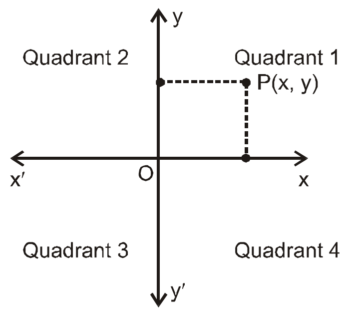
STRAIGHT LINE

1. INTRODUCTION OF COORDINATE GEOMETRY

Coordinate geometry is the combination of algebra and geometry. A french mathematician René Descartes carried out the systematic study of geometry by the use of algebra. This geometry is also called as analytical geometry. It is the branch of geometry in which algebraic equations are used to denote points, lines and curves.

2. CARTESIAN CO-ORDINATES SYSTEM

In two dimensional coordinate system, two perpendicular lines are used which forms a rectangular coordinate system. The horizontal axis is the x-axis and the vertical axis is y-axis. The point of intersection O is the origin of the coordinate system. Distances along the x-axis to the right of the origin are taken as positive, distances to the left as negative. The two perpendicular lines xox′ and yoy′ divide the plane in four regions which are called quadrants, numbered as shown in the figure.



Distances along the y-axis above the origin are positive; distances below are negative. The position of a point anywhere in the plane can be specified by two numbers, the coordinates of the point, written as (x, y). The x-coordinate (or abscissa) is the algebraic distance of the point from the y-axis in a direction parallel to the x-axis (i.e. horizontally). The y-coordinate (or ordinate) is the algebraic distance from the x-axis in a direction parallel to the y-axis (vertically). The origin O is the point (0, 0).

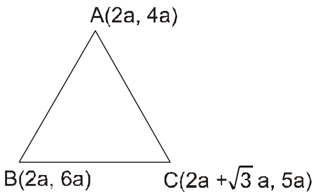
3. DISTANCE FORMULA

The distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

SOLVED EXAMPLE

Example : 1 Prove that the points (2a, 4a), (2a, 6a) and $(2a + \sqrt{3}a$, 5a) are the vertices of an equilateral triangle whose side is 2a.

Solution :



AB = BC = CA = 2a hence triangle is equilateral.

Example : 2 The number of points on x-axis which are at a distance c(c < 3) from the point (2, 3) is :-
(A) 2 (B) 1 (C) infinite (D) no point

Solution: Let a point on x-axis is $(x_1, 0)$ then its distance from the point (2, 3)

$= \sqrt{(x_1-2)^2+9} = c$ or $(x_1-2)^2 = c^2-9$

$\therefore\ x_1 - 2 = \pm\sqrt{c^2-9}$ since $c<3 \Rightarrow c^2-9<0$

$\therefore$ x_1 will be imaginary. **Ans. (D)**

Problems for Self Practice-1 :

(i) Find the value of x, if the distance between the points (x, –1) and (3, 2) is 5

(ii) Show that four points (0, –1), (6, 7) (–2, 3) and (8, 3) are the vertices of a rectangle.

Answers : (i) 7, –1

4. SECTION FORMULA

If P(x, y) divides the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then;

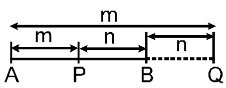
$x = \dfrac{mx_2+nx_1}{m+n}$; $y = \dfrac{my_2+ny_1}{m+n}$.

Notes :

(i) If $\dfrac{m}{n}$ is positive, the division is internal, but if $\dfrac{m}{n}$ is negative, the division is external.

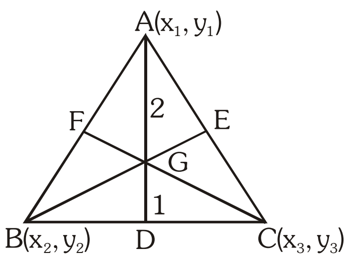
(ii) If P divides AB internally in the ratio m : n & Q divides AB externally in the ratio m: n then P & Q are said to be **harmonic conjugate** of each other w.r.t. AB.

Mathematically, $\dfrac{2}{AB} = \dfrac{1}{AP} + \dfrac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.



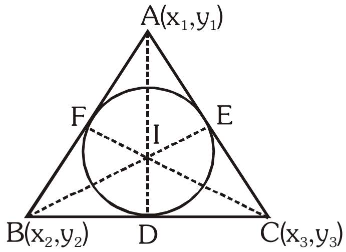
(iii) Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of any triangle ABC, then

(a) **Centroid :** The centroid is the point of intersection of the medians (line joining the mid point of sides and opposite vertices). Centroid divides each median in the ratio of 2 : 1.



Co-ordinates of centroid $G\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

(b) Incenter : The incenter is the point of intersection of internal bisectors of the angles of a triangle. Also it is a centre of the circle touching all the sides of a triangle.



Co-ordinates of incenter I $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

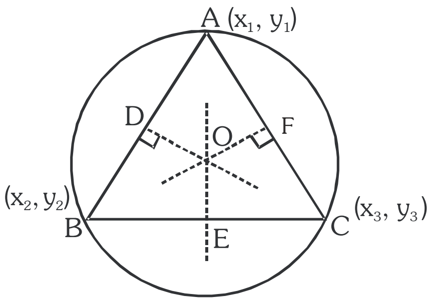
where a, b, c are the sides of triangle ABC.

Remarks :

(i) Angle bisector divides the opposite sides in the ratio of remaining sides. e.g. $\frac{BD}{DC}=\frac{AB}{AC}=\frac{c}{b}$

(ii) Incenter divides the angle bisectors in the ratio $(b+c):a,\ (c+a):b,\ (a+b):c$.

(c) Circumcenter : It is the point of intersection of perpendicular bisectors of the sides of a triangle. If O is the circumcenter of any triangle ABC, then $OA^2=OB^2=OC^2$. Also it is a centre of a circle touching all the vertices of a triangle.

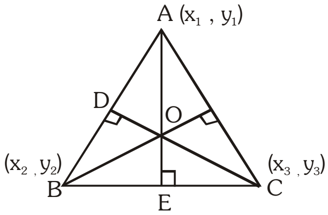


Remarks :

(i) If the triangle is right angled, then its circumcenter is the mid point of hypotenuse.

(ii) Co-ordinates of circumcenter $\left(\frac{x_1\sin 2A+x_2\sin 2B+x_3\sin 2C}{\sin 2A+\sin 2B+\sin 2C}, \frac{y_1\sin 2A+y_2\sin 2B+y_3\sin 2C}{\sin 2A+\sin 2B+\sin 2C}\right)$

(d) Orthocenter : It is the point of intersection of perpendiculars drawn from vertices on the opposite sides of a triangle and it can be obtained by solving the equation of any two altitudes.



Remarks :

(i) If a triangle is right angled, then orthocenter is the point where right angle is formed.

(ii) Co-ordinates of circumcenter $\left(\dfrac{x_1\tan A + x_2\tan B + x_3\tan C}{\tan A+\tan B+\tan C}, \dfrac{y_1\tan A + y_2\tan B + y_3\tan C}{\tan A+\tan B+\tan C}\right)$

Generals Points :

(i) If the triangle is equilateral, then centroid, incentre, orthocenter, circumcenter, coincide.

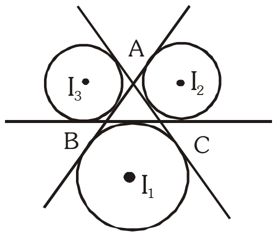
(ii) Orthocentre, centroid and circumcentre are always collinear (i.e., lie on the same line) and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1

(iii) In an isosceles triangle centroid, orthocentre, incentre & circumcentre lie on the same line.

(e) Ex-centers :

The centre of a circle which touches side BC and the extended portions of sides AB and AC is called the ex-centre of ΔABC with respect to the vertex A. It is denoted by I_1 and its coordinates are

$$I_1\left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$$



Similarly ex-centers of ΔABC with respect to vertices B and C are denoted by I_2 and I_3 respectively,

and $I_2\left(\dfrac{ax_1-bx_2+cx_3}{a-b+c}, \dfrac{ay_1-by_2+cy_3}{a-b+c}\right), I_3\left(\dfrac{ax_1+bx_2-cx_3}{a+b-c}, \dfrac{ay_1+by_2-cy_3}{a+b-c}\right)$

SOLVED EXAMPLE

Example : 3 Find the points which trisect the line segment joining the points (0, 0) and (9, 12).

Ans. **(3, 4) (6, 8)**

Solution: Let required point is P & Q

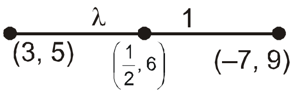
P divides in 1 : 2 $P\left(\dfrac{9+2\times 0}{1+2}, \dfrac{1\times 12+2\times 0}{1+2}\right) \equiv (3, 4)$

Q divides in 2 : 1 Hence $Q\left(\dfrac{2\times 9+1\times 0}{2+1}, \dfrac{2\times 12+1\times 0}{2+1}\right) \equiv Q\ (6, 8)$

Example : 4 In what ratio does the point $\left(\dfrac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (– 7, 9)?

Solution: Let $\left(\dfrac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (–7, 9) in λ : 1

$\dfrac{1}{2} = \dfrac{-7\lambda+3}{\lambda+1} \Rightarrow \lambda = 1 : 3$



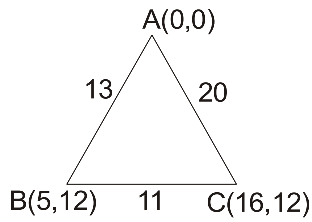
Example : 5 For triangle whose vertices are (0, 0), (5, 12) and (16, 12). Find coordinates of

(i) Centroid (ii) Circumcentre

(iii) Incentre (iv) Excentre opposite to vertex (5, 12)

Ans. (i) (7, 8) (ii) $\left(\frac{21}{2}, \frac{8}{3}\right)$ (iii) (7, 9) (iv) (27, –21)

Solution: (i) centroid $\left(\frac{0+5+16}{3}, \frac{0+12+12}{3}\right) \equiv (7, 8)$



(ii) Let coordinates of circumcentre is O (x, y).

Therefore OA = OB = OC

$\Rightarrow$ $x^2 + y^2 = (x - 5)^2 + (y - 12)^2 = (x - 16)^2 + (y - 12)^2$

$\Rightarrow$ $x^2 + y^2 = (x - 5)^2 + (y - 12)^2 \Rightarrow 10x + 24y = 16g$

$(x - 5)^2 + (y - 12)^2 = (x - 16)^2 + (y - 12)^2$

$\Rightarrow$ $2x = 21 \Rightarrow x = \frac{21}{2}, \; y = \frac{8}{3}$

(iii) $I\left(\frac{0\times 11 + 5\times 20 + 16\times 13}{13+20+11}, \frac{0\times 11 + 12\times 20 + 13\times 12}{13+20+11}\right) \equiv (7, 9)$

(iv) $I^2 = \left(\frac{-5\times 20 + 13\times 16 + 11\times 13}{-20+13+11}, \frac{-12\times 20 + 0\times 11 + 13\times 12}{-20+13+11}\right) \equiv (27, -21)$

Problems for Self Practice-2 :

(i) Find the co-ordinates of the point dividing the join of A(1, – 2) and B(4, 7) :

(a) Internally in the ratio 1 : 2

(b) Externally in the ratio of 2 : 1

(ii) The three vertices of a parallelogram taken in order are (–1, 0), (3, 1) and (2, 2) respectively. Find the co-ordinates of the fourth vertex.

(iii) The coordinates of the vertices of a triangle are (0, 1), (2, 3) and (3, 5) :

(a) Find centroid of the triangle.

(b) Find circumcentre & the circumradius.

(c) Find orthocentre of the triangle.

Answers : (i) (a) (2,1); (b) (7,16) (ii) (–2, 1)

(iii) (a) $\left(\frac{5}{3}, 3\right)$; (b) $\left(-\frac{9}{2}, \frac{15}{2}\right)$, $\frac{5\sqrt{10}}{2}$, (c) (14, – 6)

5. AREA OF TRIANGLE / POLYGON

5.1 Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of a triangle, then

$$\text{Area of } \Delta ABC = \left| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right| = \frac{1}{2} \left|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

Notes :

Area of Equilateral triangle : If altitude of any equilateral triangle is h, then its area = $\frac{h^2}{\sqrt{3}}$. If 'a' be the side of equilateral triangle, then its area = $\left(\frac{a^2\sqrt{3}}{4}\right)$.

5.2 Area of n-sided polygon formed by points (x_1, y_1) ; (x_2, y_2); ....;(x_n, y_n) taken in order is given by

$$\frac{1}{2}\left(\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \ldots\ldots\ldots\ldots \begin{vmatrix} x_{n-1} & x_n \\ y_{n-1} & y_n \end{vmatrix} + \begin{vmatrix} x_n & x_1 \\ y_n & y_1 \end{vmatrix}\right).$$

SOLVED EXAMPLE

Example : 6 If the vertices of a triangle are (1, 2), (4, –6) and (3, 5) then its area is

(A) $\frac{25}{2}$ sq. units (B) 12 sq. units (C) 5 sq. units (D) 25 sq. units

Solution: $\Delta = \frac{1}{2}\left[1(-6-5)+4(5-2)+3(2+6)\right] = \frac{1}{2}[-11+12+24] = \frac{25}{2}\text{square units}$ **Ans. (A)**

Example : 7 If the co-ordinates of two points A and B are (3, 4) and (5, –2) respectively. Find the co-ordinates of any point P if PA = PB and Area of ΔPAB = 10.

Solution : Let the co-ordinates of P be (x, y). Then

$PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow \quad (x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2$

$\Rightarrow \quad x - 3y - 1 = 0$

Now, Area of ΔPAB = 10

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \pm 10 \Rightarrow 6x + 2y - 26 = \pm 20$$

$\Rightarrow \quad 6x + 2y - 46 = 0$ or $6x + 2y - 6 = 0$

$\Rightarrow \quad 3x + y - 23 = 0$ or $3x + y - 3 = 0$

Solving $x - 3y - 1 = 0$ and $3x + y - 23 = 0$ we get x = 7, y = 2.

Solving $x - 3y - 1 = 0$ and $3x + y - 3 = 0$, we get x = 1, y = 0.

Thus, the co-ordinates of P are (7, 2) or (1, 0)

Problems for Self Practice-3 :

(i) Find the area of the triangle whose vertices are A(1,1), B(7, – 3) and C(12, 2)

(ii) Find the area of the quadrilateral whose vertices are A(1,1) B(7, – 3), C(12,2) and D(7, 21)

Answers : (i) 25 square units (ii) 132 square units ;

6. SLOPE (OR GRADIENT) OF LINE

If θ is the angle at which a straight line is inclined to the positive direction of x–axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y–axis. If $\theta = 0$, then m = 0 & the line is parallel to the x-axis.

If A (x_1, y_1) & B (x_2, y_2), $x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by : $m = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)$.

