CHAPTER 15

Triangulation

15.1 Introduction

- In survey, whether mapping is done for a large area (e.g. a country or a continent) or for a very small area, it depends on the type of framework used which should be totally error free.
- Arriving at the suitable framework for the survey of the required area is called as **horizontal control**.
- Some of the methods to establish horizontal control are:
  (a) Triangulation
  (b) Trilateration
  (c) Electronic Distance Measurement (EDM)
- Because of the reason of high level of accuracy and more economy, EDM is preferred over triangulation as far as establishing the horizontal control is concerned.
- The method of triangulation was first introduced by a Dutch citizen named **Snell**.
- **Suitability of triangulation:** Triangulation is mostly suitable for hilly areas where direct measurement of distances is not possible or where long distance sights are required to be taken. For long sights in triangulation, elevated stations are preferred.
- For plain and crowded areas, triangulation is not preferred because inter-visibility of stations get affected. On the other hand, traversing is suitable for relatively flat areas.
- **Drawbacks of triangulation:** Linear errors get accumulated (i.e. in lengths) and the direction of lines for successive lines since it depends on the computations done for the preceding line which in fact requires a number of checks.

  In order to minimize the accumulation of errors, additional base lines called as check lines (or check bases) are measured particularly in large triangulation networks.
- **Laplace stations:** Triangulation stations at which astronomical observations are also made are called as **Laplace stations**.

15.2 Importance of Triangulation

- The method of triangulation for establishing the horizontal control does not provide complete details and topographical features but is meant simply to locate a number of isolated points/stations on the plane area (i.e. land).

- Once these points have been located, other details including the topographical details are established by other means like chain traverse, plane tabling, total station surveys and so on.

15.3 Principle of Triangulation

- In triangulation, the whole area under consideration is covered with a framework of triangles. It is based on the principle that:

  "If the length and direction of one side and all the three angles of the triangle are measured precisely then the lengths and directions of the remaining two sides can be determined. The precisely measured first line is called as base line."

- Furthermore, the other two sides of the triangle (whose lengths and directions are now known) act as base lines of the other interconnected triangles. This process goes on further which gives rise to a network of triangles throughout the area to be surveyed.

  Check base: When the whole area has been covered with triangles, then at last, as a **check**, the length of one of the sides of the last triangle is also measured directly and compared with the computed value. This side is called as **check base.**

NOTE

This traditional method of triangulation in which the base lines and the check base are measured by invar tapes has now become out of vogue. Now we have more advanced methods like EDM which not only measures the base line but, now we can have more check bases in the network.

Scale error: The difference between measured and computed check bases is called as **scale error.**

15.3.1 Steps followed in Triangulation

Field work

(a) Reconnaissance

(b) Establishment and preparation of triangulation stations

(c) Measurement of base line and check base

(d) Measurement of horizontal angles

Office work

(a) Specifications of stations and signals

(b) Necessary adjustments in observations

(c) Computations of lengths of sides

(d) Computations of bearings, latitudes and departures

(e) Computation of independent co-ordinates

15.4 Triangulation System

- A system consisting of triangulation stations connected by a chain of triangles is called as triangulation system or triangulation figure.
- Most commonly used figures in triangulation system are:

  (a) Triangles (b) Quadrilaterals (c) Polygons

All these triangulation figures must satisfy the required geometrical conditions.

- But it is not possible to satisfy all the geometrical conditions due to errors involved in observations and calculations until the field measurements have been adjusted.

15.4.1 Triangulation Figure: Triangle (Δ)

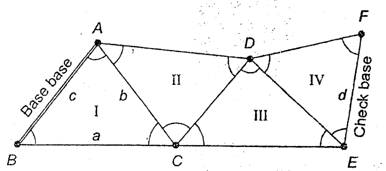
- When a narrow strip of chain is to be surveyed then a chain of triangles is very economical and rapid. e.g. Survey of a railway line or a highway. For **well-conditioned triangles,** the angles must not be less than 30° or greater than 120°.
- **Well-conditioned Triangle:** A triangle is said to be well-conditioned if its shape is such that any error in the measurement of an angle has a minimum effect on the computed lengths.

15.4.2 Triangulation Figure : Quadrilateral (◇)

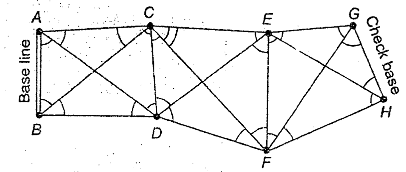
- It is a better system as compared to triangles due to the availability of various combinations of sides and angles that can be used to compute the lengths of required sides and checks can be made frequently.
- Among all the quadrilaterals, the best quadrilateral is the **square.**

15.4.3 Triangulation Figure : Polygons (⬠ ⬡ etc.)

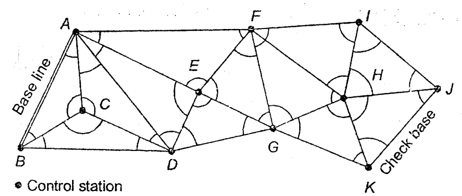
- When a wide area is to be surveyed then a pentagon or a hexagon is more economical.
- However the pace of work reduces due to more number of settings of the instrument required.
- This system provides the desired checks on the calculations.



