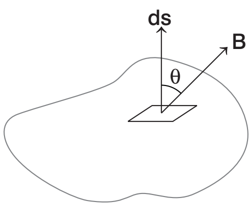
DAY TWENTY FOUR

Electromagnetic Induction

Learning & Revision for the Day

- Magnetic Flux (ϕ_B)
- Faraday's Law of Electromagnetic Induction
- Lenz's Law
- Motional Emf
- Rotational Emf
- Self-Induction
- Mutual Induction
- Combination of Inductors
- Eddy Currents

Magnetic Flux (ϕ_B)



The flux associated with a magnetic field is defined in a similar manner to that used to define electric flux. Consider an element of area $d\mathbf{s}$ on an arbitrary shaped surface as shown in figure. If the magnetic field at this element is $\mathbf{B}$, the magnetic flux through the element is,

$$d\phi_B = \mathbf{B} \cdot d\mathbf{s} = Bds \cos\theta$$

Here, $d\mathbf{s}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area ds and θ is the angle between $\mathbf{B}$ and $d\mathbf{s}$ at that element.

Magnetic flux is a scalar quantity. Outward magnetic flux is taken as positive (i.e. $\theta < 90°$) and inward flux is taken as negative (i.e. $\theta > 90°$).

SI unit of magnetic flux is 1 weber (1 Wb).

where, $1 \text{ Wb} = 1 \text{ T} \times 1 \text{ m}^2 = 1 \text{ T-m}^2$

Dimensional formula of magnetic flux is $[\text{ML}^2 \text{T}^{-2} \text{A}^{-1}]$.

Faraday's Law of Electromagnetic Induction

This law states that, the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

Induced emf, $|e| = \dfrac{d\phi_B}{dt}$

- For N turns, $|e| = N \dfrac{d\phi_B}{dt}$

  However, if we consider the direction of induced emf, then

$$e = -N \frac{d\phi_B}{dt} = -\frac{Nd(BA\cos\theta)}{dt} = \frac{-NBA(\cos\theta_2 - \cos\theta_1)}{\Delta t}$$

- If the given electric circuit is a closed circuit having a total resistance R, then the induced current,

$$I = \frac{e}{R} = -\frac{N}{R}\frac{d\phi_B}{dt}$$

Induced charge, $dq = Idt = -\frac{N}{R}d\phi_B$

and induced power, $P = \frac{e^2}{R} = \frac{N^2}{R}\left(\frac{d\phi_B}{dt}\right)^2$

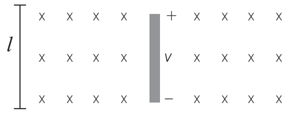
Lenz's Law

The negative sign in Faraday's equations of electromagnetic induction describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with the help of Lenz's law. This law states that **the direction of any magnetic induction effect is such as to oppose the cause of the effect**.

Later, we will see that Lenz's law is directly related to **energy conservation.**

Motional Emf

Let a conducting rod of length l be moving with a uniform velocity **v** perpendicular to a uniform magnetic field **B**, an induced emf is set up.



The magnitude of the induced emf will be

$$|e| = B\,l\,v$$

- If the rod is moving such that it makes an angle θ with the direction of the magnetic field, then

$$|e| = B\,l\,v \sin\theta$$

Hence, for the motion parallel to **B**, the induced emf is zero.
- When a conducting rod moves horizontally, then an induced emf is set up between its ends due to the vertical component of the earth's magnetic field. However, at the magnetic equator, induced emf will be zero, because $B_V = 0$.
- If during landing or taking off, the wings of an aeroplane are along the East-West direction, an induced emf is set up across the wings (due to the effect of B_H).

Motional Emf in a Loop

If a conducting rod moves on two parallel conducting rails, then an emf is induced whose magnitude is $|e| = B\,l\,v$

and the direction is given by the Fleming's right hand rule.

- Induced current, $|I| = \frac{|e|}{R} = \frac{B\,l\,v}{R}$
- Magnetic force, $F_m = BIl = \frac{B^2 l^2 v}{R}$

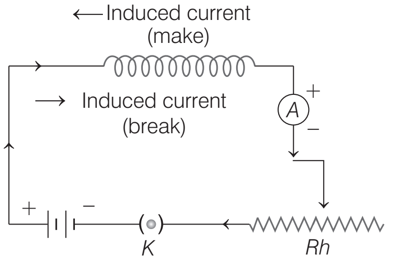
Rotational Emf

Let a conducting rod of length l rotate about an axis passing through one of its ends (that end may be fixed), with an angular velocity ω in a plane perpendicular to the magnetic field B, then an induced emf is set up between the ends of the rod, whose magnitude is given by

$$|e| = \frac{1}{2}Bl^2\omega$$

Self-Induction

Self-induction is the phenomenon due to which an induced emf is set up in a coil or a circuit whenever the current passing through it changes. The induced emf opposes the change that causes it and is thus known as **back emf**.



- Inductance is the inherent property of electrical circuits and is known as the **electrical inertia**.
- An inductor is said to be an **ideal inductor** if its resistance is zero.
- An inductor does not oppose current but opposes changes (growth or decay of current) in the circuit.

Self-Inductance

Flux linked with the coil is

$$N\phi_B \propto I \quad \text{or } N\phi_B = LI,$$

where the constant L is known as the **coefficient of self-induction** or **self-inductance** of the given coil.

It may be defined as the magnetic flux linked with the coil, when a constant current of 1 A is passed through it.

Induced emf due to self-induction,

$$e = -N\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

SI unit of inductance is **henry**.

Magnetic Potential Energy of an Inductor

- In building, a steady current in an electric circuit, some work is done by the emf of the source, against the self-inductance of the coil.

The work done, $W = \frac{1}{2}LI^2$
- The work done is stored as the magnetic potential energy of that inductor.

Thus, $U = \frac{1}{2}LI^2$

Formulae for Self-Inductance

- For a circular coil of radius R and N turns, the self-inductance,

$$L = \frac{1}{2}\mu_0 \pi N^2 R$$

- For a solenoid coil having length l, total number of turns N and cross-sectional area A,

$$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al \qquad \left[\text{where, } n = \frac{N}{l}\right]$$

- For a toroid of radius R and number of turns N,

$$L = \frac{1}{2}\mu_0 N^2 R$$

- For a square coil of side a and number of turns N,

$$L = \frac{2\sqrt{2}}{\pi}\mu_0 N^2 a$$

Mutual Induction

Mutual induction is the phenomenon due to which an emf is induced in a coil when the current flowing through a neighbouring coil changes.

