Unit - II

Matrices

Chapter at a Glance

1. **Order of Matrix :** The number of rows and columns that matrix has is called its order. By convention, rows are listed first and columns second.

   i.e., a matrix having m rows and n columns is called an $m \times n$ (read "m by n") matrix or a matrix of order $m \times n$.

2. **Comparable pair of matrices :** Two matrices are said to be comparable if each one of them contains as many rows and columns as the other *i.e.,* they are of same order.

3. **Types of matrices** :

   (a) Rectangular matrix : Any $m \times n$ matrix, where $m \neq n$

   (b) Row matrix : Any $m \times n$ matrix where $m = 1$

   (c) Column matrix : Any $m \times n$ matrix where $n = 1$

   (d) Square matrix : Any $m \times n$ matrix where $m = n$

   (e) Diagonal matrix : Any *square matrix* all of whose elements *except diagonal elements (i.e. elements along leading diagonal)*, are **zero**.

$$\begin{bmatrix} \blacksquare & 0 & \cdots & 0 \\ 0 & \blacksquare & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \blacksquare \end{bmatrix}$$

   (f) Scalar matrix : Any *diagonal matrix* in which *diagonal elements are all equal* is called a scalar matrix.

$$\begin{bmatrix} k & o & o \\ o & k & o \\ o & o & k \end{bmatrix}$$

   (g) Identity (or unit) matrix : Any *scalar matrix* in which *diagonal elements are all equal being unity* is called an identity or unit matrix (I).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   (h) Equality of matrices : Two **matrices** are **equal** if they have the same order and the corresponding elements are identical.

   Equal Matrices :

$$A = \begin{bmatrix} 4 & 13 \\ -2 & 19 \end{bmatrix}; \; B = \begin{bmatrix} 4 & 13 \\ -2 & 19 \end{bmatrix}; \; C = \begin{bmatrix} 4 & 13 \\ 19 & -2 \end{bmatrix} \text{ and } M = \begin{bmatrix} 4 & 13 \\ -2 & 19 \\ 0 & -5 \end{bmatrix}$$

   $A = B; A \neq C; A \neq M.$

   (i) Zero matrix (or null matrix) : Any matrix *all of whose elements are zero* and is denoted by $O_{m \times n}$, where $m \times n$ is the order of matrix.

4. **Operations on matrices :**

   (i) Addition of matrices : If A and B be two matrices of the same order $m \times n$, they are called conformable or compatible for the sum and their sum denoted by A + B, is a matrix of the same order $m \times n$ and is obtained by adding their corresponding elements. Thus, if A = $[a_{ij}]$ and B = $[b_{ij}]$ be two $m \times n$ matrices, then

$$\begin{aligned} A + B &= [a_{ij}]_{m \times n} + [b_{ij}]_{m \times n} \\ &= [a_{ij} + b_{ij}]_{m \times n}. \qquad 1 \le i \le m,\ 1 \le j \le n \end{aligned}$$

For example, if

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & -1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix}$$

then

$$\begin{aligned} A + B &= \begin{vmatrix} 1 & 2 & 5 \\ 3 & 4 & -1 \end{vmatrix} + \begin{bmatrix} -1 & 3 & 2 \\ 2 & 5 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 1-1 & 2+3 & 5+2 \\ 3+2 & 4+5 & -1+7 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 7 \\ 5 & 9 & 6 \end{bmatrix} \end{aligned}$$

(ii) Difference of two matrices : If A and B are two matrices of the same order then we define A – B as the sum A + (– B) *i.e.*, if A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{m \times n}$

Then $$A - B = [a_{ij} - b_{ij}]_{m \times n}.$$

(iii) Negative of a matrix : Negative of a matrix $\mathbf{A}_{m \times n}$ is defined as a ***m × n matrix*** *whose (i, j)th entry is* ***negative*** *(or* ***additive inverse****) of (i, j)th entry of* $\mathbf{A}_{m\times n}$, *i.e.*

$$-A = [-a_{ij}]_{m \times n}$$

$$A = \begin{bmatrix} 3 & -5 & 7 \\ -2 & 4 & 9 \end{bmatrix}$$

$$-A = \begin{bmatrix} -3 & 5 & -7 \\ 2 & -4 & -9 \end{bmatrix}$$

(A) Properties of matrix addition :

(a) Addition of matrices is commutative : *i.e.*, if A and B are matrices of the same order, then

$$A + B = B + A.$$

(b) Addition of matrices is associative : *i.e.*, if A and B are matrices of the same order, then

$$(A + B) + C = A + (B + C).$$

(c) Existence of additive identity : For any matrix A, there exists a null matrix 0 of the same order such that

$$A + 0 = A = 0 + A.$$

0 is called the *additive identity.*

(d) Existence of additive inverse : For any matrix A, there exists a unique matrix –A of the same order such that

$$A + (-A) = 0 = (-A) + A,$$

– A is called additive inverse or negative of A.

(e) Cancellation laws under addition : If A, B and C are three matrices of the same order, then

(a) If $A + B = A + C$

$\Rightarrow$ $B = C.$ (left cancellation law)

(b) If $B + A = C + A$

$\Rightarrow$ $B = C.$ (right cancellation law)

(f) Scalar multiplication : The scalar multiple kA (or Ak) of a matrix A by a scalar k is the matrix obtained by multiplying each element of A by k.

Thus, if $$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

then $$2A = 2\begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 2\times2 & 2\times3 & 2\times1 \\ 2\times0 & 2\times2 & 2\times(-1) \end{bmatrix} = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 4 & -2 \end{bmatrix}$$

(B) Properties of scalar multiplication :

(a) If A and B are two matrices of the same type and k, m are any numbers, then

(i) $(k + m)\,A = kA + mA$

(ii) $k\,(A + B) = kA + kB$

(b) Let A be any matrix and k_1, k_2 are any scalars, then

(i) $(k_1k_2)\,A = k_1\,(k_2A) = k_2\,(k_1A)$

(ii) $1.A = A$

(iii) $(-1)\,A = -A.$

(C) Multiplication of matrices : Multiplication of two matrices is *defined only if* ***the number of columns of the 1st matrix must equal the number of rows of the 2nd matrix***, *i.e.* Let A and B be two matrices, then the product AB is defined only if the number of columns in A = number of rows in B. Such matrices are said to be **compatible** or **conformable** for multiplication.

Now, let us see how to multiply the two matrices if multiplication is defined :

This is a "Multiply row by column" process. We multiply the entries of a row by the corresponding entries of a column and then add the products.

Thus,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times2}$$

$$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} \end{bmatrix}_{3\times2}$$

Note : From the above multiplication, we find that the product of an $m \times p$ and $p \times n$ matrix is an $m \times n$ matrix

$$\underbrace{\mathbf{A}_{m\times p} \times \mathbf{B}_{p\times n}}_{\text{inner dimensions equal}} = \underbrace{\mathbf{C}_{m\times n}}_{\text{Product of the outer dimensions}}$$

(D) Properties of Matrix Multiplication :

(a) The product of matrices is not commutative, *i.e. whenever* AB *exists,* BA *is not always defined*. **Even if both AB and BA are defined, it is not necessary that AB = BA**.

(b) The product of two matrices can be zero without either factor being a zero matrix.

(c) Cancellation law for the multiplication of real numbers is not valid for the multiplication of matrices, *i.e.* **AB = AC does not imply that B = C** (However, if A, B, C are square matrices of same type and if A is non-singular, then AB = AC implies that B = C.

(d) Matrix multiplication is associative if conformability is assured, *i.e.* **A(BC) = (AB)C**

(e) Matrix multiplication is distributive with respect to matrix addition, *i.e.* **A (B + C) = AB + AC**.

(f) For any matrix $A_{p\times n}$, we have $\mathbf{O}_{m\times p}\,\mathbf{A}_{p\times n} = \mathbf{O}_{m\times n}$ (O denotes zero matrix)

(g) Multiplicative Identity for a square matrix : $\mathbf{A}_{n\times n}\,\mathbf{I}_{n\times n} = \mathbf{A}_{n\times n}$

(h) A^2 is defined only when A is a square matrix and is equal to AA.

5. **Linear combination of matrices :** Let A, B be two matrices of the same order and k_1, k_2 be two scalars, then the matrix k_1A + k_2B is called a linear combination of the matrices A and B.