SOLVED EXAMPLE

Example : 8 What is the slope of a line whose inclination with the positive direction of x-axis is :
(i) 0º (ii) 90º (iii) 120º (iv) 150º

Solution :
(i) Here $\theta = 0°$
Slope = $\tan\theta = \tan 0° = 0$.
(ii) Here $\theta = 90°$
$\therefore$ The slope of line is not defined.
(iii) Here $\theta = 120°$
$\therefore$ Slope = $\tan\theta = \tan 120° = \tan(180° - 60°) = -\tan 60° = -\sqrt{3}$.
(iv) Here $\theta = 150°$
$\therefore$ Slope = $\tan\theta = \tan 150° = \tan(180° - 30°) = -\tan 30° = -\frac{1}{\sqrt{3}}$

Example : 9 Find the slope of the line passing through the points :
(i) (1, 6) and (– 4, 2) (ii) (5, 9) and (2, 9)

Solution :
(i) Let A = (1, 6) and B = (– 4, 2)
$\therefore$ Slope of AB = $\frac{2-6}{-4-1} = \frac{-4}{-5} = \frac{4}{5}$ $\left(\text{Using slope} = \frac{y_2 - y_1}{x_2 - x_1}\right)$
(ii) Let A = (5, 9), B = (2, 9)
$\therefore$ Slope of AB = $\frac{9-9}{2-5} = \frac{0}{-3} = 0$

Problems for Self Practice-4 :

(i) Find the value of x, if the slope of the line joining (1, 5) and (x, –7) is 4.
(ii) What is the inclination of a line whose slope is
(a) 0 (b) 1 (c) –1 (d) $-1/\sqrt{3}$

Answers : (i) – 2 (ii) (a) 0º (b) 45º (c) 135º (d) 150º

7. COLLINEARITY OF THREE GIVEN POINTS

Three given points A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) are collinear if they lie on a common line.

7.1 Conditions of collinearity

(a) Area of triangle ABC is zero i.e. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ **=** 0 (Determinant method)

(b) Slope of AB = slope of BC = slope of AC. i.e. $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{y_3 - y_1}{x_3 - x_1}$

(c) Evaluate the equation of line passing through 2 given points, if the third point satisfies the given equation of the line, then three points are collinear.

SOLVED EXAMPLE

Example : 10 Show that the points (1, 1), (2, 3) and (3, 5) are collinear.

Solution : Let (1, 1) (2, 3) and (3, 5) be the co-ordinates of the points A, B and C respectively.

Slope of AB = $\frac{3-1}{2-1}$ = 2 and Slope of BC = $\frac{5-3}{3-2}$ = 2

$\therefore$ Slope of AB = slope of BC

$\therefore$ AB & BC are parallel

$\therefore$ A, B, C are collinear because B is on both lines AB and BC.

<u>Problems for Self Practice</u>-5 :

(i) Prove that the points A(a, b + c), B(b, c + a) and C(c, a + b) are collinear (By determinant method)

(ii) Prove that the points (a, 0), (0, b) and (1, 1) are collinear if $\frac{1}{a} + \frac{1}{b} = 1$

8. EQUATION OF A STRAIGHT LINE IN VARIOUS FORMS

8.1 Point - Slope form :

$y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is m & which passes through the point (x_1, y_1).

SOLVED EXAMPLE

Example : 11 Find the equation of a line passing through (2, –3) and inclined at an angle of 135º with the positive direction of x-axis in anticlockwise direction.

Solution : Here, m = slope of the line = tan 135º = tan (90º + 45º) = – cot 45º = –1, $x_1 = 2$, $y_1 = -3$

So, the equation of the line is $y - y_1 = m(x - x_1)$

i.e. $y - (-3) = -1(x - 2)$ or $y + 3 = -x + 2$ or $x + y + 1 = 0$

<u>Problems for Self Practice</u>-6 :

(i) Find the equation of the perpendicular bisector of the line segment joining the points A(2, 3) and B (6, –5).

Answers : (i) $x - 2y - 6 = 0$

8.2 Slope-intercept form :

$y = mx + c$ is the equation of a straight line whose slope is m & which makes an intercept c on the y–axis.

SOLVED EXAMPLE

Example : 12 Find the equation of a line with slope –1 and cutting off an intercept of 4 units on negative direction of y-axis.

Solution : Here $m = -1$ and $c = -4$. So, the equation of the line is $y = mx + c$

i.e. $y = -x - 4$ or $x + y + 4 = 0$

Problems for Self Practice-7 :

(i) Find the equation of a straight line which cuts off an intercept of length 3 on y-axis and is parallel to the line joining the points (3, –2) and (1, 4).

Answers : (i) $3x + y - 3 = 0$

8.3 Two point form :

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) & (x_2, y_2).

This equation can also be written in Determinant form as $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

SOLVED EXAMPLE

Example : 13 Find the equation of the line joining the points (– 1, 3) and (4, – 2)

Solution : Here the two points are $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (4, -2)$.

So, the equation of the line in two-point form is $y - 3 = \frac{3-(-2)}{-1-4}(x + 1)$

$\Rightarrow \quad y - 3 = -x - 1 \quad \Rightarrow \quad x + y - 2 = 0$

Example : 14 Find the equation of line passing through (2, 4) & (– 1, 3).

Solution : $\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad x - 3y + 10 = 0$

Problems for Self Practice-8 :

(i) Find the equations of the sides of the triangle whose vertices are (–1, 8), (4, –2) and (–5, –3). Also find the equation of the median through (–1, 8).

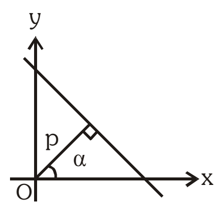
(ii) Find the equation of the passing through (– 2, 3) & (– 1, – 1).

Answers : (i) $2x + y - 6 = 0$, $x - 9y - 22 = 0$, $11x - 4y + 43 = 0$, $21x + y + 13 = 0$

(ii) $4x + y + 5 = 0$

8.4 Normal form (or Perpendicular form)

If p is the length of perpendicular on a line from the origin, and α the angle which this perpendicular makes with positive x-axis, then the equation of this line is written as : $\mathbf{x\cos\alpha + y\sin\alpha = p}$ (p is always positive) where $\mathbf{0 \le \alpha < 2\pi}$.

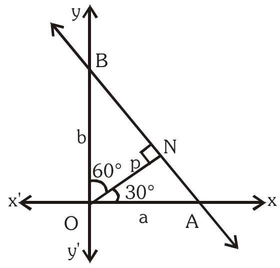


SOLVED EXAMPLE

Example : 15 Find the equation of the straight line on which the perpendicular from origin makes an angle 30° with positive x-axis and which forms a triangle of area $\left(\frac{50}{\sqrt{3}}\right)$ sq. units with the co-ordinates axes.

Solution : $\angle NOA = 30°$

Let ON = p > 0, OA = a, OB = b



In ΔONA, $\cos 30° = \frac{ON}{OA} = \frac{p}{a} \Rightarrow \frac{\sqrt{3}}{2} = \frac{p}{a}$

or $a = \frac{2p}{\sqrt{3}}$

and in ΔONB, $\cos 60° = \frac{ON}{OB} = \frac{p}{b} \Rightarrow \frac{1}{2} = \frac{p}{b}$

or b = 2p

$\because$ Area of ΔOAB $= \frac{1}{2}$ ab $= \frac{1}{2}\left(\frac{2p}{\sqrt{3}}\right)(2p) = \frac{2p^2}{\sqrt{3}}$

$\therefore$ $\frac{2p^2}{\sqrt{3}} = \frac{50}{\sqrt{3}} \Rightarrow p^2 = 25$

or p = 5

$\therefore$ Using $x\cos\alpha + y\sin\alpha = p$, the equation of the line AB is $x\cos 30° + y \sin 30° = 5$

or $x\sqrt{3} + y = 10$

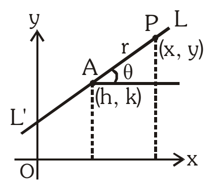
Problems for Self Practice-9 :

(i) The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150º with the positive direction (clock-wise) of y-axis. Find the equation of the line.

Answers : (i) $\sqrt{3}\,x + y - 14 = 0$

8.5 Parametric form :

To find the equation of a straight line which passes through a given point A(h, k) and makes a given angle θ with the positive direction of the x-axis. P(x, y) is any point on the line LAL'.



Let AP = r, then **$x - h = r\cos\theta$, $y - k = r\sin\theta$** & $\frac{x-h}{\cos\theta} = \frac{y-k}{\sin\theta}$ **= r** is the equation of the straight line LAL'.

Any point P on the line will be of the form $(h + r\cos\theta, k + r\sin\theta)$, where |r| gives the distance of the point P from the fixed point (h, k).

SOLVED EXAMPLE

Example: 16 Find the equation of the line through the point A(2, 3) and making an angle of 45º with the positive direction of x-axis. Also determine the length of intercept on it between A and the line x + y + 1 = 0

Solution : The equation of a line through A and making an angle of 45º with the x-axis is

$$\frac{x-2}{\cos 45^\circ} = \frac{y-3}{\sin 45^\circ} \text{ or } \frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-3}{\frac{1}{\sqrt{2}}} \quad \text{or} \quad x - y + 1 = 0$$

Suppose this line meets the line x + y + 1 = 0 at P such that AP = r. Then the co-ordinates of P are given by

$$\frac{x-2}{\cos 45^\circ} = \frac{y-3}{\sin 45^\circ} = r \Rightarrow x = 2 + r\cos 45^\circ,\ y = 3 + r\sin 45^\circ$$

$$\Rightarrow \quad x = 2 + \frac{r}{\sqrt{2}},\ y = 3 + \frac{r}{\sqrt{2}}$$

Thus, the co-ordinates of P are $\left(2+\frac{r}{\sqrt{2}}, 3+\frac{r}{\sqrt{2}}\right)$

Since P lies on x + y + 1 = 0, so $2 + \frac{r}{\sqrt{2}} + 3 + \frac{r}{\sqrt{2}} + 1 = 0$

$$\Rightarrow \quad \sqrt{2}\,r = -6 \Rightarrow r = -3\sqrt{2} \quad \Rightarrow \text{ length AP} = |r| = 3\sqrt{2}$$

Thus, the length of the intercept = $3\sqrt{\mathbf{2}}$.

<u>Problems for Self Practice</u>-10 :

(i) A straight line is drawn through the point $A\left(\sqrt{3}, 2\right)$ making an angle of π/6 with positive direction of the x-axis. If it meets the straight line $\sqrt{3x} - 4y + 8 = 0$ in B, find the distance between A and B.

Answers : (i) 6 units

8.6 General form

We know that a first degree equation in x and y, ax +by + c = 0 always represents a straight line. This form is known as general form of straight line.

(i) Slope of this line $= \frac{-a}{b} = -\frac{\text{coeff. of } x}{\text{coeff. of } y}$

(ii) Intercept by this line on x-axis = $-\frac{c}{a}$ and intercept by this line on y-axis = $-\frac{c}{b}$

(iii) To change the general form of a line to normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2+b^2}$.

Notes :

(a) Equation of a line parallel to x-axis at a distance 'a' is **y = a** or **y = – a.**

(b) Equation of x-axis is **y = 0.**

(c) Equation of a line parallel to y-axis at a distance 'b' is **x = b** or **x = – b.**

(d) Equation of y-axis is **x = 0.**

9. ANGLE BETWEEN TWO LINES

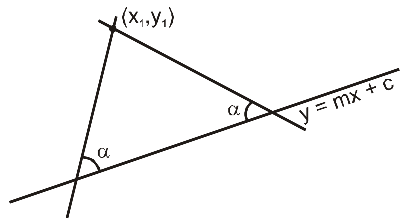
If m_1 & m_2 are the slopes of two intersecting straight lines ($m_1 m_2 \neq -1$) & θ is the acute angle between them, then $\tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$.

Notes :

(i) Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0; L_2 = 0; L_3 = 0$ where $m_1 > m_2 > m_3$, then the tangent of interior angles of the Δ ABC formed by these lines are given by,

$$\tan A = \frac{m_1 - m_2}{1+m_1m_2} \; ; \; \tan B = \frac{m_2 - m_3}{1+m_2m_3} \quad \& \tan C = \frac{m_3 - m_1}{1+m_3m_1}$$

(ii) The equation of lines passing through point (x_1, y_1) and making angle α with the line $y = mx + c$ are given by :
$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ & $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.



(iii) If equation of lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then these lines are -

(a) Parallel $\Leftrightarrow$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(b) Perpendicular $\Leftrightarrow$ $a_1a_2+b_1b_2=0$

(c) Coincident $\Leftrightarrow$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iv) (a) Equation of line parallel to line $ax + by + c = 0$
$ax + by + \lambda = 0$

(b) Equation of line perpendicular to line $ax + by + c = 0$
$bx - ay + k = 0$

Here λ, k, are parameters and their values are obtained with the help of additional information given in the problem.

SOLVED EXAMPLE

Example : 17 The acute angle between two lines is $\pi/4$ and slope of one of them is 1/2. Find the slope of the other line.

Solution : If θ be the acute angle between the lines with slopes m_1 and m_2, then $\tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$

Let $\theta = \frac{\pi}{4}$ and $m_1 = \frac{1}{2}$

$\therefore \quad \tan\frac{\pi}{4} = \left|\frac{\frac{1}{2}-m_2}{1+\frac{1}{2}m_2}\right| \Rightarrow 1 = \left|\frac{1-2m_2}{2+m_2}\right| \Rightarrow \frac{1-2m_2}{2+m_2} = +1 \text{ or } -1$

Now $\frac{1-2m_2}{2+m_2} = 1 \Rightarrow m_2 = -\frac{1}{3}$ and $\frac{1-2m_2}{2+m_2} = -1 \Rightarrow m_2 = 3$.

$\therefore$ The slope of the other line is either – 1/3 or 3

Example : 18 If the straight line 3x + 4y + 5 – k (x + y + 3) = 0 is parallel to y-axis, then the value of k is-

(A) 1 (B) 2 (C) 3 (D) 4

Solution : A straight line is parallel to y-axis, if its y - coefficient is zero, i.e. 4 – k = 0 i.e. k = 4

Problems for Self Practice-11 :

(i) Find the angle between the lines 3x + y – 7 = 0 and x + 2y – 9 = 0.

(ii) Find the line passing through the point (2, 3) and perpendicular to the straight line 4x – 3y = 10.

(iii) Find the equation of the line which has positive y-intercept 4 units and is parallel to the line 2x – 3y – 7 = 0. Also find the point where it cuts the x-axis.

(iv) Classify the following pairs of lines as coincident, parallel or intersecting :

(a) x + 2y – 3 = 0 & –3x – 6y + 9 = 0

(b) x + 2y + 1 = 0 & 2x + 4y + 3 = 0

(c) 3x – 2y + 5 = 0 & 2x + y – 5 = 0

Answers : (i) θ = 135° or 45° ;

(ii) 3x + 4y = 18 ;

(iii) 2x – 3y + 12 = 0, (–6, 0)

(iv) (a) Coincident, (b) Parallel, (c) Intersecting

10. POINT AND LINE

10.1 Length of perpendicular from a point on a line

Length of perpendicular from a point (x_1,y_1) on the line ax + by + c = 0 is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

In particular, the length of the perpendicular from the origin on the line ax + by + c = 0 is $P = \frac{|c|}{\sqrt{a^2+b^2}}$

SOLVED EXAMPLE

Example : 19 Find the distance between the line 12x – 5y + 9 = 0 and the point (2, 1)

Solution : The required distance = $\left|\frac{12\times2-5\times1+9}{\sqrt{12^2+(-5)^2}}\right| = \frac{|24-5+9|}{13} = \frac{28}{13}$

Example : 20 Find all points on x + y = 4 that lie at a unit distance from the line 4x + 3y – 10 = 0.