Mutual Inductance

Mutual inductance of a pair of coils is defined as the magnetic flux linked with one coil, when a constant current of unit magnitude, flows through the other coil.

Mathematically, $N\phi_{B_2} = MI_1$

where, M is known as the **mutual inductance** for the given pair of coils.

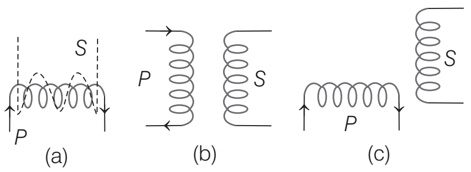
Induced emf due to mutual inductance,

$$e_2 = -N\frac{d\phi_{B_2}}{dt} = -M\frac{dI_1}{dt}$$

Hence, mutual inductance for a pair of coils is numerically equal to the magnitude of induced emf in one coil when current in the other coil changes at a rate of 1 As^{-1}.

SI unit of mutual inductance M, is **henry.**

Mutual inductance of a pair of coils is maximum, when the two coils are wound on the same frame. However, mutual inductance is negligible when the two coils are oriented mutually perpendicular to each other (see figure). In this context, we define a term **coupling coefficient** k.



Coupling coefficient is given by

$$k = \frac{\text{Magnetic flux linked with secondary coil}}{\text{Magnetic flux developed in primary coil}}$$

It is observed that $0 \le k \le 1$.

For a pair of two magnetically coupled coils of self-inductances L_1 and L_2 respectively, the mutual inductance,

$$M_{12} = M_{21} = M = k\sqrt{L_1 L_2}$$

where, k is the coupling coefficient.

Formulae for Mutual Inductance

- Assuming the coupling coefficient $k = 1$ and medium to be a free space or air. Mutual inductance of a pair of concentric circular coils is

$$M = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}$$

  where, r = radius of the coil (of small radius)
  and R = radius of the coil (of larger radius).

- For a pair of two solenoid coils, wound one over the other, $M = \frac{\mu_0 N_1 N_2 A}{l}$

  For a pair of concentric coplanar square coils,

$$M = \frac{2\sqrt{2}\,\mu_0 N_1 N_2 a^2}{\pi b}$$

  where, a = side of the smaller coil and b = side of the larger coil.

- For a given pair of coils, mutually coupled, then according to theorem of reciprocity,

$$M_{12} = M_{21} = M$$

Combination of Inductors

- If two coils of self-inductances L_1 and L_2 are placed quite far apart and are arranged in series, then their equivalent inductance,

$$L_s = L_1 + L_2$$

- If the coils are placed quite close to each other, so as to mutually affect each other, then their equivalent inductance,

$$L_s = L_1 + L_2 \pm 2M$$

  Here, M has been written with ± sign depending on the fact whether currents in the two coils are flowing in same sense or opposite sense.

- If two coils of self-inductances L_1 and L_2 are connected in parallel, then equivalent inductance L_p is given by

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_p = \frac{L_1 L_2}{L_1 + L_2}$$

Eddy Currents

Currents induced in the body of bulk of the conductors due to charge in magnetic flux linked to them, are called the eddy currents. The production of eddy currents in a metallic conductor leads to a loss of electric energy in the form of heat energy.

Eddy currents can be minimised by taking the metal (generally soft iron) core in the form of a combination of thin laminated sheets or by slotting process.

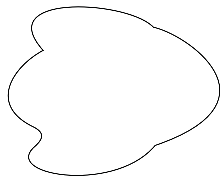
DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A square of side L metres lies in the XY-plane in a region, where the magnetic field is given by $B = B_0\,(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T, where B_0 is constant. The magnitude of flux passing through the square is

(a) $(2\,B_0\,L^2)$ Wb (b) $(3\,B_0\,L^2)$ Wb
(c) $(4\,B_0\,L^2)$ Wb (d) $(\sqrt{29}\,B_0\,L^2)$ Wb

2 As a result of change in the magnetic flux linked to the closed loop shown in the figure, an emf V(volt) is induced in the loop. The work done (joules) in taking a charge q (coulomb) once along the loop is



(a) qV (b) zero (c) $2qV$ (d) $\frac{qV}{2}$

3 The magnetic flux linked with a coil varies with time as $\phi = (3t^2 + 4t + 9)$ Wb. What is the induced emf at, $t = 2$ s?

(a) 3 V (b) 4 V (c) 9 V (d) 16 V

4 The flux linked with a coil at any instant t given by

$$\phi = 10t^2 - 50t + 250$$

Then, induced emf at $t = 3$ s is

(a) -10 V (b) 10 V (c) 190 V (d) -190 V

5 The magnetic flux through a surface varies with time as follows $\phi = 12t^2 + 7t - 3$

Here, ϕ is in milliweber and t is in seconds. What will be the induced emf at $t = 5$ s?

(a) 338 mV (b) 127 mV
(c) 105 mV (d) None of these

6 A coil of resistance 400 Ω is placed in a magnetic field. If the magnetic flux ϕ (Wb) linked with the coil varies with time t (second) as $\phi = 50t^2 + 4$.