For example, if
$$A = \begin{bmatrix} 2 & 0 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

and
$$B = \begin{bmatrix} 7 & -6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$$

Then
$$2A + 3B = 2\begin{bmatrix} 2 & 0 & 3 \\ 2 & 1 & 4 \end{bmatrix} + 3\begin{bmatrix} 7 & -6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 0 & 6 \\ 4 & 2 & 8 \end{bmatrix} + \begin{bmatrix} 21 & -18 & 9 \\ 3 & 12 & 15 \end{bmatrix}$$
$$= \begin{bmatrix} 4+21 & 0-18 & 6+9 \\ 4+3 & 2+12 & 8+15 \end{bmatrix}$$
$$= \begin{bmatrix} 25 & -18 & 15 \\ 7 & 14 & 23 \end{bmatrix}$$

is a linear combination of matrices A and B.

6. **Transpose of a matrix :** Let A = [a_{ij}] be a $m \times n$ matrix, then the matrix obtained from A by changing rows into columns and columns into rows is called the *transpose* of A and is denoted by A^T or A′. Clearly A′ is an $n \times m$ matrix whose $(j, i)^{th}$ entry = $(i, j)^{th}$ entry of A.

For example, if
$$A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ 1 & 5 \end{bmatrix}$$

then
$$AT = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 5 \end{bmatrix}$$

(A) Properties of transpose matrix :

(a) **(A′)′ = A**, (b) **(– A)′ = – A′**, (c) **(A + B)′ = A′ + B′**, (d) **(A – B)′= A′ – B′**, (e) **(AB)′ =B′ A′**, (f) **(kA)′ = k A′.**

7. **Symmetric matrix** : A square matrix A = [a_{ij}] is symmetric if $a_{ij} = a_{ji}$.

Note : (a) Symmetric matrix is always a square matrix.

(b) A necessary and sufficient condition for a matrix to be symmetric is A′ = A, where A′ denotes transpose of matrix A.

(c) Diagonal matrices are always symmetric.

Skew-symmetric matrix : A square matrix A = [a_{ij}] is skew-symmetric if $a_{ij} = - a_{ji}$. Note that *all diagonal elements of a skew-symmetric matrix is zero*. Also note that the necessary and sufficient condition for a matrix to be symmetric is A′ = – A, where A′ denotes transpose of matrix A. (A matrix which is both symmetric and skew-symmetric is called a square null matrix.)

8. **Singular and non-singular matrices :** A square matrix is singular if det [A] = 0 (or Δ = 0), otherwise it is non-singular.

9. **Nilpotent matrix :** A square matrix A is called Nilpotent if there exists a positive integer m such that $A^m = 0$. If m is the least positive integer such that $A^m = 0$, then m is called the index of the nilpotent matrix A.

10. **Adjoint of a square matrix :** Let A = [a_{ij}] be a square matrix of order n. Then the *adjoint of* A is the transpose of the matrix $[C_{ij}]_{m \times n}$ where C_{ij} is the co-factor of a_{ij} in |A|. It is denoted by adj. A.

For example, let $$A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

then $$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix}$$

Now, $C_{11} = 5$, $C_{12} = -1$, $C_{21} = -3$, $C_{22} = 2$.

Then, $$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -1 & 2 \end{bmatrix}$$

Note : (i) If A is a square matrix of order *n*, then

$$A\ (\text{adj. } A) = |A|\ I_n = (\text{adj } A)\ A.$$

(ii) If A is a square matrix of order 3, then adj $(kA) = k^2$ (adj A).

11. Relation between a square matrix and adj A : If A be any n^{th} order square matrix, then (adj A) A = A (adj A) = $|A| I_n$, where I_n is n^{th} order unit matrix.

12. If A is a $n \times n$ *non-singular matrix*, then $|\text{adj } A| = |A|^{n-1}$

13. If A and B are two *non-singular matrices of same type*, then adj (AB) = (adj B) (adj A)

14. Inverse of an n × n matrix : A^{-1} = adj A/ |A|, but *inverse of a square matrix exists if and only if* A *is non-singular, i.e.* $|A| \neq 0$

15. Properties of matrices and inverses :

(a) If A, B be two n^{th} order non-singular matrices, then AB is also non-singular and $(AB)^{-1} = B^{-1} A^{-1}$ and $(A')^{-1} = (A^{-1})'$

(b) $AA^{-1} = A^{-1}A = I$

16. Applications of matrices in solving linear equations (Martin's rule) :

Consider the three simultaneous equations in variables *x*, *y* and *z*.

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

These can be written in matrix form as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

or $$AX = B$$

where A is 3 × 3 matrix and X and B are 3 × 1 column matrices.

Now, if A is non-singular *i.e.*, $|A| \neq 0$ then we can left multiply both members of this equation by A^{-1} to obtain

$$A^{-1}AX = A^{-1}B$$

$\Rightarrow$ $$IX = A^{-1}B$$

$\Rightarrow$ $$X = A^{-1}B$$

The equations having one or more solutions are called consistent equations.

(i) If $|A| \neq 0$, the system of equations is consistent and has a unique solution.

(ii) If $|A| = 0$, the system of equations has either infinite number of solutions or no solution.

$\Rightarrow$ If $|A| = 0$ and (*adj* A)·B = 0 equations have infinitely many solutions *i.e.*, consistent but dependent.

$\Rightarrow$ If $|A| = 0$ but (*adj* A)·B $\neq 0$, equations have no solution *i.e.*, inconsistent.

Multiple Choice Questions

1. If the matrix A = $\begin{bmatrix} 5 & x & -1 \\ 4 & -2 & -3 \\ 7 & 2 & 2 \end{bmatrix}$ is a singular matrix, then value of x is :

(a) $\frac{-12}{29}$ (b) $\frac{12}{29}$

(c) $\frac{12}{19}$ (d) none of these

2. If A = $\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A is symmetric matrix, then :

(a) $x = y$ (b) $x = 0$

(c) $y = 0$ (d) $x \neq y$

3. The sum of two skew matrices is :

(a) symmetric matrix

(b) null matrix

(c) skew symmetric matrix

(d) diagonal matrix

4. If matrix A = $[a_{13}]_{2 \times 2}$, where a_{13} = 1 if $i \neq j$ and $a_{i \times j}$ = 0 if $i = j$, then A^2 is equal to :

(a) unit matrix (b) A

(c) 0 (d) none of these

5. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then A + A′ = I, then the value of α is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{3\pi}{2}$ (d) π

6. If A is a square matrix such that A^2 = A, then (I + A) – 3A is :

(a) I (b) 2A

(c) 3I (d) A

7. If matrices A and B are inverse of each other, then :

(a) AB = BA (b) AB = BA = I

(c) AB = BA = 0 (d) AB = 0, BA = I

8. Given that A = $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 5 & 9 \\ 1 & 3 & 4 \end{bmatrix}$, The value of $3a_{22} - 4a_{33}$ is :

(a) 0 (b) – 1

(c) 2 (d) none of these

9. For any square matrix A, AA^T is a :

(a) Unit matrix

(b) Symmetric matrix

(c) Skew symmetric matrix

(d) Diagonal matrix

10. Which of the following statement is correct?

(a) Every identity matrix is a scalar matrix.

(b) Every scalar matrix is an identity matrix.

(c) Every diagonal matrix is an identity matrix.

(d) A square matrix whose each element is 1 is an identity matrix.