(a) Chain of simple triangles



(b) Braced quadrilaterals



(c) Polygons with central points

Fig. 15.1 *Triangulation figures*

15.4.4 Choice of Triangulation Figure

- Of all the figures as stated above, the triangle requires the least measurement of sides and angles as compared to quadrilaterals and polygons but the only check available for the chain of triangles is that sum of all the angles of a triangle must be 180°. Thus this gives rise to the necessity of having additional checks as check bases which nullifies the advantage of economy achieved in this system.

- Quadrilaterals are suitable for narrow chain system while polygons are suitable for wide chain system.

Remember The figure should be so selected that there are two independent routes available for calculation purpose with at least one route having the well-conditioned triangles. Also the type of figure so adopted must achieve maximum progress in field with minimum efforts and field work. All sides of the figure must have comparable lengths and should not have more than 12 conditions.

15.4.5 Strength of Figure

- The accuracy of triangulation work depends on the shape of figure adopted for triangulation along with precision involved in taking the observations. This accuracy of shapes is measured in terms of **strength of figure** which decides the desired degree of accuracy.
- In triangulation, usually length of one side of the triangle is known along with included angles and the lengths of other sides are determined using sine law. Now sine of an angle changes rapidly with small angles and thus for sine of very small angle, the corresponding length of side opposite to that angle will vary more.

For example:

$$\sin 15° = 0.258819 \qquad \sin 85° = 0.996195$$
$$\sin 15°30' = 0.267238 \qquad \sin 85°30' = 0.996917$$
$$\text{Change} = 0.008419 = 8.419 \times 10^{-3} \qquad \text{Change} = 0.722 \times 10^{-3}$$

Thus for a small change of angle by 0.5°, the sine of small angle (i.e. 15°) changes much more than sine of larger angle (i.e. 85°).

15.5 System of Framework in Triangulation

(a) **Grid iron system:** Here the primary network consists of two chains of triangles, one approximately in the direction of meridian and other roughly perpendicular to the meridian. The areas enclosed are then surveyed by forming a network of small triangles. This system is used on very extensive surveys and is also adopted in India.

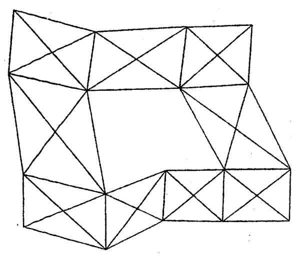


Fig. 15.2 *Grid iron system of triangulation*

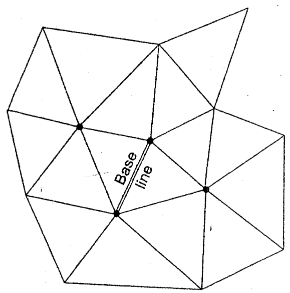


Fig. 15.3 *Central system of triangulation*

(b) **Central system:** Here the area to be surveyed is covered by a chain of triangles extending outwards in all the directions from the initial base line. A suitable system of triangles should be selected to provide an appropriate strength of the figure. It must be possible to carry out the computation by two separate routes, by well-conditioned triangles. The aim is to cover the whole area with minimum number of stations with the required degree of accuracy. This system is adopted in the **United Kingdom.**

15.6 Classification of Triangulation System

(a) **Primary or first order triangulation:** It is the highest order of triangulation and is used for very large survey areas like earth's surface for having the most accurate control in the map and for small scale mapping. It consists of making the combinations of large well-conditioned triangles with the help of very accurate instruments to be used for making the observation with every possible refinement wherever required.

(b) **Secondary or second order triangulation:** In the second order or secondary triangulation, a second series of triangles are run by fixing points at close intervals inside the primary series of triangles. Mainly it consists of forming small well-conditioned triangles with comparatively less precise instruments.

(c) **Tertiary or third order triangulation:** A third order or tertiary triangulation is employed for running a third series of triangles by fixing points inside the secondary triangles at short intervals. Here the triangles are of smallest size as compared to other two orders of triangulation.

15.6.1 Accuracy of Triangulation Networks

Table 15.1 *Accuracy of triangulation networks*

S. No.	Requirement	Triangulation Order		
		1st	2nd	3rd
1.	Length of sides of triangles	30 - 160 km	8 - 70 km	1.5 - 10 km
2.	Length of base line	5 - 20 km	2 - 5 km	0.5 - 3 km
3.	Average triangular closure	1″	3″	6″
4.	Maximum triangular closure	3″	8″	12″
5.	Instrument	Wild/Zeiss theodolite	Wild theodolite	Any theodolite with least count of 5″
6.	Limiting strength of figure	25	40	50
7.	Sets of observations with theodolite	16	8	3

15.7 Signals

- Signals are small devices which are used to define the exact location of a triangulation station so that it can be easily observed from the other stations or places.
- Signals are placed centrally over the station mark because the accuracy of triangulation system depends on the proper centering of signals on triangulation stations.
- Also signals must be truly vertical over the station.