The current in the coil at $t = 2$ s is ➜ CBSE AIPMT 2012

(a) 0.5 A (b) 0.1 A
(c) 2 A (d) 1 A

7 When a coil of cross-sectional area A and number of turns N is rotated in a uniform magnetic field B with angular velocity ω, then the maximum emf induced in the coil will be

(a) BNA (b) $\frac{BA\omega}{N}$ (c) $BNA\omega$ (d) zero

8 A rectangular coil of 20 turns and area of cross-section 25 cm^2 has a resistance of 100 Ω. If a magnetic field which is perpendicular to the plane of the coil changes at a rate of 1000 T/s, the current in the coil is

(a) 1.0 A (b) 50 A (c) 0.5 A (d) 5.0 A

9 Magnetic flux (weber) in a closed circuit of resistance 10 Ω varies with time t (second) as $\phi = 6t^2 - 5t + 1$. The magnitude of induced current at $t = 0.25$ s is

(a) 0.2 A (b) 0.6 A (c) 1.2 A (d) 0.8 A

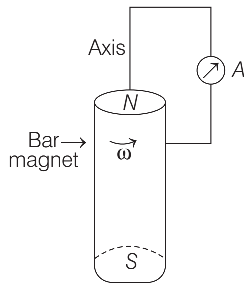
10 A circular ring of diameter 20 cm has a resistance of 0.01 Ω. The charge that will flow through the ring, if it is turned from a position perpendicular to a uniform magnetic field of 2.0 T to a position parallel to the field is about ➜ CBSE AIPMT 2010

(a) 63 C (b) 0.63 C (c) 6.3 C (d) 0.063 C

11 The magnetic flux through a circuit of resistance R changes by an amount $\Delta\phi$ in a time Δt. Then, the total quantity of electric charge q that passes any point in the circuit during the time Δt is presented by

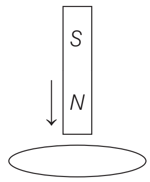
(a) $q = \frac{1}{R}\frac{\Delta\phi}{\Delta t}$ (b) $q = \frac{\Delta\phi}{R}$ (c) $q = \frac{\Delta\phi}{\Delta t}$ (d) $q = R\frac{\Delta\phi}{\Delta t}$

12 A cylindrical bar magnet is rotated about its axis shown in figure. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then,



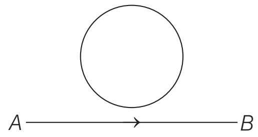
(a) a direct current flows in the ammeter A
(b) no current flows through the ammeter A
(c) an alternating sinusoidal current flows through the ammeter A with a time period $T = 2\pi/\omega$
(d) a time varying non-sinusoidal current flows through the ammeter A

13 A copper ring having a cut such as not to form a complete loop is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring, as shown in figure. The acceleration of the falling magnet is



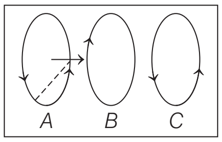
(a) g (b) less than g
(c) more than g (d) zero

14 The current from A to B is increasing in magnitude. What is the direction of induced current, if any in the loop as shown in figure.



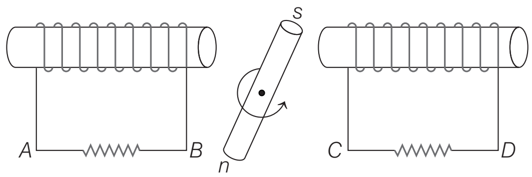
(a) No current is induced (b) Clockwise current
(c) Anti-clockwise current (d) Alternating current

15 Three identical coils A, B and C are placed with their planes parallel to one another. Coils A and C carry currents as shown in figure. Coils B and C are fixed in position and coil A is moved towards B. Then, current
induced in B is in



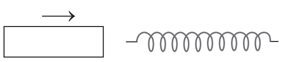
(a) clockwise current
(b) anti-clockwise current
(c) no current is induced in B
(d) current in induced only when both coils move

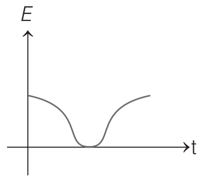
16 The magnet in figure rotates a shown on a pivot through its centre. At the instant shown, what are the directions of the induced currents

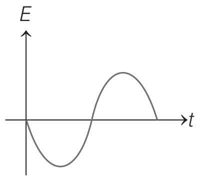


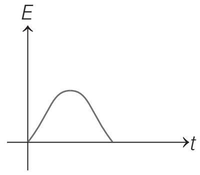
(a) A to B and C to D (b) B to A and C to D
(c) A to B and D to C (d) B to A and D to C

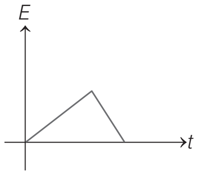
17 The variation of induced emf (E) with time (t) in a coil, if a short bar magnet is moved along its axis with a constant velocity is best represented as



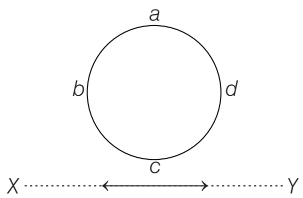
(a)


(b)


(c)


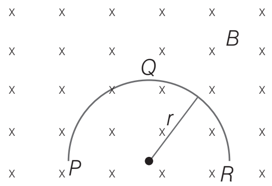
(d)


18 An electron moves on a straight line path XY as shown. The $abcd$ is a coil adjacent in the path of electron. What will be the direction of current, if any induced in the coil?
➜ CBSE AIPMT 2015



(a) $abcd$ (b) $adcb$
(c) The current will reverse its direction as the electron goes past the coil
(d) No current induced

19 A thin semi-circular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in figure. The potential difference developed across the ring when its speed is v, is



(a) zero ➜ CBSE AIPMT 2014
(b) $Bv\pi r^2/2$ and P is at higher potential
(c) πrBv and R is at higher potential
(d) $2rBv$ and R is at higher potential

20 A conducting circular loop is placed in a uniform magnetic field 0.04 T with its plane perpendicular to the magnetic field. The radius of the loop starts shrinking at 2 mm s^{-1}. The induced emf in the loop when the radius is 2 cm is ➜ CBSE AIPMT 2009

(a) $3.2\ \pi\ \mu$V (b) $4.8\ \pi\ \mu$V (c) $0.8\ \pi\ \mu$V (d) $1.6\ \pi\ \mu$V

21 A fan blade of length $\dfrac{1}{\sqrt{\pi}}$ metre rotates with frequency 5 cycle per second perpendicular to a magnetic field 10 T. The potential difference between the centre and the end of the blade is

(a) – 50 V (b) +50 V (c) – 2 V (d) + 0.02 V

22 A wire loop is rotated in a magnetic field. The frequency of change of direction of the induced emf is ➜ NEET 2013

(a) once per revolution (b) twice per revolution
(c) four times per revolution (d) six times per revolution

23 A disc of radius 0.1 m is rotating with a frequency 10 rev/s in a normal magnetic field of strength 0.1 T. Net induced emf is

(a) $2\pi \times 10^{-2}$ V (b) $\pi \times 10^{-2}$ V
(c) $\dfrac{\pi}{2} \times 10^{-2}$ V (d) None of these

24 The coefficients of self-induction of two coils are L_1 and L_2. To induce an emf of 25 V in the coils change of current of 1A has to be produced in 5 s and 50 ms, respectively. The ratio of their self-inductances $L_1 : L_2$ will be

(a) 1 : 5 (b) 200 : 1 (c) 100 : 1 (d) 50 : 1

25 When the current changes from + 2 A to – 2 A in 0.05 s, an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is

(a) 0.1 H (b) 0.2 H (c) 0.4 H (d) 0.8 H

26 In a solenoid, if number of turns is doubled, then self-inductance will become

(a) half (b) double (c) 1/4 times (d) quadruple

27 The current in a coil changes from + 10A to – 2A in 3 ms. What is the induced emf in the coil? The self-inductance of the coil is 2 mH.

