2\log_3 y - 3\log_3 z$
3. No
4. $\log_{0.1}(0.001) = 3$
5. 4
6. 1
7. 5
8. False
9. 3.3010
10. 1
11. $\dfrac{1}{512}$
12. 0.6990
13. $x + y$
14. 1/2
15. 32
16. -1
17. $\log_a m$
18. 0
19. 2; 0.2987
20. $\log_a m$
21. 11
22. $\dfrac{8}{3}$
23. 1
24. 3
25. 2.3222
26. 2
27. 0.3371
28. 16.43
29. 27
30. 2

Short Answer Type Questions

32. 4
33. $\log\left(\dfrac{x^{1/3}z^{7/2}}{y^{8/5}}\right)$
36. 1
37. 13
38. 1.8751
41. $x = 1280$
43. 13.45
44. 0.04158
45. 478

Essay Type Questions

46. 2
47. 25
48. ABCD
49. 3
50. $\log \dfrac{a^{n(n+1)}}{b^{\frac{n(n+1)}{2}}}$

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (d)
11. (d)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (a)
21. (c)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (c)		

Level 2

29. (b)	**30.** (d)	**31.** (c)	**32.** (c)	**33.** (a)	**34.** (d)	**35.** (c)	**36.** (a)	**37.** (d)	**38.** (a)
39. (b)	**40.** (b)	**41.** (a)	**42.** (a)	**43.** (a)	**44.** (d)	**45.** (b)	**46.** (b)	**47.** (a)	**48.** (c)
49. (b)	**50.** (b)	**51.** (c)	**52.** (b)						

Level 3

53. (b)	**54.** (c)	**55.** (c)	**56.** (a)	**57.** (a)	**58.** (a)	**59.** (c)	**60.** (c)	**61.** (c)	**62.** (c)
63. (b)	**64.** (d)	**65.** (c)	**66.** (d)	**67.** (c)					

CONCEPT APPLICATION

Level 1

1. Apply laws of logarithm.
2. Recall the definition of characteristic.
3. Verify all the options.
4. Apply laws of logarithm.
5. Apply laws of logarithm.
6. (i) Apply log on both the sides
   (ii) $x^3 - y^3 = 3xy(x - y) \Rightarrow (x - y)^3 = 0$.
7. (i) $\log x - \log y - \log z = \log\left(\dfrac{x}{yx}\right)$
   (ii) $\log M - \log N = \log\left(\dfrac{M}{N}\right)$ and $\log M + \log N = \log MN$.
8. (i) $\log_2 16 < \log_2 20 < \log_2 32$
   (ii) Given four options are to be compared.
   (iii) For example $\log_2 16 < \log_2 20 < \log_2 32 \Rightarrow 4 < \log_2 20 < 5$.
   (iv) $7 < 35 < 7^2$; $5^2 < 70 < 5^3$; $3^3 < 68 < 3^4$ apply logarithms to suitable bases and evaluate $\log_7 15$, $\log_5 70$ and $\log_3 68$.
9. (i) Use the identity $\log m + \log n - \log p = \log mn/p$
   (ii) Use $\log x + \log y = \log xy$ and $\log p - \log q = \log(p/q)$.
10. Find $(x + y)^3$.
11. (i) Use the identity $\dfrac{\log a}{\log b} = \log_b a$
    (ii) $\dfrac{\log x}{\log y} = 2 \Rightarrow \log_y x = 2$
    (iii) $\log_b a = n \Rightarrow a = b^n$.
12. (i) Use $\log m - \log n = \log\dfrac{m}{n}$
    (ii) Use $\log m - \log n = \log\dfrac{m}{n}$ and simplify LHS.
    (iii) Write 1 as $\log_{10} 10$ and proceed.
13. (i) Use $\log m = \log n \Rightarrow m = n$.
    (ii) Remove logarithms to set $(x + 4)^2 = 16$, then find x.
14. (i) Use $\log m = \log n \Rightarrow m = n$.
    (ii) Use, $\log_b a = N \Rightarrow a = b^N$.
15. Apply laws of logarithms.
16. (i) $\log_b a^m = m\log_b a$ and $\log_a a = 1$
    (ii) Express 81 as 3^4 and then use $\log a^m = m \log a$ and proceed until all the brackets are removed.
17. Apply laws of logarithms.
18. Find $\sqrt{14641}$.
19. Simplify the expression by applying laws of logarithms.
20. Apply laws of logarithm.
21. Apply laws of logarithm.
22. Find p and q.
23. Write 1024 as a power of 2.
24. Apply laws of logarithm.
25. Apply laws of logarithm.
26. (i) Recall the laws of logarithm.
    (ii) Use $a^{\log_a N} = N$ after simplifying the given term.
27. (i) Simplify and then use laws of logarithm.
    (ii) Take LCM and simplify.
    (iii) Use $\log_a x + \log_a y = \log_a xy$.
29. (i) Use $\log_a a = 1$ to find y and then substitute y value.
    (ii) Write $x^2 - 6x + 9$ as $(x - 3)^2$ and use $\log_a a = 1$.