Qualities of a good signal: A signal should have the following qualities:

- It should be conspicuous (i.e. noticeable) against any background. For that the signal should also have sufficient height. Roughly the height of signal (h, cm) and the distance (D, in km) is related as

$$h = 13.3\,D \qquad ...(15.1)$$

In general, the height of signal above the station mark should preferably be more than 75 cm.
- Signal must be capable of being centered accurately over the station mark.
- The signal should be of sufficient size so that it gets bisected easily.
- It must be easy to erect in minimum time.
- The signal should be free from phase errors.
- It should carry a flag on its top for distinguishing purposes.

15.7.1 Classification of Signals

(a) Opaque signals (b) Luminous signals

15.7.2 Opaque Signals

- These signals are used for triangulation system requiring relatively less accuracy and for short sight distances which normally do not exceed 30 km. These signals can be used only during the day time.
- These are cheaper than luminous signals.

The following are some of the opaque signals:

(a) **Pole signal:** It consists of a round pole painted alternatively white and black in the form of strips.

It is supported vertically over the station mark on tripod. Pole signals are quite suitable for sight distances of up to 6 km.

(b) **Target signal:** It consists of a pole carrying two square or rectangular targets placed at right angles to each other. These targets are usually made of cloth stretched on wooden frame. These signals are suitable for distances up to 30 km.

(c) **Beacon:** It consists of three poles arranged to form a tripod on which red and white clothes are tied. It can be easily centered over the station.

(d) **Plumb bob:** Here a plumb bob is hung from a slanting rod and the bob centers over the station.

(e) **Elevated signal:** It is used for plane areas where elevated towers are used as signals and are suitable for distances up to 30 to 40 km.

(f) **Pole and brush signal:** It is made of a straight pole (usually 2.5 m long) provided with a bunch of grass tied symmetrically at its top to form a sort of cross. It is many a times painted in white to make it noticeable.

(g) **Stone signpost:** Here pile of stones are heaped in a conical shape about 3 m in height and a cross shaped signal is erected on the stone heap.

15.7.3 Luminous Signals

These signals are usually used for geodetic survey being due to the reason that they are quite distinct and have clear visibility even for long distances (greater than 30 km). These signals can also be used at night. Luminous signals are of two types viz. sun signals and the night signals.

(a) **Sun signal:** It is also called as heliotrope which reflects the sun rays towards the instrument station. Heliotrope consists of plane mirrors that reflect sun rays in the required direction. A heliotrope can either be telescopic heliotrope or may be defined by a sight vane and an aperture carrying cross hairs.

(b) **Night signal:** As the name implies, this is used for making the observations at night. e.g. Magnesium lamps with parabolic reflectors, Drummond's light, various oil lamps with parabolic reflectors etc.

15.7.4 Phase of a Signal

- While taking observations on a circular opaque signal in sunlight, a part of it is illuminated and remaining part remains in shadow.
- The observer sees the illuminated portion only and thus has the tendency to bisect the illuminated part only.
- Thus the signal is not bisected at the center but at some distance away from it. This gives rise to angular error called as phase error or the **phase of the signal**. Thus phase correction needs to be applied.

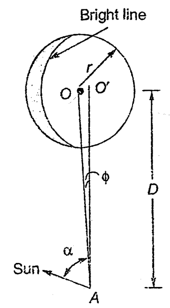


Fig. 15.4 *Phase error in signal*

There are following two cases for the phase of a signal viz.:

(a) When bright portion is bisected

(b) When bright line is sighted

(a) **When bright portion is bisected:** As shown in **Fig. 15.5** the plan of an opaque cylindrical signal at station A which is observed from station B. The sun rays strike at an angle ϕ with the vertical at the center of the signal. The portion PE is illuminated which is visible from station B. Here the point C is the midpoint of the illuminated portion PF. Thus when illuminated portion is bisected then BC is the line of sight.

Let $\angle EBA = \phi_1$

$\angle PBA = \phi_2$

$\angle CBA = \beta$

$$\text{Phase correction} = \phi_1 + \left(\frac{\phi_2 - \phi_1}{2}\right) = \left(\frac{\phi_1 + \phi_2}{2}\right)$$

Distance $AB = D$

Radius of signal $= r$

Thus, in ΔAPB, $$\tan\phi_2 = \frac{r}{D} = \phi_2 \qquad ...(15.2)$$

In ΔBEF, $$\tan\phi_1 = \frac{EF}{D} = \frac{r\cos\phi}{D} = \phi_1 \qquad ...(15.3)$$

where, ϕ = angle between the sun ray and line AB.

