2.5
CHAPTER

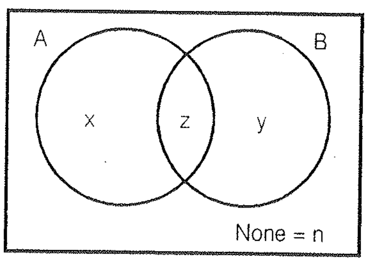
Set Theory

Venn Diagrams

Venn Diagrams are pictorial ways of representation of sets of things having different attributes. Basic understanding of regions represented by a Venn diagram is the tool for solving these questions.

If a Venn diagram is representing distribution of n attributes then total 2^n regions will be created in entire Venn diagram (including a region when particular thing will not belong to any of attributes.)

Venn-diagram involving two attributes



Clearly 4 regions are then (x, y, z & n)

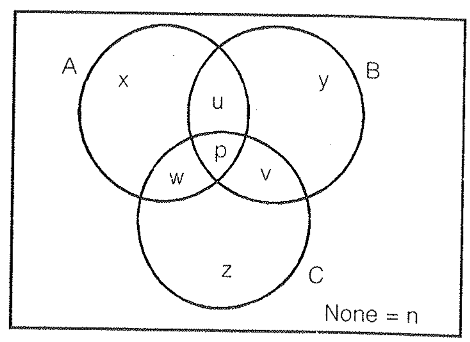
Region x can be referred as

(a) z = things that belongs to both attribute A & B
(b) x = things that belong to A but not to B
(c) y = things that belong to B but not to A
(d) n = represent region of thing which neither belongs to A nor to B.

Further observations

(a) If things belong to exactly one attribute then region = $x + y$
(b) If things belong to at least one of the attribute = exactly one + exactly two $n(A \cup B) = x + y + z$.
(c) Total sample (μ) : [(Things belongs to atleast one) + (things belong to none of attribute.)] $\mu = x + y + z + n$
(d) Things belong to both attribute $n(A \cap B) = Z$
(e) Things belong to attribute A $n(A) = x + z$
(f) Things belong to attribute Y $n(B) = y + z$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
(g) Things does not belong to attribute A = (y + n)
(h) Things does not belong to attribute B = (x + n)

Venn diagram with three attributes



Total sample sets = 3

Here X, Y, Z are different 3 attributes hence

(a) Total number of regions = $2^3 = 8$
(b) $n(A) = x + w + u + p$, things belong to only $A = x$
(c) $n(B) = u + p + v + y$, things belong to only $B = y$
(d) $n(C) = w + p + v + z$, things belong to only $C = z$
(e) Things belong to exactly one of attribute $= x + y + z$
(f) Things belong to attribute (A & B) both $n(A \cap B) = u + p$
(g) Things belong to attribute (B & C) both $n(B \cap C) = v + p$
(h) Things belong to attribute (A & C) both $n(A \cap C) = w + p$
(i) Things belong to only attribute (A & B) both = u
(j) Things belong to only attribute (B & C) both = v

(k) Things belong to only attribute (A & C) both = w

(l) Things belong to all the three attributes $n(A \cap B \cap C) = p$

(m) Things belong to all the three attributes $n(A \cap B \cap C) = p$

(n) Things belong to none of the attributes = n

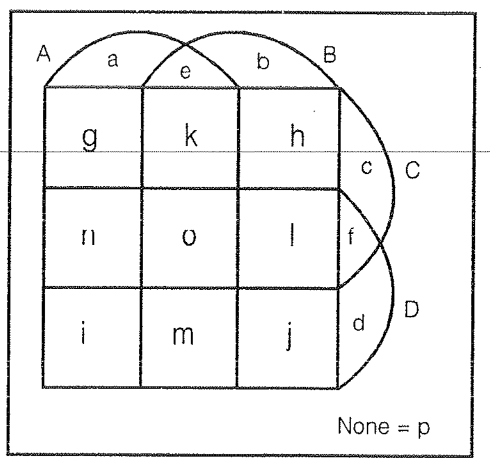
(o) Things belong to at least one of the attributes
Total sample – n = μ – n
= Exactly one + Exactly two + Exactly three
$= x + y + z + u + v + w + p$
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(p) Things does not belong to A = n + z + u + y