Solution : Note that the co-ordinates of an arbitrary point on x + y = 4 can be obtained by putting x = t (or y = t) and then obtaining y (or x) from the equation of the line, where t is a parameter.

Putting x = t in the equation x + y = 4 of the given line, we obtain y = 4 – t. So, co-ordinates of an arbitrary point on the given line are P(t, 4 – t). Let P(t, 4 – t) be the required point. Then, distance of P from the line 4x + 3y – 10 = 0 is unity i.e.

$$\Rightarrow \quad \left|\frac{4t+3(4-t)-10}{\sqrt{4^2+3^2}}\right| = 1 \quad \Rightarrow \quad |t+2| = 5$$

$$\Rightarrow \quad t + 2 = \pm 5 \quad \Rightarrow \quad t = -7 \text{ or } t = 3$$

Hence, required points are (–7, 11) and (3, 1)

Problems for Self Practice-12 :

(i) Find the length of the altitudes from the vertices of the triangle with vertices :(–1, 1), (5, 2) and (3, –1).

Answers : (i) $\frac{16}{\sqrt{13}}, \frac{8}{\sqrt{5}}, \frac{16}{\sqrt{37}}$

10.2 Position of two points with respect to a given line

Two points (x_1, y_1) and (x_2, y_2) will lie on same side or opposite side of ax + by + c = 0 according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of same or opposite sign respectively.

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of ax + by + c = 0. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non–origin side of ax + by + c = 0.

SOLVED EXAMPLE

Example: 21 Show that (1, 4) and (0, –3) lie on the opposite sides of the line x + 3y + 7 = 0.

Solution : At (1, 4), the value of x + 3y + 7 = 1 + 3(4) + 7 = 20 > 0.

At (0, – 3), the value of x + 3y + 7 = 0 + 3(–3) + 7 = –2 < 0

$\therefore$ The points (1, 4) and (0, – 3) are on the opposite sides of the given line.

Problems for Self Practice-13 :

(i) Are the points (1, – 3) and (12, 6) on the same or opposite side of the line 2x – y –10 = 0 ?

(ii) Which one of the points (5, 2), (–1, 2) and (–2, 8) lies on the side of the line 4x + 3y – 5 = 0 on which the origin lies?

Answers : (i) Opposite sides (ii) (–1, 2)

10.3 Foot of the perpendicular from a point on a line :

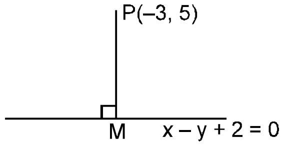
Foot of the perpendicular from a point (x_1, y_1) on the line ax + by + c = 0 is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

SOLVED EXAMPLE

Example: 22 Find the foot of perpendicular of the line drawn from P (– 3, 5) on the line x – y + 2 = 0.

Solution : Slope of PM = – 1



$\therefore$ Equation of PM is

x + y – 2 = 0 ....(i)

solving equation (i) with x – y + 2 = 0, we get co-ordinates of M (0, 2)

Aliter Here, $\frac{x+3}{1} = \frac{y-5}{-1} = -\frac{(1\times(-3)+(-1)\times5+2)}{(1)^2+(-1)^2}$

$\Rightarrow$ $\frac{x+3}{1} = \frac{y-5}{-1} = 3$

$\Rightarrow$ x + 3 = 3 $\Rightarrow$ x = 0 and y – 5 = – 3 $\Rightarrow$ y = 2

$\therefore$ M is (0, 2)

Problems for Self Practice-14 :

(i) Find the foot of perpendicular of the line drawn from (– 2, – 3) on the line 3x – 2y – 1 = 0.

Answers : (i) $\left(\frac{-23}{13}, \frac{-41}{13}\right)$

10.4 Reflection of a point about a line

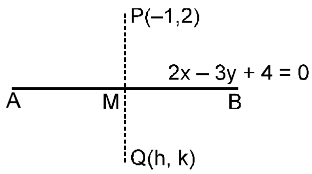
The image of a point (x_1, y_1) about the line ax + by + c = 0 is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right).$$

SOLVED EXAMPLE

Example: 23 Find the image of the point P(–1, 2) in the line mirror 2x – 3y + 4 = 0.

Solution : Let image of P is Q.



$\therefore$ PM = MQ & PQ $\perp$ AB

Let Q is (h, k)

$\therefore$ M is $\left(\frac{h-1}{2}, \frac{k+2}{2}\right)$

It lies on 2x – 3y + 4 = 0.

$\therefore$ $2\left(\frac{h-1}{2}\right) - 3\left(\frac{k+2}{2}\right) + 4 = 0.$

or 2h – 3k = 0 ....(i)

slope of PQ = $\frac{k-2}{h+1}$

PQ $\perp$ AB

$\therefore$ $\frac{k-2}{h+1} \times \frac{2}{3} = -1.$

$\Rightarrow$ 3h + 2k – 1 = 0. ....(ii) soving (i) & (ii), we get $h = \frac{3}{13}$, $k = \frac{2}{13}$

∴ Image of P(– 1, 2) is $Q\left(\frac{3}{13}, \frac{2}{13}\right)$

Aliter The image of P (– 1, 2) about the line

$2x - 3y + 4 = 0$ is $\frac{x+1}{2} = \frac{y-2}{-3} = \frac{-2[2(-1)-3(2)+4]}{2^2+(-3)^2}$

$\frac{x+1}{2} = \frac{y-2}{-3} = \frac{8}{13}$

⇒ $13x + 13 = 16 \Rightarrow x = \frac{3}{13}$ & $13y - 26 = -24 \Rightarrow y = \frac{2}{13}$

∴ image is $\left(\frac{3}{13}, \frac{2}{13}\right)$

Problems for Self Practice-15 :

(i) Find the image of the point (1, 2) in y-axis.

Ans. (i)(– 1, 2)

11. DISTANCE BETWEEN TWO PARALLEL LINES :

(a) The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $= \frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$

(**Note :** The coefficients of x & y in both equations should be same)

(b) The area of the parallelogram = $\frac{p_1 p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1x + c_1$, $y = m_1x + c_2$ and $y = m_2x + d_1$, $y = m_2x + d_2$ is given by $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.

SOLVED EXAMPLE

Example: 24 Two sides of a square lie on the lines x + y = 1 and x + y + 2 = 0. What is its area ?

Solution : Clearly the length of the side of the square is equal to the distance between the parallel lines

x + y – 1 = 0 ....(i) and x + y + 2 = 0 ....(ii)

Putting x = 0 in (i), we get y = 1. So (0, 1) is a point on line (i).

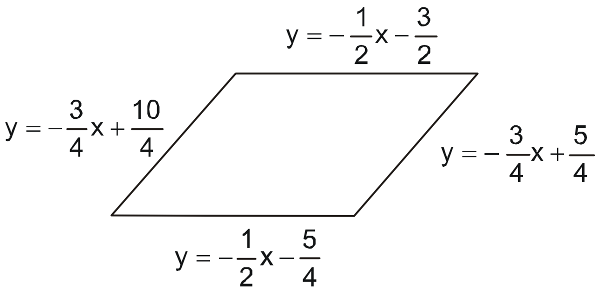
Now, Distance between the parallel lines

= length of the ⊥ from (0, 1) to x + y + 2 = 0 = $\frac{|0+1+2|}{\sqrt{1^2+1^2}} = \frac{3}{\sqrt{2}}$

Thus, the length of the side of the square is $\frac{3}{\sqrt{2}}$ and hence its area = $\left(\frac{3}{\sqrt{2}}\right)^2 = \frac{9}{2}$

Example : 25 Find the area of the parallelogram whose sides are $x + 2y + 3 = 0$, $3x + 4y - 5 = 0$, $2x + 4y + 5 = 0$ and $3x + 4y - 10 = 0$

Solution :



Here, $c_1 = -\frac{3}{2}$, $c_2 = -\frac{5}{4}$, $d_1 = \frac{10}{4}$, $d_2 = \frac{5}{4}$, $m_1 = -\frac{1}{2}$, $m_2 = -\frac{3}{4}$

$$\therefore \quad \text{Area} = \left| \frac{\left(-\frac{3}{2}+\frac{5}{4}\right)\left(\frac{10}{4}-\frac{5}{4}\right)}{\left(-\frac{1}{2}+\frac{3}{4}\right)} \right| = \frac{5}{4} \text{ sq. units}$$

Problems for Self Practice-16 :

(i) Find the distances between the following pair of parallel lines :

(a) $3x + 4y = 13$, $3x + 4y = 3$

(b) $3x - 4y + 9 = 0$, $6x - 8y - 15 = 0$

(ii) Find the area of parallelogram whose sides are given by $4x - 5y + 1 = 0$, $x - 3y - 6 = 0$, $4x - 5y - 2 = 0$ and $2x - 6y + 5 = 0$

Answers : (i) (a) 2; (b) 33/10 ; (ii) $\frac{51}{14}$ sq. units

12. EQUATION OF BISECTORS OF ANGLES BETWEEN TWO LINES

If equation of two intersecting lines are $a_1x+b_1y+c_1=0$ and $a_2x + b_2y+c_2=0$, then equation of bisectors of the angles between these lines are written as :

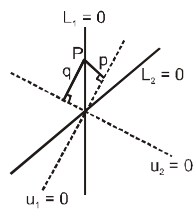
$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm \frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} \quad \text{..........(i)}$$

Equation of bisector of acute/obtuse angles :

To find the equation of the bisector of the acute or obtuse angle :

(i) Let ϕ be the angle between one of the two bisectors and one of two given lines. Then if **$\tan\phi < 1$** i.e. $\phi < 45°$ i.e. $2\phi < 90°$, the angle bisector will be bisector of acute angle.

(ii) Let $L_1 = 0$ & $L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ & $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ & $u_2 = 0$ as shown in figure. If,



$|p| < |q| \Rightarrow u_1$ is the acute angle bisector.

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.

$|p| = |q| \Rightarrow$ the lines L_1 & L_2 are perpendicular.

(iii) If $a_1a_2 + b_1b_2 < 0$, then the equation of the bisector of this obtuse angle is

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = -\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

If $a_1a_2 + b_1b_2 > 0$, then the equation of the bisector of this obtuse angle is

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = +\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Note : Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

SOLVED EXAMPLE

Example : 26 For the straight lines $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the

(i) bisector of the obtuse angle between them.

(ii) bisector of the acute angle between them.

(iii) bisector of the angle which contains origin.

(iv) bisector of the angle which contains (1, 2).

Solution : Equations of bisectors of the angles between the given lines are

$$\frac{4x+3y-6}{\sqrt{4^2+3^2}} = \pm\frac{5x+12y+9}{\sqrt{5^2+12^2}} \Rightarrow 9x - 7y - 41 = 0 \text{ and } 7x + 9y - 3 = 0$$

If θ is the acute angle between the line $4x + 3y - 6 = 0$ and the bisector

$9x - 7y - 41 = 0$, then $\tan\theta = \left|\frac{-\frac{4}{3}-\frac{9}{7}}{1+\left(\frac{-4}{3}\right)\frac{9}{7}}\right| = \frac{11}{3} > 1$

Hence

(i) bisector of the obtuse angle is $9x - 7y - 41 = 0$

(ii) bisector of the acute angle is $7x + 9y - 3 = 0$

(iii) bisector of the angle which contains origin

$$\frac{-4x-3y+6}{\sqrt{(-4)^2+(-3)^2}} = \frac{5x+12y+9}{\sqrt{5^2+12^2}} \Rightarrow 7x+9y-3=0$$

(iv) $L_1(1, 2) = 4 \times 1 + 3 \times 2 - 6 = 4 > 0$

$L_2(1, 2) = 5 \times 1 + 12 \times 2 + 9 = 38 > 0$

+ve sign will give the required bisector, $\frac{4x+3y-6}{5} = +\frac{5x+12y+9}{13}$

$\Rightarrow$ $9x - 7y - 41 = 0$.

Alternative :

Making c_1 and c_2 positive in the given equation, we get $-4x - 3y + 6 = 0$ and $5x + 12y + 9 = 0$

Since $a_1a_2 + b_1b_2 = -20 - 36 = -56 < 0$, so the origin will lie in the acute angle.

Hence bisector of the acute angle is given by

$$\frac{-4x-3y+6}{\sqrt{4^2+3^2}}=\frac{5x+12y+9}{\sqrt{5^2+12^2}} \quad \Rightarrow \quad 7x+9y-3=0$$

Similarly bisector of obtuse angle is $9x - 7y - 41 = 0$.

Problems for Self Practice-17 :

(i) Find the equations of the bisectors of the angles between the lines $x + y - 3 = 0$ and $7x - y + 5 = 0$ and state which of them bisects the acute angle between the lines.

Answers : (i) $x - 3y + 10 = 0$ (bisector of the obtuse angle) ;
$6x + 2y - 5 = 0$ (bisector of the acute angle)

13. CONCURRENCY OF LINES

(a) Three lines $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent,

if $$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_2 & c_3 \end{vmatrix}=0$$

(b) To test the concurrency of three lines, first find out the point of intersection of any two of the three lines. If this point lies on the remaining line (i.e. coordinates of the point satisfy the equation of the line) then the three lines are concurrent otherwise not concurrent.

SOLVED EXAMPLE

Example: 27 Prove that the straight lines $4x + 7y = 9$, $5x - 8y + 15 = 0$ and $9x - y + 6 = 0$ are concurrent.

Solution : Given lines are

$4x + 7y - 9 = 0$ ....(1)

$5x - 8y + 15 = 0$ ....(2)

and $9x - y + 6 = 0$ ....(3)

$$\Delta = \begin{vmatrix} 4 & 7 & -9 \\ 5 & -8 & 15 \\ 9 & -1 & 6 \end{vmatrix} = 4(-48 + 15) - 7(30 - 135) - 9(-5 + 72) = -132 + 735 - 603 = 0$$

Hence lines (1), (2) and (3) are concurrent.

Problems for Self Practice-18 :

(i) Find the value of m so that the lines $3x + y + 2 = 0$, $2x - y + 3 = 0$ and $x + my - 3 = 0$ may be concurrent.

Answers : (i) 4

14. FAMILY OF LINES

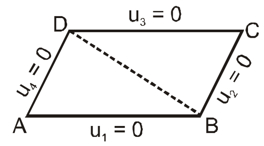
If equation of two lines be $P \equiv a_1x + b_1y + c_1 = 0$ and $Q \equiv a_2x + b_2y + c_2 = 0$, then the equation of the lines passing through the point of intersection of these lines is :

$P + \lambda Q = 0$ or $a_1x + b_1y + c_1 + \lambda(a_2x + b_2y + c_2) = 0$

The value of λ is obtained with the help of the additional informations given in the problem.

