4

Laws of Motion

TOPIC 1

Newton's Laws of Motion and Conservation of Momentum

01 The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt, so that the rocket is given an acceleration of $20\ ms^{-1}$. The gases come out at a relative speed of $500\ ms^{-1}$ with respect to the rocket [Use, $g = 10\ m/s^2$] **[2021, 26 Aug Shift-I]**

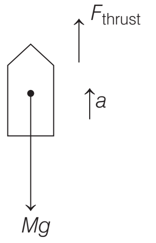
(a) $6.0 \times 10^2\ kg\ s^{-1}$ (b) $500\ kg\ s^{-1}$
(c) $10\ kg\ s^{-1}$ (d) $60\ kg\ s^{-1}$

Ans. ***(d)***

Given, $M = 1000$ kg, $a = 20\ m/s^2$

$v_{relative} = 500$ m/s, $g = 10\ m/s^2$

The given situation is shown below



$$F_{thrust} = \frac{d}{dt}(Mv_{relative})$$

$$\Rightarrow F_{thrust} = v_{relative}\left(\frac{dM}{dt}\right)$$

By Newton's second law of motion,

$$\Rightarrow \quad F_{thrust} - Mg = Ma$$

$$\Rightarrow v_{relative}\left(\frac{dM}{dt}\right) - Mg = Ma$$

$$\Rightarrow \quad 500\left(\frac{dM}{dt}\right) - 1000 \times 10 = 1000 \times 20$$

$$\Rightarrow \quad 500\left(\frac{dM}{dt}\right) = 1000\,(20 + 10)$$

$$\Rightarrow \quad 500\left(\frac{dM}{dt}\right) = 1000 \times 30$$

$$\Rightarrow \quad \frac{dM}{dt} = \frac{1000 \times 30}{500} = 60$$

$$\Rightarrow \quad \frac{dM}{dt} = 60\ kg/s$$

02 A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0\left[1 - \left(\frac{t - T}{T}\right)^2\right]$$

where, F_0 and T are constants. The force acts only for the time interval $2T$. The velocity v of the particle after time $2T$ is **[2021, 27 July Shift-II]**

(a) $\frac{2F_0T}{M}$ (b) $\frac{F_0T}{2M}$ (c) $\frac{4F_0T}{3M}$ (d) $\frac{F_0T}{3M}$

Ans. ***(c)***

Given, force on particle,

$$F = F_0\left[1 - \left(\frac{t - T}{T}\right)^2\right] \quad \ldots(i)$$

$\because$ Acceleration of ball will given as

$$a = \frac{F}{M} \quad \ldots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$a = \frac{F_0}{M}\left[1 - \left(\frac{t - T}{T}\right)^2\right]$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{F_0}{M}\left[1 - \left(\frac{t - T}{T}\right)^2\right]$$

$$\Rightarrow \quad dv = \frac{F_0}{M}\left[1 - \left(\frac{t - T}{T}\right)^2\right]$$

Integrate above equation, we get

$$\Rightarrow \quad \int dv = \frac{F_0}{M}\int_0^{2T}\left[1 - \left(\frac{t - T}{T}\right)^2\right]dt$$

[$\because$ The force acts only for time interval $2T$]

$$\Rightarrow \quad v = \frac{F_0}{M}\left[t - \frac{1}{3T^2}(t - T)^3\right]_0^{2T}$$

$$\Rightarrow \quad v = \frac{F_0}{M}\left\{\left[2T - \frac{1}{3T^2}(2T - T)^3\right] - \left[0 - \frac{(-T)^3}{3T^2}\right]\right\}$$

$$\Rightarrow \quad v = \frac{4F_0T}{3M}$$

03 A force $\mathbf{F} = (40\hat{\mathbf{i}} + 10\hat{\mathbf{j}})$ N acts on a body of mass 5 kg. If the body starts from rest its position vector $\mathbf{r}$ at time $t = 10$ s will be **[2021, 25 July Shift-II]**

(a) $(100\hat{\mathbf{i}} + 400\hat{\mathbf{j}})$ m (b) $(100\hat{\mathbf{i}} + 100\hat{\mathbf{j}})$ m
(c) $(400\hat{\mathbf{i}} + 100\hat{\mathbf{j}})$ m (d) $(400\hat{\mathbf{i}} + 400\hat{\mathbf{j}})$ m

Ans. ***(c)***

Given, force, $\mathbf{F} = 40\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$

Mass, $m = 5$ kg

Initial speed, $u = 0$

Time taken, $t = 10$ s

Let the distance travelled by the body be s.

Since, $F = ma \Rightarrow a = \frac{F}{m}$

where, a is acceleration.

$$\therefore \quad a = \frac{40\hat{\mathbf{i}} + 10\hat{\mathbf{j}}}{5} = 8\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\ m/s^2$$

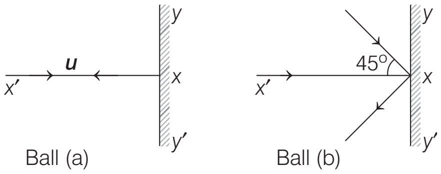
$$\text{Now,} \quad s = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad s = 0 \times 10 + \frac{1}{2} \times (8\hat{\mathbf{i}} + 2\hat{\mathbf{j}})10^2$$

$$= \frac{100}{2}(8\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = 50(8\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$$

$$= (400\hat{\mathbf{i}} + 100\hat{\mathbf{j}})\ m$$

04 Two billiard balls of equal mass 30 g strike a rigid wall with same speed of 108 km/h (as shown) but at different angles. If the balls get reflected with the same speed, then the ratio of the magnitude of impulses imparted to ball *a* and ball *b* by the wall along, *x*. direction is **[2021, 25 July Shift-I]**



Ball (a) Ball (b)

(a) 1 : 1 (b) $\sqrt{2}:1$ (c) 2 : 1 (d) $1:\sqrt{2}$

Ans. *(b)*

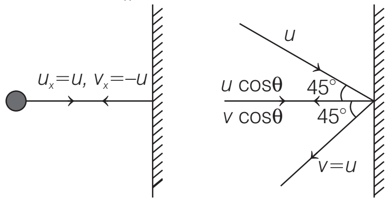
Given, mass of ball,

$m = 30\text{ g} = 30\times10^{-3}\text{ kg}$

Speed of approach, $u = 180$ kmph

$= 180\times\frac{5}{18} = 50\text{ ms}^{-1}$

and speed of approach (μ_x) = speed of separation (v_x)



Ball (a) Ball (b)

As we know that,

Impulse (I) = change in momentum (Δp)

Impulse for ball (a),

$I_a = m(v_x - u_x)$

$I_a = m(u+u) = 2mu$

$= 2\times30\times10^{-3}\times50 = 3\text{ kg-ms}^{-1}$

and impulse for ball (b),

$I_b = m(u\cos\theta + v\cos\theta)$

$\Rightarrow I_b = m(u\cos\theta + u\cos\theta)$ $(\because v = u)$

$= 2mu\cos\theta$

$= 2\times30\times10^{-3}\times50\times\cos45°$

$= 3/\sqrt{2}\text{ kg-ms}^{-1}$

$\therefore I_a : I_b = \sqrt{2}:1$

05 A bullet of 4 g mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are **[2021, 22 July Shift-II]**

(a) 0.4 kg-ms^{-1}, 0.1 ms^{-1}
(b) 0.2 kg-ms^{-1}, 0.05 ms^{-1}
(c) 0.2 kg-ms^{-1}, 0.1 ms^{-1}
(d) 0.4 kg-ms^{-1}, 0.05 ms^{-1}

Ans. *(b)*

Given, mass of bullet,

$m_B = 4\text{g} = 4\times10^{-3}\text{ kg}$

Mass of gun, $m_G = 4$ kg

Speed of bullet, $v_B = 50\text{ ms}^{-1}$

Let recoiling velocity of gun be v_G.

By using law of conservation of momentum for recoiling of gun

$m_B v_B = -m_G v_G$

$\Rightarrow v_G = -\frac{m_B v_B}{m_G} = -\frac{4\times10^{-3}\times50}{4}$

$= -50\times10^{-3} = -0.05\text{ ms}^{-1}$

Impulse (I) = Change in momentum of gun

$I = m_G(0 - V_G) = 4\times0.05 = 0.2\text{ kg ms}^{-1}$

06 A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is *x* N. The value of *x* to the nearest integer is .... **[2021, 18 March Shift-I]**

Ans. *(10)*

Given,

The mass of the bullet, $m = 0.1$ kg

The initial velocity of the bullet before hitting the wooden, $u = 10$ m/s

The final velocity of the bullet after hitting the wooden, $v = 0$

The distance travelled by the bullet before coming to the rest, $s = 50$ cm

Using the equation of the motion,

$v^2 - u^2 = 2as$

$\Rightarrow 0^2 - (10)^2 = 2a(0.50)$

$\Rightarrow a = -100\text{ m/s}^2$

The uniform retardation of the bullet is 100 m/s^2. The magnitude of the effective retarding force on the bullet,

$F = ma$

$F = 0.1(-100)$

$F = -10$ N

Hence, the value of *x* to the nearest integer is 10.

07 A body of mass 2 kg moves under a force of $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$N. It starts from rest and was at the origin initially. After 4 s, its new coordinates are (8, *b*, 20). The value of *b* is .... (Round off to the nearest integer) **[2021, 16 March Shift-II]**

Ans. *(12)*

Given,

Force, $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ N

Mass, $m = 2$ kg, Time, $t = 4$s

We know that

Force = mass × acceleration

$\Rightarrow \mathbf{F} = m\times\mathbf{a} \Rightarrow \mathbf{a} = \frac{\mathbf{F}}{m}$

$\Rightarrow \mathbf{a} = \frac{2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}}{2}$ ... (i)

From second equation of motion,

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ ... (ii)

From Eqs. (i) and (ii), we get

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\frac{(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})}{2}\cdot t^2$

$= 0 + \frac{1}{4}(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})\cdot(4)^2$

$[\because \mathbf{u} = 0 \text{ and } t = 4\text{ s}]$

$= 0 + 8\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 20\hat{\mathbf{k}}$

$= 8\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 20\hat{\mathbf{k}}$

Let $s = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$\Rightarrow x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = 8\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 20\hat{\mathbf{k}}$ ... (iii)

According to question, new coordinates are (8, *b*, 20), it means

$x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = 8\hat{\mathbf{i}} + b\hat{\mathbf{j}} + 20\hat{\mathbf{k}}$ ... (iv)

Comparing Eqs. (iii) and (iv), we get

$b = 12$

08 A boy pushes a box of mass 2 kg with a force $\mathbf{F} = (20\hat{\mathbf{i}} + 10\hat{\mathbf{j}})$ N on a frictionless surface. If the box was initially at rest, then .... m is displacement along the *X*-axis after 10 s. **[2021, 26 Feb Shift-I]**

Ans. *(500)*

Given, mass of box, $m = 2$ kg

Force, $F = 20\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$ N

Initial speed of box, $u = 0\text{ ms}^{-1}$

Time, $t = 10$ s

Let acceleration of box is *a* and displacement along *X*-axis after 10 s is s_x.

As, $F = ma \Rightarrow a = F/m$

$= \frac{20\hat{\mathbf{i}} + 10\hat{\mathbf{j}}}{2} = (10\hat{\mathbf{i}} + 5\hat{\mathbf{j}})\text{ ms}^{-2}$

By second equation of motion along X-axis,

$s_x = u_x t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}\times10\times(10)^2 = 500\text{ m}$

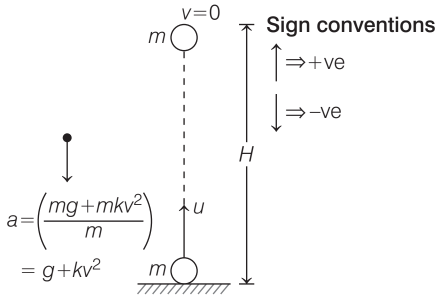
Hence, displacement along *X*-axis after 10 s is 500 m.

09 A small ball of mass *m* is thrown upward with velocity *u* from the groun(d) The ball experiences a resistive force mkv^2, where *v* is its speed The maximum height attained by the ball is **[2020, 4 Sep Shift-II]**

(a) $\frac{1}{k}\tan^{-1}\left(\frac{ku^2}{2g}\right)$ (b) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$

(c) $\frac{1}{k}\ln\left(1+\frac{ku^2}{2g}\right)$ (d) $\frac{1}{2k}\tan^{-1}\left(\frac{ku^2}{g}\right)$

Ans. *(b)*



Net force on ball = weight of ball + resistive force

i.e., $F_{net} = w + F_{resistive}$

$= (-mg) + (-mkv^2)$

$= -m(g + kv^2)$

So, net acceleration of ball,

$$a = \frac{F_{net}}{m} = \frac{-m(g+kv^2)}{m} = -(g+kv^2)$$

$$\Rightarrow v\frac{dv}{dy} = -(g+kv^2) \Rightarrow \frac{vdv}{(g+kv^2)} = -dy$$

Integrating both sides,

$$\int_u^0 \frac{v}{(g+kv^2)}dv = -\int_0^H dy \quad ...(i)$$

Let $g + kv^2 = t \Rightarrow 0 + 2kvdv = dt$

$\Rightarrow vdv = \frac{1}{2k}dt$

Lower limit

If $v = u$, then $g + ku^2 = t \Rightarrow t = g + ku^2$

Upper limit

If $v = 0$, then $g + k(0)^2 = t \Rightarrow t = g$

Putting these values in eq. (i), we get

$$\int_{g+ku^2}^{g} \frac{\frac{1}{2k}dt}{t} = -\int_0^H dy$$

$$\Rightarrow \frac{1}{2k}\int_{g+ku^2}^{g}\frac{1}{t}dt = -[y]_0^H$$

$$\Rightarrow \frac{1}{2k}[\ln(t)]_{g+ku^2}^{g} = -[H-0]$$

$$\Rightarrow \frac{1}{2k}[\ln(g) - \ln(g+ku^2)] = -[H]$$

$$\Rightarrow -\frac{1}{2k}[\ln(g) - \ln(g+ku^2)] = H$$

$$\Rightarrow H = \frac{1}{2k}[\ln(g+ku^2) - \ln(g)]$$

$$= \frac{1}{2k}\left[\ln\left(\frac{g+ku^2}{g}\right)\right]$$

$$= \frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$$

Hence, option (b) is correct.