11. If $A_{m \times n} = B_{n \times p} = C_{m \times p}$, then matrices A, B, C are :

(a) $A_{2\times3}, B_{3\times2}, C_{2\times3}$ (b) $A_{3\times2}, B_{2\times3}, C_{3\times2}$

(c) $A_{3\times3}, B_{2\times3}, C_{3\times3}$ (d) $A_{3\times2}, B_{2\times3}, C_{3\times3}$

12. If M = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - \lambda M - I_2 = 0$, then λ =

(a) – 2 (b) 2

(c) – 4 (d) 4

13. If A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then A^n =

(a) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} n & n \\ 0 & n \end{bmatrix}$

(c) $\begin{bmatrix} n & 1 \\ 0 & n \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 0 & n \end{bmatrix}$

14. If A = $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 0 & 7 \\ 6 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 2 \end{bmatrix}$, then which of the follwing is defined :

(a) AB (b) A + B

(c) A′ B′ (d) B′ A′

15. If A and B are two matrices such that A + B and AB are both defined, then :

(a) A and B are two matrices not necessary of same order

(b) A and B are square matrices of same order

(c) number of column of A = number of rows of B

(d) none of these

16. If $a_{ij} = \frac{1}{2}(3i - 2j)$ and A = $[a_{ij}]_{2\times2}$, then A is :

(a) $\begin{bmatrix} \frac{1}{2} & 2 \\ \frac{-1}{2} & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 \\ \frac{1}{2} & \frac{-1}{2} \end{bmatrix}$ (d) none of these

17. If A and B are square matrices of the same order, then (A + B) (A – B) is equal to:

(a) $A^2 - B^2$

(b) $A^2 - BA - AB - B^2$

(c) $A^2 - B^2 + BA - AB$

(d) $A^2 - BA + B^2 + AB$

18. **If A = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$, then**
   (a) only AB is defined
   (b) only BA is defined
   (c) AB and BA both are defined
   (d) AB and BA both are not defined

19. **The matrix A = $\begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ is a:**
   (a) scalar matrix (b) diagonal matrix
   (c) unit matrix (d) square matrix

20. **If A and B are symmetric matrices of the same order, then (AB′ – BA′) is a**
   (a) Skew symmetric matrix
   (b) Null matrix
   (c) Symmetric matrix
   (d) None of these

21. **If A = [2 – 3 4], B = $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, X = [1 2 3] and Y = $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, then AB + XY equals : [CBSE OD, Set-1, 2020]**
   (a) [28] (b) [24]
   (c) 28 (d) 24

22. **If $[x-1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$ = 0, then x equals: [CBSE Delhi Set-2, 2020]**
   (a) 0 (b) – 2
   (c) – 1 (d) 2

23. **Which of the following is a I_2 matrix?**
   (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
   (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

24. **Two matrices A and B are multiplied to get AB if,**
   (a) both are rectangular
   (b) both have same order
   (c) no. of columns of A is equal to no. of rows of B
   (d) no. of rows of A is equal to no. of columns of B

25. **If A is a matrix of order 3 × 4, then each row of A has:**
   (a) 3 elements (b) 12 elements
   (c) 7 elements (d) 4 elements

26. **The matrix A = $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a:**
   (a) Scalar matrix
   (b) Diagonal matrix
   (c) Unit matrix
   (d) Square matrix

27. **If A is a square matrix of order 3, such that A (adj A) = 10 I, then |adj A| is equal to:**
   (a) 1 (b) 10
   (c) 100 (d) 101

28. **The number of all possible matrices of order 2 × 3 with each entry 0 or 1 is:**
   (a) 64 (b) 12
   (c) 36 (d) None of these

29. **The matrix A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a :**
   (a) Identity matrix
   (b) Scalar matrix
   (c) Skew-symmetric matrix
   (d) Diagonal matrix

30. **For any 2 × 2 matrix, if A(adj. A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| is equal to :**
   (a) 20 (b) 100
   (c) 10 (d) 0

31. **If A is a 3 × 2 matrix, B is a 3 × 3 matrix and C is a 2 × 3 matrix, then the elements in A, B and C are respectively:**
   (a) 6, 9, 8 (b) 6, 9, 6
   (c) 9, 6, 6 (d) 6, 6, 9

32. **If a matrix has 8 elements, then which of the following will not be a possible order of the matrix?**
   (a) 1 × 8 (b) 2 × 4
   (c) 4 × 2 (d) 4 × 4

33. **Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is: [NCERT Exemplar]**
   (a) 9 (b) 27
   (c) 81 (d) 512

34. **The matrix P = $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is not a: [NCERT Exemplar]**
   (a) square matrix (b) diagonal matrix
   (c) unit matrix (d) None of these

35. **Which of the given values of x and y make the following pair of matrices equal:**
$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$
[NCERT]
   (a) $x = \frac{-1}{3}, y = 7$ (b) not possible to find
   (c) $y = 7, x = \frac{-2}{3}$ (d) $x = \frac{-1}{3}, y = \frac{-2}{3}$

36. If $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3 & 2\\4 & 3 & 1\end{bmatrix}, C=\begin{bmatrix}1\\2\end{bmatrix}$ and $D=\begin{bmatrix}4 & 6 & 8\\5 & 7 & 9\end{bmatrix}$, then which of the following is defined? **[NCERT Exemplar]**
(a) A + B (b) B + C
(c) C + D (d) B + D

37. If $\begin{bmatrix}1 & 2\\-2 & -b\end{bmatrix}+\begin{bmatrix}a & 4\\3 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 0\end{bmatrix}$, then a^2+b^2 is equal to:
(a) 20 (b) 22
(c) 12 (d) 10

38. If $A=\begin{bmatrix}0 & 2\\3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\2b & 24\end{bmatrix}$ then the values of k, a, b are respectively:
(a) – 6, – 12, – 18 (b) – 6, 4, 9
(c) – 6, – 4, – 9 (d) – 6, 12, 18

39. The product $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$ is equal to:
(a) $\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$ (b) $\begin{bmatrix}(a+b)^2 & 0\\(a+b)^2 & 0\end{bmatrix}$
(c) $\begin{bmatrix}a^2+b^2 & 0\\a^2+b^2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}a & 0\\0 & b\end{bmatrix}$

40. If the product of two matrices is a zero matrix, then :
(a) atleast one of the matrix is a zero matrix
(b) both the matrices are zero matrices
(c) it is not necessary that one of the matrices is a zero matrix
(d) None of these

41. If $A=\begin{bmatrix}2 & -1 & 3\\-4 & 5 & 1\end{bmatrix}_{2\times3}$ and $B=\begin{bmatrix}2 & 3\\4 & -2\\1 & 5\end{bmatrix}_{3\times7}$, then :
(a) only AB is defined
(b) only BA is defined
(c) AB and BA both are defined
(d) AB and BA both are not defined

42. The set of all 2 × 2 matrices which is commutative with the matrix $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ with respect to matrix multiplication is:
(a) $\begin{bmatrix}p & q\\r & r\end{bmatrix}$ (b) $\begin{bmatrix}p & q\\q & r\end{bmatrix}$
(c) $\begin{bmatrix}p-q & q\\q & r\end{bmatrix}$ (d) $\begin{bmatrix}p & q\\q & p-q\end{bmatrix}$

43. If A is matrix of order m × n and B is matrix such that AB′ and B′ A are both defined, then order of matrix B is: **[NCERT Exemplar]**
(a) $m \times n$ (b) $n \times n$
(c) $n \times m$ (d) $m \times n$

44. If A and B are square matrices of the same order and AB = 3I, then A^{-1} is equal to:
(a) 3B (b) $\frac{1}{3}B$
(c) $3B^{-1}$ (d) $\frac{1}{3}B^{-1}$

45. If $A=\frac{1}{\pi}\begin{bmatrix}\sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x)\end{bmatrix}$,

$B=\frac{1}{\pi}\begin{bmatrix}-\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x)\end{bmatrix}$, then A – B is equal to: **[NCERT Exemplar]**
(a) I (b) O
(c) 2I (d) $\frac{1}{2}I$

46. If A and B are two matrices of the order 3 × m and 3 × n, respectively, and $m = n$, then the order of matrix (5A – 2B) is: **[NCERT Exemplar]**
(a) $m \times 3$ (b) 3×3
(c) $m \times n$ (d) $3 \times n$

47. If $A=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, then A^2 is equal to: **[NCERT Exemplar]**
(a) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$
(c) $\begin{bmatrix}0 & 1\\0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

48. If matrix $A=[a_{ij}]_{2\times2}$, where $a_{ij}=\begin{cases}1 \text{ if } i\neq j\\0 \text{ if } i=j\end{cases}$ then A^2 is equal to: **[NCERT Exemplar]**
(a) I (b) A
(c) 0 (d) None of these

49. The matrix $\begin{bmatrix}1 & 2 & 4\\2 & 5 & 6\\4 & 6 & 7\end{bmatrix}$ is a **[NCERT Exemplar]**
(a) identity matrix
(b) symmetric matrix
(c) skew symmetric matrix
(d) none of these

50. $A=[a_{ij}]_{m\times n}$ is a square matrix, if
(a) $m < n$ (b) $m > n$
(c) $m = n$ (d) None of these