(q) Things belong to A or B = x + u + y + w + p + v

(r) Things belong neither A nor B = (z + n)

Venn diagrams with 4-attributes

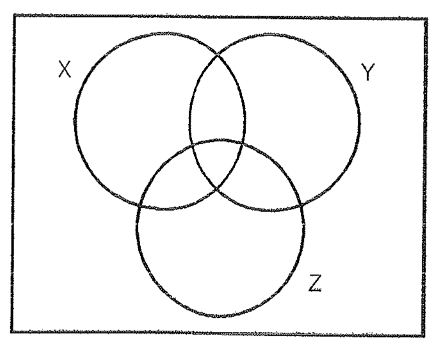


Total sample = m

1. n(A) = a + e + g + k + n + o + j + m
2. n(B) = b + e + h + k + l + o + j + m
3. n(C) = c + f + h + l + k + o + g + n
4. n(D) = d + f + j + l + m + o + j + n
5. Only A = a
6. Only B = b
7. Only C = c
8. Only D = d
9. Things belong to exactly one of attributes = a + b + c + d
10. A & B = $n(A \cap B)$ = e + k + o + m
11. A & C both = $n(A \cap C)$ = g + k + o + m
12. A & D both = $n(A \cap D)$ = n + o + i + m
13. B & C both = $n(B \cap C)$ = k + h + o + l
14. B & D both = $n(B \cap D)$ = m + j + o + l
15. C and D both = $n(C \cap D)$ = m + o + l + f
16. Only to (A & B) both = e
17. Only to (A & C) both = g
18. Only to (A & C) both = i
19. Only to (B & C) both = h
20. Only to (B & D) both = j
21. Only to (C & D) both = f
22. To exactly two out of four = e + f + g + h + i + j
23. (A, B & C) all three = k + 0 = $n(A \cap B \cap C)$
24. (B, C & D) all three = l + 0 = $n(B \cap C \cap D)$
25. (A, B & D) all three = m + 0 = $n(A \cap B \cap D)$
26. (A, C & D) all three = n + 0 = $n(A \cap C \cap D)$
27. Only to (A, B, C) all three = k
28. Only to (B, C, D) all three = l
29. Only to (A, B, D) all three = m
30. Only to (A, C, D) all three = h
31. To exactly three of four attributes = k + l + m + n
32. Belong to A, B, C, D all four = O
33. None among A, B, C, D = p

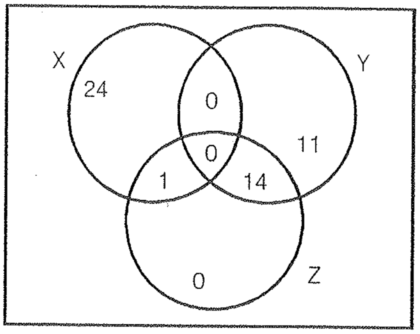
Solved Examples

Direction (Q.1 to Q. 5): These examples are based on the following diagram.



Total sample = First 50 natural numbers.
Circle X- represent even numbers in sample.
Circle Y-represent odd numbers in sample.
Circle Z-represent prime number in sample.

Solution : Following venn-diagram can be drawn.

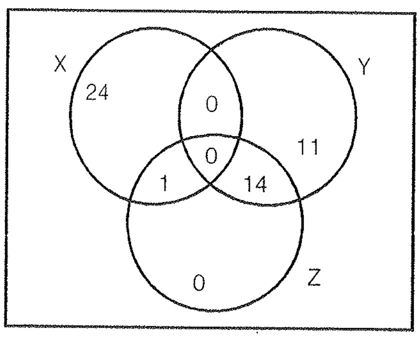


1. How many numbers belong to X only?
   (a) 25 (b) 24
   (c) 23 (d) 22

Ans. (b)

Only even number but not prime number hence 24 elements, option (b).