10 A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where $v(t)$ is its instantaneous velocity. The instantaneous acceleration of the satellite is **[2020, 5 Sep Shift-II]**

(a) $-bv^3(t)$ (b) $-\frac{bv^3}{M(t)}$

(c) $-\frac{2bv^3}{M(t)}$ (d) $-\frac{bv^3}{2M(t)}$

Ans. *(b)*

Given that, mass increases at the rate,

$$\frac{dM(t)}{dt} = bv^2(t)$$

Now, from Newton's second law,

Force = Rate of change of linear momentum

i.e., $F = \frac{dp}{dt} = \frac{d}{dt}(Mv) = M\frac{dv}{dt} + v\frac{dM}{dt}$

$$= M\frac{dv}{dt} + v[bv^2(t)] = M\frac{dv}{dt} + bv^3(t)$$

Now, force acting on satellite is zero, i.e., $F = 0$

$$\Rightarrow M\frac{dv}{dt} = -bv^3 \Rightarrow \frac{dv}{dt} = -\frac{bv^3}{M(t)}$$

∴ Acceleration of the satellite, $a = -\frac{bv^3}{M(t)}$

Hence, correct option is (b).

11 A ball is thrown upward with an initial velocity v_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where, m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is **[2019, 10 April Shift-I]**

(a) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}v_0\right)$

(b) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}v_0\right)$

(c) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}v_0\right)$

(d) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}v_0\right)$

Ans. *(b)*

Given, drag force, $F = m\gamma v^2$ ... (i)

As we know, general equation of force

$= ma$ ... (ii)

Comparing Eqs. (i) and (ii), we get

$a = \gamma v^2$

∴ Net retardation of the ball when thrown vertically upward is

$$a_{net} = -(g + \gamma v^2) = \frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{(g+\gamma v^2)} = -dt \quad ... (iii)$$

By integrating both sides of Eq. (iii) in known limits, i.e.

When the ball thrown upward with velocity v_0 and then reaches to its zenith, i.e. for maximum height at time $t = t, v = 0$

$$\Rightarrow \int_{v_0}^{0}\frac{dv}{(\gamma v^2 + g)} = \int_0^t -dt$$

$$\text{or} \quad \frac{1}{\gamma}\int_{v_0}^{0}\frac{1}{\left[\left(\sqrt{\frac{g}{\gamma}}\right)^2 + v^2\right]}dv = -\int_0^t dt$$

$$\Rightarrow \frac{1}{\gamma}\cdot\frac{1}{\sqrt{g/\gamma}}\cdot\left[\tan^{-1}\left(\frac{v}{\sqrt{g/\gamma}}\right)\right]_{v_0}^{0} = -t$$

$$\left[\because \int\frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$\Rightarrow \frac{1}{\sqrt{\gamma g}}\cdot\tan^{-1}\left(\frac{\sqrt{\gamma}v_0}{\sqrt{g}}\right) = +t$$

12 A bullet of mass 20 g has an initial speed of $1\,ms^{-1}$, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to **[2019, 10 April Shift-II]**

(a) $0.3\,ms^{-1}$ (b) $0.4\,ms^{-1}$

(c) $0.1\,ms^{-1}$ (d) $0.7\,ms^{-1}$

Ans. *(d)*

Given, resistance offered by the wall

$= F = -2.5\times10^{-2}$ N

So, deacceleration of bullet,

$$a = \frac{F}{m} = \frac{-2.5\times10^{-2}}{20\times10^{-3}} = -\frac{5}{4}ms^{-2}$$

$(\because m = 20\text{ g} = 20\times10^{-3}\text{ kg})$

Now, using the equation of motion,

$$v^2 - u^2 = 2as$$

We have, $v^2 = 1 + 2\left(-\frac{5}{4}\right)(20\times10^{-2})$

$(\because u = 1ms^{-1}$ and $s = 20\text{ cm} = 20\times10^{-2}\text{ m})$

$\Rightarrow \qquad v^2 = \frac{1}{2}$

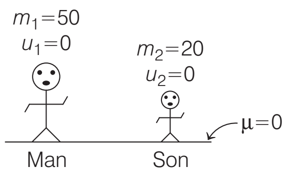
$\therefore \qquad v = \frac{1}{\sqrt{2}} \approx 0.7\,\text{ms}^{-1}$

13 A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son, so that he starts moving at a speed of $0.70\ \text{ms}^{-1}$ with respect to the man. The speed of the man with respect to the surface is **[2019, 12 April Shift-I]**

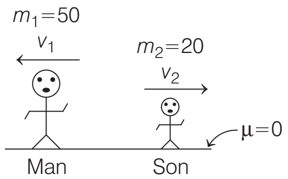
(a) $0.28\ \text{ms}^{-1}$ (b) $0.20\ \text{ms}^{-1}$
(c) $0.47\ \text{ms}^{-1}$ (d) $0.14\ \text{ms}^{-1}$

Ans. *(b)*

The given situation can be shown as below



Before collision



After collision

Using momentum conservation law,

$(\text{Total momentum})_{\text{before collision}}$
$= (\text{Total momentum})_{\text{after collision}}$

$(m_1 \times 0) + (m_2 \times 0) = m_1 v_1 + m_2 v_2$

$0 = m_1(-v_1)\hat{i} + m_2 v_2 \hat{i}$

$\Rightarrow \qquad m_1 v_1 = m_2 v_2$

$\Rightarrow \qquad 50 v_1 = 20 v_2$

$\Rightarrow \qquad v_2 = 2.5 v_1 \qquad$ ...(i)

Again, relative velocity = 0.70 m/s

But from figure, relative velocity = $v_1 + v_2$

$\therefore \qquad v_1 + v_2 = 0.7 \qquad$ ...(ii)

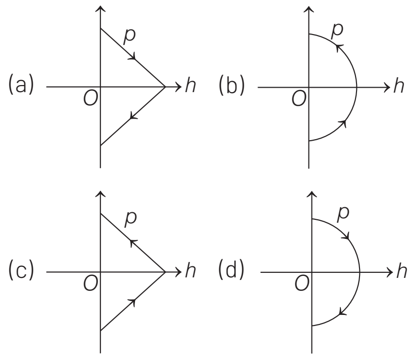
From Eqs. (i) and (ii), we get

$v_1 + 2.5 v_1 = 0.7$

$\Rightarrow \qquad v_1 (3.5) = 0.7$

$v_1 = \frac{0.7}{3.5} = 0.20\ \text{m/s}$

14 A ball is thrown vertically up (taken as + Z-axis) from the groun(d) The correct momentum-height (p-h) diagram is



[2019, 9 April Shift-I]

Ans. *(d)*

When a ball is thrown vertically upward, then the acceleration of the ball,

a = acceleration due to gravity (g) (acting in the downward direction).

Now, using the equation of motion,

$$v^2 = u^2 - 2gh$$

or $\qquad h = \frac{-v^2 + u^2}{2g} \qquad$ ...(i)

As we know, momentum, $p = mv$ or $v = p/m$

So, substituting the value of v in Eq. (i), we get

$$h = \frac{u^2 - (p/m)^2}{2g}$$

As we know that, at the maximum height, velocity of the ball thrown would be zero.

So, for the flight when the ball is thrown till is reaches the maximum height (h).

$v \rightarrow$ changes from u to 0

$\Rightarrow \qquad p \rightarrow$ changes from mu to 0

Similarly, when it reacher it's initial point, then

$h \rightarrow$ changes from $h_{\max}$ to 0

Also, $p \rightarrow$ changes from 0 to some values.

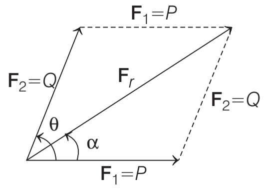
Thus, these conditions are only satisfied in the plot given in option (d).

15 Two forces P and Q of magnitude $2F$ and $3F$, respectively,are at an angle θ with each other. If the force Q is doubled, then their resultant also gets double(d) Then, the angle θ is **[2019, 10 Jan Shift-II]**

(a) 60° (b) 120°
(c) 30° (d) 90°

Ans. *(b)*

Resultant force $\mathbf{F}_r$ of any two forces $\mathbf{F}_1$ (i.e. P) and $\mathbf{F}_2$ (i.e. Q) with an angle θ between them can be given by vector addition as



$F_r^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \qquad$ ...(i)

In first case $F_1 = 2F$ and $F_2 = 3F$

$\Rightarrow \qquad F_r^2 = 4F^2 + 9F^2 + 2 \times 2 \times 3F^2 \cos\theta$

$\Rightarrow \qquad F_r^2 = 13F^2 + 12F^2 \cos\theta \qquad$ ...(ii)

In second case $F_1 = 2F$ and $F_2 = 6F$

($\because$ Force Q gets doubled)

and $\qquad F_r' = 2F_r \qquad$ (Given)

By putting these values in Eq. (i), we get

$(2F_r)^2 = (2F)^2 + (6F)^2 + 2 \times 2 \times 6F^2 \cos\theta$

$\Rightarrow \quad 4F_r^2 = 40F^2 + 24F^2 \cos\theta \qquad$ ...(iii)

From Eq. (ii) and Eq. (iii), we get;

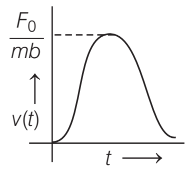
$52F^2 + 48F^2 \cos\theta = 40F^2 + 24F^2 \cos\theta$

$\Rightarrow \qquad 12 + 24\cos\theta = 0$ or $\cos\theta = -1/2$

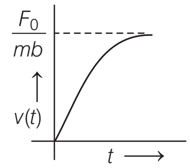
or $\qquad \theta = 120° \qquad (\because \cos 120° = -1/2)$

16 A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x-direction. Its speed $v(t)$ is depicted by which of the following curves? **[AIEEE 2012]**

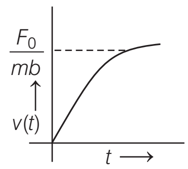
(a)



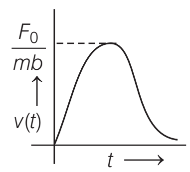
(b)



(c)



(d)



Ans. *(c)*

As the force is exponentially decreasing, so its acceleration, *i.e.*, rate of increase of velocity will decrease with time. Thus, the graph of velocity will be an increasing curl with decreasing slope with time.

$$\therefore \quad a=\frac{F}{M}=\frac{F_0}{m}e^{-bt}=\frac{dv}{dt}$$

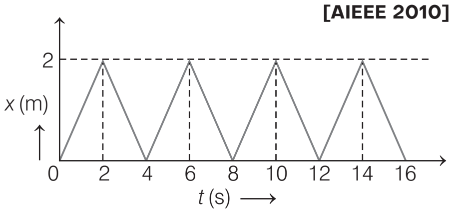
$$\Rightarrow \quad \int_0^v dv=\int_0^t \frac{F_0}{m}e^{-bt}dt$$

$$\Rightarrow \quad v=\frac{F_0}{m}\left[\left(\frac{1}{-b}\right)e^{-bt}\right]_0^t=\frac{F_0}{mb}[e^{-bt}]_t^0$$

$$=\frac{F_0}{mb}(e^0-e^{-bt})=\frac{F_0}{mb}(1-e^{-bt})$$

$$v_{max}=\frac{F_0}{mb}$$

17 The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is **[AIEEE 2010]**



(a) 0.4 N- s (b) 0.8 N-s
(c) 1.6 N-s (d) 0.2 N-s

Ans. ***(b)***

From the graph, it is a straight line, so uniform motion. Because of impulse direction of velocity changes as can be seen from the slope of the graph.

Initial velocity, $v_1=\frac{2}{2}=1\ ms^{-1}$

Final velocity, $v_2=-\frac{2}{2}=-1\ ms^{-1}$

$$\mathbf{p}_i=mv_1=0.4\ \text{N-s}$$

$$\mathbf{p}_f=mv_2=-0.4\ \text{N-s}$$

$$\mathbf{J}=\mathbf{p}_f-\mathbf{p}_i=-0.4-0.4=-0.8\ \text{N-s}$$

[$\mathbf{J}$ = impulse]

$$\therefore \quad |\mathbf{J}|=0.8\ \text{N-s}$$

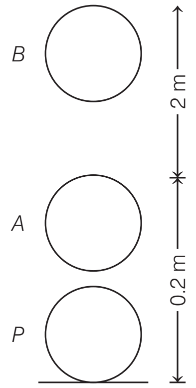
18 A ball of mass 0.2 kg is thrown vertically upwards by applying a force by han(d) If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g=10\ m/s^2$. **[AIEEE 2006]**

(a) 4 N (b) 16 N (c) 20 N (d) 22 N

Ans. ***(d)***

The situation is shown in figure. At initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B let acceleration of ball during PA be a ms^{-2} (assumed to be constant) in upward direction and velocity of ball at be is v m/s.

Then, for PA,



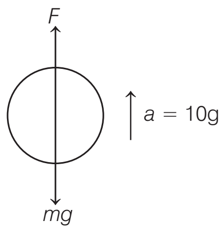
$$v^2=0^2+2a\times0.2$$

For AB,

$$0=v^2-2\times g\times2$$

$$\Rightarrow \quad v^2=2g\times2$$

From above equations,



$$a=10g=100\ ms^{-2}$$

Then, for PA, FBD of ball is

$$F-mg=ma$$

[F is the force exerted by hand on ball]

$$\Rightarrow \quad F=m(g+a)$$

$$=0.2(11g)=22\ N$$

Alternate Solution Using work-energy theorem

$$W_{mg}+W_F=0$$

$$\Rightarrow \quad -mg\times2.2+F\times0.2=0$$

or $\quad F=22$ N

19 A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to **[AIEEE 2006]**

(a) 150 N (b) 3 N
(c) 30 N (d) 300 N

Ans. ***(c)***

This is the question based on impulse-momentum theorem.

$$|F\cdot\Delta t|=|\text{Change in momentum}|$$

$$\Rightarrow \quad F\times0.1=|p_f-p_i|$$

As the ball will stop after catching

$p_i=mv_i=0.15\times20=3$, $p_f=0$

$$\Rightarrow \quad F\times0.1=3\Rightarrow F=30\ N$$

20 A particle of mass 0.3 kg is subjected to a force $F=-kx$ with $k=15\ Nm^{-1}$. What will be its initial acceleration, if it is released from a point 20 cm away from the origin? **[AIEEE 2005]**

(a) 3 ms^{-2} (b) 15 ms^{-2}
(c) 5 ms^{-2} (d) 10 ms^{-2}

Ans. ***(d)***

Given, $m=0.3$ kg, $x=20$ cm

and $k=15$ N/m

given $F=-kx$ ...(i)

and $F=ma$ ...(ii)

$\therefore \quad ma=-kx$

$$\Rightarrow \quad a=-\frac{15}{0.3}\times20\times10^{-2}$$

$$\Rightarrow \quad a=-\frac{15}{3}\times2=-10\ ms^{-2}$$

Negative sign indicates that acceleration is always towards the mean position.