51. **If A = $\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, then A^{16} is .... matrix.**

(a) unit matrix (b) diagonal matrix
(c) null matrix (d) none of these

52. **If matrix A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 - kA = 0$, then the value of k is ....**

(a) 1 (b) 2
(c) – 2 (d) none of these

53. **If x and y two matrices such that $x - y = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $x + y = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, then y matrix is ....**

(a) $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix}$ (d) none of these

54. **$A^2 - A + I = 0$, then the inverse of A is ....**

(a) A (b) A + I
(c) I – A (d) A – I

55. **The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is ....**

(a) diagonal matrix (b) symmetric matrix
(c) skew symmetre (d) scalar matrix

56. **If the transpose of a square matrix give the same matrix is known as .... matrix.**

(a) Skew symmetric
(b) Symmetric matrix
(c) Scalar matrix
(d) none of these

57. **In .... the number of rows and columns are not necessarily equal.**

(a) diagonal matrix
(b) scalar matrix
(c) rectangular matrix
(d) none of these

58. **A diagonal matrix is said to be .... if its diagonal elements are equal (other than unity).**

(a) Scalar matrix
(b) Skew symmetric matrix
(c) Column matrix
(d) none of these

59. **For any matrix A, A – A′ is .... matrix.**

(a) Skew symmetric matrix
(b) Symmetric matrix
(c) Diagonal matrix
(d) none of these

60. **If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = (1 + x)(1 - x)$, then f(A) is .....**

(a) $-4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $-8\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) none of these

61. **If A = $\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ is the sum of a symmetric matrix B and skew symmetric matrix C, then B is ....**

(a) B = $\begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$ (b) B = $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 5 & -2 \\ 2 & 2 & 0 \end{bmatrix}$

(c) B = $\begin{bmatrix} 0 & 6 & 7 \\ -6 & 2 & -5 \\ -7 & 5 & 1 \end{bmatrix}$ (d) B = $\begin{bmatrix} 0 & 6 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 0 \end{bmatrix}$

62. **If A is any square matrix, then which of the following is not symmetric ?**

(a) $A - A^T$ (b) $A + A^T$
(c) AA^T (d) A^TA

63. **The system of linear equations $x + y + z = 2$, $2x + y - z = 3$ and $3x + 2y - kz = 4$ has a unique solution if ....**

(a) $k = 0$ (b) $-1 < k < 1$
(c) $-2 < x < 2$ (d) $k \neq 0$

64. **If matrix A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is ....**

(a) 2 (b) – 2
(c) 1 (d) 3

65. **If A is a matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then the order of matrix B is ....**

(a) $m \times m$ (b) $n \times n$
(c) $n \times m$ (d) $m \times n$

Assertion and Reason based questions

Choose the correct option :

(a) Both (A) and (R) are true and R is the correct explanation A.
(b) Both (A) and (R) are true but R is not correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

66. **Assertion (A) :** If A = $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then adj (adj A) = A.

Reason (R) : $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$, A be n rowed non-singular matrix

67. **Assertion (A) :** If $A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$, then

$$A^{-1} = \begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix}$$

Reason (R) : The inverse of a diagonal matrix is a diagonal matrix.

68. **Assertion (A) :** The rank of a unit matrix of order $n \times n$ is n.

Reason (R) : The rank of a non-singular matrix of order $n \times n$ is not n.

69. **Assertion (A) :** The matrix $\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{bmatrix}$ is a diagonal matrix.

Reason (R) : $A = [a_{ij}]$ is a square matrix such that $a_{ij} = 0 \ \forall \ i \neq j$, then A is called diagonal matrix.

70. **Assertion (A) :** The inverse of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{pmatrix}$ does not exist.

Reason (R) : The matrix A is singular.

71. **Assertion (A) :** If A is a matrix of order $n \times n$, then det $(kA) = k^n$ det (A) or $|kA| = k^n |A|$.

Reason (R) : If B is a matrix obtained from A by multiplying any row or column by a scalar k, then det B = k det A or $|B| = k|A|$.

72. **Assertion (A) :** The matrix

$$A = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{pmatrix}$$

is an orthogonal matrix.

Reason (R) : If A and B are orthogonal, then AB is also orthogonal.

73. **Assertion (A) :** If A is a skew-symmetric matrix of order 3×3, then det (A) = 0 or $|A| = 0$.

Reason (R) : If A is a square matrix, then det (A) = det (A′) = det (– A′).

74. **Assertion (A) :** The inverse of $A = \begin{pmatrix} 3 & 4 \\ 3 & 5 \end{pmatrix}$ does not exist.

Reason (R) : The matrix A is non-singular.

75. **Assertion (A) :** If a matrix of order 2×2, commutes with every matrix of order 2×2, then it is scalar matrix.

Reason (R) : A scalar matrix of order 2×2 commutes with every 2×2 matrix.

76. **Assertion (A) :** The determinant of a matrix $A = [a_{ij}]_{5\times5}$ where $a_{ij} + a_{ij} = 0$ for all i and j is zero.

Reason (R) : The determinant of a skew symmetric matrix of odd order is zero.

Competency based questions

77. **A firm produces three products P_1, P_2 and P_3 requiring the mix-up of three materials M_1, M_2 and M_3 per unit requirement of each product for each material (in units) is represented by matrix A as follows :**

$$A = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array}\begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix} \end{array}$$

The per unit cost of material M_1, M_2 and M_3 is ₹ 5, ₹ 10 and ₹ 5 respectively.

Attempt the following questions:

(i) If C represents the matrix showing cost of material, the matrix C will be expressed as :

(a) $C = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$ (b) $C = [5 \ \ 10 \ \ 15]$

(c) $C = \begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix}$ (d) $C = \begin{bmatrix} 15 \\ 10 \\ 20 \end{bmatrix}$

(ii) Expressed matrix B in the order of 3×1 if the firm produces 100 units of each product :

(a) $\begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}$ (b) $[100 \ \ 100 \ \ 100]$

(c) $\begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$ (d) $\begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$

(iii) Find the total requirement of each material for 100 units of each product :

(a) $\begin{bmatrix} 900 \\ 800 \\ 800 \end{bmatrix}$ (b) $\begin{bmatrix} 90 \\ 80 \\ 80 \end{bmatrix}$

(c) $\begin{bmatrix} 190 \\ 200 \\ 80 \end{bmatrix}$ (d) $\begin{bmatrix} 800 \\ 900 \\ 800 \end{bmatrix}$

(iv) Find per unit cost of production of each product as per the unit cost of material given above :

(a) $\begin{bmatrix} 45 \\ 65 \\ 60 \end{bmatrix}$ (b) $\begin{bmatrix} 65 \\ 60 \\ 45 \end{bmatrix}$

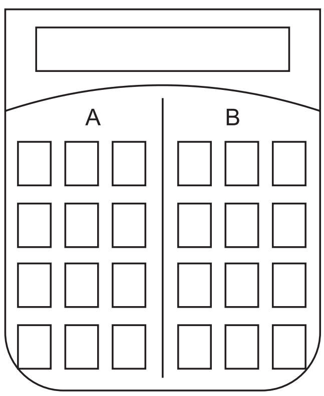
(c) $\begin{bmatrix} 60 \\ 65 \\ 40 \end{bmatrix}$ (d) $\begin{bmatrix} 50 \\ 60 \\ 70 \end{bmatrix}$

(v) Find the total cost of production if the firm produces 200 units of each product :

(a) 28,000 (b) 30,000

(c) 34,000 (d) 32,000

78. A concert is organised to earn the revenue of ₹ 1,80,000 which can be distributed among the needy people during the period of Covid-19. The concert hall has 4,000 seats which are divided into two sections A and B. The cost of a ticket in Section A is ₹ 50 and that of Section B is ₹ 40. All the seats are occupied.



Attempt the following questions :

(i) If x and y are number of seats of Section A and B respectively, then write the equation related to number of seats :

(a) $x - y = 4000$ (b) $x + y = 400$

(c) $x + y = 4000$ (d) $2x + y = 4000$

(ii) What will be relation between x and y for the total revenue ?