Note :

(i) If $u_1 = ax + by + c$, $u_2 = a'x + b'y + d$, $u_3 = ax + by + c'$, $u_4 = a'x + b'y + d'$ then $u_1 = 0; u_2 = 0; u_3 = 0; u_4 = 0$ form a parallelogram.
The diagonal BD can be given by $u_2u_3 - u_1u_4 = 0$.



(ii) The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some real λ and μ.
[For getting the values of λ & μ compare the coefficients of x, y & the constant terms].

SOLVED EXAMPLE

Example : 28 Find the equation to the straight line passing through the point (3, 2) and the point of intersection of the lines $2x + 3y = 1$ and $3x - 4y = 6$.

Solution: By family of lines $(2x + 3y - 1) + \lambda(3x - 4y - 6) = 0$ it passes through (3, 2)

$(6 + 6 - 1) + \lambda(9 - 8 - 6) = 0 \quad \Rightarrow \quad \lambda = \frac{11}{5}$

$(2x + 3y - 1) + \frac{11}{5}(3x - 4y - 6) = 0 \quad \Rightarrow \quad 43x - 29y = 71$

<u>Problems for Self Practice</u>-19 :

(i) Find the equations of the line which pass through the point of intersection of the lines $4x - 3y = 1$ and $2x - 5y + 3 = 0$ and is equally inclined to the coordinate axes.

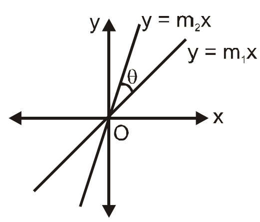
(ii) Find the equation of the line through the point of intersection of the lines $3x - 4y + 1 = 0$ & $5x + y - 1 = 0$ and perpendicular to the line $2x - 3y = 10$.

Answers : (i) $x + y = 2$, $x = y$; (ii) $69x + 46y - 25 = 0$

15. PAIR OF STRAIGHT LINES

(A) A Pair of straight lines through origin

(i) A homogeneous equation of degree two, "$ax^2 + 2hxy + by^2 = 0$" always represents a pair of straight lines passing through the origin if :



(a) $h^2 > ab \Rightarrow$ lines are real & distinct .

(b) $h^2 = ab \Rightarrow$ lines are coincident .

(c) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection i.e. (0, 0)

This equation is obtained by multiplying the two equations of lines $(m_1x - y)(m_2x - y) = 0$

$\Rightarrow \quad m_1m_2x^2 - (m_1 + m_2)xy + y^2 = 0$

(ii) If $y = m_1x$ & $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;

$m_1 + m_2 = -\frac{2h}{b}$ & $m_1 m_2 = \frac{a}{b}$.

(iii) If θ is the acute angle between the pair of straight lines represented by,

$ax^2 + 2hxy + by^2 = 0$, then $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$.

(iv) The condition that these lines are :

(a) at right angles to each other is $a + b = 0$. i.e. co–efficient of x^2 + co–efficient of $y^2 = 0$.

(b) coincident is $h^2 = ab$.

(c) equally inclined to the axis of x is $h = 0$.i.e. coeff. of $xy = 0$.

Note that a homogeneous equation of degree n represents at most n straight lines passing through origin.

(v) The equation to the pair of straight lines bisecting the angles between the straight lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$.

Note :

(i) If a = b, the bisectors are $x^2 - y^2 = 0$ i.e. $x - y = 0$, $x + y = 0$

(ii) If h = 0, the bisectors are $xy = 0$ i.e. $x = 0$, $y = 0$.

(iii) The two bisectors are always at right angles, since we have coefficient of x^2 + coefficient of $y^2 = 0$

(vi) Pair of straight lines perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$ and through origin are given by $bx^2 - 2hxy + ay^2 = 0$.

(vii) The product of the perpendiculars drawn from the point (x_1, y_1) on the lines $ax^2 + 2hxy + by^2 = 0$ is

$$\left|\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}\right|$$

(B) General equation of second degree representing a pair of Straight lines

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, i.e. if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

Such an equation is obtained again by multiplying the two equation of lines
$(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only.

i.e., If θ be the angle between the lines, then $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a+b}$

Obviously these lines are

(1) Parallel, if $\Delta = 0$, $h^2 = ab$ or if $h^2 = ab$ and $bg^2 = af^2$

(2) Perpendicular, if a + b =0 i.e. coeff. of x^2 + coeff. of y^2 = 0.

(iii) The product of the perpendiculars drawn from the origin to the lines

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\left|\frac{c}{\sqrt{(a-b)^2 + 4h^2}}\right|$

SOLVED EXAMPLE

Example: 29 For what value of λ does the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represent a pair of straight lines ? Find their equations, point of intersection, acute angle between them and pair of angle bisector.

Ans. $\lambda = 2$, $3x - y + 2 = 0$, $4x - 2y + 1 = 0$, point of intersection $\left(-\frac{3}{2}, -\frac{5}{2}\right)$,

$\tan^{-1}\left(\frac{1}{7}\right)$, $2x^2 + 4xy - 2y^2 + 16x - 4y + 7 = 0$.

Solution : $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$

This represents pair of straight lines if $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ we get $\lambda = 2$

Now $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = (6x - 2y + p)(2x - y + q)$

compair both sides $2p + 6q = 11 \Rightarrow -p - 2q = -5$

solving both we get $p = 4$, $q = \frac{1}{2}$

Hence required lines are $6x - 2y + 4 = 0 \Rightarrow 3x - y + 2 = 0$

$\Rightarrow \quad 2x - y + \frac{1}{2} = 0 \Rightarrow 4x - 2y + 1 = 0$

solving both equations we get point of intersection $\left(-\frac{3}{2}, -\frac{5}{2}\right)$

Now angle between both lines $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{3-2}{1+3\times 2}\right| = \frac{1}{7} \Rightarrow \theta = \tan{-1}\ \frac{1}{7}$

Now equation of pair of angle bisector $\dfrac{\left(x+\frac{3}{2}\right)^2 - \left(y+\frac{5}{2}\right)^2}{12-2} = \dfrac{\left(x+\frac{3}{2}\right)\left(y+\frac{5}{2}\right)}{-5}$

$\Rightarrow \quad 2x^2 + 4xy - 2y^2 + 16x - 4y + 7 = 0$

Problems for Self Practice-20 :

(i) Find the combined equation of the straight lines passing through the point (1, 1) and parallel to the lines represented by the equation $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ and find the angle between them.

Answers : (i) $x^2 - 5xy + 4y^2 + 3x - 3y = 0$, $\tan^{-1}\left(\frac{3}{5}\right)$

16. HOMOGENIZATION :

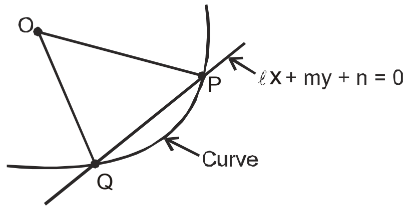
It is the process of obtaining equation of the lines joining the points of intersection of a line and a curve to the origin.

(a) Let the equation of curve be :

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ...(i)

and straight line be

$lx + my + n = 0$ ...(ii)



Now joint equation of line OP and OQ joining the origin and points of intersection P and Q can be obtained by making the equation (i) homogenous with the help of equation of the line. Thus required equation is given by

$$ax^2+2hxy+by^2+2(gx+fy)\left(\frac{\ell x+my}{-n}\right)+c\left(\frac{\ell x+my}{-n}\right)^2=0$$

$\Rightarrow$ $(an^2 + 2gln + cl^2)x^2 + 2(hn^2 + gmn + fln + clm)xy + (bn^2 + 2fmn + cm^2)y^2 = 0$ ......(iii)

All points which satisfy (i) and (ii) simultaneously, will satisfy (iii)

(b) Any second degree curve through the four points of intersection of $f(x, y) = 0$ & $xy = 0$ is given by $f(x, y) + \lambda xy = 0$ where $f(x, y) = 0$ is also a second degree curve.

SOLVED EXAMPLE

Example: 30 Find the equation of the straight lines joining the origin to the points of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.

Ans : $4alx^2 + 4amxy + ny^2 = 0$; $4al + n = 0$

Solution : $\frac{lx+my}{-n} = 1 \Rightarrow y^2 = 4ax \times 1 \Rightarrow y^2 = 4ax\left(\frac{\ell x+my}{-n}\right) \Rightarrow 4alx^2 + 4amxy + ny^2 = 0$

for perpendicular $4al + n = 0$

Problems for Self Practice-21 :

(i) Find the angle subtended at the origin by the intercept made on the curve $x^2 - y^2 - xy + 3x - 6y + 18 = 0$ by the line $2x - y = 3$.

(ii) Find the equation of the straight lines joining the origin to the points of intersection of the line $3x + 4y - 5 = 0$ and the curve $2x^2 + 3y^2 = 5$.

Answers : (i) $\theta = \pm\tan^{-1}\frac{4}{7}$ (ii) $x^2 - y^2 - 24xy = 0$

Exercise # 1

PART-I : SUBJECTIVE QUESTIONS

SECTION (A) : Distance Formula, Section Formula.

A-1. (i) Show that the points (12, 8), (–2, 6) and (6, 0) are the vertices of a right angled triangle.

(ii) Prove that the points A(–4, –1), B(–2, –4), C(4, 0), D(2, 3) are the vertices of a parallelogram. Is this parallelogram a rectangle?

(iii) Are the points A(0, –1), B(2, 1), C(0, 3), D(–2, 1) form a rhombus? Is this rhombus a square?

A-2 (i) Find the ratio in which the point (9, 6) divides the line segment joining the points (5, – 2) and (8, 4).

(ii) Find the harmonic conjugate of the point R(5, 1) with respect to points P(2, 10) and Q(6, – 2).

A-3. Find the ratio in which the line segment joining of the points (1, 2) and (– 2, 3) is divided by the line $3x + 4y = 7$

SECTION (B) : Area of triangle/Polygon, collinearity.

B-1. The point A divides the join of the points (–5, 1) and (3, 5) in the ratio k:1 and coordinates of points B and C are (1, 5) and (7, –2) respectively. If the area of ΔABC be 2 units, then find the value of k.

B-2. Find the area of the pentagon whose vertices are A(1, 1), B(7, 21), C(7, –3), D(12, 2) and E(0, –3).

B-3. Find the possible values of k so that the points (k, 2 – 2k), (1 – k, 2k) and (–k –4, 6 – 2k) be collinear.

B-4. Prove that the co-ordinates of the vertices of an equilateral triangle can not all be rational.

SECTION (C) : Equation of Straight line in various forms, Angle between two lines.

C-1. The coordinate of the midpoints of the sides of a triangle ABC are (6, –1), (–1, –2) and (1, –4). Find the length and equations of its sides.

C-2. The line joining two points A (2, 0) and B (3, 1) is rotated about A in the anticlock wise direction through an angle of 30º. Find the equation of the line in the new position.

C-3. Find the equation of a straight line passing through the origin and making with x – axis an acute angle twice the size of the angle made by the line $y = \frac{1}{3}x$ with the x – axis.

C-4. Find the equations to the straight lines pass through the point (2, 3) and inclined at an angle of 45° to the line $3x + y = 5$.

C.-5 The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$,then find equation of other line.

C-6. Find the equation of the lines joining the origin to the points of trisection of the portion of the line $3x + y = 12$ intercepted between the coordinate axes.

C-7. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

C-8. The vertex A of triangle ABC is given to be (1, 3) and the medians BE and CF are $x - 2y + 1 = 0$ & $y - 1 = 0$. Determine the equation of its sides.

C-9. Two consecutive sides of a parallelogram are $4x + 5y = 0$ & $7x + 2y = 0$. If the equation to one diagonal is $11x + 7y = 9$, find the equation to the other diagonal.

C-10. Equation of a line which passes through point A(2, 3) and makes an angle of 45° with x axis. If this line meet the line $x + y + 1 = 0$ at point P then find the distance AP.

C-11. One diagonal of a square is the portion of the line $7x + 5y = 35$ intercepted by the coordinate axes, obtain the extremities of the other diagonal.

C-12. Through the point P(4, 1) a line is drawn to meet the line $3x - y = 0$ at Q where $PQ = \frac{11}{2\sqrt{2}}$. Determine the equation of line.

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre

D-1. For triangle whose vertices are (0, 0), (5, 12) and (16, 12). Find coordinates of
(i) Centroid
(ii) Circumcentre
(iii) Incentre
(iv) Excentre opposite to vertex (5, 12)

D-2. Find the co-ordinates of the othocentre of the triangle whose sides are
$3x - 2y = 6$, $3x + 4y + 12 = 0$ and $3x - 8y + 12 = 0$.

D-3. If $(b_2 - b_1)(b_3 - b_1) + (a_2 - a_1)(a_3 - a_1) = 0$ then prove that the circumcentre of the triangle having vertices (a_1, b_1), (a_2, b_2) and (a_3, b_3) is $\left(\frac{a_2 + a_3}{2}, \frac{b_2 + b_3}{2}\right)$

D-4. If two vertices of a triangle are (–2, 3) & (5, – 1), the orthocentre lies at the origin and the centroid on the line $x + y = 7$, then find the third vertex.

D-5. The vertices of a triangle are $\left(pq, \frac{1}{pq}\right), \left(qr, \frac{1}{qr}\right)$ and $\left(rq, \frac{1}{rp}\right)$ where p, q, r are the roots of the equation $x^3 - 3x^2 + 6x + 1 = 0$. Find the coordinates of centroid of the triangle.

D-6. Two vertices of a triangle are (4, –3) & (–2, 5). If the orthocentre of the triangle is at (1, 2), find the coordinates of the third vertex.

Section (E) : Position of a Point, Perpendicular Distance, Foot of Perpendicular, Image of a Point.

E-1. Find the position of the origin with respect to the triangle whose sides are
$x + 1 = 0$, $3x - 4y - 5 = 0$, $5x + 12y - 27 = 0$.

E-2. Find the coordinates of the points on the line $x + 5y = 13$ at a distance of 2 units from the line $12x - 5y + 26 = 0$

E-3. If p and p' be the lengths of perpendicualrs from origin to the lines $x \sec\theta - y \operatorname{cosec}\theta = 2$ and $x \cos\theta - y \sin\theta = 2 \cos 2\theta$ respectively then find the value of $4p^2 + p'^2$.

E-4. Find coordinates of the foot of perpendicular, image and equation of perpendicular drawn from the point (2, 3) to the line $y = 3x - 4$.

E-5. Three lines $x + 2y + 3 = 0$, $x + 2y - 7 = 0$ and $2x - y - 4 = 0$ form 3 sides of two squares. Find the equation of remaining sides of these squares.

E-6. Find the area of the parallelogram fomed by the lines $2x - 3y + 5 = 0$, $3x - 2y - 5 = 0$, $2x - 3y + 15 = 0$, $3x - 2y - 10 = 0$.

Section (F) : Angle Bisectors, Condition of Concurrency, Family of Lines.