(a) 8 V (b) 4 V (c) 0.8 V (d) 0.4 V

28 The inductance of a coil is proportional to

(a) its length (b) the number of turns
(c) the resistance of coil
(d) the square of the number of turns

29 A long solenoid has 1000 turns. When a current of 4A flows through it, the magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is → NEET 2016

(a) 3 H (b) 2 H
(c) 1 H (d) 4 H

30 Two coils of self-inductances 2 mH and 8 mH are placed so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is

(a) 10 mH (b) 6 mH
(c) 4 mH (d) 16 mH

31 Two coils *X* and *Y* are placed in a circuit such that a current changes by 3 A in coil *X* and the magnetic flux changes of 1.2 Wb occurs in *Y*. The value of mutual inductance of the coils is

(a) 0.2 H (b) 0.4 H
(c) 0.6 H (d) 3.6 H

32 The cause of production of eddy currents is

(a) the motion of a conductor in a varying magnetic field
(b) the motion of a insulator in a varying magnetic field
(c) current flowing in a conductor
(d) current flowing in an insulator

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A square loop of wire of side 5 cm is lying on a horizontal table. An electromagnet above and to one side of the loop is turned on, causing a uniform magnetic field downward at an angle of 60° to the vertical as shown in figure. The magnetic induction is 0.50 T. The average induced emf in the loop, if the field increases from zero to its final value in 0.2 s is

(a) 5.4×10^{-3} V (b) 3.12×10^{-3} V
(c) 0 (d) 0.25×10^{-3} V

2 A coil having *n* turns and resistance $R\ \Omega$ is connected with a galvanometer of resistance $4R\ \Omega$. This combination is moved in time *t* second. From a magnetic field (W_1) Wb to (W_2) Wb. The induced current in the circuit is

(a) $\frac{W_2 - W_1}{5Rnt}$ (b) $\frac{-n(W_2 - W_1)}{5Rt}$

(c) $\frac{-(W_2 - W_1)}{Rnt}$ (d) $-\frac{n(W_2 - W_1)}{Rt}$

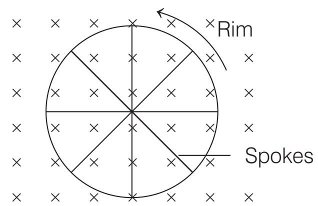
3 A conducting circular loop is placed in a uniform magnetic field $B = 0.025$ T with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of 1 mm s^{-1}. The induced emf when the radius is 2 cm, is → CBSE AIPMT 2010

(a) $2\pi\ \mu V$ (b) $\pi\mu V$ (c) $\frac{\pi}{2}\mu V$ (d) $2\ \mu V$

4 A rectangular, a square, a circular and an elliptical loop, all in the *XY*-plane, are moving out of a uniform magnetic field with a constant velocity, $\mathbf{v} = v\hat{\mathbf{i}}$. The magnetic field is directed along the negative *Z*-axis direction. The induced emf, during the passage of these loops, out of the field region, will not remain constant for → CBSE AIPMT 2009

(a) the rectangular, circular and elliptical loops
(b) the circular and the elliptical loops
(c) only the elliptical loop
(d) any of the four loops

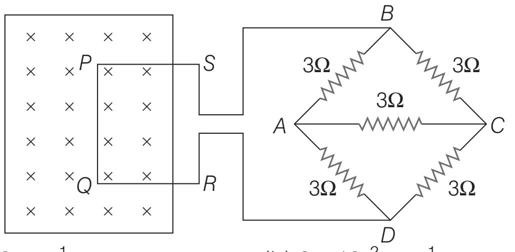
5 A bicycle wheel of radius 0.5 m has 32 spokes. It is rotating at the rate of 120 revolutions per minute, perpendicular to the horizontal component of the earth's magnetic field $B_H = 4 \times 10^{-5}$ T. The emf induced between the rim and the centre of the wheel will be



(a) 6.28×10^{-5} V (b) 4.8×10^{-5} V
(c) 6×10^{-5} V (d) 1.6×10^{-5} V

6 A square metal wire loop *PQRS* of side 10 cm and resistance $1\ \Omega$ is moved with a constant velocity v_c in a uniform magnetic field of induction $B = 2\ \text{Wbm}^{-2}$, as shown in figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to net work *ABCD* of resistors each of value $3\ \Omega$. The resistance of the lead wires *SB* and *RD* are negligible. The speed of the loop so as to have a steady current of 1 mA in the loop is

➡ CBSE AIPMT 2010



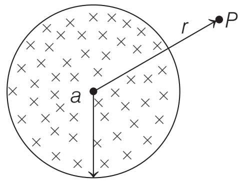
(a) $2\ \text{ms}^{-1}$
(b) $2 \times 10^{-2}\ \text{ms}^{-1}$
(c) $20\ \text{ms}^{-1}$
(d) $200\ \text{ms}^{-1}$

7 A long solenoid of diameter 0.1 m has 2×10^4 turns per metre. At the centre of the solenoid, a coil of 100 turns and radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid reduces at a constant rate to 0 A from 4 A in 0.05 s. If the resistance of the coil is $10\pi^2\ \Omega$, the total charge flowing through the coil during this time is

➡ NEET 2017

(a) $32\pi\ \mu\text{C}$
(b) $16\ \mu\text{C}$
(c) $32\ \mu\text{C}$
(d) $16\pi\ \mu\text{C}$

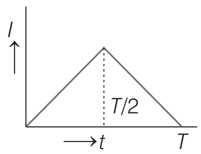
8 A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in figure. The magnitude of induced electric field at point P at a distance r from the centre of the circular region



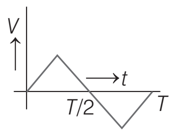
(a) is zero
(b) decreases as $1/r$
(c) increases as r
(d) decreases as $1/r^2$

9 The current (I) in the inductance is varying with time according to the plot shown in figure. ➡ CBSE AIPMT 2012

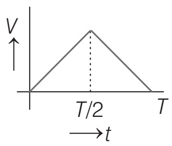
Which one of the following is the correct variation of voltage with time in the coil?