$\therefore$ Initial acceleration, $a=10\ ms^{-2}$

21 A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it, can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most? **[AIEEE 2004]**

(a) 1 (b) 4
(c) 2 (d) 3

Ans. ***(d)***

Force exerted by machine gun on man's hand in firing a bullet = Change in momentum per second on a bullet or rate of change of momentum

$$=\left(\frac{40}{1000}\right)\times1200=48\ N$$

Force exerted by man on machine gun

= Force exerted on man by machine gun

= 144 N

Hence, number of bullets fire(d) $=\frac{144}{48}=3$

22 A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 ms^{-2}. Then, the initial thrust of the blast is **[AIEEE 2003]**

(a) 3.5×10^5 N (b) 7.0×10^5 N
(c) 14.0×10^5 N (d) 1.75×10^5 N

Ans. ***(a)***

Here, thrust force is responsible to accelerate the rocket, so initial thrust of the blast

$$=ma=3.5\times10^4\times10=3.5\times10^5\ N$$

23 Two forces are such that the sum of their magnitudes is 18 N and their resultant which has magnitude 12 N, is perpendicular to the smaller force. Then, the magnitudes of the forces are **[AIEEE 2002]**

(a) 12 N, 6 N (b) 13 N, 5 N
(c) 10 N, 8 N (d) 16 N, 2 N

Ans. ***(b)***

The sum of the two forces

$$A + B = 18 \quad \text{...(i)}$$

$$12 = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{...(ii)}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \quad \tan 90° = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \quad \cos\theta = \frac{-A}{B} \quad \text{...(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$A = 5\text{ N},\ B = 13\text{ N}$$

Since, $A + B = 18$ and $\cos\theta = -\frac{A}{B}$

Squaring both sides in Eq. (i), we get

$$144 = A^2 + B^2 + 2AB\cos\theta$$

On putting $\cos\theta = \frac{-A}{B}$ in above equation, we get

$$144 = A^2 + B^2 + 2AB\left(-\frac{A}{B}\right)$$

$$= A^2 + B^2 - 2A^2$$

$$\Rightarrow \quad 144 = B^2 - A^2$$

$$\Rightarrow \quad 144 = (B - A)\cdot(B + A)$$

On putting $B + A = 18$, we have

$$B - A = \frac{144}{18} = 8$$

Solving $B - A = 8$

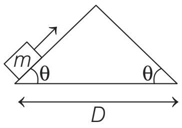
$B + A = 18$

We have, $B = 13\text{ N}$, $A = 5\text{ N}$

TOPIC 2

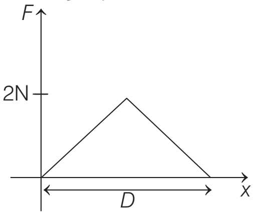
Equilibrium of a Particle and Common Forces in Mechanics

24 An object of mass m is being moved with a constant velocity under the action of an applied force of 2 N along a frictionless surface with following surface profile. **[2021, 1 Sep Shift-II]**

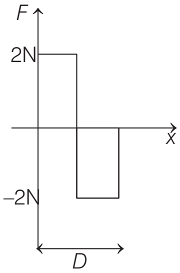


The correct applied force *versus* distance graph will be

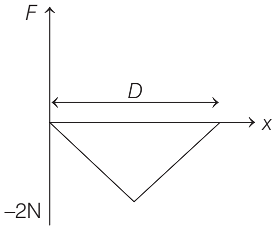
(a)



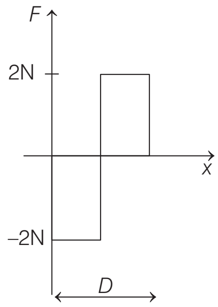
(b)



(c)



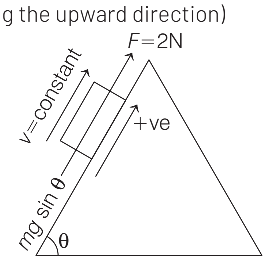
(d)



Ans. ***(b)***

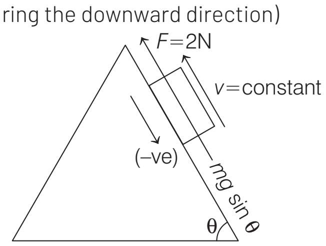
Let's draw the free body diagram,

(During the upward direction)



$F = 2\text{N} = (+\text{ ve})$ constant

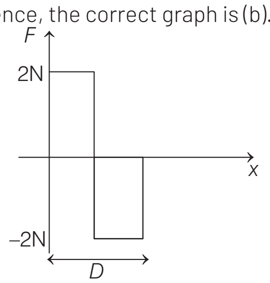
(During the downward direction)



$F = 2\text{N} = (-\text{ ve})$ constant

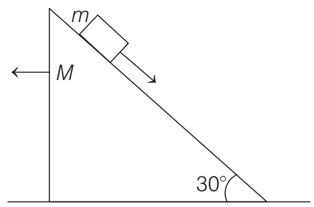
During the upward motion, the force is positive constant and during the downward motion the force is negative constant.

Hence, the correct graph is (b).



25 A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is [Given, $m = 8$ kg, $M = 16$ kg]

Assume all the surfaces shown in the figure to be frictionless. **[2021, 1 Sep Shift-II]**



(a) $\frac{4}{3}g$ (b) $\frac{6}{5}g$ (c) $\frac{3}{5}g$ (d) $\frac{2}{3}g$

Ans. ***(d)***

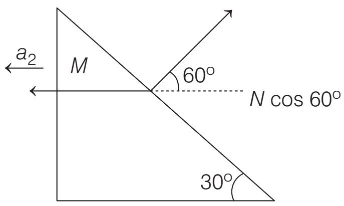
Here, both block and wedge are moving.

Consider the acceleration of the block with respect to the wedge is a_1 and the acceleration of the wedge is a_2.

Given, mass of the wedge, $M = 16$ kg

and mass of block, $m = 8$ kg

Let's draw the free body diagram of the wedge,



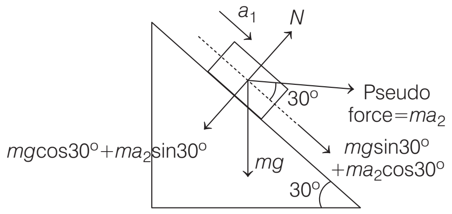
In the x-directions,

$$N\cos 60° = Ma_2$$

$$N(0.5) = 16 \times a_2$$

$$\Rightarrow \quad N = 32a_2$$

Now, draw the free body diagram of the block with respect to the wedge.



Along the perpendicular to the inclined plane,

$$N = 8g\cos30° - 8a_2\sin30°$$
$$32a_2 = 4\sqrt{3}g - 4a_2$$
$$36a_2 = 4\sqrt{3}g \Rightarrow a_2 = \frac{\sqrt{3}}{9}g$$

Along the inclined plane,

$$mg\sin30° + ma_2\cos30° = ma_1$$
$$8g\times\frac{1}{2} + 8\times\frac{\sqrt{3}}{9}g\times\frac{\sqrt{3}}{2} = 8a_1$$
$$\Rightarrow \quad a_1 = \frac{2g}{3}$$

$\therefore$ The acceleration of the block with respect to the wedge is $2g/3$.

26 A car is moving on a plane inclined at 30° to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is ....
(Take, $g = 10\ ms^{-2}$) **[2021, 31 Aug Shift-I]**

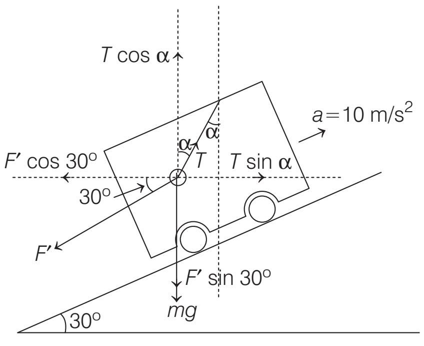
Ans. *(30)*

Given, Angle of inclination, $\theta = 30°$

Acceleration, $a = 10\ ms^{-2}$

Acceleration due to gravity, $g = 10\ ms^{-2}$

According to the question the car and bob is as shown below,



Here, F' is the pseudo force acting on the bob when we considered it from car's frame and T is the tension on the string.

In equilibrium,

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

$$\Rightarrow \quad F'\cos30° = T\sin\alpha$$
$$\frac{ma\cos30°}{\sin\alpha} = T \quad \text{...(i)}$$

where, m is the mass of the bob.

$$F'\sin30° + mg = T\cos\alpha$$
$$\Rightarrow ma\sin30° + mg = \frac{ma\cos30°}{\sin\alpha}(\cos\alpha)$$

[$\because$ using Eq. (i)]

$$\Rightarrow \quad a\sin30° + g = \frac{a\cos30°}{\sin\alpha}(\cos\alpha)$$
$$10\times\frac{1}{2} + 10 = \frac{10\times\sqrt{3}}{2}\cot\alpha$$
$$\Rightarrow \quad \frac{1}{2} + 1 = \frac{\sqrt{3}}{2}\cot\alpha$$
$$\Rightarrow \quad \frac{3}{2} = \frac{\sqrt{3}}{2}\cot\alpha$$
$$\text{or} \quad \cot\alpha = \sqrt{3}$$
$$\Rightarrow \quad \alpha = 30°$$

27 **Statement I** If three forces $\mathbf{F}_1 \cdot \mathbf{F}_2$ and $\mathbf{F}_3$ are represented by three sides of a triangle and $\mathbf{F}_1 + \mathbf{F}_2 = -\mathbf{F}_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement II A triangle made up of three forces $\mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the most appropriate answer from the options given below. **[2021, 31 Aug Shift-II]**

(a) Statement I is false but statement II is true.
(b) Statement I is true but statement II is false.
(c) Both statement I and statement II are false.
(d) Both statement I and statement II are true.

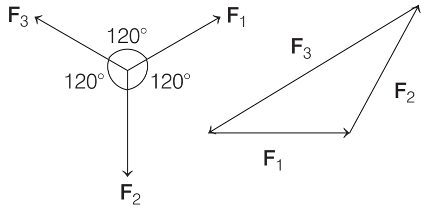
Ans. *(d)*

Three forces $\mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$ are acting on a body and if this body is in equilibrium, then resultant of these three forces must be zero i.e. $\mathbf{F}_{net} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$$\Rightarrow \quad \mathbf{F}_1 + \mathbf{F}_2 = -\mathbf{F}_3$$

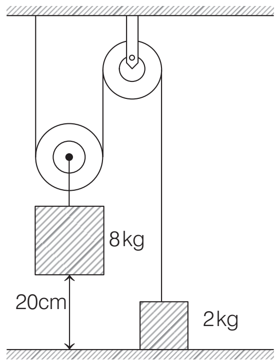
This situation can be shown graphically by three concurrent forces at 120° with each others.

or, by three forces in the same order along three sides of a triangle.



Hence, both statement I and statement II are true.

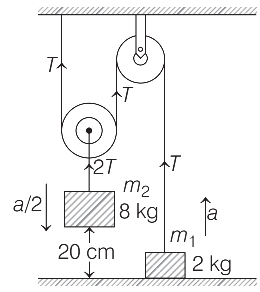
28 The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest.
(Use, $g = 10\ m/s^2$)
[2021, 27 Aug Shift-II]



(a) 0.34 s (b) 0.2 s
(c) 0.25 s (d) 0.4 s

Ans. *(d)*

According to the given figure, and free body diagram



masses of two bodies $m_1 = 2$ kg $m_2 = 8$ kg,

Acceleration of mass $m_1 = a$

Acceleration of mass $m_2 = \frac{a}{2}$

Tension $= T$

Distance between m_2 and ground $= 20$ cm $= 0.2$ m

Initial velocity $u = 0$

Equation of motion of 2kg block,

$$\therefore \quad T - 2g = 2a \quad \text{...(i)}$$

Equation of motion of 8 kg block,

and $8g - 2T = 8\frac{a}{2} \Rightarrow 4g - T = 2a$ ...(ii)

From Eqs. (i) and (ii),

$$T - 2g = 4g - T$$

$\Rightarrow$ $2T = 6g \Rightarrow T = 3g$

Substituting the value in Eq. (i) we get

$$3g - 2g = 2a \Rightarrow g = 2a$$

$\Rightarrow$ $a = \frac{g}{2} = 5\,\text{ms}^{-2}$

$\therefore$ $a_1 = 5\,\text{ms}^{-2}$ and $a_2 = \frac{5}{2}\,\text{ms}^{-2}$

Since, $s = ut + \frac{1}{2}at^2$

$\therefore$ $\frac{20}{100} = 0 + \frac{1}{2}\frac{5}{2} \times t^2 \Rightarrow t^2 = \frac{20 \times 4}{5 \times 100}$

$\Rightarrow$ $t = \frac{2}{5} = 0.4\,\text{s}$

29 A body of mass m is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$. if the time of ascent is half of the time of descent. The value of x is ....

[2021, 20 July Shift-II]

Ans. *(3)*

Let us assume that,

T_A be the time of ascent of body and T_D be the time of descent of body.