F-1. Find equations of acute and obtuse angle bisectors of the angle between the lines $4x + 3y - 7 = 0$ and $24x + 7y - 31 = 0$. Also comment in which bisector origin lies.

F-2. Prove that the straight line $5x + 4y = 0$, passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$.

F-3. Prove that each member of the family of straight lines
$(3\sin\theta + 4\cos\theta)x + (2\sin\theta - 7\cos\theta)y + (\sin\theta + 2\cos\theta) = 0$ (θ is a parameter) passes through a fixed point.

F-4. A ray of light is sent along the line $x - 2y - 3 = 0$. Upon reaching the line mirror $3x - 2y - 5 = 0$, the ray is reflected from it. Find the equation of the line containing the reflected ray.

F-5. If the algebraic sum of perpendiculars from n given points on a variable straight line is zero then prove that the variable straight line passes through a fixed point.

F-6. Find the equation of the line through the point of intersection of the line $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$.

F-7. Find the equation of a straight line which passes through the point of intersection of the straight lines $x + y - 5 = 0$ and $x - y + 3 = 0$ and perpendicular to the straight line intersecting x-axis at the point $(-2, 0)$ and the y-axis at the point $(0, -3)$,

Section (G) : Locus Problems.

G-1. Find the locus of the centroid of a triangle whose vertices are $(a \cos \alpha, -a \sin \alpha)$, $(b \sin \alpha, b \cos \alpha)$ and $(-1, 0)$, where 'α' is the parameter.

G-2. Find the locus of point of intersection of the lines $x \cos t - y \sin t = a$ and $x \sin t + y \cos t = b$, where t is a parameter.

G-3. The ends of the hypotenuse of a right angled triangle are (5, 0) and (0, 5), then find the locus of third vertex of triangle.

G-4. A(a, 0) and B(–a, 0) are two fixed points of ΔABC. If its vertex C moves in such a way that $\cot A + \cot B = \lambda$, where λ is a constant, then find the locus of the point C.

G-5. Two points A and B move on the positive direction of x-axis and y-axis respectively, such that OA + OB = K. Show that the locus of the foot of the perpendicular from the origin O on the line AB is $(x + y)(x^2 + y^2) = Kxy$.

G-6. Find the locus of the circumcentre of a triangle whose two sides are along the co-ordinate axes and third side passes through the point of intersection of the lines $ax + by + c = 0$ and $\ell x + my + n = 0$.

G-7. A variable line is drawn through O, to cut two fixed straight lines L_1 and L_2 in A_1 and A_2, respectively. A point A is taken on the variable line such that $\frac{m+n}{OA} = \frac{m}{OA_1} + \frac{n}{OA_2}$. Show that the locus of A is a straight line passing through the point of intersection of L_1 and L_2 where O is being the origin.

Section (H) : Pair of Straight Lines.

H-1. Show that the equation $6x^2 - 5xy + y^2 = 0$ represents a pair of distinct straight lines, each passing through the origin. Find the separate equations of these lines.

H-2. Find the angle between the pair of straight lines $4x^2 + 24xy + 11y^2 = 0$

H-3. Find the equation of the bisectors of the angle between the lines represented by $3x^2 - 5xy + 4y^2 = 0$

H-4. Find the equations to the pair of lines through the origin which are perpendicular to the lines represented by $2x^2 - 7xy + 3y^2 = 0$.

H-5. Prove that the equation $2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$ represents a pair of straight lines. Find the co-ordinates of their point of intersection.

H-6. Prove that the angle between the lines joining the origin to the points of intersection of the straight line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is $\tan^{-1}\frac{2\sqrt{2}}{3}$.

H-7. Find the equation of the lines joining the origin to the points of intersection of the curve $2x^2 + 3xy - 4x + 1 = 0$ and the line $3x + y = 1$.

H-8. The chord $\sqrt{6}y = \sqrt{8}px + \sqrt{2}$ of the curve $py^2 + 1 = 4x$ subtends a right angle at origin then find the value of p.

PART-II : OBJECTIVE QUESTIONS

Section (A) : Distance Formula, Section Formula.

A-1. The distance between the point $P(a\cos\alpha,\ a\sin\alpha)$ and $Q(a\cos\beta,\ a\sin\beta)$, where a > 0 & $\alpha > \beta$, is -

(A) $4a\sin\frac{\alpha-\beta}{2}$ (B) $2a\sin\frac{\alpha+\beta}{2}$ (C) $2a\sin\frac{\alpha-\beta}{2}$ (D) $2a\cos\frac{\alpha-\beta}{2}$

A-2. The coordinates of the base BC of an isosceles triangle ABC are given B (1, 3) and C (–2, 7). Which of the following points can be the possible coordinates of the vertex A ?

(A) $\left(-7, \frac{1}{8}\right)$ (B) (1, 6) (C) $\left(-\frac{1}{2}, 5\right)$ (D) $\left(\frac{5}{6}, 8\right)$

A-3. Which of the following sets of points from an equilateral triangle ?

(A) (1, 0), (4, 0), (7, –1) (B) (0, 0), $\left(\frac{3}{2}, \frac{4}{3}\right)$, $\left(\frac{4}{3}, \frac{3}{2}\right)$

(C) $\left(\frac{2}{3}, 0\right)$, $\left(0, \frac{2}{3}\right)$, (1, 1) (D) None of these

A-4. A line segment AB is divided internally and externally in the same ratio (> 1) at P and Q respectively and M is mid point of AB.

Statement-1: MP, MB, MQ are in G.P.

Statement-2 : AP, AB and AQ are in HP.

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1

(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1Pro

(C) STATEMENT-1 is true, STATEMENT-2 is false

(D) STATEMENT-1 is false, STATEMENT-2 is true

(E) Both STATEMENTS are false

Section (B) : Area of triangle, collinearity.

B-1. Find the area of the triangle formed by the mid points of sides of the triangle whose vertices are (2, 1), (– 2, 3), (4, – 3)

(A) 1.5 sq. units (B) 3 sq. units (C) 6 sq. units (D) 12 sq. units

B-2. If a vertex of a triangle is (1,1) and the mid–points of two sides through this vertex are (–1,2) and (3,2), then the centroid of the triangle is :

(A) $\left(\frac{1}{3},\frac{7}{3}\right)$ (B) $\left(1,\frac{7}{3}\right)$ (C) $\left(-\frac{1}{3},\frac{7}{3}\right)$ (D) $\left(-1,\frac{7}{3}\right)$

B-3. Let A(h, k), B(1, 1) and C(2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of triangle is 1, then the set of values which 'k' can take is given by

(A) {1, 3} (B) {0, 2} (C) {–1, 3} (D) {–3, – 2}

B-4. The vertices of a quadrilateral are (6, 3), (–3, 5), (4, –2) and (x, 3x) and are denoted by A, B, C and D, respectively. Find the sum of all the possible values of x so that the area of triangle ABC is double the area of triangle DBC.

(A) $\frac{11}{8}$ (B) 1 (C) $-\frac{3}{8}$ (D) $\frac{7}{4}$

B-5. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are

(A) $\left(\frac{4}{3},3\right)$ (B) $\left(3,\frac{2}{3}\right)$ (C) $\left(3,\frac{4}{3}\right)$ (D) $\left(\frac{4}{3},\frac{2}{3}\right)$

Section (C) : Equation of Straight line in various forms, Angle between two lines.

C-1. A straight line through P (1, 2) is such that its intercept between the axes is bisected at P. Its equation is :

(A) x + 2y = 5 (B) x – y + 1 = 0 (C) x + y – 3 = 0 (D) 2x + y – 4 = 0

C-2. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is :

(A) 3x – 4y + 7 = 0 (B) 4x + 3y = 24 (C) 3x + 4y = 25 (D) x + y = 7

C-3. Let P = (–1, 0) Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. The equation of the bisector of the $\angle$PQR is

(A) $\sqrt{3}\ x + y = 0$ (B) $x + \frac{\sqrt{3}}{2}\ y = 0$ (C) $\frac{\sqrt{3}}{2}\ x + y = 0$ (D) $x + \sqrt{3}\ y = 0$

C-4. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept – 4. Then a possible value of k is

(A) – 4 (B) 1 (C) 2 (D) – 2

C-5. If the x intercept of the line y = mx + 2 is greater than 1/2 then the gradient of the line lies in the interval-

(A) (–1,0) (B) (–1/4,0) (C) $(-\infty,-4)$ (D) (–4,0)

C-6. A line L passes through the points (1, 1) and (0, 2) and another line M which is perpendicular to L passes through the point (0, –1/2).The area of the triangle formed by these lines with y-axis is -

(A) 25/8 (B) 25/16 (C) 25/4 (D) 25/32

C-7. In a ΔABC, side AB has the equation 2x + 3y = 29 and the side AC has the equation x + 2y = 16. If the mid point of BC is (5, 6), then the equation of BC is
(A) 2x + y = 16 (B) x + y = 11 (C) 2x – y = 4 (D) x + y = 10

C-8. A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha \left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is :
(A) y (cos α – sin α) – x (sin α – cos α) = a (B) y (cos α + sin α) + x (sin α – cos α) = a
(C) y (cos α + sin α) + x (sin α + cos α) = a (D) y (cos α + sin α) + x (cos α – sin α) = a

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre

D-1. Coordinates of the vertices of a triangle ABC are (12,8), (–2,6) and (6,0) then the **correct** statement is-
(A) triangle is right but not isosceles
(B) triangle is isosceles but not right
(C) triangle is obtuse
(D) the product of the abscissa of the centroid, orthocenter and circumcenter is 160.

D-2. The orthocenter of the triangle ABC is 'B' and the circumcenter is 'S' (a,b). If A is the origin then the co-ordinates of C are :
(A) (2a,2b) (B) $\left(\frac{a}{2},\frac{b}{2}\right)$ (C) $\left(\sqrt{a^2+b^2},0\right)$ (D) none

D-3. The medians of a triangle meet at (0,–3) and its two vertices are at (–1,4) and (5,2). Then the third vertex is at -
(A) (4,15) (B) (–4,–15) (C) (–4,15) (D) (4,–15)

D-4. If in triangle ABC, A ≡ (1,10), circumcenter ≡ $\left(-\frac{1}{3},\frac{2}{3}\right)$ and orthocenter ≡ $\left(\frac{11}{3},\frac{4}{3}\right)$ then the co-ordinates of mid-point of side opposite to A is-
(A) (1,–11/3) (B) (1,5) (C) (1,–3) (D) (1,6)

D-5. The vertices of a triangle are A(0, –6), B(–6, 0) and C(1,1) respectively, then coordinates of the ex-centre opposite to vertex A is :
(A) $\left(\frac{-3}{2},\frac{-3}{2}\right)$ (B) $\left(-4,\frac{3}{2}\right)$ (C) $\left(\frac{-3}{2},\frac{3}{2}\right)$ (D) (–4, 6)

D-6. The co-ordinates of the orthocentre of the triangle bounded by the lines, 4x – 7y + 10 = 0; x + y = 5 and 7x + 4y = 15 is-
(A) (2,1) (B) (–1,2) (C) (1,2) (D) (1,–2)

D-7. A triangle has the lines y = m_1x and y = m_2x for two of its sides, where m_1, m_2 are the roots of the equation x^2 + ax – 1 = 0, then the orthocentre of triangle is, .
(A) (0, 0) (B) (1, 0) (C) (– 2, 1) (D) none of these

D-8. A triangle ABC with vertices A (– 1, 0),B (– 2, 3/4) & C (– 3, – 7/6) has its orthocentre H. Then the orthocentre of triangle BCH will be :
(A) (– 3, – 2) (B) (1, 3) (C) (– 1, 2) (D) none of these

Section (E) : Position of a Point, Perpendicular Distance, Area of Parallelogram, Foot of Perpendicular, Image of a Point.

E-1. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then 'a' belongs to :

(A) $(3, \infty)$ (B) $\left(\frac{1}{2}, 3\right)$ (C) $\left(-3, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{2}\right)$

E-2. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + a\,by + 1 = 0$ $\forall$ $a \in R$, $b > 0$ are :

(A) $b \in (2, 4)$ (B) $b \in (0, 2)$ (C) $b \in [0, 2]$ (D) $(2, \infty)$

E-3. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is

(A) 133 (B) 190 (C) 233 (D) 105

E-4. Find distance of point A (2, 3) measured parallel to the line $x - y = 5$ from the line $2x + y + 6 = 0$.

(A) $\frac{13}{3}$ (B) $\frac{13}{3\sqrt{2}}$ (C) $\frac{13\sqrt{2}}{3}$ (D) None of these

E-5. The point on the line $3x + 4y - 1 = 0$ which is nearest from the origin is

(A) $\left(\frac{4}{25}, \frac{3}{25}\right)$ (B) $\left(\frac{-4}{25}, \frac{-3}{25}\right)$ (C) $\left(\frac{3}{25}, \frac{4}{25}\right)$ (D) $\left(\frac{-3}{25}, \frac{-4}{25}\right)$

E-6. Two straight lines $x + 2y = 2$ and $x + 2y = 6$ are given, then find the equation of the line parallel to given lines and divided distance between lines in the ratio 2 : 1 internally

(A) $3x + 6y + 8 = 0$ (B) $3x + 6y = 14$ (C) $3x + 6y + 14 = 0$ (D) $3x + 2y = 10$

E-7. A ray of light passing through the point A(1,2) is reflected at a point B on the x-axis and then passes through (5,3). Then the equation of AB is :

(A) $5x + 4y = 13$ (B) $5x - 4y = -3$

(C) $4x + 5y = 14$ (D) $4x - 5y = -6$

E-8. The equations of the perpendicular bisector of the sides AB and AC of a ΔABC are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. If the point A is (1, –2), then the equation of the line BC is :

(A) $14x + 23y = 40$ (B) $14x - 23y = 40$ (C) $23x + 14y = 40$ (D) $23x - 14y = 40$

Section (F) : Angle Bisectors, Condition of Concurrency, Family of Lines.

F-1. Number of values of λ such that lines $x + 2y = 3$, $3x - y = 1$ and $\lambda x + y = 2$ can not form a triangle, is

(A) 0 (B) 1 (C) 2 (D) 3

F-2. If non–zero numbers a,b,c are in HP, then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point.

That point is :

(A) $\left(1, -\frac{1}{2}\right)$ (B) (1, – 2) (C) (–1, – 2) (D) (–1, 2)

F-3. If the lines ax + by + p = 0, $x\cos\alpha + y\sin\alpha - p = 0$ $(p \neq 0)$ and $x\sin\alpha - y\cos\alpha = 0$ are concurrent and the first two lines include an angle $\frac{\pi}{4}$, then $a^2 + b^2$ is equal to -

(A) 1 (B) 2 (C) 4 (D) p^2

F-4. The equation of bisectors of two lines L_1 & L_2 are $2x - 16y - 5 = 0$ and $64x + 8y + 35 = 0$. If the line L_1 passes through $(-11, 4)$, the equation of acute angle bisector of L_1 & L_2 is :

(A) $2x - 16y - 5 = 0$ (B) $64x + 8y + 35 = 0$
(C) $2x + 16y + 5 = 0$ (D) $2x + 16y - 5 = 0$

F-5. Consider the family of lines $5x + 3y - 2 + \lambda_1 (3x - y - 4) = 0$ and $x - y + 1 + \lambda_2(2x - y - 2) = 0$. Equation of a straight line that belong to both families is -

(A) $25x - 62y + 86 = 0$ (B) $62x - 25y + 86 = 0$
(C) $25x - 62y = 86$ (D) $5x - 2y - 7 = 0$

Section (G) : Locus Problems.