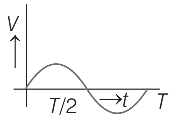
(a)



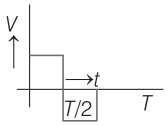
(b)



(c)

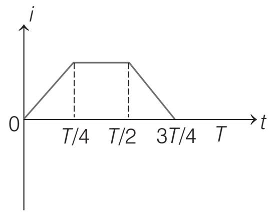


(d)

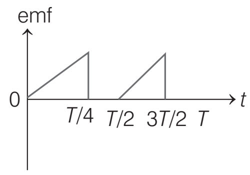


10 The current i in a coil varies with time as shown in the figure. The variation of induced emf with time would be

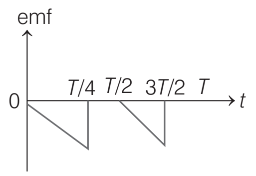
➡ CBSE AIPMT 2011



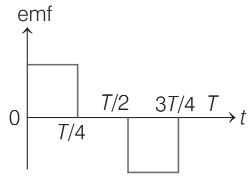
(a)



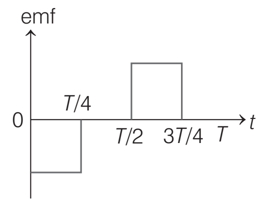
(b)



(c)



(d)



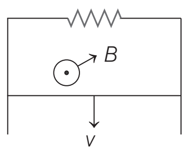
11 In a uniform magnetic field of induction B, a wire in the form of semi-circle of radius r rotates about the diameter of the circuit with angular frequency ω. If the total resistance of the circuit is R, the mean power generated per rotation of rotation is

(a) $\dfrac{B\pi r^2\omega}{2R}$
(b) $\dfrac{(B\pi r^2\omega)^2}{8R}$
(c) $\dfrac{(B\pi r\omega)^2}{2R}$
(d) None of these

12 A loop made of straight edges has six corners at $A(0,0,0)$, $B(L,0,0)$, $C(L,L,0)$, $D(0,L,0)$, $E(0,L,L)$ and $F(0,0,L)$. A magnetic field $B = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ T is present in the region. The flux passing through the loop *ABCDEFA* (in that order) is

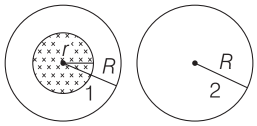
(a) $B_0 L^2$ Wb
(b) $2 B_0 L^2$ Wb
(c) $\sqrt{29}\, B_0 L^2$ Wb
(d) $4 B_0 L^2$ Wb

13 A conductor of length l and mass m can slide along a pair of vertical metal guides connected by a resistor R. A uniform magnetic field of strength B normal to the plane of page is directed outwards. The steady speed of fall of rod is



(a) $\dfrac{mgR}{B^2l^2}$
(b) $\dfrac{mg}{B^2l^2R}$
(c) $\dfrac{B^2l^2}{mgR}$
(d) $\dfrac{mgB}{l^2R}$

14 A uniform magnetic field is restricted within a region of radius r. The magnetic field changes with time at a rate $\frac{d\mathbf{B}}{dt}$. Loop 1 of radius $R > r$ encloses the region r and loop 2 of radius R is outside the region of magnetic field as shown in the figure. Then, the emf generated is ➜ NEET 2016



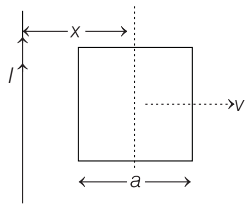
(a) zero in loop 1 and zero in loop 2

(b) $-\frac{d\mathbf{B}}{dt}\pi r^2$ in loop 1 and $-\frac{d\mathbf{B}}{dt}\pi r^2$ in loop 2

(c) $-\frac{d\mathbf{B}}{dt}\pi R^2$ in loop 1 and zero in loop 2

(d) $-\frac{d\mathbf{B}}{dt}\pi r^2$ in loop 1 and zero in loop 2

15 A conducting square frame of side a and a long straight wire carrying current I are located in the same plane as shown in the figure. The frame moves to the right with a constant velocity v. The emf induced in the frame will be proportional to ➜ CBSE AIPMT 2015



(a) $\frac{1}{x^2}$ (b) $\frac{1}{(2x-a)^2}$ (c) $\frac{1}{(2x+a)^2}$

(d) $\frac{1}{(2x-a)(2x+a)}$

16 The magnetic potential energy stored in a certain inductor is 25 mJ, when the current in the inductor is 60 mA. This inductor is of inductance

(a) 1.389 H (b) 138.88 H (c) 0.138 H (d) 13.89 H

ANSWERS

SESSION 1	**1** (c)	**2** (a)	**3** (d)	**4** (a)	**5** (b)	**6** (a)	**7** (c)	**8** (c)	**9** (a)	**10** (c)
	11 (b)	**12** (b)	**13** (a)	**14** (b)	**15** (a)	**16** (a)	**17** (b)	**18** (c)	**19** (d)	**20** (a)
	21 (a)	**22** (b)	**23** (b)	**24** (c)	**25** (a)	**26** (d)	**27** (a)	**28** (d)	**29** (c)	**30** (c)
	31 (b)	**32** (a)								
SESSION 2	**1** (b)	**2** (b)	**3** (b)	**4** (b)	**5** (a)	**6** (b)	**7** (c)	**8** (b)	**9** (d)	**10** (d)
	11 (b)	**12** (b)	**13** (a)	**14** (c)	**15** (d)	**16** (d)				

Hints and Explanations

SESSION 1

1 Here, $\mathbf{A} = L^2\,\mathbf{k}$

and $\mathbf{B} = B_0\,(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$\phi = ?$