2. How many numbers belong to Y only



   (a) 14 (b) 25
   (c) 11 (d) 13

Ans. (c)

All odd numbers which are not prime 11 elements.

3. How many elements are these in $(Y \cap Z)$?
   (a) 14 (b) 12
   (c) 24 (d) 13

Ans. (a)

All odd prime number till 50 are 14.

4. How many elements are there in $(X \cap Z)$?
   (a) 0 (b) 1
   (c) 2 (d) 3

Ans. (b)

2 is the only even & prime number. Hence only one element.

5. How many elements are there which do not belong to Z?
   (a) 35 (b) 25
   (c) 34 (d) 14

Ans. (a)

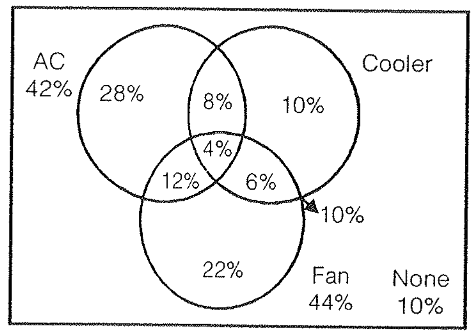
All non prime number till 50, hence 35 numbers which are non-prime.

Direction (Q.6 to Q. 10): In summer a survey was conducted in a colony to know how many houses are equipped with fans, AC's & coolers 224 houses have coolers. 10% of the houses have coolers & fan. 42% houses have AC's. 44% of the houses have fans. 10% of houses have none of three. 22% have only fans & 12% of the houses have AC's & coolers.

Sol.

Since maximum data 18 is % terms we can comfortably work by assuming sample size to be 100%.

And we can draw following venn-diagram.



Means 28% of houses have cooler

$$\frac{28}{100} \times (T) = 224$$

$$T = \frac{100 \times 224}{28} = 800$$

Total house surveyed = 800

Now all question can be answered.

6. How many houses have fans only?
   (a) 352 (b) 176
   (c) 80 (d) 320

Ans. (b)

22% have fans only $22 \times \frac{800}{100} = 176$

Hence option (b)

7. How many houses have at most two devices among AC, fan & coolers?
   (a) 768 (b) 800
   (c) 400 (d) 608

Ans. (a)

At most two devices:

= Total no. of houses – No. of houses will 3 devices

$$= 800 - \frac{800 \times 4}{100} = 768$$

Hence option (a)

8. How many total houses were surveyed?

(a) 640 (b) 768

(c) 600 (d) 800

Ans. (d)

Total houses surveyed 800

Hence option (d)

9. How many houses do not have fan?

(a) 176 (b) 352

(c) 448 (d) 288

Ans. (c)

Over all 44% houses have fans

Hence 56% don't have fans.

Hence $56 \times \frac{800}{100} = 448$

Hence option (c)

10. What is the ratio of number of houses that have exactly 2 out of 3 appliances to those who have at most one appliance.

(a) 13 : 35 (b) 10 : 35

(c) 3 : 7 (d) 4 : 7

Ans. (a)

No. of houses having 2 out of 3 = 26% of population

No. of houses having at most one appliance = 70%

Ratio = 13 : 35

Hence option (a)

I Practice Exercise: I

Direction (Q.1 to Q. 5): The following table gives partial information about the number of students of a school who like Hockey & Cricket. There are 800 students in the school.

	Hockey	Cricket	Both	Total
Boys	100			300
Girls		170		
Total				800

of the total students, 50% like Cricket, 30% like Hockey & 20% like both the games. 20% of the girls like both the games.

1. The number of girls who like Cricket is

(a) 140 (b) 170

(c) 190 (d) 230

2. The number of boys who like both games is

(a) 20 (b) 40

(c) 60 (d) 100

3. How many boys like none of the games?

(a) 0 (b) 90

(c) 170 (d) 190

4. How many students like Hockey?

(a) 0 (b) 320

(c) 160 (d) 240

5. How many girls like at least one of the game?

(a) 270 (b) 360

(c) 290 (d) 230

Direction (Q.6 to Q. 8): During the annual day celebrations of a college 225 students participated. Out of them 100 played Cricket, 105 played Hockey, 95 played Football, 20 played none of these three, 45 students played exactly two of the above three sports, 20 student played all three game.