From second equation of motion, we have

$$s = ut + \frac{1}{2}at^2$$

During the time of ascent, the distance covered by the body,

$$s = \frac{1}{2}a_A t_A^2 \quad \text{...(i)}$$

and during the time of descent, the distance covered by the body,

$$s = \frac{1}{2}a_D t_D^2 \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{1}{2}a_A t_A^2 = \frac{1}{2}a_D t_D^2$$

$\Rightarrow$ $a_A t_A^2 = a_D t_D^2 \Rightarrow \frac{t_A^2}{t_D^2} = \frac{a_D}{a_A}$

$\Rightarrow$ $\frac{t_A^2}{t_D^2} = \frac{g\sin\theta - \mu g\cos\theta}{g\sin\theta + \mu g\cos\theta}$

$\Rightarrow$ $\frac{t_A}{t_D} = \sqrt{\frac{g\sin\theta - \mu g\cos\theta}{g\sin\theta + \mu g\cos\theta}}$

$$= \sqrt{\frac{g\sin 30° - \mu g\cos 30°}{g\sin 30° + \mu g\cos 30°}}$$

$\Rightarrow$ $\frac{1}{2} = \sqrt{\frac{1-\sqrt{3}\mu}{1+\sqrt{3}\mu}}$ $\left\{\because \frac{t_A}{t_D} = \frac{1}{2}\text{(given)}\right\}$

$\Rightarrow$ $1 + \sqrt{3}\mu = 4 - 4\sqrt{3}\mu$

$\Rightarrow$ $\sqrt{3}\mu + 4\sqrt{3}\mu = 4 - 1$

$\Rightarrow$ $\mu(\sqrt{3} + 4\sqrt{3}) = 3 \Rightarrow \mu(5\sqrt{3}) = 3$

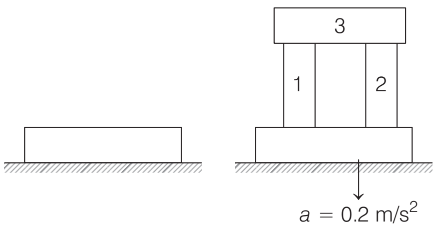
$\Rightarrow$ $\mu = \frac{\sqrt{3}}{5}$ ....(iii)

According to question, the coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$, therefore, on comparing it with Eq. (iii), we can write $x = 3$.

30 A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s^2. The normal reaction R by the floor, if mass of the iron cylinders are equal and of 20 kg each, is ...... N.

[Take, $g = 10\,\text{m/s}^2$ and $\mu_s = 0.2$]

[2021, 20 July Shift-I]

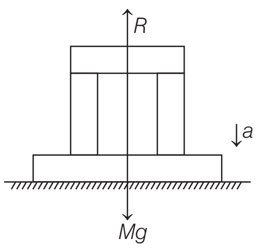


(a) 716 (b) 686 (c) 714 (d) 684

Ans. *(b)*

The force equation in vertical direction is

$$Mg - R = Ma$$



where, M = collective mass of block and all three iron cylinders

$= 10 + 3 \times 20 = 70\,\text{kg}$

a = acceleration of block $= 0.2\,\text{ms}^{-2}$

$g = 10\,\text{ms}^{-2}$ and $\mu_s = 0.2$

and R = normal reaction

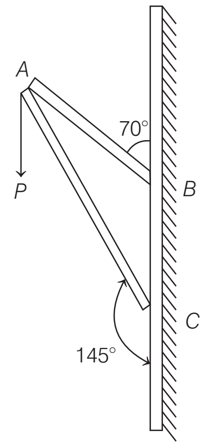
Force along verticle axis $Mg - R = Ma$

$\therefore$ $70g - R = 70 \times 0.2$

$\Rightarrow$ $R = 70 \times 10 - 14$

$= 700 - 14 = 686\,\text{N}$

31 Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force **P** of magnitude 100 N is applied at point A of the frame.



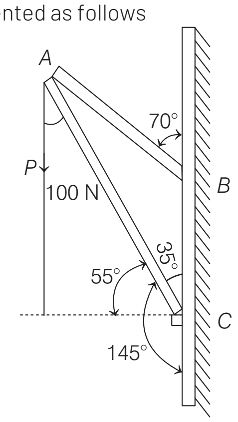
Suppose the force is **P** resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is x N. The value of x, to the nearest integer, is ....

[Given, $\sin(35°) = 0.573$, $\cos(35°) = 0.819$, $\sin(110°) = 0.939$, $\cos(110°) = -0.342$]

[2021, 16 March Shift-I]

Ans. *(82)*

If the force **P** of magnitude 100 N is resolved parallel to the arms AB and AC of the frame, the above figure will be represented as follows



Component of force along $AC = 100\cos 35°\,\text{N}$

$= 100 \times 0.819\,\text{N} = 81.9\,\text{N} \approx 82\,\text{N}$

This is the required magnitude of the resolved component along the arm AC.

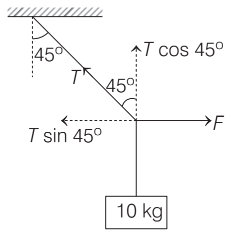
Compare with given in question, $x = 82$.

32 A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then, F equals (Take, $g = 10\,\text{ms}^{-2}$ and the rope to be massless) **[2020, 7 Jan Shift-II]**

(a) 75 N (b) 70 N (c) 100 N (d) 90 N

Ans. *(c)*

Given situation is as shown below.



We resolve tension T in string into vertical and horizontal components.

For equilibrium,

$$F = T\sin 45° \quad \ldots\ldots\text{(i)}$$

and $$Mg = T\cos 45° \quad \ldots\ldots\text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

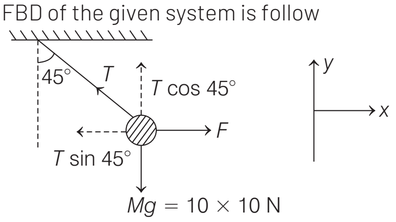
$$\frac{F}{Mg} = \tan 45° \text{ or } F = Mg = 10 \times 10 = 100\text{ N}$$

33 A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the mass, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is (Take, $g = 10\,\text{ms}^{-2}$) **[2019, 9 Jan Shift-II]**

(a) 70 N (b) 200 N
(c) 100 N (d) 140 N

Ans. *(c)*

FBD of the given system is follow



Let T = tension in the rope.

For equilibrium condition of the mass,

$\Sigma F_x = 0$ (force in x-direction)

$\Sigma F_y = 0$ (force in y-direction)

When $\Sigma F_x = 0$, then

$$\because \quad F = T\sin 45° \quad \ldots\text{(i)}$$

When $\Sigma F_y = 0$, then

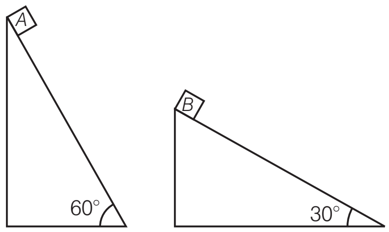
$$Mg = T\cos 45° \quad \ldots\text{(ii)}$$

Using Eqs. (i) and (ii), we get

$$\Rightarrow \frac{F}{Mg} = \frac{T\sin 45°}{T\cos 45°} \Rightarrow \frac{F}{Mg} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\Rightarrow F = Mg = 10 \times 10 = 100\text{ N}$$

34 Two frictionless inclined plane making the angles 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? **[AIEEE 2010]**



(a) $4.9\,\text{ms}^{-2}$ in horizontal direction
(b) $9.8\,\text{ms}^{-2}$ in vertical direction
(c) zero
(d) $4.9\,\text{ms}^{-2}$ in vertical direction

Ans. *(d)*

Force applying on the block

$$F = mg\sin\theta$$

or $$mg\sin\theta = ma$$

$\therefore$ $$a = g\sin\theta$$

where, a is along the inclined plane.

$\therefore$ Vertical component of acceleration is $g\sin^2\theta$.

$\therefore$ Relative vertical acceleration of A with respect to B is

$$g(\sin^2 60° - \sin^2 30°) = \frac{g}{2} = 4.9\,\text{ms}^{-2}$$

[in vertical direction]

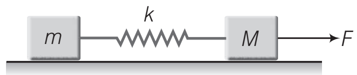
35 A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then, a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m. **[AIEEE 2007]**

(a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{m}$

(c) $\frac{mF}{(m+M)}$ (d) $\frac{MF}{(m+M)}$

Ans. *(c)*

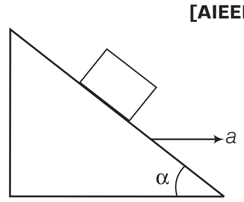
Acceleration of system, $a = \frac{F}{m+M}$



So, force acting on mass,

$$F = ma = \frac{mF}{m+M}$$

36 A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then, a is equal to **[AIEEE 2005]**

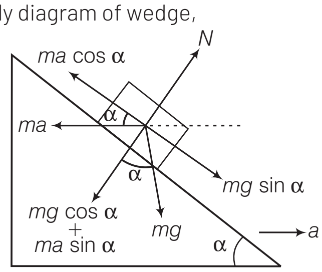


(a) $\frac{g}{\tan\alpha}$ (b) g cosec

(c) g (d) $g\tan\alpha$

Ans. *(d)*

In the frame of wedge, the force diagram of block is shown in figure. From free body diagram of wedge,

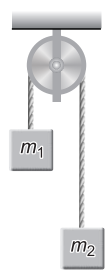


For block to remain stationary,

$$ma\cos\alpha = mg\sin\alpha$$

or $$a = g\tan\alpha$$

37 Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift is free to move ? ($g = 9.8\,\text{ms}^{-2}$) **[AIEEE 2004]**



(a) $0.2\ ms^{-2}$ (b) $9.8\ ms^{-2}$
(c) $5\ ms^{-2}$ (d) $4.8\ ms^{-2}$

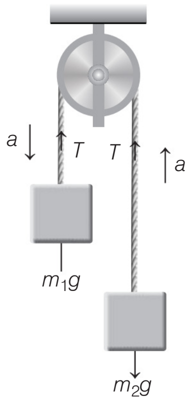
Ans. *(d)*

On releasing, the motion of the system will be according to figure.

$$m_1 g - T = m_1 a \quad ...(i)$$

and $T - m_2 g = m_2 a$ ...(ii)

On solving, $a = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right) g$ ...(iii)



Here, $m_1 = 5$ kg, $m_2 = 4.8$ kg, $g = 9.8\ ms^{-2}$

$$\therefore \quad a = \left(\frac{5 - 4.8}{5 + 4.8}\right) \times 9.8$$

$$= \frac{0.2}{9.8} \times 9.8 = 0.2\ ms^{-2}$$

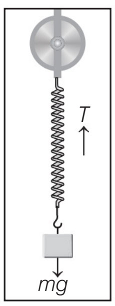
38 A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of $5\ ms^{-2}$, the reading of the spring balance will be **[AIEEE 2003]**

(a) 24 N (b) 74 N (c) 15 N (d) 49 N

Ans. *(a)*

In stationary position, spring balance reading $= mg = 49$

or $m = \dfrac{49}{9.8} = 5$ kg



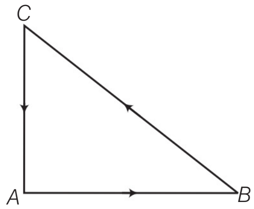
When lift moves downward,

$$mg - T = ma$$

Reading of balance,

$$T = mg - ma$$
$$= 5(9.8 - 5)$$
$$= 5 \times 4.8 = 24.0\ \text{N}$$

39 Three forces start acting simultaneously on a particle moving with velocity **v**. These forces are represented in magnitude and direction by the three sides of a Δ *ABC* (as shown). The particle will now move with velocity **[AIEEE 2003]**



(a) less than **v**
(b) greater than **v**
(c) $|v|$ in the direction of largest force *BC*
(d) **v**, remaining unchanged

Ans. *(d)*

Resultant force is zero, as three forces acting on the particle can be represented in magnitude and direction by three sides of a triangle in same order. Hence, by Newton's 2nd law $\left(F = m\dfrac{dv}{dt}\right)$, the velocity (v) of particle will be same.

40 A block of mass *M* is pulled along a horizontal frictionless surface by a rope of mass *m*. If a force *P* is applied at the free end of the rope, the force exerted by the rope on the block is **[AIEEE 2003]**

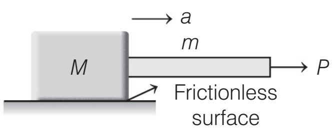
(a) $\dfrac{Pm}{M + m}$ (b) $\dfrac{Pm}{M - m}$

(c) P (d) $\dfrac{PM}{M + m}$

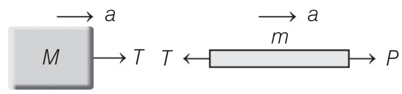
Ans. *(d)*

Let acceleration of system (rope + block) be *a* along the direction of applied force. Then,

$$a = \frac{P}{M + m}$$



Draw the FBD of block and rope as shown in figure.



where, *T* is the required parameter.

For block, $T = Ma$

$$\Rightarrow \quad T = \frac{MP}{M + m}$$

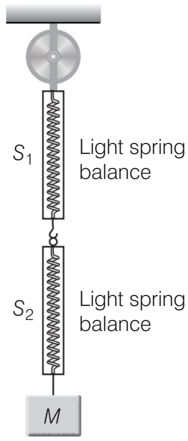
41 A light spring balance hangs from the hook of the other light spring balance and a block of mass *M* kg hangs from the former one. Then, the true statement about the scale reading is **[AIEEE 2003]**

(a) Both the scales read *M* kg each
(b) The scale of the lower one reads *M* kg and of the upper one zero
(c) The reading of the two scales can be anything but the sum of the readings will be *M* kg
(d) Both the scales read *M*/2 kg

Ans. *(a)*

The arrangement is shown in figure.

Now, draw the free body diagram of the spring balances and block.



For equilibrium of block, $T_1 = Mg$

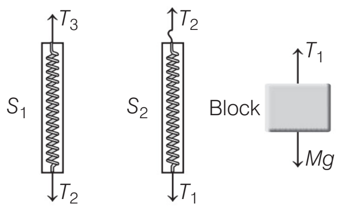
where, T_1 = Reading of S_2

For equilibrium of S_2, $T_2 = T_1$

where, T_2 = Reading of S_1

For equilibrium of S_1,

$$T_2 = T_3$$



Hence, $T_1 = T_2 = Mg$

So, both scales read *M* kg.

42 A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively **[AIEEE 2002]**

(a) g, g (b) $g-a, g-a$
(c) $g-a, g$ (d) a, g

Ans. *(c)*

Apparent weight of ball,

$w' = w - R$ [where, R=normal reaction]

$R = ma$ [acting upward]

$w' = mg - ma = m(g-a)$

Hence, apparent acceleration in the lift is $g-a$. Now, if the man is standing stationary on the ground, then the apparent acceleration of the falling ball is g.

43 When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed, then the acceleration of the particle is **[AIEEE 2002]**

(a) $\frac{F_1}{m}$ (b) $\frac{F_2F_3}{mF_1}$ (c) $\frac{(F_2-F_3)}{m}$ (d) $\frac{F_2}{m}$

Ans. *(a)*

The particle remains stationary under the acting of three forces $\mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$, it means resultant force is zero,

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$$

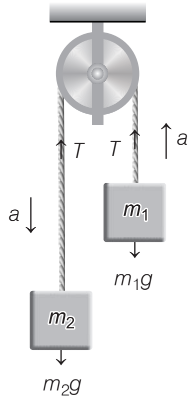
Since, in second cases F_1 is removed (in terms of magnitude we are talking now), the forces acting are F_2 and F_3 the resultant of which has the magnitude as F_1, so acceleration of particle is $\frac{F_1}{m}$ in the direction opposite to that of $\mathbf{F}_1$.