(a) $5x + 4y = 18000$ (b) $50x + 40y = 18000$

(c) $5x + 4y = 1800$ (d) $x + y = 180$

(iii) If equations are to be solved through the matrix method in the form of AX = B, then what will be matrix A and matrix B :

(a) $A = \begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix}, B = \begin{bmatrix} 4000 \\ 18000 \end{bmatrix}$

(b) $A = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4000 \\ 18000 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 5 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 18000 \\ 4000 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix}, B = \begin{bmatrix} 4000 \\ 18000 \end{bmatrix}$

(iv) The value of X will be equal to $A^{-1}B$, then A^{-1} :

(a) $\begin{bmatrix} -4 & 1 \\ -5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & 1 \\ 5 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -5 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 5 \\ -1 & -1 \end{bmatrix}$

(v) How many seats are there in Section A and Section B of Concert Hall ?

(a) 2000, 4000 (b) 4000, 2000

(c) 2000, 2000 (d) 6000, 4000

79. A mobile shop purchases three types of mobiles namely LG, I-phone and samsung and sells them in two malls of Gurugram. The annual sales in units are given below :

	LG	**I - phone**	**Samsung**
Mall I	**20**	**15**	**35**
Mall II	**10**	**20**	**30**

If the sale prices of per unit of LG phone is ₹ 20,000, I-phone ₹ 50,000 and Samsung is ₹ 15,000 respectively and costs per unit for LG ₹ 15,000, I-phone ₹ 40,000 and Samsung ₹ 12,000 respectively. From the above case study give the answers of the following questions :

(i) Represent the matrix Q as quantity sold :

(a) $Q = \begin{bmatrix} 20 & 15 & 35 \\ 10 & 20 & 30 \end{bmatrix}$

(b) $Q = \begin{bmatrix} 200 & 150 & 350 \\ 100 & 200 & 300 \end{bmatrix}$

(c) $Q = \begin{bmatrix} 20 & 10 \\ 15 & 20 \\ 35 & 30 \end{bmatrix}$

(d) $Q = \begin{bmatrix} 200 & 100 \\ 150 & 200 \\ 350 & 300 \end{bmatrix}$

(ii) Express the matrix S as total sales price of the item:

(a) $S = \begin{bmatrix} 20,000 \\ 50,000 \\ 15,000 \end{bmatrix}$

(b) S = [20,000 50,000 15,000]

(c) $S = \begin{bmatrix} 50,000 \\ 20,000 \\ 15,000 \end{bmatrix}$

(d) S = [2,000 5,000 1,500]

(iii) Express the matrix C as the total cost price of each item :

(a) $C = [15{,}000 \;\; 40{,}000 \;\; 12{,}000]$

(b) $C = \begin{bmatrix} 15{,}000 \\ 40{,}000 \\ 12{,}000 \end{bmatrix}$

(c) $C = [1{,}500 \;\; 4{,}000 \;\; 1{,}200]$

(d) $C = \begin{bmatrix} 12{,}000 \\ 15{,}000 \\ 40{,}000 \end{bmatrix}$

(iv) Express the matrix P as profit earned on each item :

(a) $P = [3{,}000 \;\; 10{,}000 \;\; 5{,}000]$

(b) $P = [5{,}000 \;\; 10{,}000 \;\; 3{,}000]$

(c) $P = \begin{bmatrix} 5{,}000 \\ 10{,}000 \\ 3{,}000 \end{bmatrix}$

(d) $P = \begin{bmatrix} 3{,}000 \\ 5{,}000 \\ 10{,}000 \end{bmatrix}$

(v) What is the total amount of profit in Mall I and Mall II if all goods are sold ?

(a) $\begin{bmatrix} 35{,}500 \\ 34{,}000 \end{bmatrix}$ (b) $\begin{bmatrix} 3{,}40{,}000 \\ 3{,}55{,}000 \end{bmatrix}$

(c) $[34{,}000 \;\; 35{,}500]$ (d) $\begin{bmatrix} 3{,}55{,}000 \\ 3{,}40{,}000 \end{bmatrix}$

80. Two trust A and B receive ₹ 70,000 and ₹ 55,000 respectively from central government to award prize to persons of a district in three fields defence, health services and education. Trust A awarded 10, 5 and 15 persons in the field of defence, health services and education respectively, while B awarded 15, 10 and 5 persons respectively. All three prizes amount of ₹ 6000.

Solve the questions on the above information :

(i) If x, y, z represents the amount of individual prize then write the linear equations :

(a)
$$\begin{aligned} x + y + z &= 6{,}000 \\ 10x + 5y + 15z &= 70{,}000 \\ 15x + 10y + 5z &= 55{,}000 \end{aligned}$$

(b)
$$\begin{aligned} x + 10y + 15z &= 6{,}000 \\ x + 5y + 10z &= 70{,}000 \\ x + 15y + 5z &= 55{,}000 \end{aligned}$$

(c)
$$\begin{aligned} x - y + z &= 6{,}000 \\ 5y + 15z &= 70{,}000 \\ 15x + 5z &= 55{,}000 \end{aligned}$$

(d)
$$\begin{aligned} x - y - z &= 6{,}000 \\ x - 5y + 10z &= 70{,}000 \\ 15x + 10y + 5z &= 55{,}000 \end{aligned}$$

(ii) If all the equations are written in the form of AX = B then find |A| :

(a) 100 (b) 175

(c) 75 (d) 200

(iii) Find adj A :

(a) $\begin{bmatrix} -125 & 5 & 10 \\ 175 & -10 & -5 \\ 25 & 5 & -5 \end{bmatrix}$

(b) $\begin{bmatrix} -125 & 175 & 25 \\ 5 & -10 & 5 \\ 10 & -5 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 175 & -10 & -5 \\ 25 & 5 & -5 \\ -125 & 5 & 10 \end{bmatrix}$

(d) $\begin{bmatrix} -125 & 5 & 10 \\ 175 & -10 & -5 \\ 25 & 5 & 5 \end{bmatrix}$

(iv) Find A^{-1} :

(a) $\frac{1}{75}\begin{bmatrix} -125 & 5 & 10 \\ 175 & -10 & -5 \\ 25 & 5 & -5 \end{bmatrix}$

(b) $\frac{1}{75}\begin{bmatrix} -125 & 175 & 25 \\ 5 & -10 & 5 \\ 10 & -5 & -5 \end{bmatrix}$

(c) $\frac{1}{75}\begin{bmatrix} 175 & -10 & -5 \\ 25 & 5 & -5 \\ -125 & 5 & 10 \end{bmatrix}$

(d) $\frac{1}{75}\begin{bmatrix} -125 & 5 & 10 \\ 175 & -10 & -5 \\ 25 & 5 & 5 \end{bmatrix}$

(v) Find the value of each prize :

(a) ₹ 2000, ₹ 3000, ₹ 1000

(b) ₹ 2000, ₹ 1000, ₹ 3000

(c) ₹ 1000, ₹ 2000, ₹ 3000

(d) ₹ 3000, ₹ 2000, ₹ 1000

81. Three shopkeepers Ram Lal, Shyam Lal and Ghansham are using polythene bags handmade bags (prepared by prisoners), and newspaper's envelope as carry bags. It is found that the shopkeepers Ram Lal Shyam Lal and Ghansham are using (20, 30, 40), (30, 40, 20) and (40, 20, 30) polythene bags handmade bags and newspapers envelopes respectively. The shopkeepers Ram Lal, Shyam Lal and Ghansham spent ₹ 250, ₹ 270 and ₹ 200 on these carry bags respectively.

(i) What is the cost of one polythene bag?

(a) ₹ 1 (b) ₹ 2

(c) ₹ 3 (d) ₹ 5

(ii) What is the cost of one handmade bag?

(a) ₹ 1 (b) ₹ 2

(c) ₹ 3 (d) ₹ 5

(iii) What is the cost of one newspaper bag?

(a) ₹ 1 (b) ₹ 2

(c) ₹ 3 (d) ₹ 5

(iv) Keeping in mind the social conditions, which shopkeeper is better?

(a) Ram Lal (b) Shyam Lal

(c) Ghansham (d) None of these

(v) Keeping in mind the environmental conditions, which shopkeeper is better?