6. If 45 students played only Cricket, then find the number of students who played Football & Hockey only

(a) 15 (b) 10

(c) 20 (d) 25

7. If 20 students played both Cricket & Football but not Hockey, find the number of students who played only Hockey

(a) 50 (b) 55

(c) 45 (d) 60

8. If 50 students played Hockey but not Football, find the number of students who played Cricket or Hockey

(a) 110 (b) 120

(c) 135 (d) 140

Direction (Q.9 to Q.13): In an apartment, a survey was conducted among 600 people to find out readership of 3 magazines India today, Business today and Front line. It is known that 200 people read at least two of these magazines. 460 people read either India today or Front line. 360 read exactly one among three. 160 read neither India today nor Business today. 260 read the India today or Business today but not Front line.

9. How many people read at least one of the other two magazines along the Front line?
   (a) 140 (b) 180
   (c) 200 (d) 220

10. How many people read all three magazines?
   (a) 140 (b) 120
   (c) 80 (d) Can't determined

11. How many people read India today only?
   (a) 100 (b) 120
   (c) 140 (d) 160

12. How many people read none of three magazines?
   (a) 60 (b) 40
   (c) 30 (d) None of these

13. How many people read exactly one among three magazines?
   (a) 360 (b) 300
   (c) 200 (d) 260

Direction (Q.14 to Q. 21): In a party there were 70 people who took tea. There were 80 people who took coffee. If there were total 120 people in the party then.

14. What is the maximum possible no. of people who took none of these two?
   (a) 60 (b) 30
   (c) 40 (d) None of these

15. What is the maximum possible number of people who took exactly one of these two drink?
   (a) 80 (b) 90
   (c) 120 (d) 100

16. What is the maximum possible number of people who took both of the tea and coffee?
   (a) 80 (b) 90
   (c) 70 (d) 100

17. What is the maximum possible number of people who took at least one of these two drinks?
   (a) 120 (b) 100
   (c) 90 (d) None of these

18. What is the minimum possible number who took both the drinks?
   (a) 40 (b) 30
   (c) 50 (d) None of these

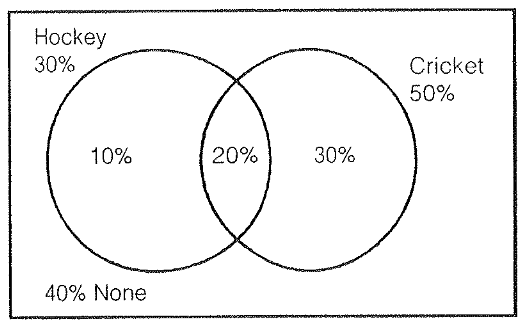
19. What is the minimum possible no. of people who took at least one of the two drink?
   (a) 70 (b) 40
   (c) 80 (d) None of these

20. What is the minimum possible number of people, who took at most one of the two drinks?
   (a) 40 (b) 50
   (c) 60 (d) 70

21. What is the minimum possible number of people who took none of these two drinks?
   (a) 20 (b) 30
   (c) 40 (d) None of these

□□□□

Solutions



Q.1 to Q.5

Number of girls who like both games

$= 300 \times \frac{20}{100} = 60$

Number of students linking none of two games is 40%

Hence $\frac{40}{100} \times 800 = 320$ students like none of the game.