44 A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is **[AIEEE 2002]**

(a) 8 : 1 (b) 9 : 7
(c) 4 : 3 (d) 5 : 3

Ans. *(b)*

As the string is inextensible, both masses have the same acceleration a. Also, the pulley is massless and frictionless, hence the tension at both ends of the string is the same. Suppose, the mass m_2 is greater than mass m_1, so the heavier mass m_2 is accelerating downward and the lighter mass m_1 is accelerating upwards.



Therefore, by Newton's 2nd law

$$T - m_1 g = m_1 a \quad \text{...(i)}$$

and $$m_2 g - T = m_2 a \quad \text{...(ii)}$$

After solving Eqs. (i) and (ii), we get

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)} \cdot g = \frac{g}{8} \quad \text{[given]}$$

So, $$\frac{g}{8} = \frac{m_2(1 - m_1/m_2)}{m_2(1 + m_1/m_2)} \cdot g \quad \text{...(iii)}$$

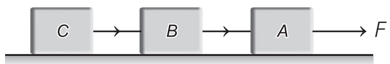
Let $\frac{m_1}{m_2} = x$

Thus, Eq. (iii) becomes

$$\frac{1-x}{1+x} = \frac{1}{8} \text{ or } x = \frac{7}{9} \text{ or } \frac{m_2}{m_1} = \frac{9}{7}$$

So, the ratio of the masses is 9 : 7.

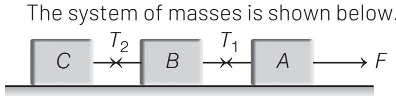
45 Three identical blocks of masses $m = 2$ kg are drawn by a force $F = 10.2$ N with an acceleration of 0.6 ms^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C? **[AIEEE 2002]**



(a) 9.2 (b) 7.8
(c) 4 (d) 9.8

Ans. *(b)*

The system of masses is shown below.



From the figure,

$$F - T_1 = ma \quad \text{...(i)}$$

and $$T_1 - T_2 = ma \quad \text{...(ii)}$$

Eq. (i) gives,

$$10.2 - T_1 = 2 \times 0.6$$

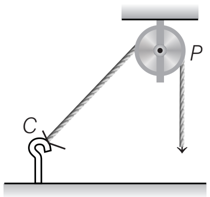
$\Rightarrow$ $$T_1 = 10.2 - 1.2 = 9 \text{ N}$$

Again, from Eq. (ii), we get

$$9 - T_2 = 2 \times 0.6$$

$\Rightarrow$ $$T_2 = 9 - 1.2 = 7.8 \text{ N}$$

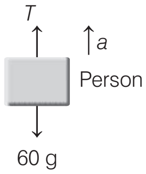
46 One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope? **[AIEEE 2002]**



(a) 16 (b) 6 (c) 4 (d) 80

Ans. *(c)*

The free body diagram of the person can be drawn as



Let the person move up with an acceleration a, then

$$T - 60g = 60a$$

$\Rightarrow$ $$a_{max} = \frac{T_{max} - 60g}{60}$$

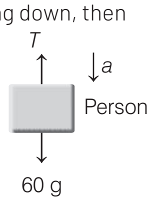
or $$a_{max} = \frac{360 - 60g}{60} \rightarrow -\text{ve value}$$

That means, it is not possible to climb up on the rope.

Even in this problem, it is not possible to remain at rest on rope.

Hence, no option is correct.

But, if they will ask for the acceleration of climbing down, then



$$60g - T = 60a$$

$\Rightarrow$ $$60g - T_{max} = 60a_{min}$$

or $$a_{min} = \frac{60g - 360}{60} = 4 \text{ ms}^{-2}$$

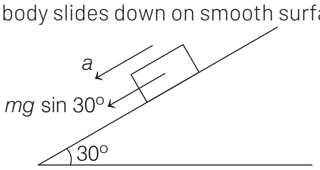
TOPIC 3

Friction

47 When a body slides down from rest along a smooth inclined plane making an angle of 30° with the horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time αT, where α is a constant greater than 1. The coefficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2 - 1}{\alpha^2}\right)$, where x is .... **[2021, 1 Sep Shift-II]**

Ans. *(3)*

Let's draw the free body diagram when body slides down on smooth surface



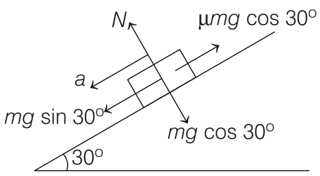
For smooth surface,

$$ma = mg \sin 30^\circ$$
$$a = g \sin 30^\circ = g/2$$

Distance covered by the block on the smooth surface in time T,

$$s = ut + \frac{1}{2}at^2$$
$$s = 0 + \frac{1}{2}\left(\frac{g}{2}\right)T^2$$
$$\Rightarrow \quad s = \left(\frac{g}{4}\right)T^2 \quad \text{...(i)}$$

Now, let's draw the free body diagram when body slides down on rough surface



For rough surface,

$$ma = mg \sin 30^\circ - \mu\, mg \cos 30^\circ$$
$$a = g \sin 30^\circ - \mu\, g \cos 30^\circ$$

Distance covered by the block on the rough surface in time αT,

$$s = ut + \frac{1}{2}at^2$$
$$s = 0 + \frac{1}{2}(g \sin 30^\circ - \mu\, g \sin 30^\circ)\, t^2$$
$$s = \frac{g}{4}(1 - \sqrt{3}\mu)(\alpha T)^2 \quad \text{...(ii)}$$

Distance covered by the block is same for both the case,

$$\Rightarrow \quad \frac{g}{4}(1 - \sqrt{3}\mu)(\alpha T)^2 = \frac{g}{4}T^2$$

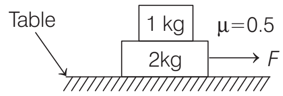
[from Eq. (i) & Eq. (ii)]

$$\Rightarrow \quad 1 - \sqrt{3}\mu = \frac{1}{\alpha^2} \Rightarrow \mu = \left(\frac{\alpha^2 - 1}{\alpha^2}\right)\frac{1}{\sqrt{3}}$$

Comparing with $\mu = \left(\frac{\alpha^2 - 1}{\alpha^2}\right)\frac{1}{\sqrt{x}}$

The value of the $x = 3$.

48 The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is ....N. (Take, $g = 10\,\text{ms}^{-2}$) **[2021, 26 Aug Shift-II]**



Ans. *(15)*

Given, coefficient of static friction, $\mu = 0.5$

Value of acceleration due to gravity, $g = 10\,\text{ms}^{-2}$

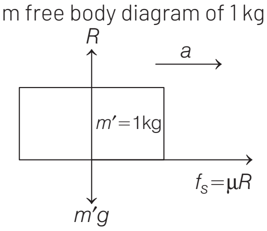
For complete system to move together,

$$F = ma$$

Here, m is total mass.

$$F = (1 + 2)\, a = 3a \quad \text{...(i)}$$

From free body diagram of 1 kg block,



Balance forces in horizontal direction,

$$F_s = \mu R = m'a \quad \text{...(ii)}$$

Balance forces in vertical direction,

$$R = m'g$$

Put value of R in Eq. (ii),

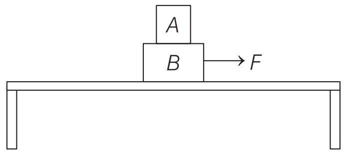
$$\mu m'g = m'a$$
$$\Rightarrow \quad 0.5 \times 1 \times 10 = 1 \times a \Rightarrow a = 5\,\text{ms}^{-2}$$

Put the value of a in Eq. (i), we get

$$F = 3 \times 5 = 15\,\text{N}$$

Thus, the maximum horizontal force required to move block together is 15 N.

49 Two blocks A and B of masses $m_A = 1\,\text{kg}$ and $m_B = 3\,\text{kg}$ are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is [Take, $g = 10\,\text{m/s}^2$] **[2019, 10 April Shift-II]**



(a) 12 N (b) 16 N (c) 8 N (d) 40 N

Ans. *(b)*

Acceleration a of system of blocks A and B is

$$a = \frac{\text{Net force}}{\text{Total mass}} = \frac{F - f_1}{m_A + m_B}$$

where, f_1 = friction between B and the surface

$$= \mu(m_A + m_B)g$$

So, $$a = \frac{F - \mu(m_A + m_B)g}{(m_A + m_B)} \quad \text{...(i)}$$

Here, $\mu = 0.2$, $m_A = 1\,\text{kg}$, $m_B = 3\,\text{kg}$, $g = 10\,\text{ms}^{-2}$

Substituting the above values in Eq. (i), we have

$$a = \frac{F - 0.2(1 + 3) \times 10}{1 + 3}$$
$$a = \frac{F - 8}{4} \quad \text{...(ii)}$$

Due to acceleration of block B, a pseudo force F' acts on A.

This force F' is given by

$$F' = m_A a$$

where, a is acceleration of A and B caused by net force acting on B.

For A to slide over B; pseudo force on A, i.e. F' must be greater than friction between A and B.

$$\Rightarrow \quad m_A a \geq f_2$$

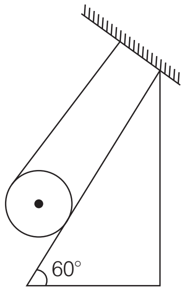
We consider limiting case,

$$m_A a = f_2 \Rightarrow m_A a = \mu(m_A)g$$
$$\Rightarrow \quad a = \mu g = 0.2 \times 10 = 2\,\text{ms}^{-2} \quad \text{...(iii)}$$

Putting the value of a from Eq. (iii) into Eq. (ii), we get

$$\frac{F - 8}{4} = 2$$
$$\therefore \quad F = 16\,\text{N}$$

50 A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is

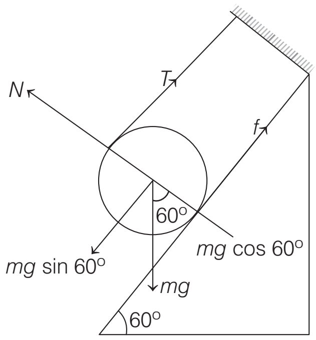


(The coefficient of static friction, μ_s, is 0.4) **[2021, 18 March Shift-II]**

(a) $\frac{7}{2}mg$ (b) $5\,mg$ (c) $\frac{mg}{5}$ (d) 0

Ans. *(c)*

Let's draw the free body diagram of the solid cylinder.



Using the condition of the equilibrium of the cylinder;

In the direction of inclined plane,

$T + f - mg\sin 60° = 0$

In the perpendicular direction of inclined plane,

$N - mg\cos 60° = 0$

$\Rightarrow N = mg\cos 60°$

The frictional force between the rough surface and cylinder is

$f = \mu_s N$

$\Rightarrow f = 0.4\,mg\cos 60° \Rightarrow f = 0.2\,mg$

or $f = \frac{mg}{5}$

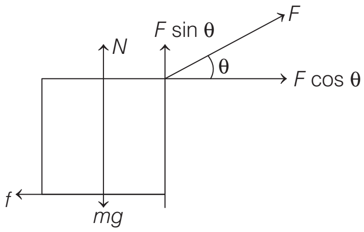
51 A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $1/\sqrt{3}$. It is desired to make the body move by applying the minimum po ssible force F newton. The value of F will be ....
(Round off to the nearest integer)
(Take, $g = 10\,ms^{-2}$)
[2021, 17 March Shift-II]

Ans. *(5)*

Given, mass of the body, $m = 1$ kg

Coefficient of static friction, $\mu = 1/\sqrt{3}$

Let's draw the free body diagram of the block



Using the condition of the equilibrium

In x-direction,

$$F\cos\theta = f = \phi = \mu N \quad \text{... (i)}$$

In y-direction,

$$F\sin\theta + N = mg \quad \text{... (ii)}$$

$$\Rightarrow \quad N = mg - F\sin\theta$$

Substituting the value of N in Eq. (i), we get

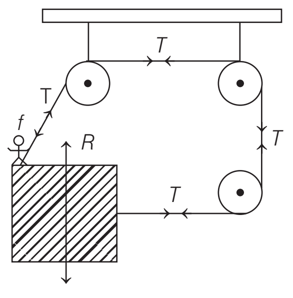
$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta} \Rightarrow F = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Substituting the values in the above equation, we get

$$F = \frac{\frac{1}{\sqrt{3}} \times 10}{\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}} \Rightarrow F = 5\,\text{N}$$

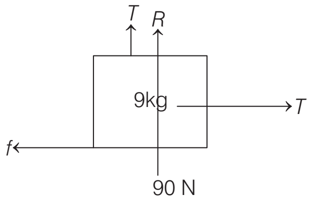
Hence, the body move by applying minimum possible force of 5 N. So, the value of F will be 5.

52 A boy of mass 4 kg is standing on a piece of wood having mass 5 kg . If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope, so that the piece of wood does not move from its place is .... N. (Round off to the nearest integer)
(Take, $g = 10\,ms^{-2}$)
[2021, 17 March Shift-II]



Ans. *(30)*

The free body diagram for the wooden block is shown below



Using the condition of the equilibrium,

In the x-direction, the summation of all the forces is to be zero.

$f - T = 0$

$\mu R - T = 0$

$T = 0.5\,R$ ...(i)

In the y-direction, , the summation of all the forces is to be zero.

$T + R - 90 = 0$

$\Rightarrow 0.5\,R + R - 90 = 0 \Rightarrow R = 60$ N

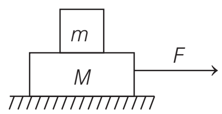
Hence, the normal force on the wooden block is 60 N.

Using the Eq. (i),

$T = 0.5(60) = 30$ N

Hence, the maximum value of the tension in the rope, so that wooden block will not move is 30N.