As, $\phi = \mathbf{B}\cdot\mathbf{A} = B_0\,(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot L^2\,\hat{\mathbf{k}}$

$\therefore\quad \phi = (4B_0\,L^2)\,\text{Wb}$

2 $\therefore$ Induced emf

$$= \frac{\text{Work done in taking a charge } q \text{ once along the loop}}{\text{charge } (q)}$$

i.e. $\quad V = \frac{W}{q} \Rightarrow W = Vq$

3 Given, $\phi = (3t^2 + 4t + 9)\,\text{Wb}$

$\therefore\ e = \frac{d\phi}{dt}$

$= \frac{d(3t^2 + 4t + 9)}{dt}$

$= 6t + 4$

At $t = 2$, we get $e = 16\,\text{V}$

4 Here, $\phi = 10t^2 - 50t + 250$

$\therefore\ e = \frac{-d\phi}{dt} = \frac{-d}{dt}(10t^2 - 50t + 250)$

$= -(20t - 50)$

At $t = 3\text{s}$, $e = -(20 \times 3 - 50) = -10\text{ V}$

5 $e = -\frac{d\phi}{dt} = 24\,t + 7$

At $t = 5\text{s}$, we get $e = 24 \times 5 + 7 = 127\text{ mV}$

6 Induced emf in a coil is given by

$$E = \left|-\frac{d\phi}{dt}\right|$$

Given, $\quad \phi = 50t^2 + 4$

and resistance, $\quad R = 400\,\Omega$

So, $E = \left|-\frac{d\phi}{dt}\right|_{t=2} = |100t|_{t=2} = 200\text{ V}$

So, current in the coil will be

$\therefore\quad I = \frac{E}{R} = \frac{200}{400} = \frac{1}{2} = 0.5\,\text{A}$

7 $\quad e = -\frac{d\phi}{dt} = \frac{d}{dt}(NAB\cos\omega t)$

or $\quad e = NAB\,\omega\sin\omega t = e_0\sin\omega t$

$\therefore\quad e_0 = NAB\,\omega$

8 The emf of coil, $e = -\frac{d\phi}{dt}$

and current, $I = \frac{e}{R}$

So, the current in the coil,

$I = \frac{1}{R}\frac{d\phi}{dt} = \frac{-1}{R}\frac{d}{dt}(NBA) = -\frac{NA}{R}\frac{dB}{dt}$

$= -\frac{20 \times (25 \times 10^{-4})}{100} \times 1000$

$= 0.5\,\text{A}$

9 Induced emf,

$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(6t^2 - 5t + 1) = -12t + 5$

At $t = -0.25\,\text{s}$, $e = -12(\ 0.25) + 5 = 2\text{ V}$

Induced current, $I = \frac{e}{R} = \frac{2}{10} = 0.2\text{ A}$

10 $\therefore\ q = \frac{d\phi}{R} = \frac{BA(\cos\,0° - \cos\,90°)}{R}$

$= \frac{B\pi r^2(1-0)}{R} = \frac{B\pi r^2}{R}$

$= \frac{2 \times 3.143 \times (10^{-1})^2}{0.01}$

$= 6.286\text{ C} = 6.3\text{ C}$

11 From Faraday's second law, emf induced in the circuit,

$$e = \frac{\Delta\phi}{\Delta t}$$

If R is the resistance of the circuit, then

$$I = \frac{e}{R} = \frac{\Delta\phi}{R\Delta t}$$

Thus, charge passes through the circuit,

$$q = I \times \Delta t \Rightarrow q = \frac{\Delta\phi}{R\Delta t} \times \Delta t \Rightarrow q = \frac{\Delta\phi}{R}$$

12 As there is no change in magnetic flux associated with the circuit, no current is induced in the circuit. The ammeter A shows no deflection.

13 Though emf is induced in the copper ring, but there is no induced current, because of cut in the ring. Hence, nothing opposes the free fall of the magnet. Therefore, $a = g$.

14 Magnetic flux through the loop is upwards and it is increasing due to increasing current along AB. Current induced in the loop should have magnetic flux in the downward direction, so that to oppose the increase in flux. Therefore, current induced in the loop is clockwise.

15 As coil A is moved closer to B, field due to A intercepting B is increasing. Induced current in B must oppose this increase. Hence, the current in B must be clockwise.

16 In the rotation of magnet, N pole moves closer to coil CD and S pole moves closer to coil AB. As per Lenz's law, N pole should develop at the end corresponding to C. Induced current flows from C to D. Again, S pole should develop at the end corresponding to B. Therefore, induced current in the coil flows from A to B.

17 Polarity of emf will be opposite in the two cases, while entering and while leaving the coil. Only in option (b) polarity is changing.

18 First current develops in direction of $abcd$, but when electron moves away, then magnetic field inside loop decreases and current changes its direction.

19 For emf, $e = Bv(L_{eff}) = Bv \times (2r) = 2Bvr$

$[\because L_{eff} = \text{diameter} = 2r]$

R will be at higher potential, we can find it by using right hand rule.

20 Magnetic field, $B = 0.04$ T and rate of change of radius of coil due to shrinkage,

$$\frac{-dr}{dt} = 2\ \text{mm s}^{-1}$$

Induced emf, $e = \frac{-d\phi}{dt} = -B\frac{dA}{dt}$

$$= -B\frac{d(\pi r^2)}{dt} = -B\pi 2r\frac{dr}{dt}$$

Now, if $r = 2$ cm

$$e = -0.04 \times \pi \times 2 \times 2 \times 10^{-2} \times 2 \times 10^{-3}$$

$$= 3.2\ \pi\mu\text{V}$$

21 $\int_{V_0}^{V} de = -\int_0^a Bx\,\omega\,dx$

$$V - V_0 = -\frac{1}{2}Ba^2\omega$$

$$= -\frac{1}{2} \times 10 \times 5 \times 2\pi \times \frac{1}{\pi} = -50\ \text{V}$$

22 Flux will change two times per revolution.

23 Net induced emf due to disc is

$$e = \frac{1}{2}B\omega l^2$$

$$= \frac{1}{2} \times 0.1 \times 2\pi \times 10 \times 0.1 \times 0.1$$

$$= \pi \times 10^{-2}\ \text{V}$$

24 $e = L\frac{dI}{dt}$ or $L \propto dt$

$$\therefore \frac{L_1}{L_2} = \frac{dt_1}{dt_2} = \frac{5}{50 \times 10^{-3}} = 100:1$$

25 Use, $e = L\frac{dI}{dt}$

Here, $e = 8$ V, $dI = 2\text{ A} - (-2\text{ A})$

$= 4$ A, $dt = 0.05$ s

Hence, $L = 0.1$ H

26 The self-inductance of the solenoid is

$$L = \frac{\mu_0 N^2 A}{l}$$

When $N' = 2N$, then

$$L' = \frac{\mu_0 (2N)^2 A}{l} = \frac{4\mu_0 N^2 A}{l} = 4L$$

Hence, when number of turns is doubled, then self-inductance becomes quadruple.