(a) Ram Lal (b) Shyam Lal

(c) Ghansham (d) None of these

Solutions

1. (a) $\because$ Matrix A is singular

$$\therefore \quad |A| = 0$$
$$\Rightarrow 5[-4+6] - x(8+21) - 1(8+14) = 0$$
$$\Rightarrow 10 - 29x - 22 = 0$$
$$\Rightarrow -12 - 29x = 0$$
$$\Rightarrow -29x = 12$$
$$\Rightarrow x = \frac{-12}{29}$$

2. (a) For symmetric matrix

$$A' = A$$
$$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$$
$$x = y$$

3. (c) For skew symmetric matrix

$$A' = -A$$
$$B' = -B$$
$$(A+B)' = A' + B'$$
$$= -A - B$$
$$= -(A+B)$$

4. (a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (given)

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

5. (b) $A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$$A + A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$
$$A + A' = \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix}$$
$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix}$$
$$\therefore \quad 2\cos\alpha = 1$$
$$\Rightarrow \quad \cos\alpha = \frac{1}{2}$$
$$\Rightarrow \quad \alpha = \frac{\pi}{3}$$

6. (a) $(I + A)^2 - 3A$

$$= I + A^2 + 2A - 3A$$
$$= I + A + 2A - 3A$$
$$= I$$

7. (b) $AB = BA = I$

8. (b) $3a_{22} - 4a_{33} = 3 \times 5 - 4 \times 4$

$$= 15 - 16$$
$$= -1$$

9. (b) $(AA^T)^T = (A^T)^T.(A)^T$

$$= AA^T$$

$\Rightarrow AA^T$ is a symmetric matrix

10. (a) Every identity is a matrix is a scalar matrix.

11. (d) $A_{3\times 2}$, $B_{2\times 3}$, $C_{3\times 3}$

12. (d) $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$$M^2 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$$

Given, $M^2 - \lambda M - I_2 = 0$

$$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} \lambda & 2\lambda \\ 2\lambda & 3\lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$
$$\begin{bmatrix} 4 & 8 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} \lambda & 2\lambda \\ 3\lambda & 3\lambda \end{bmatrix}$$
$$\lambda = 4$$

13. (a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$A^2 = A \times A$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$
$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\vdots \quad \vdots \quad \vdots$$

$$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

14. (c) Order of A′ $= 3 \times 3$
Order of B′ $= 3 \times 2$
So, A′ B′ is possible.

15. (b) A and B are the square a matrices of same order.

16. (b)

$$a_{ij} = \frac{1}{2}(3i - 2j)$$

$$a_{11} = \frac{1}{2}(3-2) = \frac{1}{2}$$

$$a_{12} = \frac{1}{2}(3-4) = -\frac{1}{2}$$

$$a_{21} = \frac{1}{2}(6-2) = 2$$

$$a_{22} = \frac{1}{2}(6-4) = 1$$

So, $$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

17. (c) $A^2 - B^2 + BA - AB$
$(A + B)(A - B) = A(A - B) + B(A - B)$
$= A^2 - AB + BA - B^2$.

18. (c) AB and BA both are defined
Let $A = [a_{ij}]_{2\times3}$ and $B = [b_{ij}]_{3\times2}$
Both AB and BA are defined.
So the correct option is (c).

19. (d) square matrix
As number of rows and columns are equal.

20. (a) Skew symmetric matrix
Since $(AB' - BA') = (AB')' - (BA')'$
$= (BA' - AB')$
$= -(AB' - BA')$.

21. (a) [28]

$$AB + XY = [2\ -3\ 4]\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} + [1\ 2\ 3]\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= [8] + [20] = [28]$$

So the correct option is (a).

22. (b) – 2
Given :

$$[x - 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$$

$$\Rightarrow \quad [x + 2 \quad 0] = [0 \quad 0]$$

$$\Rightarrow \quad x + 2 = 0$$

$$\Rightarrow \quad x = -2$$

So the correct option is (b).

23. (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

24. (c) no. of columns of A is equal to no. of rows of B.

25. (d) 4 elements
Since, 3×4 signifies that there are 3 rows and 4 columns. Then each row has 4 elements.

26. (a) Scalar matrix
All elements except diagonal are zero.

27. (c) 100
Given: $A(\text{adj } A) = 10\,I$
We know, $A.\text{adj } A = |A|.I$
$\therefore \quad |A| = 10$
$\because \quad |\text{adj } A| = |A|^{n-1}$
$\therefore \quad |\text{adj } A| = |A|^{3-1} = 10^2 = 100$

28. (a) 64
In 2×3 matrix, number of elements are 6.
Each place could have 2 elements
$\therefore$ Total possible matrices $= 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 8 \times 8 = 64$

29. (d) Diagonal matrix

30. (c) 10

$$\because \quad A(\text{adj. } A) = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 10I = 10\,AA^{-1}$$

$$\therefore \quad (\text{adj. } A) = 10\,A^{-1}$$

$$\Rightarrow \quad \text{adj. } A = \frac{10 \text{ adj. } A}{|A|}$$

$$\Rightarrow \quad |A| = 10$$

31. (b) 6, 9, 6
The number of elements in $m \times n$ matrix is equal to mn.

32. (d) 4×4
We know that if a matrix is of order $m \times n$, then it has mn elements. Thus, to find all possible orders of a matrix with 8 elements, we will find all ordered pairs of natural numbers, whose product is 8. Thus, all possible ordered pair are (1, 8), (2, 4), (4, 2).

33. (d) 512
Number of entries in 3×3 matrix is 9. Since, each entry has 2 choices, namely 2 or 0. Therefore, number of possible matrices $= \underbrace{2 \times 2 \times 2 ...... \times 2}_{9 \text{ times}} = 2^9 = 512$

34. (c) unit matrix

If square matrix in which all diagonals elements are 1 and rest are 0, is called unit matrix.

35. (b) not possible to find

Consider, $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

$\Rightarrow 3x + 7 = 0; 5 = y - 2; y + 1 = 8; 2 - 3x = 4$

$\Rightarrow x = \frac{-7}{3}, y = 7; y = 7; x = \frac{-2}{3}$

36. (d) B + D

Only B + D is defined because matrices of the same order can only be added.

37. (a) 20

We have,

$$\begin{bmatrix} 1 & 2 \\ -2 & -b \end{bmatrix} + \begin{bmatrix} a & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+1 & 6 \\ 1 & 2-b \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$$

$\Rightarrow a + 1 = 5, 2 - b = 0$

$\Rightarrow a = 4, b = 2$

$\Rightarrow a^2 + b^2 = 20.$

38. (c) – 6, – 4, – 9

$$kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\Rightarrow k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$-4k = 24$

$k = -6$

$\Rightarrow 3a = 2 \times -6$

$\Rightarrow 3a = -12$

$\Rightarrow a = -4$

and $3k = 2b$

$3 \times -6 = 2b$

$b = -9.$

39. (a) $\begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & -ab+ba \\ -ba+ab & b^2+a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

40. (c) it is not necessary that one of the matrices is a zero matrix

Let $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$. Then,

AB = O but A and B are non-zero.

41. (c) AB and BA both are defined

Let $A = [a_{ij}]_{2 \times 3}$ and $B = [b_{ij}]_{3 \times 2}$.

Since, number of columns of A = number of rows of B

$\therefore$ AB is defined

Also, as number of columns of B = number of rows of A.

$\therefore$ BA is defined.

Hence, both AB and BA are defined.

42. (d) $\begin{bmatrix} p & q \\ q & p-q \end{bmatrix}$

Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ be a matrix which commute with matrix $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

Then, AB = BA

$$\Rightarrow \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p+q & p \\ r+s & r \end{bmatrix} = \begin{bmatrix} p+r & q+s \\ p & q \end{bmatrix}$$

Here, both matrices are equal, so we equate the corresponding elements.

$\therefore p + q = p + r, p = q + s, r + s = p$ and $r = q$

$\Rightarrow r = q$ and $s = p - q$

$$\therefore \quad A = \begin{bmatrix} p & q \\ q & p-q \end{bmatrix}$$

Hence, the required set is $\begin{bmatrix} p & q \\ q & p-q \end{bmatrix}$.

43. (d) $m \times n$

Let $A = [a_{ij}]_{m \times n}$

and $B = [b_{ij}]_{p \times q}$

$\therefore B' = [b_{ji}]_{q \times p}$

Now, AB′ is defined, so $n = q$

and BA is also defined, so $p = m$

$\therefore$ Order of B is $m \times n$.