Thus, $$\beta = \left(\frac{1}{2}\right)\left(\frac{r}{D} + \frac{r}{D}\cos\phi\right)$$

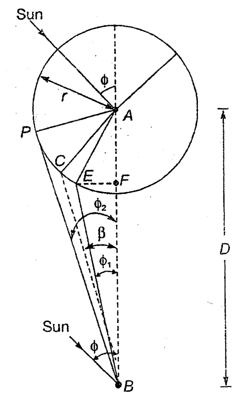


Fig. 15.5 *Phase error when bright portion is bisected*

$$= \left(\frac{1}{2}\right)\left(\frac{r}{D}\right)(1+\cos\phi)$$

$$= \left(\frac{r}{2D}\right)\left[2\cos^2\left(\frac{\phi}{2}\right)\right]$$

$$= \left(\frac{r}{D}\right)\cos^2\left(\frac{\phi}{2}\right) \text{ radians} \qquad \text{....(15.4)}$$

$$= \frac{r\cos^2(\phi/2)}{D\sin 1''} \text{ seconds} \qquad \text{...(15.5)}$$

$$= 206265\left[\frac{r\cos^2(\phi/2)}{D}\right]$$

It is to be noted that 1 rad $= \frac{180}{\pi}$ deg = 206265 seconds.

(b) **When bright line is sighted:** As shown in **Fig. 15.6** line *BP* is the line of sight when the bright line at *P* is sighted from station *B*. Here $\angle PBA$ is the phase correction which is equal to β. Sun rays *SP* and $S_1\beta$ are parallel.

$$\angle BPS = 180° - (\theta - \beta) \qquad (\because SP \parallel S_1B)$$

Line *PF* bisects $\angle BPS$.

Thus,

$$\angle BPF = \frac{\angle BPS}{2}$$

$$= \frac{[180° - (\theta - \beta)]}{2}$$

$$= 90° - \frac{(\theta - \beta)}{2} \qquad \text{...(15.6)}$$

Now *APF* is a straight line and thus,

$$\angle BPA = 180° - \angle BPF$$

$$= 180° - \left[90° - \frac{(\theta-\beta)}{2}\right]$$

$$= 90° + \frac{(\theta-\beta)}{2} \qquad \text{...(15.7)}$$

In ΔBAP,

$$\angle BAP = 180° - \beta - \left[90° + \frac{(\theta-\beta)}{2}\right]$$

$$= 90° - \frac{(\theta+\beta)}{2}$$

$$\simeq 90° - \frac{\theta}{2} \qquad \text{...(15.8)}$$

Draw perpendicular *PG* on *AB* from P meeting *AB* at *G*.

$$\tan\beta = \frac{PG}{D} = \frac{r}{D}\sin\left(90° - \frac{\theta}{2}\right)$$

$$= \frac{r}{D}\cos\left(\frac{\theta}{2}\right) \qquad \text{...(15.9)}$$

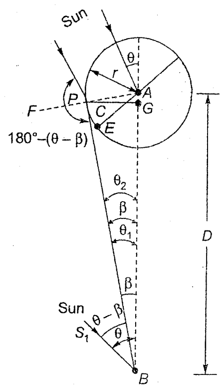


Fig. 15.6 *Phase error when bright line is bisected*

$$\beta \simeq \frac{r}{D}\cos\left(\frac{\theta}{2}\right) \text{ radians}$$

$$= \frac{r}{D}\cos\left(\frac{\theta}{2}\right)\sin 1'' \text{ seconds} \qquad \text{...(15.10)}$$

$$= 206265\left[\frac{r}{D}\cos\left(\frac{\theta}{2}\right)\right] \qquad \text{...(15.11)}$$

15.8 Selection of Triangulation Stations

(a) Triangulation stations should be placed at the most elevated ground points and at the top of hills so that sights through undisturbed atmosphere may be made possible.

(b) As far as possible, the triangulation stations should be inter-visible.

(c) The triangulation stations should form well-conditioned triangles i.e. isosceles triangle with base angle of about 56° or the triangle should be equilateral. In no case, the angles of the triangle should be less than 30° and greater than 120°.

(d) The various triangulation stations should be easily accessible with adequate supply of food and water.

(e) The triangulation stations should be so selected that the lengths of sight are neither too small nor too long. For small length of sight, error due to centering and bisection is large. For long sights, the signal is not distinct for accurate bisection.

(f) Triangulation stations should be at the commanding locations so as to serve as a control for subsidiary triangulation and for possible future extension of the triangulation programme.

(g) The triangulation stations should be so selected that cost of clearing the obstructions (like trees, bushes, poles etc.) is the minimum.

(h) The triangulation stations should be so selected that line of sight should not pass over towns, factories, and furnaces etc. which result in irregular refraction.

(i) The line of sight should not graze the ground but should preferably remain 2 m to 3 m above the ground to minimize the irregular atmospheric refraction.

15.9 Base Line

It is the most important one in triangulation on which the entire system of triangulation frame work depends. Thus it needs to be measured very precisely. Apart from that, base line measurement is used to assess the scale of triangulation.

15.9.1 Site Selection for Base Line

The accuracy of triangulation depends on the accuracy of base line measurement. The following factors must be accounted for, while selecting the site for base line:

(a) As far as possible, the site must be level or at most gently undulating. Very high undulating ground should be avoided. If the ground is sloping then slope should be gentle and uniform.

(b) The site must be free of obstructions throughout the base length.

(c) The ground should be firm and smooth.

(d) The two ends of the base line should be intervisible.

(e) The site should be such that well shaped (as far as possible well-conditioned) triangles can be obtained while connecting its end stations to the main triangulation stations.

(f) The site should be such that minimum length of base line as specified is available.