53 Two blocks ($m = 0.5$ kg and $M = 4.5$ kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is 3/7. Then, the maximum horizontal force that can be applied on the larger block so that the blocks move together is ...... N.
(Round off to the nearest integer. Take, $g = 9.8\,ms^{-2}$)



[2021, 17 March Shift-I]

Ans. *(21)*

When both the blocks move together as a system, then acceleration of this system will be given as

$$a = \frac{F}{m+M} \quad \text{...(i)}$$

Frictional force on mass,

$$f = ma \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$f = m\left(\frac{F}{m+M}\right)$$

For no slipping, $f \le \mu mg$ [$\because \mu$ being the coefficient of static friction]

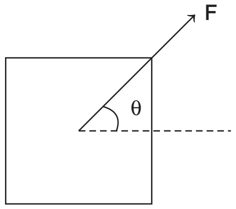
$\Rightarrow \quad m\left(\dfrac{F}{m+M}\right) \le \mu mg \Rightarrow$

$F \le \mu(m+M)g$

$\therefore \quad F_{\max} = \dfrac{3}{7}(0.5+4.5)\times 9.8 \Rightarrow F_{\max} = 21\,\text{N}$

54 A block of mass m slides along a floor, while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_k. Then, the block's acceleration a is given by (g is acceleration due to gravity)

[2021, 16 March Shift-I]



(a) $-\dfrac{F}{m}\cos\theta - \mu_k\left(g - \dfrac{F}{m}\sin\theta\right)$

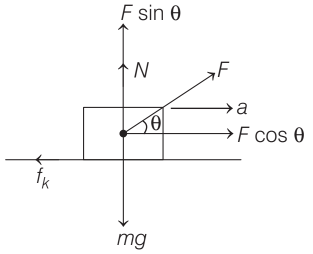
(b) $\dfrac{F}{m}\cos\theta - \mu_k\left(g - \dfrac{F}{m}\sin\theta\right)$

(c) $\dfrac{F}{m}\cos\theta - \mu_k\left(g + \dfrac{F}{m}\sin\theta\right)$

(d) $\dfrac{F}{m}\cos\theta + \mu_k\left(g - \dfrac{F}{m}\sin\theta\right)$

Ans. ***(b)***

The diagram and the required components of force on given block are shown below



From the above diagram,

$$N = mg - F\sin\theta \quad \text{...(i)}$$

where, N = normal force

and $F\cos\theta - f_k = ma$

$\Rightarrow F\cos\theta - \mu_k N = ma$ ...(ii)

where, f_k = kinetic friction force.

From Eq. (i) and Eq. (ii), we get

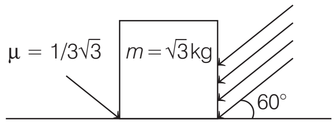
$$F\cos\theta - \mu_k(mg - F\sin\theta) = ma$$

$$\Rightarrow \quad a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right)$$

This is the required acceleration of the block.

55 As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $1/3\sqrt{3}$. The critical force to be applied on the vertical surface as shown at an angle $60°$ with horizontal such that it does not move, will be $3x$. The value of x will be .... [$g = 10\ \text{ms}^{-2}$; $\sin 60° = \dfrac{\sqrt{3}}{2}$; $\cos 60° = \dfrac{1}{2}$]

[2021, 26 Feb Shift-I]



Ans. ***(3.33)***

Given, mass of block, $m = \sqrt{3}$ kg

Coefficient of friction, $\mu = 1/3\sqrt{3}$

According to diagram,

Let F be the force applied on the body,

w be the weight $(= mg)$,

N be the normal reaction.

Friction force $f = \mu N$

For no movement of body along X-axis, net force along X-axis should be zero.

If, F_y be the net force along y-axis then it will also be zero because body is not accelerating at all.

$\therefore \quad N = F\sin 60° + mg$

$\Rightarrow \quad N = \dfrac{\sqrt{3}}{2}F + 10\sqrt{3}$ ...(i)

Similarly, $F_x = F\cos 60° - \mu N = 0$

From Eq. (i), we get

$\Rightarrow \quad \dfrac{F}{2} - \dfrac{1}{3\sqrt{3}}\left(\dfrac{\sqrt{3}}{2}F + 10\sqrt{3}\right) = 0$

$\Rightarrow \quad \dfrac{F}{2} = \dfrac{F}{6} + \dfrac{10}{3} \Rightarrow \dfrac{F}{2} - \dfrac{F}{6} = \dfrac{10}{3}$

$\Rightarrow \quad F = 10\,\text{N}$

Given, $\quad F = 3x$

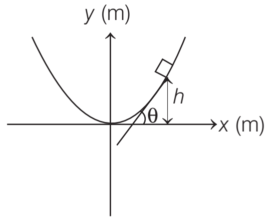
$\Rightarrow \quad x = \dfrac{10}{3} = 3.33$

56 An inclined plane is bent in such a way that the vertical cross-section is given by $y = \dfrac{x^2}{4}$ where, y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is .... cm.

[2021, 24 Feb Shift-I]

Ans. ***(25)***

The graph for given equation is shown below



At maximum height, the slope of tangent drawn,

$$\tan\theta = \frac{dy}{dx} = \frac{2x}{4} = \frac{x}{2} \quad \left[\because y = \frac{x^2}{4}\right]$$

$\Rightarrow \quad 0.5 = \dfrac{x}{2} \quad (\because \mu = \tan\theta)$

$\Rightarrow \quad x = 1\,\text{m}$

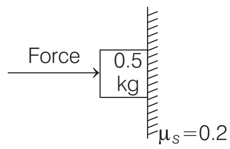
$\therefore \quad y = \dfrac{x^2}{4} = \dfrac{1}{4} = 0.25\,\text{m} = 25\,\text{cm}$

57 The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be .... N. [Take, $g = 10\ \text{ms}^{-2}$]

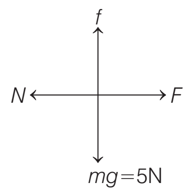
[2021, 24 Feb Shift-I]

Ans. ***(25)***

Given, coefficient of static friction, $\mu_s = 0.2$



Various forces acting on block are shown below



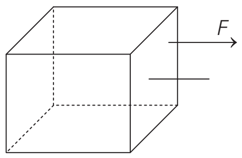
Frictional force $\le mg$

$\Rightarrow N \times 0.2 \le 5$

$\Rightarrow \quad N \le 25$

$\therefore$ Magnitude of horizontal force,

$F = N = 25\,\text{N}$

58 

Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is ....

[2020, 7 Jan Shift-II]

Ans. *(75)*

According to the given situation;

When the minimum force F capable to topple the block is applied, then the block will be on the verge of toppling.

As the block is not moving, we have friction f such that

$$f = F \quad \text{... (i)}$$

Also, note that reaction N acts from point A as block is at the verge of toppling.

To maintain the equilibrium, net torque about centre of mass C is zero.

$$\Rightarrow \quad F \cdot b + f\left(\frac{a}{2}\right) = N\left(\frac{a}{2}\right)$$

Using result of Eq. (i), we get

$$f\left(\frac{a}{2} + b\right) = N\left(\frac{a}{2}\right)$$

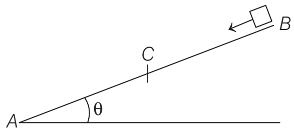
Now, $f = \mu mg$ and $N = mg$

$$\therefore \quad \mu mg\left(\frac{a}{2} + b\right) = mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad \mu\left(\frac{a}{2} + b\right) = \frac{a}{2}$$

As, $\quad \mu = 0.4 \therefore \frac{b}{a} = \frac{3}{4}$

Hence, $\quad 100 \times \frac{b}{a} = 100 \times \frac{3}{4} = 75$

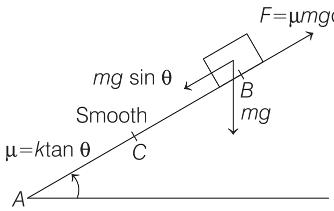
59 

A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k \tan\theta$. Then, the value of k is.........

[2020, 2 Sep Shift-I]

Ans. *(3)*

Different forces acting on the inclined plane are shown below,



As block stops at point A this means work done by component of weight down the plane is dissipated in doing work against friction.

$$\Rightarrow \quad mg\sin\theta(AB) = \mu mg\cos\theta(AC)$$

$$\Rightarrow \quad mg\sin\theta(3AC) = \mu mg\cos\theta(AC)$$

$$\Rightarrow \quad 3\tan\theta = \mu$$

Given, $\quad \mu = k\tan\theta$

Comparing both, we get

$$\Rightarrow \quad k = 3$$

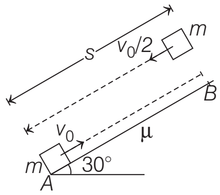
60 A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$, the nearest integer to I is ....

[2020, 3 Sep Shift-II]

Ans. *(346)*

Let μ be the coefficient of kinetic friction between the block and the inclined plane.

The free-body diagram of the given situation is shown below,



While going from A to B, the acceleration of the block is

$$a_1 = g\sin 30° + \mu g\cos 30°$$

$$\Rightarrow \quad a_1 = \frac{g}{2} + \frac{\mu g\sqrt{3}}{2}$$

$$\Rightarrow \quad a_1 = 5 + 5\sqrt{3}\mu \quad \text{...(i)} \; (g = 10\ \text{ms}^{-2})$$

Let s be the distance between A and *(b)*

From third equation of motion,

$$v_0^2 - 0 = 2a_1 s$$

$$\Rightarrow \quad s = \frac{v_0^2}{2a_1} = \frac{v_0^2}{2(5 + 5\sqrt{3}\mu)} \quad \text{...(ii)}$$

[Using Eq. (i)]

When the block comes back to its initial position (i.e., from B to A), its velocity is $\frac{v_0}{2}$.

So, the acceleration of the block while coming from B to A is

$$a_2 = g\sin 30° - \mu g\cos 30°$$

$$\Rightarrow \quad a_2 = \frac{g}{2} - \frac{\mu g\sqrt{3}}{2}$$

$$\Rightarrow \quad a_2 = 5 - 5\sqrt{3}\mu \quad \text{...(iii)}$$

Again, using third equation of motion, we get

$$\left(\frac{v_0}{2}\right)^2 - 0 = 2a_2 s \Rightarrow \frac{v_0^2}{4} = 2a_2 s$$

$$\Rightarrow \quad s = \frac{v_0^2}{8a_2}$$

$$\Rightarrow \quad s = \frac{v_0^2}{8(5 - 5\sqrt{3}\mu)} \quad \text{...(iv)}$$

[Using Eq. (iii)]

Equating Eqs. (ii) and (iv), we get

$$\frac{v_0^2}{2(5 + 5\sqrt{3}\mu)} = \frac{v_0^2}{8(5 - 5\sqrt{3}\mu)}$$

$$\Rightarrow \quad 4(5 - 5\sqrt{3}\mu) = 5 + 5\sqrt{3}\mu$$

$$\Rightarrow \quad 20 - 20\sqrt{3}\mu = 5 + 5\sqrt{3}\mu$$

$$\Rightarrow \quad 25\sqrt{3}\mu = 15$$

$$\Rightarrow \quad \mu = \frac{15}{25\sqrt{3}} = \frac{3}{5\sqrt{3}} \Rightarrow \mu = \frac{\sqrt{3}}{5}$$

So, $\quad \frac{I}{1000} = \frac{\sqrt{3}}{5} = 0.346$

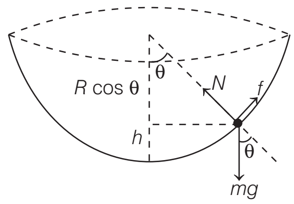
$$\Rightarrow \quad I = 0.346 \times 1000 = 346$$

61 An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is (Take, $g = 10\text{ms}^{-2}$) **[2020, 6 Sep Shift-I]**

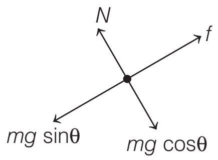
(a) 0.20 m (b) 0.45 m
(c) 0.60 m (d) 0.80 m

Ans. *(a)*

Let h be maximum height up to which insect crawls up the ditch. The free body diagram is shown,



Resolving the components of force along tangential and radial direction



For balancing, $mg\cos\theta = N$

$$mg\sin\theta = f_{max} = \mu N$$

$$\Rightarrow \quad mg\sin\theta = \mu mg\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\mu mg}{mg}$$

$$\tan\theta = \mu = 0.75 = \frac{3}{4} \Rightarrow \cos\theta = \frac{4}{5}$$

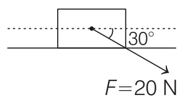
(From Pythagoras' theorem)

From diagram, $h = R - R\cos\theta$

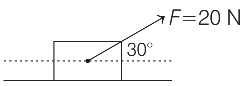
$$= R - R\left(\frac{4}{5}\right) = \frac{R}{5} \quad \text{or} \quad h = \frac{1}{5} = 0.20\,\text{m}$$

62 A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20$ N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block, the floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be (Take, $g = 10\,\text{ms}^{-2}$)

[2019, 12 April Shift-II]



(A)

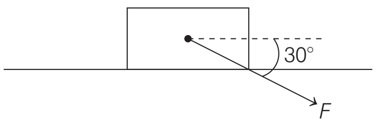


(B)

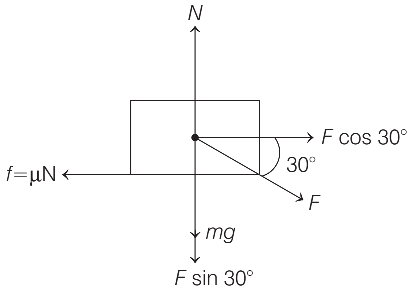
(a) $0.4\,\text{ms}^{-2}$ (b) $3.2\,\text{ms}^{-2}$
(c) $0.8\,\text{ms}^{-2}$ (d) $0\,\text{ms}^{-2}$

Ans. *(c)*

Case I Block is pushed over surface



Free body diagram of block is



In this case, normal reaction,

$$N = mg + F\sin 30°$$

$$= 5 \times 10 + 20 \times \frac{1}{2} = 60\,\text{N}$$

[Given, $m = 5$ kg, $F = 20$ N]