44. (b) $\frac{1}{3}B$

$AB = 3I$

$\Rightarrow \frac{1}{3}(AB) = I$

$\Rightarrow A\left(\frac{1}{3}B\right) = I$

$\Rightarrow A^{-1} = \frac{1}{3}B$

45. (d) $\frac{1}{2}I$

We have,

$$B = \begin{bmatrix} -\frac{1}{\pi}\cos^{-1} x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi} \\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & -\frac{1}{\pi}\tan^{-1}\pi x \end{bmatrix} \text{ and}$$

$$A = \begin{bmatrix} \frac{1}{\pi}\sin^{-1} x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi} \\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & \frac{1}{\pi}\cot^{-1}\pi x \end{bmatrix}$$

$\therefore$ A – B

$$= \begin{bmatrix} \frac{1}{\pi}(\sin^{-1} x\pi + \cos^{-1} x\pi) & 0 \\ 0 & \frac{1}{\pi}(\cot^{-1}\pi x + \tan^{-1}\pi x) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\pi}.\frac{\pi}{2} & 0 \\ 0 & \frac{1}{\pi}.\frac{\pi}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2}I$$

46. (d) $3 \times n$

$A_{3 \times m}$ and $B_{3 \times n}$ are two matrices. If $m = n$, then A and B have same orders as $3 \times n$ each, so the order of (5A – 2B) should be same as $3 \times n$.

47. (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^2 = A.\,A$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

48. (a) I

We have, $A = [a_{ij}]_{2 \times 2}$,

where $a_{ij} = \begin{cases} 1, & i \neq j \\ 0, & i = j \end{cases}$

$\Rightarrow$ $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

and $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

49. (b) symmetric matrix

We have

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 6 \\ 4 & 6 & 7 \end{bmatrix}$$

$\Rightarrow$ $A' = A$

So, the given matrix is a scalar matrix.

50. (c) $m = n$

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

51. (b) Both (A) and (R) are true but R is not correct explanation of A.

adj (adj A) = $|A|^{n-2}$ A

Here, $n = 3$

$\therefore$ (adj) (adj A) = | A | A ...(i)

Now, $|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$= 3(-3+4) + 3(2) + 4(-2)$

$= 1$

From equation (i),

adj (adj) A = A

52. (b) $A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

Given, $A^2 = kA$

$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$

$k = 2$

53. (c) $x - y = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$

$x + y = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

On subtracting, we get

$-2y = \begin{bmatrix} 2 & 4 \\ -4 & -4 \end{bmatrix}$

$y = \begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix}$

54. (c) $A^2 - A + I = 0$

On multiplying by A^{-1}

$A^2 A^{-1} - A.A^{-1} + IA^{-1} = 0$

$A - I + A^{-1} = 0$

$A^{-1} = I - A$

55. (c) $A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & -12 & 0 \end{bmatrix}$

$= -\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & 12 & 0 \end{bmatrix}$

$A' = -A$

$\therefore$ A is skew symmetric matrix.

56. (b) Symmetrix matrix

57. (c) rectangular matrix

58. (a) Scalar matrix

59. (a) skew symmetric matrix

$$(A - A')' = A' - (A')' = A' - A = -(A - A')$$

60. (a)

$$f(x) = 1 - x^2$$

$$f(A) = I - A^2$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} = -4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

61. (a)

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

Here, $\frac{1}{2}(A + A^1)$ is the symmetric matrix say B

$$B = \frac{1}{2}(A + A')$$

$$A' = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 12 & 14 \\ 12 & 4 & 10 \\ 14 & 10 & 2 \end{bmatrix}$$

$$\frac{1}{2}(A + A') = B = \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$$

62. (a)

$$B = A - A^T$$

$$B' = (A - A^T)' = A^T - (A^T)^T = A^T - A = -(A - A^T) \neq B$$

So, B is not a symmetric matrix

63. (d) for unique solution

$$(A) \neq 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & -k \end{vmatrix} \neq 0$$

$$1(-k + 2) - 1(-2k + 3) + 1(4 - 3) \neq 0$$

$$2 - k + 2k - 3 + 1 \neq 0$$

$$k \neq 0$$

64. (a)

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^2 = 2A$$

Hence, $k = 2$

65. (d) $m \times n$

66. (b) $\text{adj}\,(\text{adj}\,A) = |A|^{n-2} A$

Here, $n = 3$

$\therefore$ (adj) (adj A) = | A | A ...(i)

Now, $|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$$= 3(-3 + 4) + 3(2) + 4(-2)$$

$$= 1$$

From equation (i),

adj (adj) A = A

67. (b)

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

$\therefore$ $|A| = abc$

and $\text{adj}\,A = \begin{pmatrix} bc & 0 & 0 \\ 0 & ca & 0 \\ 0 & 0 & ab \end{pmatrix}$

$\therefore$ $A^{-1} = \dfrac{\text{adj}\,A}{|A|} = \begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix}$

68. (c) In a unit matrix of order $n \times n$,

Number of non-zero rows = n.

$\therefore$ Rank is n.

The rank of non-singular matrix of order $n \times n$ is n.

$\because$ For non-singular, $|A| \neq 0$.

69. (d) $A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{bmatrix}$ is not a diagonal matrix.

Since, A is not a square matrix.

70. (a) $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{vmatrix} = 0$

$1(14-12) - 1(7-3) + 1(4-2)\ 2 - 4 + 2 = 0$

[$\because$ A is singular]

$\therefore A^{-1}$ does not exist.

71. (a) If $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}$

$\therefore \quad kA = \begin{pmatrix} k_{11} & k_{12} & k_{13} & \ldots & k_{1n} \\ k_{21} & k_{22} & k_{23} & \ldots & k_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ ka_{m1} & ka_{m2} & ka_{m3} & \ldots & ka_{mn} \end{pmatrix}$

$\therefore \quad |kA| = k^n |A|$

72. (b) $AA' = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{pmatrix} \cdot \frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}$

$= \frac{1}{9}\begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$

$\therefore A$ is orthogonal.

Also, if A and B are orthogonal, then AB is orthogonal.

73. (c) $A = \begin{pmatrix} 0 & -c & b \\ c & 0 & a \\ -b & -a & 0 \end{pmatrix}$

$A = -A'$ [$\because A$ is skew-symmetric]

$\therefore \quad \det(A) = \det(-A')$

$= -\det(A')$

$= -\det A$

$\therefore \quad \det A = 0$

$\because \quad \det A' = \det(-A')$ is not true.

$\therefore \quad \det(-A') = (-1)^3 \det(A') = -\det A'$.

74. (d) $\because |A| = \begin{vmatrix} 3 & 4 \\ 3 & 5 \end{vmatrix} = 15 - 12 = 3 \neq 0$

$\therefore$ A is non-singular matrix.

$\therefore A^{-1}$ is exist.

75. (b) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, B = \begin{pmatrix} x & y \\ z & u \end{pmatrix}$

$\Rightarrow \quad AB = BA$

$\Rightarrow \quad \begin{pmatrix} ax+bz & ay+bu \\ cx+dz & cy+du \end{pmatrix} = \begin{pmatrix} ax+cy & bx+dy \\ az+cu & bz+du \end{pmatrix}$

On comparing, then

$ax + bz = ax + cy$

$\Rightarrow \quad bz = cy$

$\Rightarrow \quad \frac{z}{c} = \frac{y}{b} = \lambda$ (say)

$\therefore \quad y = b\lambda, z = c\lambda$ ...(i)

$ay + bu = bx + dy$

$\Rightarrow \quad ab\lambda + bu = bx + bd\lambda$ [From (i)]

$\Rightarrow \quad a\lambda + u = x + d\lambda = k$ (let)

For $\lambda = 0, y = 0, z = 0, u = k, x = k$,

Then, $B = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ = scalar matrix.

Also, if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$,

Then, $AB = BA = \begin{pmatrix} ak & bk \\ ck & dk \end{pmatrix} = kA$.

76. (a) $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ -a_{12} & a_{22} & a_{23} & a_{24} & a_{25} \\ -a_{13} & -a_{23} & a_{33} & a_{34} & a_{35} \\ -a_{14} & -a_{24} & -a_{34} & a_{44} & a_{45} \\ -a_{15} & -a_{25} & -a_{35} & -a_{45} & a_{55} \end{pmatrix}$

$A^T = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ -a_{12} & a_{22} & a_{23} & a_{24} & a_{25} \\ -a_{13} & -a_{23} & a_{33} & a_{34} & a_{35} \\ -a_{14} & -a_{24} & -a_{34} & a_{44} & a_{45} \\ -a_{15} & -a_{25} & -a_{35} & -a_{45} & a_{55} \end{pmatrix} = -A$

The determinant of skew-symmetric matrix of odd order is 0. Hence, both assertion and reason are correct and reason is the correct explanation for assertion.