Level 2

29. Find the value of 16^{12} by applying logarithm.

30. Find p.

31. Apply laws of logarithm.

32. (i) Recall the laws of logarithm.

(ii) $p = \dfrac{\log a^y}{\log x^y}$. Use $\log p^q = q \log p$.

33. (i) Use the identities $\log mn = \log m + \log n$, $\log \dfrac{m}{n} = \log m - \log n$ and $\log x^n = n \log x$

(ii) $\log\left(\dfrac{\sqrt[3]{x^2}\cdot y}{\sqrt[5]{z^2}}\right) \log\left(\left(\dfrac{x^{2/3}\cdot y}{z^{2/5}}\right)\right)$

(iii) Use $\log\left(\dfrac{a}{b}\right) = \log a - \log b$ and $\log ab = \log a + \log b$ simplify.

34. (i) $\log x = 0 \Rightarrow x = 1$

(ii) $\log x = 0 \Rightarrow x = 1$

(iii) Use $\log_b a = \dfrac{1}{n}\log_b a$

(iv) $x = a \log n \Rightarrow a^x = n$.

35. Find x.

36. Apply laws of logarithm.

37. Find a.

38. Substitute the value of $a^{\log b}$.

39. Apply laws of logarithm.

40. (i) $\log(a) = \log(b) \Rightarrow a = b$

(ii) $\log(x + y) = \log xy$

(iii) $\Rightarrow x + y = xy$.

41. (i) $a^{\log b} = b^{\log a}$

(ii) $5^{\log 2} = 2^{\log 5}$

(iii) $a^m = a^n \Rightarrow m = n$.

42. (i) $a^{\log b} = b^{\log a}$

(ii) $x^m = x^n \Rightarrow m = n$.

43. Apply laws of logarithm and solve for x.

44. $\log_{49}3 \times \log_9 7 \times \log_2 8 = x$

$$\frac{\log 3}{2\log 7} \times \frac{\log 7}{2\log 3} \times \frac{3\log 2}{\log 2} = x$$