G-1. Let A(2,–3) and B(–2,1) be vertices of a ΔABC. If the centroid of ΔABC moves on the line $2x + 3y = 1$, then the locus of the vertex C is-

(A) $2x + 3y = 9$ (B) $2x - 3y = 7$ (C) $3x + 2y = 5$ (D) $3x - 2y = 3$

G-2. A point P(x,y) moves so that the sum of the distance from P to the coordinate axes is equal to the distance from P to the point A(1,1). The equation of the locus of P in the first quadrant is -

(A) $(x + 1)(y + 1) = 1$ (B) $(x + 1)(y + 1) = 2$
(C) $(x - 1)(y - 1) = 1$ (D) $(x - 1)(y - 1) = 2$

G-3. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of 'c' is :

(A) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (B) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(C) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (D) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

G-4. A and B are any two points on the positive x and y axis respectively satisfying 2(OA) + 3(OB) = 10. If P is the middle point of AB then the locus of P is-

(A) $2x + 3y = 5$ (B) $2x + 3y = 10$ (C) $3x + 2y = 5$ (D) $3x + 2y = 10$

G-5. A stick of length 10 units rests against the floor and a wall of a room. If the stick begins to slide on the floor then the locus of its middle point is :

(A) $x^2 + y^2 = 2.5$ (B) $x^2 + y^2 = 25$ (C) $x^2 + y^2 = 100$ (D) none

Section (H) : Pair of Straight Lines.

H-1. If $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, then λ is equal to -

(A) 4 (B) 3 (C) 2 (D) 1

H-2. If the equation $ax^2 - 6xy + y^2 + 2gx + 2fy + c = 0$ represents a pair of lines whose slopes are m and m^2, then sum of all possible values of a is-

(A) 17 (B) –19 (C) 19 (D) –17

H-3. If the line $y = mx$ bisects the angle between the lines $ax^2 + 2h\,xy + by^2 = 0$ then m is a root of the quadratic equation :

(A) $hx^2 + (a-b)x - h = 0$ (B) $x^2 + h(a-b)x - 1 = 0$

(C) $(a-b)x^2 + hx - (a-b) = 0$ (D) $(a-b)x^2 - hx - (a-b) = 0$

H-4. The straight lines joining the origin to the points of intersection of the line $2x + y = 1$ and curve $3x^2 + 4xy - 4x + 1 = 0$ include an angle :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

PART-III : MATCH THE COLUMN

1.

	Column-I		Column-II
(A)	If the slope of one of the lines represented by $ax^2 + 10xy + y^2 = 0$ is four times the slope of the other line, then a =	(p)	2
(B)	The equation of second degree $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ represents a pair of straight lines. The distance between them is	(q)	4
(C)	The combined equation of the bisectors of the angle between the lines represented by $(x^2 + y^2)\sqrt{3} = 4xy$ is $4y^2 - \lambda x^2 = 0$ then value of λ is	(r)	6
(D)	The line $(K + 1)^2 x + ky - 2K^2 - 2 = 0$ passes through a point (α, β) regardless of value K. Then $(\alpha - \beta)$ equals :	(s)	16

2.

	Column-I		Column-II
(A)	The points J(2,–2), K(8,4), L(5,7) and M(–1,1) constitute the vertices of a	(p)	square
(B)	The points J(0,–1), K(2,1), L(0,3) and M(–2,1) are the vertices of a	(q)	rectangle
(C)	The points J(3,–5), K(–5,–4), L(7,10), M(15,9) are the vertices of a	(r)	trapezium
(D)	The points J(–3,4), K(–1,0), L(1,0) and M(3,4) are the vertices of a	(s)	parallelogram
		(t)	cyclic quadrilateral

Exercise # 2

PART-I : OBJECTIVE QUESTIONS

1. If P lies on the line y = x and Q lies on y = 2x and |PQ| = 4 then the equation for the locus of the mid point of PQ is

(A) $25x^2 - 8xy + 13y^2 = 4$ (B) $15x^2 - 4xy + 12y^2 = 4$

(C) $25x^2 - 6xy + 13y^2 = 4$ (D) none of these

2. In a triangle ABC, E and F are the points on AC and AB respectively dividing each of them in the ratio of 1 : 2 internally . The equation of the line DE which is perpendicular to AC is 2x – y – 1 = 0 and the equation of the line DF, which is perpendicular to AB is x + y – 5 = 0. If vertex A is (2, 3) then the area of triangle DBC is

(A) 13sq. units (B) 30sq. units (C) 15sq. units (D) 27sq. units

3. The equations of perpendicular of the sides AB & AC of Δ ABC are x – y – 4 = 0 and 2x – y – 5 = 0 respectively. If the vertex A is (– 2, 3) and circumcenter is $\left(\frac{3}{2}, \frac{5}{2}\right)$, then

(A) equation of median of side AB is x – y + 1 = 0

(B) centroid of triangle ABC is (3, 1)

(C) vertex C is (2, 0)

(D) Area of triangle ABC is 12.

4. On the portion of the straight line 2x + 3y –12 = 0 intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co–ordinates :

(A) (3, 3) (B) (5, 5) (C) (2, 3) (D) (4, 5)

5. The base of a triangle is the axis of x and its other two sides are given by the equations

$y = \left(\frac{1+\alpha}{\alpha}\right)x + (1+\alpha)$ and $y = \left(\frac{1+\beta}{\beta}\right)x + (1+\beta)$. Locus of its orthocenter is

(A) x – y = 0 (B) x – 2y = 0 (C) x + 2y = 0 (D) x + y = 0

6. A rectangle PQRS has its side PQ parallel to the line y = 5x and the vertices P, Q and S on the lines y = 2, x = 3, and x = – 3 respectively. Locus of the vertex R is

(A) 5x – 24y + 10 = 0 (B) 5x – 24y + 85 = 0

(C) 24x – 5y + 88 = 0 (D) 24x – 5y + 85 = 0

7. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at the points P and Q. Find the absolute minimum value of OP + OQ as L varies, where O is the origin.

(A) 18 (B) 9 (C) 20 (D) none of these

8. The area of parallelogram whose two sides are y = x + 3, 2x – y + 1 = 0 and remaining two sides are passing through (0, 0), is

(A) 1.5 (B) 3 (C) 6 (D) 2

9. If the quadrilateral formed by the line ax + by + c = 0, a'x + b'y + c = 0, ax + by + c' = 0, a'x + b'y + c' = 0 has perpendicular diagonals then

(A) $b^2 + c^2 = b'^2 + c'^2$ (B) $c^2 + a^2 = c'^2 + a'^2$ (C) $a^2 + b^2 = a'^2 + b'^2$ (D) none of these

10. Two ends A & B of a straight line segment of constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB is completed. Then find locus of the foot of the perpendicular drawn from P to AB.

(A) $x^{2/3} + y^{2/3} = c^{2/3}$ (B) $x^{2/3} + y^{2/3} = c^{1/3}$

(C) $x^{1/3} + y^{1/3} = c^{2/3}$ (D) $x^{1/3} + y^{1/3} = c^{1/3}$

11. The point A(–4, –1) undergoes following transformations successively :

(i) reflection about line y = –x

(ii) translation through a distance of 3 units in the positive direction of x-axis

(iii) rotation through an angle 105° in anti-clockwise direction about origin O.

Then the final position of point A is

(A) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (B) $\left(-2, 7\sqrt{2}\right)$ (C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D) $\left(-2\sqrt{6}, 2\sqrt{2}\right)$

12. The equation of the internal bisector of $\angle BAC$ of ΔABC with vertices A(5, 2), B(2, 3) and C(6, 5) is

(A) 2x + y + 12 = 0 (B) x + 2y – 12 = 0

(C) 2x + y – 12 = 0 (D) 2x – y – 12 = 0

13. The vertices B and C of a triangle ABC lie on the line 3y = 4x and y = 0 respectively and the side BC passes through the point $\left(\frac{2}{3}, \frac{2}{3}\right)$. If ABOC is a rhombus, O being the origin, then the coordinates of A are .

(A) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (B) (1, 0) (C) $\left(\frac{8}{5}, \frac{4}{5}\right)$ (D) (2, 3)

14. Locus of the image of the point (1, 2) in the line $(x - y + 2) + \lambda (2x + 3y + 1) = 0$ where λ is parameter is

(A) $x^2 + y^2 + 10x + 6y - 27 = 0$ (B) $x^2 + y^2 - 10x + 6y - 27 = 0$

(C) $x^2 + y^2 - 10x - 6y - 27 = 0$ (D) $x^2 + y^2 + 10x - 6y - 27 = 0$

15. The locus of circumcentre of the triangle formed by vertices A((–pq – p – q), –(1 + p) (1 + q)), B (pq + p – q, (1 + p) (1 + q), C (pq + q – p, (1 + p) (1 + q)) is

(A) y + x = 0 (B) y – x = 0 (C) $x^2 + y^2 = 1$ (D) xy = 1

16. If the lines given by the equation $5x^2 + 14xy + 8y^2 - 50x - 40y = 0$ and $2x + y + \lambda = 0$ are concurrent then λ is equal to

(A) 0 (B) 2 (C) 5 (D) 1

17. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameter of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector, then :

(A) $3a^2 + 2ab + 3b^2 = 0$ (B) $3a^2 + 10ab + 3b^2 = 0$

(C) $3a^2 - 2ab + 3b^2 = 0$ (D) $3a^2 - 10ab + 3b^2 = 0$

18. If the slope of one of the lines represented by $ax^2 + 4xy + by^2 = 0$ be the n^{th} power of the other, then $(ab^n)^{\frac{1}{n+1}} + (a^n b)^{\frac{1}{n+1}}$ is equal to

(A) –4 (B) 4 (C) 2 (D) – 2

PART-II : SUBJECTIVE QUESTIONS

1. Given a $\triangle ABC$ with unequal sides. P is the set of all points which is equidistant from B & C and Q is the set of all point which is equidistant from sides AB and AC. Then $n(P \cap Q)$ equals

2. A point moves in the x-y plane such that the sum of its distances from two mutually perpendicular lines is always equal to 5, then find the area enclosed by the locus of the point.

3. The interval in which 'a' lies so that $(a^2, a + 1)$ is a point in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin is $(\alpha, 0) \cup \left(\frac{\beta}{6}, \frac{\gamma}{2}\right)$. Find $\alpha + \beta + \gamma$.

4. The base BC of triangle ABC passes through a fixed point P(1 , 1) and its sides AB and AC are respectively bisected at right angle by the lines $x + y = 0$ and $y = 9x$. The locus of the vertex A is $\lambda(x^2 + y^2) + 9x - y = 0$. Find λ.

5. Drawn from the point (1, – 2) are two mutually perpendicular straight lines forming an isosceles triangle together with the straight line $3x - 4y - 1 = 0$. Then find the area of the triangle.

6. On the straight line $y = x + 2$, a point (a, b) is such that the sum of the square of distances from the straight lines $3x - 4y + 8 = 0$ and $3x - y - 1 = 0$ is least, then find value of $11(a + b)$.

7. A is a vriable point on x-axis and B(0,b) is a fixed point. An equilateral triangle ABC is completed with vertex C away from origin. If the locs of the pont C is $\alpha x + \beta y = b$, then $\alpha^2 + \beta^2$ is

8. A light beam emanating from the point A(3, 10) reflects from the straight line $2x + y - 6 = 0$ and then passes through the point B(4, 3). The equation of the reflected beam is $x + 3y - \lambda = 0$, then find the value of λ.

9. The equation of the bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$ which contains the point (– 1, 4) is $ax + 54y = 242$. Find 'a'.

10. A ray of light is sent from the point (1, 4). Upon reaching the x-axis, the ray is reflected from the point (3, 0). This reflected ray is again reflected by the line $x + y = 5$ and intersect y-axis at $P(0, \lambda)$. Find the value of $2\lambda + 1$.

11. Find the area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$

12. Two lines (L_1 and L_2) are drawn from point (α, α) making an angle 45° with the lines $L_3 \equiv x + y - f(\alpha) = 0$ and $L_4 \equiv x + y + f(\alpha) = 0$. L_1 intersects L_3 and L_4 at A and B and L_2 intersects L_3 and L_4 at C and D respectively ($|2\alpha| > |f(\alpha)|$). The area of trapezium ABDC is independent of α. If $f(\alpha) = \lambda\alpha^q$, where λ is a constant, then |q| is

13. If distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is $25\sqrt{2}$, then find the value of |a|.

14. If the straight lines joining the origin and the points of intersection of the curve

$5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis, then find the value of | k |.

15. The portion of the line $ax + by - 1 = 0$, intercepted between the lines $ax + y + 1 = 0$ and $x + by = 0$ subtends a right angle at the origin and the condition in a and b is $\lambda a + b + b^2 = 0$, then find value of λ.

PART - III : ONE OR MORE THAN ONE CORRECT OPTIONS

1. If the coordinates of the vertices of a triangle are rational number then which of the following points of the triangle will always have rational coordinates ?