Force of function, $f = \mu N$

$$= 0.2 \times 60 \quad [\because \mu = 0.2]$$

$$= 12\,\text{N}$$

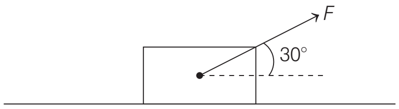
So, net force causing acceleration (a_1) is

$$F_{net} = ma_1 = F\cos 30° - f$$

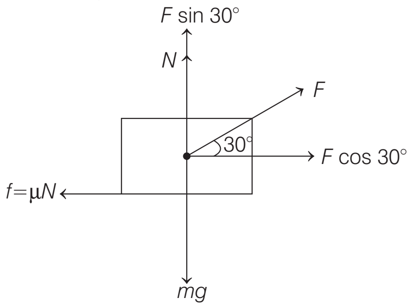
$$\Rightarrow \quad ma_1 = 20 \times \frac{\sqrt{3}}{2} - 12$$

$$\therefore \quad a_1 = \frac{10\sqrt{3} - 12}{5} \approx 1\,\text{ms}^{-2}$$

Case II Block is pulled over the surface



Free body diagram of block is,



Net force causing acceleration is

$$F_{net} = F\cos 30° - f$$

$$= F\cos 30° - \mu N$$

$$\Rightarrow F_{net} = F\cos 30° - \mu(mg - F\sin 30°)$$

If acceleration is now a_2, then

$$a_2 = \frac{F_{net}}{m}$$

$$= \frac{F\cos 30° - \mu(mg - F\sin 30°)}{m}$$

$$= \frac{20 \times \frac{\sqrt{3}}{2} - 0.2\left(5 \times 10 - 20 \times \frac{1}{2}\right)}{5}$$

$$= \frac{10\sqrt{3} - 8}{5}$$

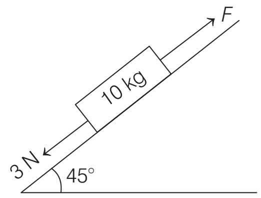
$$\Rightarrow a_2 \approx 1.8\,\text{ms}^{-2}$$

So, difference $= a_2 - a_1$

$$= 1.8 - 1 = 0.8\,\text{ms}^{-2}$$

63 A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force F, such that the block does not move downward? (Take, $g = 10\,\text{ms}^{-2}$)

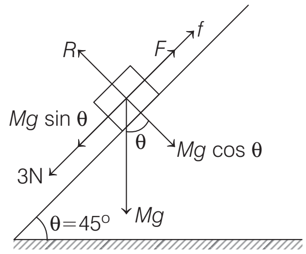
[2019, 9 Jan Shift-I]



(a) 32 N (b) 25 N
(c) 23 N (d) 18 N

Ans. *(a)*

Free body diagram, for the given figure is as follows,



For the block to be in equilibrium i.e., so that it does not move downward, then $\Sigma f_x = 0$

$$\therefore \quad 3 + Mg\sin\theta - F - f = 0$$

or $\quad 3 + Mg\sin\theta = F + f$

As, frictional force, $\quad f = \mu R$

$$\therefore \quad 3 + Mg\sin\theta = F + \mu R \quad \text{...(i)}$$

Similarly, $\quad \Sigma f_y = 0$

$$-Mg\cos\theta + R = 0$$

or $\quad Mg\cos\theta = R \quad$ ...(ii)

Substituting the value of 'R' from Eq. (ii) to Eq. (i), we get

$$3 + Mg\sin\theta = F + \mu(Mg\cos\theta) \quad \text{...(iii)}$$

Here, $M = 10$ kg, $\theta = 45°$, $g = 10\,\text{m/s}^2$

and $\mu = 0.6$

Substituting these values is Eq. (iii), we get

$3+(10\times 10\sin 45^\circ)$

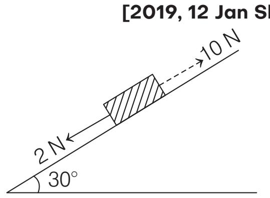
$-(0.6\times 10\times 10\cos 45^\circ)=F$

$\Rightarrow\ F=3+\frac{100}{\sqrt{2}}-\frac{60}{\sqrt{2}}=3+\frac{40}{\sqrt{2}}$

$=3+20\sqrt{2}=31.8\text{ N}$ or $F\simeq 32\text{ N}$

64 A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is (Take, $g=10\text{m/s}^2$)

[2019, 12 Jan Shift-II]

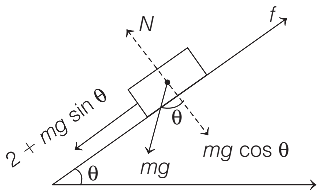


(a) $\frac{2}{3}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{1}{2}$

Ans. *(b)*

Block does not move upto a maximum applied force of 2N down the inclined plane.

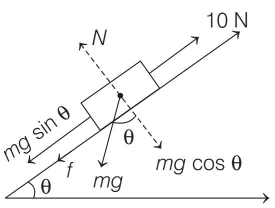


So, equating forces, we have

$2+mg\sin\theta=f$

or $2+mg\sin\theta=\mu\, mg\cos\theta$ ...(i)

Similarly, block also does not move upto a maximum applied force of 10 N up the plane.



Now, equating forces, we have

$mg\sin\theta+f=10\text{ N}$

or $mg\sin\theta+\mu mg\cos\theta=10$ ...(ii)

Now, solving Eqs. (i) and (ii), we get

$mg\sin\theta=4$ ...(iii)

and $\mu\, mg\cos\theta=6$ ...(iv)

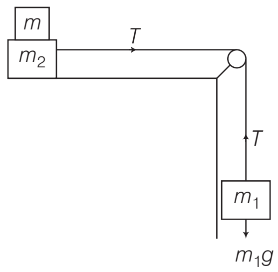
Dividing, Eqs. (iii) and (iv) we get

$\mu\cot\theta=\frac{3}{2}$

$\Rightarrow\ \mu=\frac{3\tan\theta}{2}=\frac{3\tan 30^\circ}{2}\Rightarrow\mu=\frac{\sqrt{3}}{2}$

65 Two masses $m_1=5$ kg and $m_2=10$ kg connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is

[JEE Main 2018]



(a) 18.3 kg (b) 27.3 kg

(c) 43.3 kg (d) 10.3 kg

Ans. *(b)*

Motion stops when pull due to $m_1\leq$ force of friction between m and m_2 and surface.

$\Rightarrow\ m_1g\leq\mu(m_2+m)g$

$\Rightarrow\ 5\times 10\leq 0.15(10+m)\times 10$

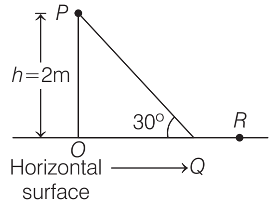
$\Rightarrow\ m\geq 23.33\text{ kg}$

Here, nearest value is 27.3 kg

So, $m_{min}=27.3\text{ kg}$

66 A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle and the rough track equals μ. The particle is released, from rest , from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x(=QR)$, are respectively close to

[JEE Main 2016]

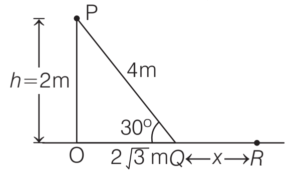


(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m

(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

Ans. *(c)*

Energy lost over path $PQ=\mu\, mg\cos\theta\times 4$



Energy lost over path $QR=\mu\, mgx$

i.e. $\mu\, mg\cos 30^\circ\times 4=\mu\, mgx\quad(\because\theta=30^\circ)$

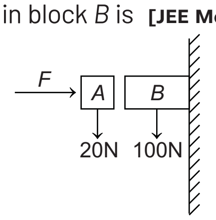
$x=2\sqrt{3}=3.45\text{ m}$

From Q to R energy loss is half of the total energy loss.

i.e. $\mu\, mgx=\frac{1}{2}\times mgh\ \Rightarrow\ \mu=0.29$

The values of the coefficient of friction μ and the distance $x(=QR)$ are 0.29 and 3.5.

67 Given in the figure are two blocks A and B of weight 20 N and 100 N respectively. These are being pressed against a wall by a force F as shown in figure. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall in block B is **[JEE Main 2015]**

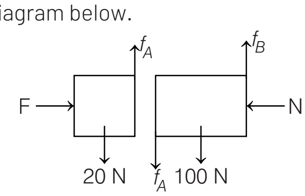


(a) 100 N (b) 80 N (c) 120 N (d) 150 N

Ans. *(c)*

Key Idea In vertical direction, weights are balanced by frictional forces.

Consider FBD of block A and B as shown in diagram below.



As the blocks are in equilibrium, balance forces are in horizontal and vertical direction.

For the system of blocks $(A+B)$.

$$F = N$$

For block A, $f_A = 20$ N and for block B,

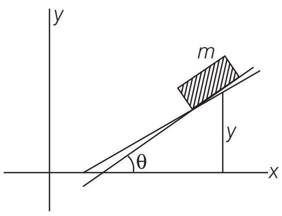
$$f_B = f_A + 100 = 120 \text{ N}$$

68 A block of mass m is placed on a surface with a vertical cross-section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is **[JEE Main 2014]**

(a) $\frac{1}{6}$ m (b) $\frac{2}{3}$ m (c) $\frac{1}{3}$ m (d) $\frac{1}{2}$ m

Ans. *(a)*

A block of mass m is placed on a surface with a vertical cross-section, then



$$\tan = \frac{dy}{dx} = \frac{d\left(\frac{x^3}{6}\right)}{dx} = \frac{x^2}{2}$$

At limiting equilibrium, we get

$$\mu = \tan\theta, \; 0.5 = x^2/2$$

$\Rightarrow$ $x^2 = 1 \Rightarrow x = \pm 1$

Now, putting the value of x in $y = x^3/6$, we get

When $x = 1$	When $x = -1$
$\therefore \; y = \frac{(1)^3}{6} = \frac{1}{6}$	$y = \frac{(-1)^3}{6} = \frac{-1}{6}$

So, the maximum height above the ground at which the block can be placed without slipping is $\frac{1}{6}$ m.

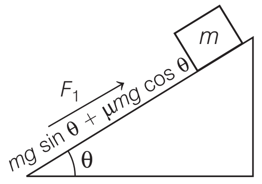
69 The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$, then the ratio F_1/F_2 is **[AIEEE 2011]**

(a) 4 (b) 1 (c) 2 (d) 3

Ans. *(d)*

$F_1 = mg(\sin\theta + \mu\cos\theta)$ [as body just in position to move up, friction force downward]

$F_2 = mg(\sin\theta - \mu\cos\theta)$
[as body just in position to slide down, friction upward]



$$\therefore \quad \frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}$$

$$= \frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = 3$$

70 Consider a car moving on a straight road with a speed of 100 ms^{-1}. The distance at which car can be stopped, is $[\mu_k = 0.5]$ **[AIEEE 2005]**

(a) 800 m (b) 1000 m
(c) 100 m (d) 400 m

Ans. *(b)*

From Newton's equations, we have

$$v^2 = u^2 - 2as$$

Given, $v = 0$ [car is stopped]

As friction provide the retardation

$$a = \mu g, v = 100 \text{ ms}^{-1}$$

$\therefore (100)^2 = 2\mu\, gs$

$$\Rightarrow \quad s = \frac{100 \times 100}{2 \times 0.5 \times 10} = \frac{100 \times 100}{5 \times 2} = 1000 \text{ m}$$

71 A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is **[AIEEE 2005]**

(a) $\mu_k = 1 - \frac{1}{n^2}$ (b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

(c) $\mu_s = 1 - \frac{1}{n^2}$ (d) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$

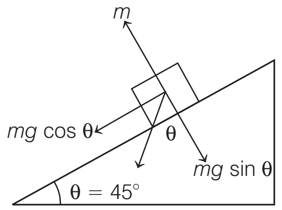
Ans. *(a)*

When friction is absent

$$ma_1 = mg\sin\theta$$

$$a = g\sin\theta$$

$$\therefore \quad s_1 = \frac{1}{2} a_1 t_1^2 \quad \text{...(i)}$$



When friction is present, friction is in opposite to the direction of motion

$$a_2 = g\sin\theta - \mu_k\, g\cos\theta$$

$$\therefore \quad s_2 = \frac{1}{2} a_2 t_2^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 t_2^2$$

$$\Rightarrow \quad a_1 t_1^2 = a_2 (nt_1)^2 \quad [\because t_2 = nt_1]$$

or $a_1 = n^2 a_2$

$$\Rightarrow \quad \frac{a_2}{a_1} = \frac{g\sin\theta - \mu_k g\cos\theta}{g\sin\theta} = \frac{1}{n^2}$$

or $\frac{g\sin 45° - \mu_k g\cos 45°}{g\sin 45°} = \frac{1}{n^2}$

or $1 - \mu_k = \frac{1}{n^2}$

or $\mu_k = 1 - \frac{1}{n^2}$

72 The upper half of an inclined plane with inclination ϕ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is given by **[AIEEE 2005]**

(a) $2\sin\phi$ (b) $2\cos\phi$
(c) $2\tan\phi$ (d) $\tan\phi$

Ans. *(c)*

According to work-energy theorem,

Work done = Change in kinetic energy

$$W = \Delta K = 0$$

$\Rightarrow$ Work done by friction + Work done by gravity = 0

$$\Rightarrow \quad -(\mu\, mg\cos\phi)\frac{l}{2} + mgl\sin\phi = 0$$

or $\frac{\mu}{2}\cos\phi = \sin\phi$

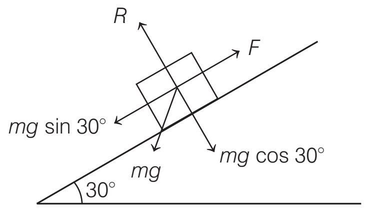
or $\mu = 2\tan\phi$

73 A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is $(g = 10 \text{ m/s}^2)$ **[AIEEE 2004]**

(a) 2.0 (b) 4.0
(c) 1.6 (d) 2.5

Ans. *(a)*

Let mass of the block be m.



Frictional force in rest position

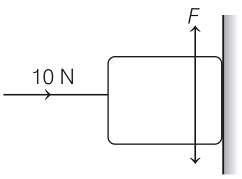
$$F = mg \sin 30°$$

[this is static frictional force and may be less than the limiting frictional force]

$\therefore \quad 10 = m \times 10 \times \frac{1}{2}$

or $\quad m = \frac{2 \times 10}{10} = 2\text{ kg}$

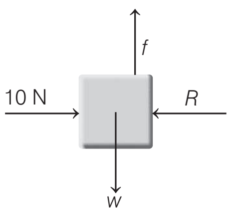
74 A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is **[AIEEE 2003]**



(a) 20 N (b) 50 N (c) 100 N (d) 2 N

Ans. ***(d)***

Let R be the normal contact force by wall on the block.