Total number of students who like both game

$= 800 \times \frac{20}{100} = 160$

Total number of students who like Cricket

$= 800 \times \frac{50}{100} = 400$

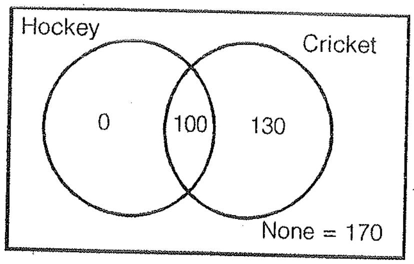
Total number of students who like Hockey

$= 800 \times \frac{30}{100} = 240$

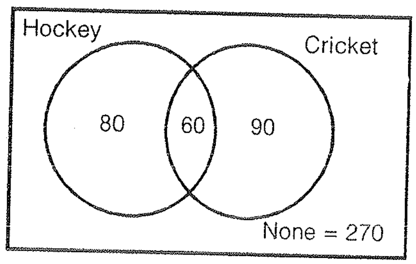
Now we are easily able to complete the table

	Hockey	Cricket	Both	Total No. of Students	Like none two of games
Boys	100	230	100	300	170
Girls	140	170	60	500	270
Total	240	400	160	800	440

Boys =

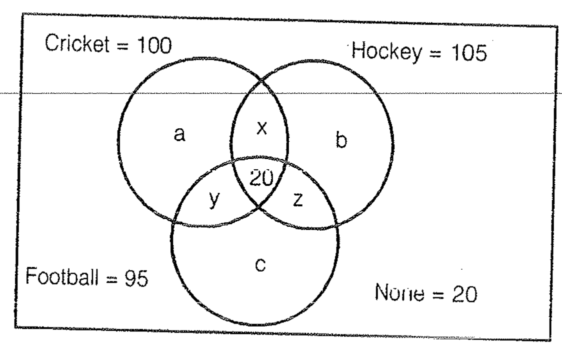


Girls =



1. (c) No. of girls like cricket = 170.
2. (d) No. of boys like both games = 100.
3. (c) No. of boys like none of the game = 170.
4. (d) No. of students like hockey = 240.
5. (d) Number of girls who like at least one of the game = 230.

Question. No. 6 to 8



No. of students who played at least one out of three games

$= 225 - 20 = 205$

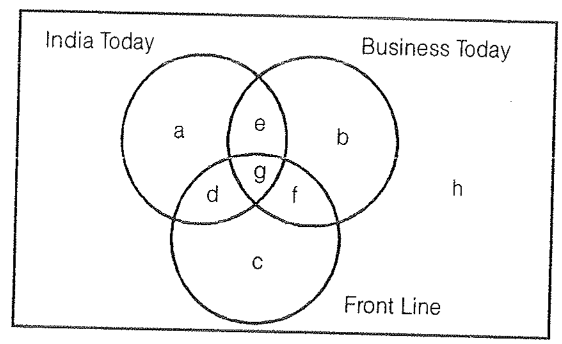
$x + y + z = 45$ (as 45 students played exactly two of the three games)

6. (b) $a = 45$ (given)
   $45 + x + y + 20 = 100$
   $x + y = 35$
   $z = 45 - (x + y) = 10$
   Hockey and Football only = 10
   Option (b).

7. (d) $y = 20$
   $x + z + b + 20 = 105$ (Total Hockey = 105)
   $x + y + z = 45$
   $x + z = 45 - 20 = 25$
   $b + 20 + 25 = 105$
   $b = 105 - 45 = 60$
   Hence option (d)

8. (b) $x + b = 50$
   $x + b + z + 20 = 105$
   $\Rightarrow z = 35$
   $x + y + z = 45$
   $\Rightarrow x + y = 10$
   $a + x + y + 20 = 100$
   $\Rightarrow a - 70$
   $a + x + b = 70 + 50 = 120$
   Hence option (b)

Question 9 to 13



$a + b + c + d + e + f + g + h = 600$

$e + d + g + f = 200$

$a + b + c = 360 \Rightarrow h = 40$ ...(i)

$a + d + c + e + f + g = 460$

$b + h + 460 = 600$

$b = 100$ ...(ii)

$c = 160$ ...(iii)

$a = 100$ ...(iv)

$a + e + b = 260$

$e \Rightarrow 260 - (a + b) = 60$ ...(v)

$e + f + g = 460 - (a + b + c)$

$= 460 - 360 = 100$

$d = 200 - 100 = 100$ ...(vi)

9. (a) $d + g + e + f = 200$, $e = 60$ from (v)
   No. of people required
   $= d + g + f = 200 - 60 = 140.$

10. (d) Can't be determined.