27 $\therefore e = L\frac{dI}{dt} = 2 \times 10^{-3} \times \frac{10-(-2)}{3 \times 10^{-3}} = 8\ \text{V}$

28 From the relation, $L = \mu_0 n^2 lA$

$$\Rightarrow \quad L = \mu_0 \frac{N^2}{l}A \qquad \left[\because n = \frac{N}{l}\right]$$

$$\Rightarrow \quad L \propto N^2$$

Thus, inductance of a coil is directly proportional to square of the number of turns.

29 Given, number of turns of solenoid, $N = 1000$

Current, $I = 4$ A

Magnetic flux, $\phi_B = 4 \times 10^{-3}$ Wb

$\because$ Self-inductance of solenoid is given by

$$L = \frac{\phi_B \cdot N}{I} \quad \ldots\text{(i)}$$

Substitute the given values in Eq. (i), we get

$$L = \frac{4 \times 10^{-3} \times 1000}{4} = 1\ \text{H}$$

30 $\because M_{max} = \sqrt{L_1 L_2}$

Given, $L_1 = 2$ mH, $L_2 = 8$ mH

$$M_{max} = \sqrt{2 \times 8} = \sqrt{16} = 4\ \text{mH}$$

31 $\therefore \phi = MI$

Hence, $d\phi = M\,dI$

Here, $d\phi = 1.2$ Wb, $dI = 3$ A

Hence, $M = 0.4$ H

32 The cause of production of eddy currents is the motion of a conductor in a varying magnetic field.

SESSION 2

1 $\therefore e = \frac{d\phi}{dt} = \frac{(NBA\cos\theta - 0)}{t}$

$$= \frac{1 \times 50 \times 25 \times 10^{-4}\cos 60° - 0}{0.2}$$

$$e = 3.12 \times 10^{-3}\ \text{V}$$

2 The rate of change of flux or emf induced in the coil is

$$e = -n\frac{d\phi}{dt}$$

Induced current,

$$I = \frac{e}{R'} = -\frac{n}{R'}\frac{d\phi}{dt}$$

Given, $R' = R + 4R = 5R$,

$d\phi = W_2 - W_1$, $dt = t$

$$\therefore \quad I = -\frac{n}{5R}\frac{(W_2 - W_1)}{t}$$

3 Magnetic flux ϕ linked with magnetic field $\mathbf{B}$ and area $\mathbf{A}$ is given by

$$\phi = \mathbf{B} \cdot \mathbf{A} = |\mathbf{B}||\mathbf{A}|\cos\theta$$

Here, $\theta = 0°$

So, $\phi = BA = B\pi r^2$

Now, induced emf, $|e|$

$$= \left|\frac{-d\phi}{dt}\right| = B\pi(2r)\frac{dr}{dt}$$

$$= 0.025 \times \pi \times 2 \times 2 \times 10^{-2} \times 1 \times 10^{-3}$$

$$= \pi\mu\text{V}$$

4 Area coming out per second from the magnetic field is not constant for elliptical and circular loops, so induced emf, during the passage out of these loops, from the field region will not remain constant.

5 Induced emf is $e_0 = \frac{Ba^2\omega}{2}$

where, a is length of each spoke.

$$n = \frac{120}{60} = 2$$

$$\therefore \quad \omega = 2\pi n = 4\pi\frac{\text{rad}}{\text{s}}$$

$\Rightarrow \quad a = 0.5$ m

$$e = \frac{Ba^2\omega}{2}$$

$$= 6.28 \times 10^{-5}\ \text{V}$$

6 Wheatstone bridge is balanced. Current through AC is zero. Effective resistance R of bridge is

$$\frac{1}{R}=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}\Rightarrow R=3\ \Omega$$

Total resistance $=1+3=4\ \Omega$

Induced emf, $e=IR=Blv$

$$\therefore\quad v=\frac{IR}{Bl}=\frac{1\times10^{-3}\times4}{2\times0.1}$$

$$=2\times10^{-2}\ \text{ms}^{-1}$$

7 Given, resistance of the solenoid,

$$R=10\pi^2\ \Omega$$

Radius of second coil, $r=10^{-2}$ m

$\Delta t=0.05$ s, $\Delta i=4-0=4$A

Charge flowing through the coil is given by

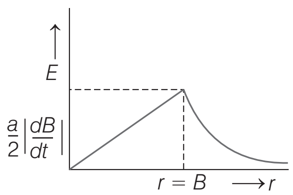
$$\Delta q=\left(\frac{\Delta\phi}{\Delta t}\right)\frac{1}{R}(\Delta t)$$

$$=\mu_0N_1N_2\pi r^2\left(\frac{\Delta i}{\Delta t}\right)\frac{1}{R}\Delta t$$

$$=4\pi\times10^{-7}\times2\times10^4\times100\times\pi\times(10^{-2})^2\times\left(\frac{4}{0.05}\right)\times\frac{1}{10\pi^2}\times0.05$$

$$=32\times10^{-6}\ \text{C}=32\ \mu\text{C}$$

8 For $r\geq a$, $\int\mathbf{E}\cdot d\mathbf{l}$



$$\Rightarrow\left|\frac{d\phi}{dt}\right|=A\left|\frac{dB}{dt}\right|\Rightarrow E(2\pi r)=\pi a^2\left|\frac{dB}{dt}\right|$$

$$E=\frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$\therefore$ Induced electric field, $E\propto\frac{1}{r}$.