$R = 10$ N

$f_L = w$ and $f_L = \mu R$

$\therefore \ \mu R = w$ or $w = 0.2 \times 10 = 2$ N

75 A marble block of mass 2 kg lying on ice when given a velocity of 6 ms^{-1} is stopped by friction in 10 s. Then, the coefficient of friction is **[AIEEE 2003]**

(a) 0.02 (b) 0.03 (c) 0.06 (d) 0.01

Ans. ***(c)***

Let coefficient of friction be μ, then retardation will be μg. From equation of motion, $v = u + at$

$\Rightarrow \quad 0 = 6 - \mu g \times 10$

or $\quad \mu = \frac{6}{100} = 0.06$

TOPIC 4
Dynamics of Circular Motion

76 A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$. The speed of particle will be **[2021, 26 Aug Shift-II]**

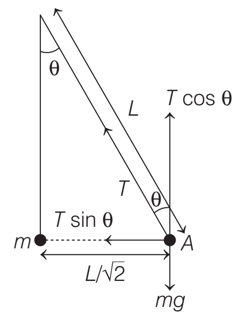
(a) $\sqrt{rg}$ (b) $\sqrt{2rg}$

(c) $2\sqrt{rg}$ (d) $\sqrt{\frac{rg}{2}}$

Ans. ***(a)***

Given, radius of horizontal circle, $r = L/\sqrt{2}$

Figure illustrating the particle of mass m moving in a horizontal circle, while suspended from a ceiling is shown



In equilibrium condition at point A,

$$T \cos\theta = mg \quad \text{...(i)}$$

$$T \sin\theta = \frac{mv^2}{r} \quad \text{...(ii)}$$

Divide Eq. (ii) by Eq. (i),

$$\tan\theta = v^2 / rg$$

$$\Rightarrow \quad v = \sqrt{rg \tan\theta} \quad \text{(iii)}$$

Now, from figure, we can write

$$\sin\theta = \frac{L/\sqrt{2}}{L} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$$

Substituting the value of θ in Eq. (iii), we get

$$v = \sqrt{rg \tan 45°} = \sqrt{rg}$$

Thus, the value of speed of particle is $v = \sqrt{rg}$.

77 Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B respectively, then **[2021, 20 July Shift-II]**

(a) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$

(b) $T_A = T_B$

(c) $T_A > T_B$ (if $m_A > m_B$)

(d) $T_A > T_B$ (if $r_A > r_B$)

Ans. ***(b)***

As per question, a binary star system of two stars A and B with masses m_A and m_B are revolving in a circular orbit of radii r_A and r_B, respectively.

It means both stars A and B will have same angular velocity because to remain perfectly align w.r.t. each other they need to cover equal angular displacement in equal time intervals.

Also, we know that

Time period, $T = \frac{2\pi}{\omega}$

$\because T_A$ and T_B are the time periods of stars A and B respectively, therefore, we can write

$$T_A = \frac{2\pi}{\omega} \quad \text{...(i)}$$

and $$T_B = \frac{2\pi}{\omega} \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we can say

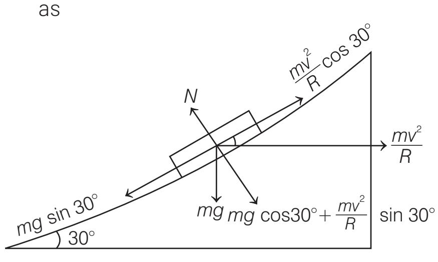
$$T_A = T_B$$

78 The normal reaction N for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is ...... $\times 10^3$ kg-m/s^2. [Take, $\mu_s = 0.2$] **[2021, 20 July Shift-I]**

(a) 10.2 (b) 7.2 (c) 12.4 (d) 6.96

Ans. ***(a)***

The given situation can be represented as



Equating forces perpendicular to the inclined plane,

$$N = mg \cos 30° + \frac{mv^2}{R} \sin 30°$$

$$\Rightarrow \quad N - mg \cos 30° = \frac{mv^2}{R} \sin 30° \quad \text{...(i)}$$

Equating forces along the inclined plane,

$$mg\sin 30° + \mu_s N = \frac{mv^2}{R}\cos 30° \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{N - mg\cos 30°}{mg\sin 30° + \mu_s N} = \tan 30°$$

$$\left[\because \cos 30° = \frac{\sqrt{3}}{2} \text{ and } \sin 30° = \frac{1}{2}\right]$$

$$\Rightarrow \frac{N - mg(\sqrt{3}/2)}{mg(1/2) + (0.2)N} = \frac{1}{\sqrt{3}}$$

$$N\sqrt{3} - mg\frac{\sqrt{3}}{2}\cdot\sqrt{3} = \frac{mg}{2} + 0.2N$$

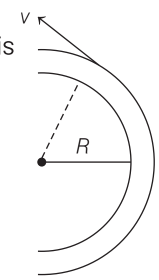
$$\Rightarrow (\sqrt{3} - 0.2)N = mg\frac{(1+3)}{2} = 2mg$$

$$\Rightarrow \quad N = \frac{2\,mg}{\sqrt{3} - 0.2} = \frac{2 \times 800 \times 10}{1.532}$$

$$= 10.44 \times 10^3 \text{ V}$$

Therefore, $N = 10.2 \times 10^3$ kg-m/s^2

79 A modern grand-prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift f_L acting downwards on the car is



(Assume forces on the four tyres are identical and g = acceleration due to gravity) **[2021, 17 March Shift-I]**

(a) $m\left(\frac{v^2}{\mu_s R} + g\right)$ (b) $m\left(\frac{v^2}{\mu_s R} - g\right)$

(c) $m\left(g - \frac{v^2}{\mu_s R}\right)$ (d) $-m\left(g + \frac{v^2}{\mu_s R}\right)$

Ans. *(b)*

We know that, static friction force,

$f_s = \mu_s N$

where, μ_s is the coefficient of static friction and N is the normal force acting on the body. As the car is travelling on a circular track, so centripetal force is also acting on it.

$$\Rightarrow \quad f_C = \frac{mv^2}{R}$$

In limiting condition,

$$f_s = f_C \Rightarrow \mu_s N = \frac{mv^2}{R} \Rightarrow N = \frac{mv^2}{\mu_s R} \quad \text{...(i)}$$

The magnitude of negative lift f_L acting downwards on the car is given by

$$f_L = mg - N = mg - \frac{mv^2}{\mu_s R}$$

$$\Rightarrow f_L = m\left(g - \frac{v^2}{\mu_s R}\right) \Rightarrow f_L = -m\left(\frac{v^2}{\mu_s R} - g\right)$$

$$\Rightarrow \quad |f_L| = m\left(\frac{v^2}{\mu_s R} - g\right)$$

80 **Statement I** A cyclist is moving on an unbanked road with a speed of $7\,\text{kmh}^{-1}$ and takes a sharp circular turn along a path of radius of 2 m without reducing the spee(d) The static friction coefficient is 0.2. The cyclist will not slip and pass the curve ($g = 9.8$ m / s^2)

Statement II If the road is banked at an angle of 45°, cyclist can cross the curve of 2 m radius with the speed of $18.5\,\text{kmh}^{-1}$ without slipping.

In the light of the above statements, choose the correct answer from the options given below. **[2021, 16 March Shift-II]**

(a) Statement I is false and statement II is true.
(b) Statement I is true and statement II is false.
(c) Both statement I and statement II are false.
(d) Both statement I and statement II are true.

Ans. *(d)*

The maximum speed of cyclist on turn of unbanked road without slipping is given as

$$v_{max} = \sqrt{\mu g R} = \sqrt{0.2 \times 10 \times 2} = 2\,\text{ms}^{-1}$$

$$[\because \mu = 0.2 \text{(given)}]$$

Given, speed = 7 km/h

$$= \frac{7000}{3600}\text{ms}^{-1} = \frac{70}{36} = 1.94\text{ ms}^{-1}$$

As given speed is lesser than v_{max}, so the cyclist will not slip. Therefore, Statement I is true.

As per Statement II, angle of banking, $\theta = 45°$

We know that, for banked road,

$$v_{max} = \sqrt{\frac{gR(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$$

$$\text{and} \quad v_{min} = \sqrt{\frac{gR(\tan\theta - \mu)}{1 + \mu\tan\theta}}$$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{10 \times 2(0.2 + \tan 45°)}{1 - 0.2\tan 45°}}$$

$$\text{and} \quad v_{min} = \sqrt{\frac{10 \times 2(1 - 0.2)}{1 + 0.2}}$$

$$\Rightarrow \quad v_{max} = 5.47\text{ ms}^{-1}$$

$$\text{and} \quad v_{min} = 3.65\text{ ms}^{-1}$$

$$\because v = 18.5\text{ km/h} = \frac{18.5 \times 1000}{5600} = 5.13\text{ ms}^{-1}$$

$$[\because \text{given speed} = 18.5\text{ kmh}^{-1}]$$

As, $v_{min} < v < v_{max}$, so the cyclist will not slip.

∴Statement II is also true.

Hence, option (d) is the correct.

81 A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is **[2021, 16 March Shift-I]**

(a) 0.0314 N (b) 9.859×10^{-2} N
(c) 6.28×10^{-3} N (d) 9.859×10^{-4} N

Ans. *(d)*

The normal force by the side walls of the groove will be equal to the centripetal force acting on it.

$$\text{i.e.} \quad N = \frac{mv^2}{r} \quad \text{...(i)}$$

where, $r = 20$ cm $= 0.2$ m

$$m = 200\text{ g} = 200 \times 10^{-3}\text{kg}$$

$$\text{and} \quad v = r\omega = \frac{2\pi r}{T} = \frac{2\pi \times 0.2}{40}\text{ m/s}$$

Substituting the given values in Eq. (i), we get

$$N = \frac{(200 \times 10^{-3}) \times \left(\frac{2\pi \times 0.2}{40}\right)^2}{0.2}$$

$$\approx 9.859 \times 10^{-4}\text{N}$$

82 A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by **[26 Feb 2021 Shift-I]**

(a) $T \propto R^2$ (b) $T \propto R^{3/2}$
(c) $T \propto R^{5/2}$ (d) $T \propto R^{4/3}$

Ans. *(a)*

Given, radius of circle = R

Central fictitious force is, $F \propto \frac{1}{R^3}$.

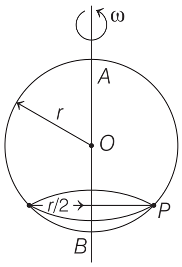
Let T be the time period of revolution, m, ω be the mass and angular velocity of Earth.

$\because \quad F = m\omega^2 R \propto \dfrac{1}{R^3}$

$\Rightarrow \quad \omega^2 \propto \dfrac{1}{R^4} \Rightarrow \omega = \dfrac{2\pi}{T} \propto \dfrac{1}{R^2}$

$\Rightarrow \quad T \propto R^2$

83 A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then, the value of ω^2 is equal to **[2019, 12 April Shift-II]**



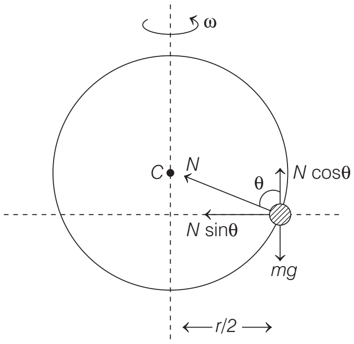
(a) $\dfrac{\sqrt{3}g}{2r}$ (b) $2g/(r\sqrt{3})$

(c) $(g\sqrt{3})/r$ (d) $2g/r$

Ans. ***(b)***

Key Idea For revolution in a circular path, there should be a force which balances the necessary centripetal force.

Let N = normal reaction of wire loop acting towards centre.



Then, component $N\cos\theta$ balances weight of bead,

$\Rightarrow \quad N\cos\theta = mg \quad$ ...(i)

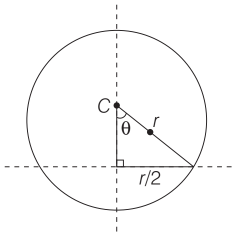
and component $N\sin\theta$ provides necessary centripetal pull on the bead,

$\Rightarrow \quad N\sin\theta = m\left(\dfrac{r}{2}\right)\omega^2 \quad$ ...(ii)

From Eqs. (i) and (ii), we have

$$\tan\theta = \frac{r\omega^2}{2g} \quad \text{...(iii)}$$

Now, from geometry of figure,



$$\tan\theta = \frac{\frac{r}{2}}{\sqrt{r^2 - \left(\frac{r}{2}\right)^2}} = \frac{r}{2\left(\frac{\sqrt{3}}{2}\right)r} = \frac{1}{\sqrt{3}} \quad \text{...(iv)}$$

Put this value in Eq. (iii), we get

$$\omega^2 = \frac{2g}{\sqrt{3}\,r}$$

84 A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then : **[JEE Main 2018]**

(a) $T \propto R^{3/2}$ for any n

(b) $T \propto R^{\frac{n}{2}+1}$

(c) $T \propto R^{(n+1)/2}$

(d) $T \propto R^{n/2}$

Ans. ***(c)***

$\because$ Force = Mass $\times$ Acceleration $= m\omega^2 R$

and given, $F \propto \dfrac{1}{R^n} \Rightarrow F = \dfrac{k}{R^n}$

So, we have $\dfrac{k}{R^n} = m\left(\dfrac{2\pi}{T}\right)^2 R$

$\Rightarrow \quad T^2 = \dfrac{4\pi^2 m}{k} \cdot R^{n+1} \Rightarrow T \propto R^{\frac{n+1}{2}}$

85 An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular spee(d) The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, F_1 / F_2 is **[AIEEE 2005]**

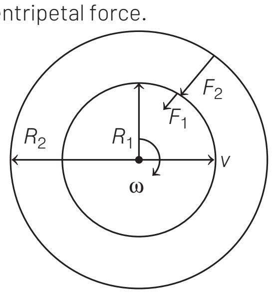
(a) $\dfrac{R_2}{R_1}$ (b) $\left(\dfrac{R_1}{R_2}\right)^2$

(c) 1 (d) $\dfrac{R_1}{R_2}$

Ans. ***(d)***

Since, ω is constant, so no net force or torque is acting on ring.

The force experienced by any particle is only along radial direction or we can say the centripetal force.



The force experienced by inner part, $F_1 = m\omega^2 R_1$

and the force experienced by outer part,

$$F_2 = m\omega^2 R_2 \Rightarrow \frac{F_1}{F_2} = \frac{R_1}{R_2}$$

86 The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is **[AIEEE 2002]**

(a) 60 (b) 30

(c) 15 (d) 25

Ans. ***(b)***

As centrifugal force is balanced by the centripetal force *i.e.*, frictional force.

Using the relation

$$\frac{mv^2}{r} = \mu R, \quad R = mg$$

$\Rightarrow \quad \dfrac{mv^2}{r} = \mu mg \quad$ or $\quad v^2 = \mu rg$

$\Rightarrow \quad v^2 = 0.6 \times 150 \times 10$

or $\quad v = 30\ ms^{-1}$