$$\frac{3}{4} = x \Rightarrow \frac{4x}{3} = \frac{4}{3}\times\frac{3}{4} = 1.$$

45. $\log_{a-b}(a^3 - b^3) - \log_{a-b} a^2 + ab + b^2$

$$= \log_{a-b}\left(\frac{a^3 - b^3}{a^2 + ab + b^2}\right)$$

$$= \log_{a-b}\frac{(a-b)(a^2+ab+b^2)}{(a^2+ab+b^2)}$$

$$= \log_{a-b}(a-b) = 1.$$

46. Given, $\log_3 2 = x$

$$\frac{\log_{10} 72}{\log_{10} 24} = \log_{24} 72 = \log_{24}(24\times 3)$$

$$= \log_{24} 24 + \log_{24} 3 = 1 + \frac{\log 3}{\log 24}$$

$$= 1 + \frac{\log_3 3}{\log_3 24}$$

$$= 1 + \frac{1}{\log_3(3\times 8)}$$

$$= 1 + \frac{1}{\log_3 3 + \log_3 8}$$

$$= 1 + \frac{1}{1 + \log_3 2^3}$$

$$= 1 + \frac{1}{1 + 3\log_3 2}$$

$$= \frac{1 + 3\log_3 2 + 1}{1 + 3\log_3 2} = \frac{2+3x}{1+3x}.$$

47. $\log\left(\dfrac{1}{2}\right) + \log\left(\dfrac{2}{3}\right) + \cdots + \log\left(\dfrac{99}{100}\right)$

$$= \log 1 - \log 2 + \log 2 - \log 3 + \cdots + \log 99 - \log 100$$

$$= \log 1 - \log 100 = 0 - \log 10^2$$

$$= -2\log 10 = -2.$$

48. Given $x^2 - y^2 = 1$

Applying log on both the sides, we get

$(x^2 - y^2) = \log 1$

$\Rightarrow \log[(x + y)(x - y)] = 0$

HINTS AND EXPLANATION

$\Rightarrow \log(x + y) + \log(x - y) = 0$

$\Rightarrow \log(x + y) = -\log(x - y)$

$\frac{\log(x+y)}{\log(x-y)} = -1$

$\Rightarrow \log_{(x-y)} (x + y) = -1.$

49. $3^{\log_3 5} + 5^{\log_x 3} = 8$

$\Rightarrow 5 + 5^{\log_x 3} = 8$

$\Rightarrow 5^{\log_x 3} = 3$

$3^{\log_x 5} = 3$

$\Rightarrow \log_x 5 = 1$

$\Rightarrow x = 5.$

50. As $\log_2 1 = 0$, $\log_2 1 \cdot \log_3 2 \cdot \log_4 3 \ldots \log_{199} 200 = 0$

51. Given $x^2 + y^2 - 3xy = 0$

$x^2 + y^2 - 2xy = xy$

Applying log on both sides

$\log(x - y)^2 = \log xy$

$2\log(x - y) = \log xy$

$\Rightarrow \frac{\log(x-y)}{\log xy} = \frac{1}{2}$

$\Rightarrow \log_{xy}(x - y) = \frac{1}{2}.$

52. Let $x = 3^{100}$

$\Rightarrow \log x = \log 3^{100}$

$\Rightarrow \log x = 100 \log 3$

$\Rightarrow \log x = 100\ (0.4771)$

$\Rightarrow \log x = 47.71$

The characteristic is 47

The number of digits in 3^{100} is 48.

Level 3

53. Apply laws of logarithm.

54. (i) $\log_a x = b \Rightarrow a^b = x$.

(ii) Write $\sqrt{x^2 - 14x + 49} = 16 + 1$

$= 17$ and solve for x.

(iii) Then verify for what values of x, $\log f(x)$ is defined.

55. (i) Find the value of p, q, r then find pqr.

(ii) Express p, q and r in terms of k.

(iii) Substitute the above values in $pqr = 100$ and find k.

56. (i) Express as product of powers of 2 and 3 then use the values of log 2 and log 3

(ii) $\log 150 = 2\log 5 + \log 2 + \log 3$.

(iii) Also, $\log 5 = \log 10 - \log 2$.

57. (i) Apply laws of logarithm.

(ii) Use $\log\left(\frac{a}{b}\right) = \log a - \log b$ and $\log a^m = m\log a$.