In a smooth and flat land, there is no problem in selecting a site for base line but in rough terrain, the choice of selecting the base line is very limited.

For triangulation system of India, a total of nine base lines are used. The lengths of these lines vary from 10.67 miles to 13 miles. Another tenth base line is of length 2.83 miles.

15.10 Measurement of Base Line

(a) Rigid bar (Colby apparatus) method
(b) Tapes and wires (flexible wires) method
(c) Hunter's short base method
(d) Tacheometric method
(e) EDM instruments method

- Invar tapes were used for the measurement of base lines. The length of invar tapes varies from 30 m to as long as 100 m.
- Hunter's short base method uses four chains, each of 22 yards (or 20117 mm) that are linked together.
- The tacheometric method of base line measurement is used only for highly undulating terrains.
- The task of measurement of base line has become highly simplified with long range measurements and very high accuracy with the advent of Electronic Distance Measurement (EDM) instruments.

15.11 Corrections to Base Line Measurement

(a) Correction for standardization
(b) Correction for slope
(c) Correction for pull
(d) Correction for sag
(e) Correction for temperature
(f) Correction for misalignment
(g) Reduction of length to MSL (altitude correction)

15.11.1 Correction for Standardization

- If the actual length of the base line is not equal to the designated length, then the distance measured will not conform to the designated length as given on the tape. Here the correction for standardization has to be applied.
- If actual length l is shorter than the designated length l_o then distance measured will be larger than the correct distance and hence correction will be **negative** or error will be **positive.**
- On the other hand, if actual length l is larger than the designated length l_o then the distance measured will be smaller and hence correction will be **positive** or error will be **negative.**

Thus, error per chain length $= l_o - l$

Total error in measured length L is $(l_0 - l)\frac{L}{l}$ ...(15.12)

Correction per base line length $=$ − error per chain length $= -(l_o - l)$

Total correction in measured length L is $-(l_0 - l)\frac{L}{l}$ ...(15.13)

15.11.2 Correction for Slope

Let, L = Sloped distance i.e. distance measured along the slope
D = Horizontal projection of the measured sloped length L
θ = Angle of slope

The slope correction is,

$$C_s = \sqrt{(L^2 - h^2)} - L$$

$$\approx -\frac{h^2}{2L} - \frac{h^4}{8L^3} \quad ...(15.14)$$

where, h = Difference in elevations of the two ends of the tape

Neglecting second term of Eq. (15.14)

$$C_s \simeq -\frac{h^2}{2L} \quad ...(15.15)$$

This correction is always **negative.**

15.11.3 Correction for Pull

Whenever the pull applied on the tape differs from the standardised of tape then measured length will not represent the actual length and correction needs to be applied.

This correction is given by,

$$C_p = (P - P_0)\frac{L}{AE} \quad ...(15.16)$$

Where, P = Applied pull
P_0 = Standardised pull designated for the tape
L = Measured length from tape
A = Cross-sectional area of tape
E = Modulus of elasticity of tape material

$$= \begin{cases} 2\times10^5 \text{ N/mm}^2 & \text{for steel} \\ 1.5\times10^5 \text{ N/mm}^2 & \text{for invar tape} \end{cases}$$

This correction is positive when applied pull (P) is greater than standardised pull (P_0)

15.11.4 Correction for Sag

- Whenever a tape is held between two supports then due to its self weight, tape takes the shape of a catenary.
- If the tape has been standardised for catenary then no correction for sag is needed. This happens when pull applied to tape while taking the measurement is equal to standardised pull.
- However if the designated length of tape refers to length on a flat surface then sag correction is needed because the straight chord length between two points is always less than the curved distance between the same two points. Due to this, sag correction is always negative and is given by

$$C_s = -\frac{w^2 l^3}{24P^2} \quad ...(15.17)$$

where C_s = Sag correction, w = Weight of tape per unit length, P = Pull applied to tape

15.11.5 Correction for Temperature

All tapes are standardised at a particular temperature and temperature at the time of measurement may or may not be the same as that of standardised temperature. This call for correction to be applied to the measured length which is given by

$$C_t = \alpha(T - T_0)L \qquad ...(15.18)$$

where α = Coefficient of thermal expansion of tape material

$$= \begin{bmatrix} 1.6\times10^{-6}\ ^\circ C^{-1} & \text{for steel} \\ 1.2\times10^{-6} \text{ to } 0.37\times10^{-6}\ ^\circ C^{-1} & \text{for invar} \end{bmatrix}$$

The temperature correction is positive when temperature at the time of measurement exceeds the standardised temperature.

15.11.6 Correction for Misalignment

The distance between the two points cannot be measured directly then an alternate path has to be followed as shown in **Fig. 15.7**.

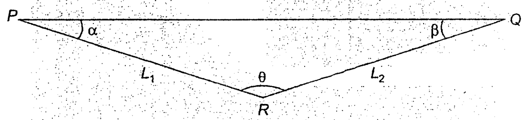


Fig. 15.7 *Correction for misalignment*

In **Fig. 15.7**, the distance between the points *P* and *Q* is measured through *PRQ* instead of *PQ* directly.