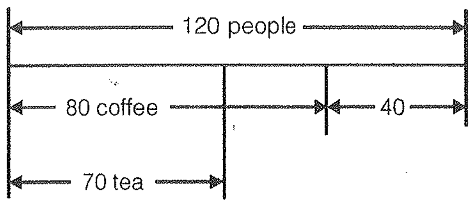
11. (a) India today only = a = 100.

12. (b) None of three $\Rightarrow h = 40$.

13. (a) $a + b + c = 360$.

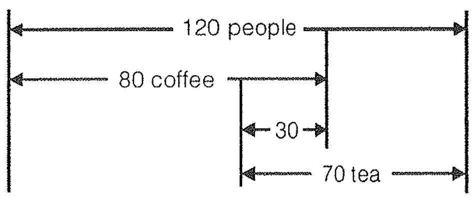
Question 14 to 21.

14.(c)



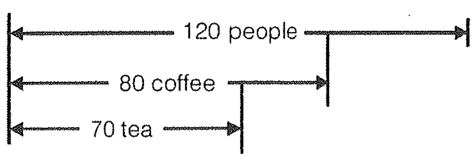
Maximum possible no. of people took none of these two.
(120 – 80) = 40. Option (c)

15.(b) To maximize the no. of only one drink taken try to minimize both and none.



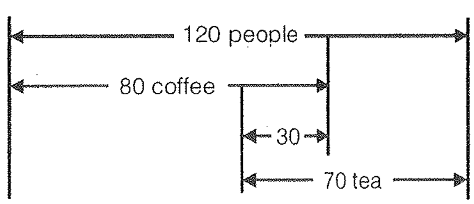
Maximum number of only one drink, taken
= (80 – 30) + (70 – 30) = 90. Option (b)

16.(c)



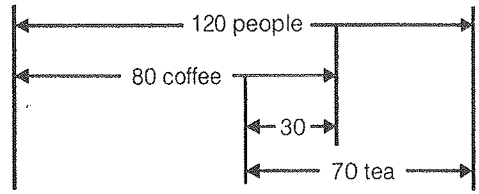
Maximum no. of people who took both
= 70. Option (c)

17.(a)



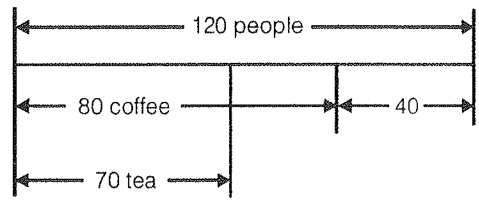
To maximize at least one of two means we have to minimize the none.
Hence 120. Option (a)

18.(b)



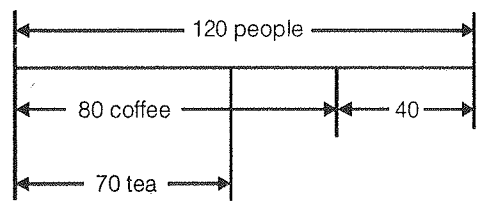
Answer 30. Option (b)

19.(c)



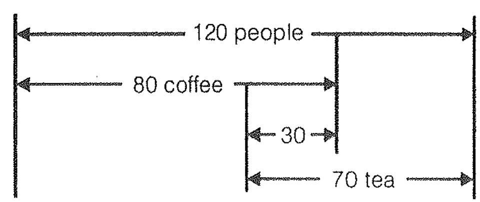
To minimize the no. of people who took at least one, we need to maximize the number of people who took none. Option (c).

20.(b) To minimize the no. of people who took at most one. I need to maximize who took both.



Maximum 70 people can take both drinks. Hence min. 50 people are here who will take at most one drink. Option (b)

21.(d)



Answer zero. Option (d)