9 For inductor, as we know induced voltage for $t=0$ to $t=T/2$,

$$V=L\frac{dI}{dt}=L\frac{d}{dt}\left(\frac{2I_0t}{T}\right)=\text{constant}$$

For $t=T/2$ to $t=T$,

$$V=L\frac{dI}{dt}=\left(\frac{-2I_0t}{T}\right)=-\text{constant}$$

So, answer can be represented with graph (d).

10 As we know that,

emf induced, $e=-L\dfrac{di}{dt}$

During 0 to $\dfrac{T}{4}$, $\dfrac{di}{dt}=$ constant

So, $e=$ negative

For $\dfrac{T}{4}$ to $\dfrac{T}{2}$, $\dfrac{di}{dt}=0$

So, $e=0$

For $\dfrac{T}{2}$ to $\dfrac{3T}{4}$, $\dfrac{di}{dt}=$ constant

So, $e=$ positive

11 $\because\phi=BA\cos\theta$

$$=\frac{1}{2}B\pi r^2\cos\omega t\qquad\left[\because A=\frac{1}{2}\pi r^2\right]$$

$$e_{\text{induced}}=-\frac{d\phi}{dt}=-\frac{d}{dt}\left(\frac{1}{2}B\pi r^2\cos\omega t\right)$$

$$=\frac{1}{2}B\pi r^2\omega\sin\omega t$$

$$\therefore\quad\rho=\frac{e^2_{\text{induced}}}{R}=\frac{B^2\pi^2r^4\omega^2\sin^2\omega t}{4R}$$

Hence, $\rho_{\text{mean}}=\langle p\rangle=\dfrac{B^2\pi^2r^4\omega^2}{4R}\dfrac{1}{2}$

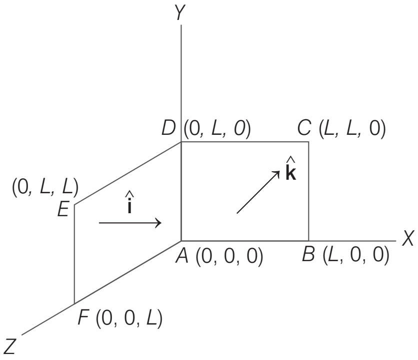
$$=\frac{(B\pi r^2\omega)^2}{8R}$$

12 Here, $\mathbf{B}=B_0(\hat{\mathbf{i}}+\hat{\mathbf{k}})$

Area of vector of $ABCD=L^2\ \hat{\mathbf{k}}$

Area of vector of $DEFA=L^2\ \hat{\mathbf{i}}$

Total area of vector, $\mathbf{A}=L^2(\hat{\mathbf{i}}+\hat{\mathbf{k}})$

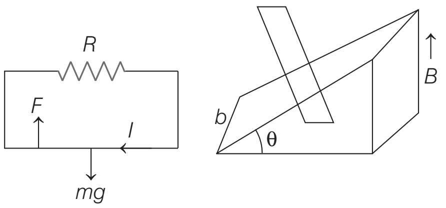


Total magnetic flux,

$$\phi=\mathbf{B}\cdot\mathbf{A}=B_0(\hat{\mathbf{i}}+\hat{\mathbf{k}})\cdot L^2(\hat{\mathbf{i}}+\hat{\mathbf{k}})$$

$$=B_0L^2(1+1)=2B_0L^2\ \text{Wb}$$

13 Induced emf, $e=-\dfrac{d\phi}{dt}$, $I=\dfrac{e}{R}=\dfrac{Blv}{R}$



Resultant magnetic force,

$$F_m=IlB.\Rightarrow F_m=\frac{B^2l^2v}{R}$$

and is directed upwards,

$$F_m=mg$$

$$\Rightarrow\quad F_m=\frac{B^2l^2v}{R}=mg\Rightarrow v=\frac{mgR}{B^2l^2}$$

14 Induced emf in the region is given by

$$|e|=\frac{d\phi}{dt}$$

where, $\phi=\mathbf{B}A=\pi R^2\mathbf{B}$

$$\Rightarrow\quad\frac{d\phi}{dt}=-\pi R^2\frac{d\mathbf{B}}{dt}$$

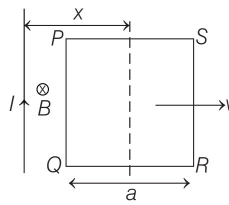
Rate of change of magnetic flux associated with loop 1,

$$e_1=-\frac{d\phi_1}{dt}=-\pi R^2\frac{d\mathbf{B}}{dt}$$

Similarly, $e_2=$ emf associated with loop 2

$$=-\frac{d\phi_2}{dt}=0\qquad[\because\phi_2=0]$$

15



Potential difference across PQ is

$$V_P-V_Q=B_1(a)v=\frac{\mu_0I}{2\pi\left(x-\frac{a}{2}\right)}av$$

Potential difference across side RS of frame is

$$V_S-V_R=B_2(a)v=\frac{\mu_0I}{2\pi\left(x+\frac{a}{2}\right)}av$$

Hence, the net potential difference in the loop will be

$$V_{\text{net}}=(V_P-V_Q)-(V_S-V_R)$$

$$=\frac{\mu_0Iav}{2\pi}\left[\frac{1}{\left(x-\frac{a}{2}\right)}-\frac{1}{\left(x+\frac{a}{2}\right)}\right]$$

$$=\frac{\mu_0Iav}{2\pi}\left(\frac{a}{\left(x-\frac{a}{2}\right)\left(x+\frac{a}{2}\right)}\right)$$

Thus, $V_{\text{net}}\propto\dfrac{1}{(2x-a)(2x+a)}$

16 Given, magnetic potential energy stored in an inductor,

$$U=25\ \text{mJ}=25\times10^{-3}\ \text{J}$$

Current in an inductor, $I_0=60$ mA $=60\times10^{-3}$ A

As, the expression for energy stored in an inductor is given as

$$U=\frac{1}{2}LI_0^2$$

where, L is the inductance of the inductor.

Substituting the given values in above equation., we get

$$(25\times10^{-3})=\frac{1}{2}\times L\times(60\times10^{-3})^2$$

$$\Rightarrow\quad L=\frac{2\times25\times10^{-3}}{3600\times10^{-6}}=\frac{500}{36}$$

or $L=13.89$ H