The correction for misalignment is given by,

$$\begin{aligned} C_m &= PQ - PRQ \\ &= PQ - (PR + RQ) \\ &= (L_1 \cos\alpha + L_2 \cos\beta) - (L_1 + L_2) \\ &= -[L_1 (1 - \cos\alpha) + L_2 (1 - \cos\beta)] \end{aligned} \qquad ...(15.19)$$

This correction is always negative.

Check: For check purpose, the distance *PQ* can be computed as

$$\cos\theta = \frac{PR^2 + RQ^2 - PQ^2}{2(PR)(RQ)}$$

$$\Rightarrow \qquad PQ^2 = PR^2 + RQ^2 - 2(PR)(RQ)\cos\theta$$

$$PQ = \sqrt{PR^2 + RQ^2 - 2PR.RQ\cos\theta}$$

where $\theta = 180^\circ - (\alpha + \beta)$

15.11.7 Reduction of Length to MSL/Altitude Correction

Often lengths are required to reported with respect to Mean Sea Level (MSL). As shown in **Fig. 15.8**, let distance *PQ* is measured at a place whose altitude for MSL is 'h'.

It is required to report the distance *PQ* at mean sea level.

Let, L = Distance between the points *P* and *Q* which are 'h' meters above the MSL

L_0 = Distance of *PQ* reduced to MSL i.e., P_1Q_1

R = Radius of earth (= 6367 km approx.)

Thus central angle is,

$$\theta = \frac{L_0}{R} = \frac{L}{R+h}$$

$$\Rightarrow \qquad L_0 = \left(\frac{R}{R+h}\right)L$$

$\therefore$ Altitude correction,

$$\begin{aligned} C_h &= L_0 - L \\ &= \left(\frac{R}{R+h} - 1\right)L \\ &= -\frac{Lh}{R+h} \end{aligned} \qquad ...(15.20)$$

When, $h \gg R$ then,

$$\therefore \qquad C_h \simeq -\frac{Lh}{R} \qquad ...(15.21)$$

Altitude correction is always negative.

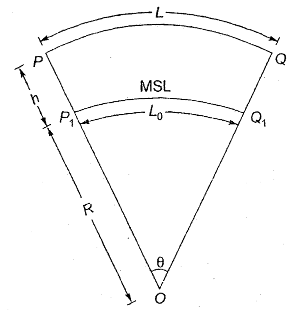


Fig. 15.8 *Reduction of measured length to MSL*

15.12 Computations in Triangulation

1. Adjustment of the observed angles.
2. Lengths computation.
3. Computation of bearings and latitudes, departures of all the sides.
4. Calculation of independent co-ordinates.

15.12.1 Adjustment of the Observed Angles

In a triangulation network, all the angles of the network need to be adjusted first before carrying out any further computations. All the observed/measured angles are corrected so that they satisfy all the geometric conditions of the figure and stations like :Among various methods of adjustments, the theory of least squares is the most accurate one.

15.12.2 Lengths Computations

- As shown in **Fig. 15.9**, length *AB* is the **base line** and further calculations are done to determine the positions of stations *C* and *D*.
- In order to locate the positions of stations *C* and *D*, the solution of two triangles is needed.
- Let the two triangles selected be ΔABC and ΔACD. The length of the side *AC* is determined from the ΔABC. Now the known length *AC* is used to determine the length of sides *AD* and *CD*.
- In order to check the accuracy of the field work, the length *CD* is also determined by an alternative path. For this, two triangles selected are ΔABD and ΔBCD. The length of the side *BD* is determined from ΔABD which is then used in ΔBCD for calculating the side *CD*.

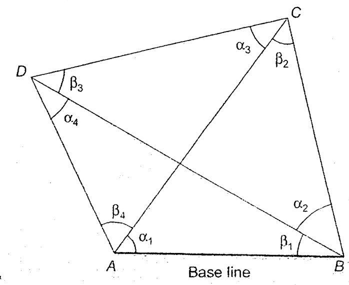


Fig. 15.9 *Lengths Computation*

15.12.3 Computation of Bearings and Latitudes, Departures of All the Sides

- Triangulation is performed to establish the horizontal position of the various triangulation stations relative to each other (i.e. other triangulation stations) and with reference to a particular selected datum.
- By knowing the bearing of a line and the angles, the bearing of all other sides can be determined. The latitudes and departures can be computed as:

$$\text{Latitude} = L\cos\theta$$
$$\text{Departure} = L\sin\theta$$

where L is the length of the side and θ is the bearing of the side.

15.12.4 Calculation of Independent Co-ordinates

By knowing the co-ordinates of the base line stations and latitudes, departures of all the sides, the independent co-ordinates of all the other stations can be computed.