(A) centroid (B) incentre (C)circumcentre (D) orthocentre

2. The points (– 2, 0), $\left(-1, \frac{1}{\sqrt{3}}\right)$ and (cos 4θ, sin 4θ) are collinear, for θ equal to

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{3\pi}{4}$ (D) $\frac{2\pi}{3}$

3. One side of a rectangle lies along the line 3x – 2y – 1 = 0. Two of its vertices are (3, 4) and (–1, 4). Then the equations of other sides are :

(A) 3x – 2y + 11 = 0 (B) 2x + 3y – 10 = 0

(C) 3x – 2y + 11 = 0 (D) 2x + 3y –18 = 0

4. In a triangle ABC vertex A is the point (–4, 5). Its altitudes are AD, BE and CF. The coordinates of the points D,E,F are respectively $\left(\frac{16}{5}, \frac{-23}{5}\right)$, (4, 1) and (–1, –4). The coordinates of the other vertices of the triangle are

(A) (0, 7) (B) (– 8, 1) (C) (0, –7) (D) (8, –1)

5. If the points $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ are collinear for three distinct values a, b, c and a ≠ 1, b ≠ 1 and c ≠ 1, then

(A) abc = $\frac{3m+n}{\ell}$ (B) a + b + c = $\frac{m}{\ell}$

(C) ab + bc + ac = $-\frac{n}{\ell}$ (D) abc– (ab + bc + ac) + 3 (a + b + c) = 0

6. A line making an angle of 60° with the positive direction of x-axis and passing through the point P($\sqrt{3}$, 0) cuts the parabola $y^2 = x + 4$ at A and B, then

(A) PA + PB = $\frac{2}{3}\sqrt{49+12\sqrt{3}}$ (B) |PA – PB| = $\frac{2}{3}$

(C) (PA) (PB) = $\frac{4(4+\sqrt{3})}{3}$ (D) $\frac{1}{PA} + \frac{1}{PB} = \frac{\sqrt{49+12\sqrt{3}}}{2(4+\sqrt{3})}$

7. In a triangle ABC, co-ordinates of A are (1, 2) and the equations to the medians through B and C are x + y = 5 and x = 4 respectively. Then the co-ordinates of B and C will be

(A) (– 2, 7), (4, 3) (B) (7, – 2), (4, 3)

(C) (2, 7), (– 4, 3) (D) (2, – 7), (3, – 4)

8. A lines $3x + 2y - 6 = 0$ and a variable line cuts the x-axis and y-axis in A, B, and A', B' respectively such that OA + OB = OA' + OB' (where 'O' is the origin). Locus of the point of intersection of AB' and A'B is

(A) a straight line whose is slope is 1

(B) a straight line passing through (3, 2)

(C) a straight line whose y intercept is 5

(D) a straight line whose perpendicular distance from origin is $\frac{1}{\sqrt{2}}$

9. The coordinates of BC of a triangle ABC are B (1, 3) and C(–2, 7). If triangle ABC is equilateral, then coordinates of A are

(A) $\left(2\sqrt{3}-\frac{1}{2},\frac{3\sqrt{3}}{2}+5\right)$ (B) $\left(2\sqrt{3}+\frac{1}{2},\frac{3\sqrt{3}}{2}-5\right)$

(C) $\left(-2\sqrt{3}-\frac{1}{2},\frac{-3\sqrt{3}}{2}+5\right)$ (D) $\left(-2\sqrt{3}+\frac{1}{2},\frac{3\sqrt{3}}{2}-5\right)$

10. Find the equation of the line passing through the point (2, 3) & making intercept of length 2 units between the lines $y + 2x = 3$ & $y + 2x = 5$.

(A) $3x - 4y = 18$ (B) $x = 2$ (C) $3x + 4y = 18$ (D) $x + 2 = 0$

11. If A(3, 4) and B(–5, –2) are the extremities of the base of an isosceles triangle ABC with $\cot C = \frac{1}{2}$, then vertex C can be

(A) $\left(\frac{3\sqrt{5}-1}{2},-1-2\sqrt{5}\right)$ (B) $\left(\frac{-3\sqrt{5}-5}{2},3+2\sqrt{5}\right)$

(C) $\left(\frac{3\sqrt{5}-1}{2},3-2\sqrt{5}\right)$ (D) $\left(\frac{-3\sqrt{5}+5}{2},2\sqrt{5}-1\right)$

12. Given two straight lines AB and AC whose equations are $3x + 4y = 5$ and $4x - 3y = 15$ respectively. The possible equation of line BC through (1, 2), such that ΔABC is isosceles, is $L_1 \equiv ax + by + c = 0$ & $L_2 \equiv dx + ey + f = 0$ where a, b, c, d, e, f $\in$ I, and a, d > 0.

(A) c + f = 4

(B) c + f = 2

(C) A straight line through P(2, 3), inclined at an angle of 60° with positive Y-axis in clockwise direction. The co-ordinates of one of the points on it at a distance 4 units from point P is $(2 + 2\sqrt{3}, 5)$

(D) A straight line through P(2, 3), inclined at an angle of 60° with positive Y-axis in clockwise direction. The co-ordinates of one of the points on it at a distance 4 units from point P is $(2 + 3\sqrt{2}, 3)$

13. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. The locus of the point P is

(A) $y = 2x + 1$ (B) $y = 2x - 1$ (C) $y = -2x - 1$ (D) $y = -2x + 1$

14. If one side of a square in parallel to $12x + 5y = 0$ & its area being 36 while centre being $(1, -1)$, then find equation of sides of parllelogram.

(A) $12x + 5y + 8 = 0$
(B) $5x - 12y + 22 = 0$
(C) $12x + 5y - 22 = 0$
(D) $5x - 12y - 56 = 0$

15. The sides of a triangle are the straight line $2x + 3y = 1$, $3x - y = 2$ and $x + y = 4$. Then which of the following statement(s) is/are correct?

(A) Triangle is acute angled triangle
(B) Triangle is obtuse angled triangle
(C) orthocentre lies out side the triangle
(D) circumcentre lies in side the triangle

16. A ΔABC is formed by the lines $x - y + 1 = 0$, $2x - 7y - 8 = 0$ and $3x + 2y - 12 = 0$. If the points $P(\alpha, 1)$ and $Q(0, \beta)$ always lie inside the ΔABC, then ;

(A) $\alpha \in \left(0, \frac{10}{3}\right)$
(B) $\beta \in \left(-\frac{8}{7}, 1\right)$
(C) $\alpha \in \left(-\frac{10}{3}, 0\right)$
(D) $\left(1, \frac{8}{7}\right)$

17. If the point $(\lambda, \lambda + 1)$ lies inside the ΔABC, where $A \equiv (0, 3)$, $B \equiv (-2, 0)$ and $C \equiv (6, 1)$, then λ can be

(A) –0.5
(B) 0
(C) 1
(D) 1.2

18. Find the equations of the sides of a triangle having $(4, -1)$ as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

(A) $2x - y + 3 = 0$
(B) $x + 2y - 6 = 0$
(C) $2x + y - 7 = 0$
(D) $x - 2y - 6 = 0$

19. The diagonals of a rhombus ABCD intersect at the point (1, 2) and its two sides are parallel to the line $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the vertex A is situated on the y-aixs, the possible coordinates of A are

(A) (0, 4)
(B) (1, – 2)
(C) (2, 0)
(D) $\left(0, \frac{5}{2}\right)$

20. The equation of the bisectors of the angle between two lines

$y - b = \frac{2m_1}{1 - m_1^2}(x - a)$ and $y - b = \frac{2m_2}{1 - m_2^2}(x - a)$ is

(A) $y - b = \frac{m_1 - m_2}{1 + m_1 m_2}(x - a)$
(B) $y - b = \frac{m_1 + m_2}{1 - m_1 m_2}(x - a)$
(C) $y - b = \frac{1 - m_1 m_2}{m_1 + m_2}(x - a)$
(D) $y - b = \frac{m_1 m_2 - 1}{m_1 + m_2}(x - a)$

21. Three lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent if

(A) $a + b + c = 0$
(B) $a^3 + b^3 + c^3 = 3abc$
(C) $a^2 + b^2 + c^2 = ab + bc + ca$
(D) $a = 5, b = 6, c = -11$

22. $A \equiv (4, 2)$ and $B \equiv (2, 4)$ are two given points and a point P on the line $3x + 2y + 10 = 0$ is given then which of the following is/are true.

(A) (PA + PB) is minimum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$
(B) (PA + PB) is maximum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$
(C) |PA – PB | is maximum when (– 22, 28)
(D) (PA – PB) is minimum when P(– 22, 28).

23. The equation $9x^3 + 9x^2 y - 45x^2 = 4y^3 + 4xy^2 - 20y^2$ represents 3 straight lines, two of which passes through origin. Then the area of the triangle formed by these lines is

(A) Divisible by 60 (B) Divisible by 3

(C) Divisible by 4 (D) Divisible by 6

24. If equation of the line pair through the origin and perpendicular to the line pair $xy - 3y^2 + y - 2x + 10 = 0$ is $\lambda xy + \mu x^2 = 0$ (where λ & μ are co-prime) then

(A) $\lambda = 1$ (B) $\mu = 5$ (C) $|\lambda - \mu| = 4$ (D) $\lambda + \mu = 4$

PART - IV : COMPREHENSION

Comprehenssion # 1 (Q. NO. 1 to 3)

Let ABC be a triangle such that the coordinates of the vertex A are (– 3, 1). Equation of the median through B is $2x + y - 3 = 0$ and equation of the angular bisector of C is $7x - 4y - 1 = 0$.

1. coordinates of C are

(A) (0, –1/4) (B) (7, 12) (C) (3, 5) (D) none of these

2. Equation of AC is

(A) $2x - 3y + 9 = 0$ (B) $3x + 2y - 19 = 0$ (C) $x - 2y + 7 = 0$ (D) $x + y + -8 = 0$

3. Slope of BC is

(A) $\frac{1}{18}$ (B) 18 (C) – 18 (D) $-\frac{1}{18}$

Comprehension # 2 (Q.No. 4 to 6)

If vertices of triangle are $P(p_1, p_2)$, $Q(q_1, q_2)$, $R(r_1, r_2)$, then area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix}$ and if P, Q, R are collinear, then $\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix} = 0$.

On the basis of above answer the following question.

4. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find equation of median through A.

(A) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

(B) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$

(D) None of these

5. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find equation of line through A and parallel to BC

(A) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

(B) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$

(D) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$

6. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find the equation of angle bisector through A

(A) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(B) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(D) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Comprehension # 3 (Q.No. 7 to 9)

A variable straight line is drawn from the origin to cut the two given straight lines $2x + 3y = 1$ and $x + 4y = 1$ in L and M respectively.

7. The locus of the point N on the variable line such that $2(ON) = OL + OM$, is

(A) $(2x + 3y)(x + 4y) = 1$

(B) $\frac{1}{2x+3y} + \frac{1}{x+4y} = 2$

(C) $3x + 7y = 2$

(D) $3x + 7y = 1$

8. The locus of the point N on the variable line such that $(ON)^2 = (OL) \cdot (OM)$, is

(A) $(2x + 3y)(x + 4y) = 1$

(B) $\frac{1}{2x+3y} + \frac{1}{x+4y} = 2$

(C) $3x + 7y = 2$

(D) $(2x + 3y)(x + 4y) = 2$

9. The locus of the point N on the variable line such that $\frac{2}{ON} = \frac{1}{OL} + \frac{1}{OM}$, is

(A) $(2x + 3y)(x + 4y) = 1$

(B) $\frac{1}{2x+3y} + \frac{1}{x+4y} = 2$

(C) $3x + 7y = 2$

(D) $3x + 7y = 1$

Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. A straight line **L** through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If **L** also intersects the x-axis, then the equation of **L** is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

2. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then **[JEE (Advanced) 2013, Paper-1, (2, 0)/60]**

(A) $a + b - c > 0$ (B) $a - b + c < 0$ (C) $a - b + c > 0$ (D) $a + b - c < 0$

3. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**

PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is **[AIEEE - 2010 (8, –2), 144]**

(1) $\sqrt{17}$ (2) $\frac{17}{\sqrt{15}}$ (3) $\frac{23}{\sqrt{17}}$ (4) $\frac{23}{\sqrt{15}}$

2. The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE - 2011, I(4, –1), 120]**

Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

3. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval : **[AIEEE - 2011, II(4, –1), 120]**

(1) $(0, \infty)$ (2) $[1, \infty)$ (3) $(-1, \infty)$ (4) $(-1, 1]$

4. If A(2, –3) and B(–2, 1) are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is : **[AIEEE - 2011, II(4, –1), 120]**

(1) $x - y = 1$ (2) $2x + 3y = 1$ (3) $2x + 3y = 3$ (4) $2x - 3y = 1$

5. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE-2012, (4, –1)/120]**

(1) $\frac{29}{5}$ (2) 5 (3) 6 (4) $\frac{11}{5}$

6. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE-2012, (4, –1)/120]**

(1) $-\frac{1}{4}$ (2) – 4 (3) – 2 (4) $-\frac{1}{2}$

7. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[AIEEE - 2013, (4, –1),360]**

(1) $y = x + \sqrt{3}$ (2) $\sqrt{3}y = x - \sqrt{3}$ (3) $y = \sqrt{3}x - \sqrt{3}$ (4) $\sqrt{3}y = x - 1$

8. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[AIEEE - 2013, (4, –1),360]**

(1) $2 + \sqrt{2}$ (2) $2 - \sqrt{2}$ (3) $1 + \sqrt{2}$ (4) $1 - \sqrt{2}$

9. Let PS be the median of the triangle with vertices P(2, 2), Q (6, – 1), and R (7, 3). The equation of the line passing through (1, – 1) and parallel to PS is : **[JEE(Main) 2014, (4, – 1), 120]**

(1) 4x + 7y + 3 = 0 (2) 2x – 9y – 11 = 0 (3) 4x – 7y – 11 = 0 (4) 2x + 9y + 7 = 0

10. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines 4ax + 2ay + c = 0 and 5bx + 2by + d = 0 lies in the fourth quadrant and is equidistant from the two axes then : **[JEE(Main) 2014, (4, – 1), 120]**

(1) 3bc – 2ad = 0 (2) 3bc + 2ad = 0 (3) 2bc – 3ad = 0 (4) 2bc + 3ad = 0

11. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is **[JEE(Main) 2015, (4, – ¼), 120]**

(1) 901 (2) 861 (3) 820 (4) 780

12. Two sides of a rhombus are along the lines, x – y + 1 = 0 and 7x – y – 5 = 0. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus ? **[JEE(Main)-2016, (4, –1), 120]**

(1) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (2) (–3, –9) (3) (–3, –8) (4) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

13. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[JEE(Main)-2017, (4, –1), 120]**

(1) $\left(2, \frac{1}{2}\right)$ (2) $\left(2, -\frac{1}{2}\right)$ (3) $\left(1, \frac{3}{4}\right)$ (4) $\left(1, -\frac{3}{4}\right)$

14. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is: **[JEE(Main)-2018, (4, –1), 120]**

(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$ (3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

15. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

[JEE(Main)-2018, (4, –1), 120]

(1) $2x + 3y = xy$ (2) $3x + 2y = xy$ (3) $3x + 2y = 6xy$ (4) $3x + 2y = 6$

16. Consider the set all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true? **[JEE(Main)-2019, Online (09-01-19) P-1 (4, –1),120]**

(1) The lines are not concurrent

(2) The lines are all parallel

(3) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(4) Each of the line passes through the origin

17. Let the equations of two of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is : **[JEE(Main)-2019, online (10-01-19), P-2 (4, –1), 120]**

(1) $26x - 122y - 1675 = 0$ (2) $26x + 61y + 1675 = 0$

(3) $122y - 26x - 1675 = 0$ (4) $122y + 26x + 1675 = 0$

18. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :

[JEE(Main)-2019, online (08-04-19), P-1 (4, –1), 120]

(1) 1^{st} and 2^{nd} quadrants (2) 4^{th} quadrant

(3) 1^{st}, 2^{nd} and 4^{th} quadrant (4) 1^{st} quadrant

19. Suppose that the points (h,k), (1,2) and (–3,4) lie on the line L_1. If a line L_2 passing through the points (h,k) and (4,3) is perpendicular to L_1, then $\frac{k}{h}$ equals : **[JEE(Main)-2019, online (08-04-19), P-2 (4, –1), 120]**

(1) 3 (2) $-\frac{1}{7}$ (3) $\frac{1}{3}$ (4) 0

20. Let A(1, 0), B(6, 2) and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is __________. **[JEE(Main)-2020, online (07-01-20), P-1 (4, –1), 120]**

21. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is :

[JEE(Main)-2020, online (07-01-20), P-2 (4, –1), 120]

(1) $2x - 3y = 0$ (2) $7x - 5y = 0$ (3) $5x - 7y = 0$ (4) $3x - 2y = 0$

22. Let two points be A(1,–1) and B(0,2). If a point P(x',y') be such that the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is **[JEE(Main)-2020, online (08-01-20), P-1 (4, –1), 120]**

(1) 1 (2) 4 (3) 3 (4) –3

Answers

Exercise # 1

PART - I

SECTION-(A)

A-1. (ii) Yes (iii) yes
A-2 (i) 4 : 1 externally (ii) (8, – 8)
A-3. 4 : 1 internally

SECTION-(B)

B-1. $7, \frac{31}{9}$ **B-2.** $\frac{137}{2}$ sq.units

B-3. $\frac{1}{2}, -1$

SECTION-(C)

C-1. 3x – 5y – 7 = 0; x + y + 7 = 0; x – 7y – 29 = 0
$2\sqrt{34}$, $4\sqrt{2}$, $10\sqrt{2}$

C-2. $y = (2 + \sqrt{3})(x - 2)$
C-3. 3x –4y = 0
C-4. x + 2y – 8 = 0; 2x – y – 1 = 0
C.-5 5x + 5y – 3 = 0
C-6. $y = \frac{3x}{2}$; y = 6x
C-7. $x + 5y + 5\sqrt{2} = 0$ or $x + 5y - 5\sqrt{2} = 0$
C-8. x – y + 2 = 0; x + 2y – 7 = 0; x – 4y – 1 = 0
C-9. x – y = 0
C-10. $3\sqrt{2}$
C-11. (6, 6), (–1, 1)
C-12. x + y = 5, x – 7y + 3 = 0
C-13. 7x + 24y + 182 = 0 or x = –2

SECTION-(D)

D-1. (i) (7, 8) (ii) $\left(\frac{21}{2}, \frac{8}{3}\right)$
(iii) (7, 9) (iv) (27, –21)

D-2. orthocentre $\left(-\frac{1}{6}, -\frac{23}{9}\right)$.