77. (i) (c) $C = \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array}\begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix}$

(ii) (a) $B = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array}\begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}$

(iii) (d) The total requirement of each material is given by $A'B$.

$A' = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 2 & 4 \\ 1 & 5 & 2 \end{bmatrix}$

$A'B = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 2 & 4 \\ 1 & 5 & 2 \end{bmatrix}\begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 800 \\ 900 \\ 800 \end{bmatrix}$

(iv) (a) The per unit cost production of each product is given by

$$AC = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 45 \\ 65 \\ 60 \end{bmatrix}$$

(v) (c) Total cost of production of 200 units

$$= [200\ \ 200\ \ 200]\begin{bmatrix} 45 \\ 65 \\ 60 \end{bmatrix}$$

$= 9{,}000 + 13{,}000 + 12{,}000$

$=$ ₹ 34,000.

78. (i) (b) Let x and y be the no. of seats in section A and section B respectively.

$\therefore \quad x + y = 4000$

(ii) (a) $50x + 40y = 1{,}80{,}000$

$5x + 4y = 18{,}000$

(iii) (a) Writing these equation in matrix form AX = B

where $\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4000 \\ 18000 \end{bmatrix}$

So, $A = \begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4000 \\ 18000 \end{bmatrix}$

(iv) (b) $X = A^{-1}B$

$$\because \quad A^{-1} = \frac{\text{adj A}}{|A|}$$

$$|A| = \begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = 4 - 5 = -1$$

$$\therefore \quad \text{adj A} = \begin{bmatrix} 4 & -1 \\ -5 & 1 \end{bmatrix}$$

So, $$A^{-1} = \frac{1}{-1}\begin{bmatrix} 4 & -1 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & -1 \end{bmatrix}.$$

(v) **(c)** $$X = -1\begin{bmatrix} 4 & -1 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} 4000 \\ 18000 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 1 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 4000 \\ 18000 \end{bmatrix}$$

$$= \begin{bmatrix} 16000 + 18000 \\ 20000 - 18000 \end{bmatrix} = \begin{bmatrix} 2000 \\ 2000 \end{bmatrix}$$

$\therefore \quad x = 2000$ seats in Sec. A

$y = 2000$ seats in Sec. B.

79. (i) (a) $$Q = \begin{bmatrix} 20 & 15 & 35 \\ 10 & 20 & 30 \end{bmatrix}$$

(ii) (a) $$S = \begin{array}{r} \text{LG} \\ \text{IP} \\ \text{Samsung} \end{array}\begin{bmatrix} 20{,}000 \\ 50{,}000 \\ 15{,}000 \end{bmatrix}$$

(iii) (b) $$C = \text{Total Cost} = \begin{array}{r} \text{LG} \\ \text{IP} \\ \text{Samsung} \end{array}\begin{bmatrix} 15{,}000 \\ 40{,}000 \\ 12{,}000 \end{bmatrix}$$

(iv) (c) P = Profit Matrix = S – C

$$P = \begin{bmatrix} 20{,}000 \\ 50{,}000 \\ 15{,}000 \end{bmatrix} - \begin{bmatrix} 15{,}000 \\ 40{,}000 \\ 12{,}000 \end{bmatrix} = \begin{bmatrix} 5{,}000 \\ 10{,}000 \\ 3{,}000 \end{bmatrix}$$

(v) (d) Total profit in each mall = QP

$$= \begin{bmatrix} 20 & 15 & 35 \\ 10 & 20 & 30 \end{bmatrix}\begin{bmatrix} 5,000 \\ 10,000 \\ 3,000 \end{bmatrix}$$

$$= \begin{bmatrix} 1,00,000 + 1,50,000 + 1,05,000 \\ 50,000 + 2,00,000 + 90,000 \end{bmatrix}$$

$$= \begin{bmatrix} 3,55,000 \\ 3,40,000 \end{bmatrix}.$$

80. (i) (a) x, y, z represents the amount of individual prize awarded then

$$x + y + z = 6000$$
$$10x + 5y + 15z = 70{,}000$$
$$15x + 10y + 5z = 55000$$

(ii) (c) From Ans. (i), we have

$$\begin{bmatrix} 1 & 1 & 1 \\ 10 & 5 & 15 \\ 15 & 10 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 70000 \\ 55000 \end{bmatrix}$$

$$AX = B$$

On comparing, we get

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 5 & 15 \\ 15 & 10 & 5 \end{bmatrix}$$

$$\therefore \quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 5 & 15 \\ 15 & 10 & 5 \end{vmatrix}$$

$= \text{l}(25 - 150) - 1(50 - 225) + (100 - 75)$

$= 75$

(iii) (a) $$\text{adj A} = \begin{bmatrix} -125 & 5 & 10 \\ 175 & -10 & -5 \\ 25 & 5 & -5 \end{bmatrix}$$

(iv) (a) Since $|A| \neq 0$

So, A^{-1} exists.

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{Adj A}$$

$$A^{-1} = \frac{1}{75}\begin{bmatrix} -125 & 5 & 10 \\ 175 & -10 & -5 \\ 25 & 5 & -5 \end{bmatrix}$$

(v) (b) $X = \frac{1}{75}\begin{bmatrix} -125 & 5 & 10 \\ 175 & -10 & -5 \\ 25 & 5 & -5 \end{bmatrix}\begin{bmatrix} 6000 \\ 70,000 \\ 55,000 \end{bmatrix}$

$= \frac{1}{75}\begin{bmatrix} 1,50,000 \\ 75,000 \\ 2,25,000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$

$\therefore$ $x =$ ₹ 2000, $y =$ ₹ 1000, $z =$ ₹ 3000.

81. Let the cost of polythene bag, handmade bag and newspaper bag envelops be x, y and z respectively.

$$20x + 30y + 40z = 250$$

or $$2x + 3y + 4z = 25$$

Similiarly

$$3x + 4y + 2z = 27$$

$$4x + 2y + 32 = 20$$

$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 25 \\ 27 \\ 20 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$|A| = 2(18) - 3\,(1) + 4(-10)$
$= 16 - 3 - 40$
$= -27$

$\text{Adj } A = \begin{bmatrix} 8 & -1 & -10 \\ -1 & -10 & 8 \\ -10 & 8 & -1 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj}A}{|A|}$

$= \frac{-1}{27}\begin{bmatrix} 8 & -1 & -10 \\ -1 & -10 & 8 \\ -10 & 8 & -1 \end{bmatrix}$

$X = A^{-1} B$

$= \frac{-1}{27}\begin{bmatrix} 8 & -1 & -10 \\ -1 & -10 & 8 \\ -10 & 8 & -1 \end{bmatrix}\begin{bmatrix} 25 \\ 27 \\ 20 \end{bmatrix}$

$= \frac{-1}{27}\begin{bmatrix} 200 - 27 - 200 \\ -25 - 270 + 160 \\ 250 + 216 - 20 \end{bmatrix}$

$= \frac{-1}{27}\begin{bmatrix} -27 \\ -135 \\ -54 \end{bmatrix}$

$x = 1$, $y = 5$ and $z = 2$.

(i) (a) 1
(ii) (d) 5
(iii) (b) 2
(iv) (b) Shyam Lal
(v) (a) Ram Lal

Word of Advice

1. Several students made errors while applying properties of inverse trigonometric functions, *converting one inverse trigonometric function into another equivalent inverse trigonometric function* and in simplification.
2. Many students made errors in applying the formula of $\sin^{-1} A + \sin^{-1} B$. Also, errors took place in simplifying and solving higher degree algebraic equations. Some students converted all terms into a particular inverse function form, for example $\tan^{-1}$ and could not handle the resulting equations.
3. A few students not only wrote incorrect formula for $\tan^{-1} x + \tan^{-1} y$ but also made errors while simplifying the expression.
4. Several students made errors while converting $\sec^{-1} x$ and $\text{cosec}^{-1} x$ to its correct form.