Illustrative Examples

Example 15.1 Observations were taken from station P to signal at station Q. The distance between P and Q is 12.5 km and diameter of signal at station Q was 20 cm. The sun rays make an angle of 50° with line PQ. Calculate the phase correction if observations were made:
(a) on the bright portion (b) on the bright line

Solution:

(a) When observation is made on the bright portion

$$\beta = \frac{206265}{D} r\cos^2\left(\frac{\phi}{2}\right)$$

Here, $\phi = 50°$
$D = 12.5 \text{ km} = 12500 \text{ m}$
$r = 10 \text{ cm} = 0.1 \text{ m}$

$$\therefore \quad \beta = \frac{206265}{12500}(0.1)\cos^2\left(\frac{50}{2}\right)$$
$$= 1.355° = 1°\,21°\,19.43''$$

(b) When observation is made on bright line

$$\beta = \frac{206265}{D} r\cos\left(\frac{\theta}{2}\right)$$

Here also, $\theta = 50°$
$D = 12.5 \text{ km} = 12500 \text{ m}$
$r = 10 \text{ cm} = 0.1 \text{ m}$

$$\therefore \quad \beta = \frac{206265}{12500}(0.1)\cos\left(\frac{50}{2}\right)$$
$$= 1.496° = 1°\,29'\,43.86''$$

Example 15.2 A base line was measured with a steel tape of designated length 30 m at 20°C at a pull of 100 N. The measured length of base line was 1543 m. The field temperature was 31.5°C and the pull applied was 130 N. Find the correct length of base line. The cross-sectional area of tape is 2 mm², coefficient of thermal expansion of steel is 2.5×10^{-6} °C^{-1} and $E = 2 \times 10^5$ N/mm².

Solution:

Measured length of base line = 1543 m

$$\text{Temperature correction } (c_t) = \alpha(T - T_0)\,L$$
$$= 2.5 \times 10^{-6}(31.5 - 20)\,1543 = 0.04436 \text{ m}$$

$$\text{Pull correction } (c_p) = (P - P_0)\frac{L}{AE}$$
$$= (130 - 100)\frac{1543}{2 \times 2 \times 10^5} = 0.115725 \text{ m}$$

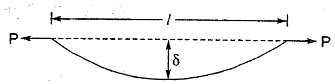
$$\therefore \quad \text{Total correction } (c) = c_t + c_p$$
$$= 0.04436 + 0.115725$$
$$= 0.160085 \text{ m} = 0.16 \text{ m}$$

$$\therefore \quad \text{Actual length of base line} = 1543 + 0.16 = 1543.16 \text{ m}$$

Example 15.3 A 20 m long tape when suspended between two ends shows a sag of 20.11 cm at mid-span. The pull applied was 105 N. What is the weight of the tape?

Solution:

$$P = \frac{wl^2}{8\delta} \quad \text{where } \delta = \text{central dip}$$



$$\Rightarrow \quad d = \frac{wl^2}{8P}$$

$$\Rightarrow \quad 20.11 \times 10^{-2} = \frac{w(20)^2}{8 \times 105}$$

$$\Rightarrow \quad w = 0.422 \text{ N/m}$$

$$\therefore \quad \text{Total weight of tape} = 0.422 \times 20 = 8.44 \text{ N}$$

Example 15.4 While measuring three segments of a base line, the following data were obtained.

Segment No.	Length (m)	Temp. (°C)	Pull (kg)	Difference in level of supports (m)
1.	28.084	18	7	0.3
2.	29.936	20	8	0.7
3.	30.115	28	10	0.5

The 30 m steel tape was standardized at 25°C under a pull of 5 kg. What are the correct lengths of segments? The tape weight 1.2 kg. Unit weight of steel is 7800 kg/m³ and coefficient of thermal expansion is 1.1×10^{-5} °C^{-1}. Take E = 2.1×10^4 kg/mm².

Solution:

Segment No. 1

Measured length = 28.084 m
Temperature = 18°C
Pull = 7 kg
Difference in level of supports = 0.3 m

$$\text{Correction for pull } (C_p) = (P-P_0)\frac{L}{AE}$$

Weight of tape = γ_{AL}

$\Rightarrow$ 1.2 kg = (7800 kg/m³) A (30 m)

$\Rightarrow$ $A = 5.128 \times 10^{-6}$ m²

$$C_p = \frac{(7-5)30}{5.128\times10^{-6}\times2.1\times10^4\times10^6}$$
$$= 5.572 \times 10^{-4} \text{ m} = 0.0005572 \text{ m}$$

Correction for temperature (C_t) = $\alpha L \Delta T$
$= 1.1 \times 10^{-5} \times 28.084 \times (18-25) = -0.00216$ m

Correction for difference in level supports (C_h)

$$= -\frac{h^2}{2L} = -\frac{0.3^2}{2\times28.084} = -0.0016 \text{ m}$$

$$\text{Correction for sag } (C_s) = -\frac{W^2L}{24P^2} = -\frac{(1.2)^2\times30}{24\times7^2} = -0.037 \text{ m}$$

$\therefore$ Corrected length = 28.084 + 0.0005572 − 0.00216 − 0.0016 − 0.037 = 28.044 m

Segment No. 2

Measured length = 29.936 m
Temperature = 20°C
Pull = 8 kg
Difference in level of supports = 0.7 m

$$\text{Correction for pull } (C_p) = (P-P_0) = \frac{(8-5)30}{5.128\times10^{-6}\times2.1\times10^4\times10^6} = 0.00084 \text{ m}$$