D-4. $\left(\frac{64}{11}, \frac{112}{11}\right)$ **D-5.** (2, –1)

D-6. (33, 26)

SECTION-(E)

E-1. Inside **E-2.** $\left(1, \frac{12}{5}\right), \left(-3, \frac{16}{5}\right)$

E-3. 4

E-4. Foot $\left(\frac{23}{10}, \frac{29}{10}\right)$, Image $\left(\frac{13}{5}, \frac{14}{5}\right)$,
x + 3y – 11 = 0
E-5. 2x – y + 6 = 0, 2x – y – 14 = 0.
E-6. 10sq. units

SECTION-(F)

F-1. acute 2x + y – 3 = 0, obtuse x – 2y + 1 = 0, origin lies in obtuse angle bisector.
F-4. 2y – 29x + 31 = 0.
F-6. 2x + y – 5 = 0
F-7. 2x – 3y + 10 = 0

SECTION-(G)

G-1. $(3x + 1)^2 + 9y^2 = a^2 + b^2$
G-2. $x^2 + y^2 = a^2 + b^2$
G-3. $x^2 + y^2 - 5x - 5y = 0$.
G-4. $y\lambda = 2a$
G-6. $2xy(ma - b\ell) + x(an - \ell c) + y(mc - bn) = 0$.

SECTION-(H)

H-2. $\tan^{-1}\left(\frac{4}{3}\right), \pi - \tan^{-1}\left(\frac{4}{3}\right)$
H-3. $5x^2 - 2xy - 5y^2 = 0$
H-4. y + 3x = 0 and 2y + x = 0
H-7. $x^2 - y^2 - 5xy = 0$
H-8. $\frac{-9 \pm \sqrt{33}}{8}$.

PART - II

SECTION-(A)

A-1.	(C)	**A-2.**	(A)
A-3.	(D)	**A-4.**	(A)

SECTION-(B)

B-1.	(A)	**B-2.**	(B)
B-3.	(C)	**B-4.**	(B)
B-5.	(C)		

SECTION-(C)

C-1.	(D)	**C-2.**	(B)
C-3.	(A)	**C-4.**	(A)
C-5.	(D)	**C-6.**	(B)
C-7.	(B)	**C-8.**	(D)

SECTION-(D)

D-1.	(D)	**D-2.**	(A)
D-3.	(B)	**D-4.**	(A)
D-5.	(D)	**D-6.**	(C)
D-7.	(A)	**D-8.**	(D)

SECTION-(E)

E-1.	(B)	**E-2.**	(B)
E-3.	(B)	**E-4.**	(C)
E-5.	(C)	**E-6.**	(B)
E-7.	(A)	**E-8.**	(A)

SECTION-(F)

F-1.	(D)	**F-2.**	(B)
F-3.	(B)	**F-4.**	(A)
F-5.	(D)		

SECTION-(G)

G-1.	(A)	**G-2.**	(B)
G-3.	(A)	**G-4.**	(A)
G-5.	(B)		

SECTION-(H)

H-1.	(C)	**H-2.**	(B)
H-3.	(A)	**H-4.**	(A)

PART - III

1. (A) $\rightarrow$ (s), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (r)

2. (A)$\rightarrow$(q,s,t),(B) $\rightarrow$(p,q,s,t), (C)$\rightarrow$(s), (D) $\rightarrow$ (r,t)

Exercise # 2

PART - I

1.	(C)	**2.**	(D)
3.	(A)	**4.**	(B)
5.	(D)	**6.**	(C)
7.	(A)	**8.**	(B)
9.	(C)	**10.**	(A)
11.	(D)	**12.**	(C)
13.	(C)	**14.**	(A)
15.	(A)	**16.**	(C)
17.	(A)	**18.**	(A)

PART - II

1.	2	**2.**	50
3.	1	**4.**	4
5.	4	**6.**	52
7.	4	**8.**	13
9.	42	**10.**	0
11.	2	**12.**	1
13.	5	**14.**	1
15.	2		

PART - III

1.	(A,C,D)	**2.**	(B,D)
3.	(A,B,D)	**4.**	(C,D)
5.	(A,D)	**6.**	(A,B,C,D)
7.	(B)	**8.**	(B,C)
9.	(A,C)	**10.**	(B,C)
11.	(A,B)	**12.**	(A,C)
13.	(A,D)	**14.**	(A,B,C,D)
15.	(B,C)	**16.**	(A,B)
17.	(A,B,C,D)	**18.**	(A,C,D)
19.	(A,D)	**20.**	(B,D)
21.	(A,B,D)	**22.**	(A,C)
23.	(B,D)	**24.**	(A,D)

PART - IV

1.	(C)	**2.**	(A)
3.	(B)	**4.**	(B)
5.	(A)	**6.**	(C)
7.	(B)	8.	(A)
9.	(C)		

Exercise # 3

PART - I

1.	(B)	**2.**	(A) or (C) or Bonus
3.	6		

PART - II

1.	(3)	**2.**	(3)
3.	(2)	**4.**	(2)
5.	(3)	**6.**	(3)
7.	(2)	**8.**	(2)
9.	(4)	**10.**	(1)
11.	(4)	**12.**	(4)
13.	(1)	**14.**	(2)
15.	(2)	**16.**	(3)
17.	(1)	**18.**	(1)
19.	(3)	**20.**	5
21.	(3)	**22.**	(3)

1. Find the acute angle between two straight lines passing through the point M(– 6, – 8) and the points in which the line segment 2x + y + 10 = 0 enclosed between the co–ordinate axes is divided in the ratio 1 : 2 : 2 in the direction from the point of its intersection with the x – axis to the point of intersection with the y – axis.

2. The vertices of a triangle OBC are O(0,0) B(–3,–1) and C(–1,–3). If $\lambda x + \mu y + \sqrt{2} = 0$ is the equation of line parallel to BC and intersecting the sides OB and OC, whose perpendicular distance from the point (0,0) is $\frac{1}{2}$, then find the value of $\lambda + \mu$.

3. A triangle is formed by the lines whose equations are AB : x + y – 5 = 0, BC : x + 7y – 7 = 0 and CA : 7x + y + 14 = 0. Find the bisector of the interior angle at A and the exterior angle at C. Determine the nature of the interior angle at B and find the equation of the bisector.

4. The coordinates of the feet of $\perp$ from the vertices of a Δ on the opposite sides are (20, 25), (8, 16) and (8, 9). Then find the coordinates of centroid of the Δ

5. Show that the orthocentre of Δ formed by the straight lines, $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is a point (x', y') such that $\frac{x'}{\ell} = \frac{y'}{m} = \frac{a+b}{am^2 - 2h\ell m + b\ell^2}$.

6. An equilateral triangle PQR is formed where P(1, 3) is fixed and Q is moving point on line x = 3. If the locus of R is $(x - 2) = \lambda (y - 3 \pm \sqrt{3})$, then find λ^2.

7. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, meets the coordinate axes in A & B. If the locus of the mid point of AB is the curve $\lambda xy(a + b) = ab(x + y)$, then find λ.

8. From the vertices A, B, C of a trinagle ABC, perpendicular AD, BE, CF are drawn to any stright line. Show that the perpendiculars from D, E, F to BC, CA, AB respectively are concurrent.

9. Let in ΔPAB, A is (0, 0), B is (a, 0) and P is variable such that $\angle$PBA is equal to three times $\angle$PAB the find then, find the locus of P.

10. The sides of a triangle are $L_r \equiv x\cos \alpha_r + y \sin \alpha_r - p_r = 0$ for r = 1, 2, 3.
Show that its orthocentre is given by
$L_1 \cos (\alpha_2 - \alpha_3) = L_2 \cos (\alpha_3 - \alpha_1) = L_3 \cos (\alpha_1 - \alpha_2)$.

11. Find the condition so that the lines joining the origin to the other two points of intersection of the curves $ax^2+2hxy+by^2+2gx = 0$ and $a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$ will be at right angles to one another.

12. A point moves so that the distance between the feet of the perpendiculars from it on the lines $ax^2 + 2h\,xy + by^2 = 0$ is a constant 2 d. Show that the equation to its locus is,
$(x^2 + y^2)(h^2 - ab) = d^2 \{(a - b)^2 + 4h^2\}$.

13. Let P is any point inside the triangle ABC of side lengths 6, 5 ,5 units and p_1, p_2, p_3 be the lengths of perpendiculars drawn from P to the sides of trinagle. Find the integer which is just less then the maximum value of $p_1 p_2 p_3$.

14. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, represents a pair of straight lines, prove that the third pair of straight lines (excluding $xy = 0$) passing through the points where these meet the axes is

$$ax^2 - 2hxy + by^2 + 2gx + 2fy + c + \frac{4fg}{c} \cdot xy = 0.$$

15. Show that the pair of lines given by $a^2x^2 + 2h(a + b)xy + b^2y^2 = 0$ is equally inclined to the pair given by $ax^2 + 2hxy + by^2 = 0$.

16. All the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrence & if not, give reasons.

17. The straight lines $(A^2 - 3B^2)x^2 + 8AB\,xy + (B^2 - 3A^2)y^2 = 0$ form a Δ with the line $Ax + By + C = 0$, then prove that

(i) Area of Δ is $\dfrac{C^2}{\sqrt{3}(A^2 + B^2)}$

(ii) Δ is equilateral

(iii) The orthocentre of Δ does not lie on one of its vertices

18. A variable line cuts the line $2y = x - 2$ and $2y = -x + 2$ in points A and B respectively. If A lies in first quadrant, B lies in 4th quadrant and area of ΔAOB is 4, then find the locus of

(i) mid point of AB

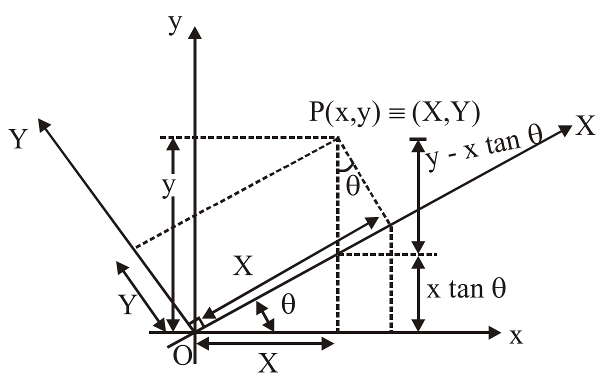
(ii) centroid of ΔOAB

19. Let A lies on $3x - 4y + 1 = 0$. B lies on $4x + 3y - 7 = 0$ and C is $(-2, 5)$. If ABCD is rhombus, then find locus of D.

20. Let D is point on line $\ell_1 : x + y - 2 = 0$ and S $(3, 3)$ is fixed point. ℓ_2 is the line perpendicular to DS and passing through S. If M is another point on line ℓ_1 (other than D), then find the locus of point of intersection of ℓ_2 and angle bisector of $\angle MDS$.

Comprehension (Q.21 & 22)

If coordinate system xy is being rotated through an angle θ in anti clock wise direction about the origin as shown in the diagram, Coordinate of P(x, y) has been change to P(X, Y) in new coordinate system XY, then x, y, X, Y are related as given below.



$X = x \sec\theta + (y - x\tan\theta)\sin\theta$ and $Y = (y - x\tan\theta)\cos\theta$

$= x\sec\theta + y\sin\theta - \dfrac{x\sin^2\theta}{\cos\theta}$ $= y\cos\theta - x\sin\theta$

$\boxed{X = x\cos\theta + y\sin\theta}$ $\boxed{y = -x\sin\theta + y\cos\theta}$

21. If the axes are rotated through 60° in anticlockwise direction about origin. find co-ordinates of point (2, 6) in new coordinate axes.

22. If axes are rotated through an acute angle in clockwise direction about origin so that equation $x^2 + 2xy + y^2 - 2x + 2y = 0$ becomes free from xy in its new position, then find equation in new position

23. The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co–ordinates is δ, find the combined equation of these straight lines.

Answers

1. $\pi/4$ **2.** 4 **3.** $12x + 6y - 11 = 0$; $8x + 8y + 7 = 0$; $3x + 6y - 16 = 0$;

4. $\left(\dfrac{70}{3}, \dfrac{35}{3}\right)$ **6.** 3 **7.** $\lambda = 2$ **9.** $4x^3 - 4xy^2 - 3ax^2 + ay^2 = 0$

11. $g(a' + b') = g'(a + b)$ **13.** 3 **16.** (1, – 2), yes (1/3, – 2/3)

18. (i) $(x - 1)^2 - 4y^2 = 9$ (ii) $(x - 2/3)^2 + 4y^2 = 4$

19. $25((x + 2)^2 + (y - 5)^2) = (3x - 4y + 1)^2$ **20.** $(x - 3)^2 + (y - 3)^2 = \left(\dfrac{x + y - 2}{\sqrt{2}}\right)^2$

21. $(1 + 3\sqrt{3}, -\sqrt{3} + 3)$ **22.** $x^2 + \sqrt{2}\,y = 0$

23. $(y_1^2 - \delta^2)x^2 - 2x_1y_1xy + (x_1^2 - \delta^2)y^2 = 0$