58. $\frac{1}{1+\log_{ab} c} = \frac{1}{\log_{ab} ab + \log_{ab} c} = \frac{1}{\log_{ab}{abc}}$

$= \log_{abc} ab \quad \frac{1}{1+\log_{ac} b} = \frac{1}{\log_{ac} ac + \log_{ac} b}$

$= \frac{1}{\log_{ac} abc} = \log_{abc} ac \quad \frac{1}{1+\log_{bc} a} +$

$\frac{1}{\log_{bc} bc + \log_{bc} a} = \frac{1}{\log_{bc} abc} \log_{abc} bc$

Hence the value of the required expression

$= \log_{abc} ab + \log_{abc} ac + \log_{abc} bc$

$= \log_{abc}[(ab)(ac)(bc)] = \log_{abc}(abc)^2 = 2.$

59. Given that, $x^2 + y^2 = z^2$

$\Rightarrow z^2 - x^2 = y^2$ (1)

$\therefore \frac{1}{\log_{z+x} y} + \frac{1}{\log_{(z-x)} y}$

$= \log_y(z + x) + \log_y(z - x)$

$= \log_y(z^2 - x^2) = \log_y y^2$ {from (1)}

$= 2.$

60. Let $x = 2^{1024}$

$\Rightarrow \log x = \log 2^{1024}$

$= 1024 \log 2 = 1024(0.301)$

$\Rightarrow \log x = 308.22$

$\Rightarrow$ The characteristic is 308

$\therefore$ The number of digits in 2^{1024} is 309.

61. Given $x^2 - y^2 = 1$

Applying log on both the sides, we get $\log(x^2 - y^2) = \log 1$

$\Rightarrow \log[(x+y)(x-y)] = 0$

$\Rightarrow \log(x+y) + \log(x-y) = 0$

$\Rightarrow \log(x+y) = -\log(x-y)$

$\frac{\log(x+y)}{\log(x-y)} = -1$

$\Rightarrow \log_{(x-y)}(x+y) = 1.$

62. Given, $x^2 + y^2 - 3xy = 0$

$x^2 + y^2 - 2xy = xy$

Applying log on both sides,

$\log(x-y)^2 = \log xy$

$2\log(x-y) = \log xy$

$\Rightarrow \frac{\log(x-y)}{\log xy} = \frac{1}{2}$

$\Rightarrow \log_{xy}(x-y) = \frac{1}{2}.$

63. $2^{\log_3 9} + 25^{\log_9 3} = 8^{\log_x 9}$

$\Rightarrow 2^{\log_3 3^2} + 25^{\log_{3^2} 3} = 8^{\log_x 9}$

$\Rightarrow 2^2 + 25^{1/2} = 8^{\log_x 9}$

$\Rightarrow 9 = 8^{\log_x 9}$

$\Rightarrow \log_x 9 = \log_8 9$

$\Rightarrow x = 8.$

64. $\log_a x = m$, $\log_b x = n$

$$\log_{\left(\frac{a}{b}\right)} x = \frac{\log x}{\log\left(\frac{a}{b}\right)} = \frac{\log_x x}{\log_x\left(\frac{a}{b}\right)}$$

$$= \frac{1}{\log_x a - \log_x b}$$

$$= \frac{1}{\frac{1}{\log_x a} - \frac{1}{\log_b x}}$$

$$= \frac{1}{\frac{1}{m} - \frac{1}{n}} = \frac{1}{\left(\frac{n-m}{mn}\right)} = \frac{mn}{n-m}.$$

65. $$\frac{\log_5 6}{\log_5 2 + 1} = \frac{\log_5 6}{\log_5 2 + \log_5 5} = \frac{\log_5 6}{\log_5(2\times 5)}$$

$$= \frac{\log_5 6}{\log_5 10} \log_{10} 6.$$

66.

$x = \log_3 27$	$y = \log_9 27$
$\Rightarrow x = \log_3 3^3$	$y = \log_{3^2} 3^3$
$\Rightarrow x = 3$	$y = \frac{3}{2}$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{3} + \frac{1}{\left(\frac{3}{2}\right)} = \frac{1}{3} + \frac{2}{3} + \frac{3}{3} = 1.$$

67. $\log_6 x + 2\log_{36} x + 3\log_{216} x = 9$

$\Rightarrow \log_6 x + \log_6 x + \log_6 x = 9$

$\Rightarrow 3\log_6 x = 9 \Rightarrow \log_6 x = 3$

$\Rightarrow x = 6^3$

$\therefore x = 216.$