Correction for temperature (C_t) = $\alpha L \Delta T$
$= 1.1 \times 10^{-5} \times 29.936 \times (20-25) = -0.00165$ m

Correction for difference in level supports (C_h)

$$= -\frac{h^2}{2L} = -\frac{0.7^2}{2\times29.936} = -0.00818 \text{ m}$$

$$\text{Correction for sag } (C_s) = -\frac{W^2L}{24P^2} = -\frac{(1.2)^2\times30}{24\times8^2} = -0.028 \text{ m}$$

$\therefore$ Corrected length = 29.936 + 0.00084 − 0.00165 − 0.00818 − 0.028 = 29.899 m

Segment No. 3

Measured length = 30.115 m
Temperature = 28°C
Pull = 10 kg
Difference in level of supports = 0.5 m

$$\text{Correction for pull } (C_p) = (P-P_0)\frac{L}{AE} = \frac{(10-5)30}{5.128\times10^{-6}\times2.1\times10^4\times10^6} = 0.00139 \text{ m}$$

Correction for temperature (C_t) = $\alpha L \Delta T$
$= 1.1 \times 10^{-5} \times 30.115 \times (28-25) = 0.000994$ m

Correction for difference in level supports (C_h)

$$= -\frac{h^2}{2L} = -\frac{0.5^2}{2\times30.115} = -0.00415 \text{ m}$$

$$\text{Correction for sag } (C_s) = -\frac{W^2L}{24P^2} = -\frac{(1.2)^2\times30}{24\times10^2} = -0.018 \text{ m}$$

$\therefore$ Corrected length = 30.115 + 0.00139 + 0.000994 − 0.00415 − 0.018 = 30.095 m

Example 15.5 Prove that for a flexible, inextensible and uniform tape of total weight 2 W, hanging freely between the two supports at the same level at a tension T at each end, the horizontal distance between the supports is,

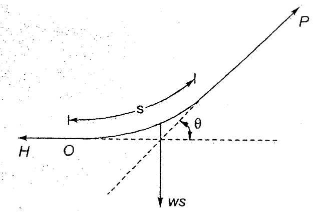
$$\frac{H}{w}\log\left(\frac{T+W}{T-W}\right)$$

where, H = Horizontal tension at centre of tape
w = weight per unit length of tape.

Solution:

Let O be the origin at lowest point of tape.

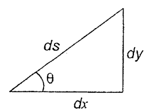
Resolving the forces,



$$P\sin\theta = ws \quad \text{...(i)}$$
$$P\cos\theta = H \quad \text{...(ii)}$$
$$\therefore \quad \tan\theta = \frac{ws}{H} \quad \text{...(iii)}$$

Differentiating (iii) w.r.t. x,

$$\sec^2\theta\frac{d\theta}{dx} = \frac{w}{H}\frac{ds}{dx} \quad \text{...(iv)}$$



From elemental triangle

$$\cos\theta = \frac{ds}{dx}$$

$$\Rightarrow \quad \sec\theta = \frac{ds}{dx} \quad \text{...(v)}$$

Substituting (v) in (iv),

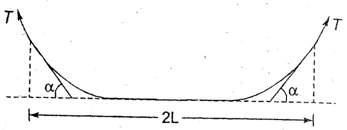
$$\sec^2\theta\frac{d\theta}{dx} = \frac{w}{H}\sec\theta$$

$$\Rightarrow \quad \sec\theta\frac{d\theta}{dx} = \frac{w}{H} \quad \text{...(vi)}$$

Let α = inclination of tangents at supports

2L = Horizontal distance between the supports

Integrating (vi)



$$\int_0^\alpha \sec\theta \, d\theta = \int_0^L \frac{w}{H} dx$$

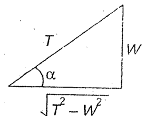
$$\Rightarrow \quad \ln(\sec\theta + \tan\theta)\Big|_0^\alpha = \frac{w}{H}(L-0)$$

$$\Rightarrow \quad L = \frac{H}{w}\ln(\sec\alpha + \tan\alpha)$$

Now,

$$T\sin\alpha = \frac{2w}{2} = W$$

$$\Rightarrow \quad \sin\alpha = \frac{W}{T}$$



$$\therefore \quad \sec\alpha = \frac{T}{\sqrt{T^2-W^2}}$$

and

$$\tan\alpha = \frac{W}{\sqrt{T^2-W^2}}$$

$$\therefore \quad L = \frac{H}{w}\ln\left[\frac{T}{\sqrt{T^2-w^2}} + \frac{W}{\sqrt{T^2-w^2}}\right]$$

$$\Rightarrow \quad L = \frac{H}{w}\ln\left[\sqrt{\frac{T+w}{T-w}}\right] = \frac{H}{2w}\ln\left(\frac{T+t}{T-w}\right)$$

$\therefore$ Distance between the supports $= 2L = \frac{H}{w}\ln\left(\frac{T+w}{T-w}\right)$

Objective Brain Teasers

Q.1 Consider the following statements:
(i) Triangulation is another system of multiplying the ground control points.
(ii) The triangulation system of quadrilaterals is most suitable for railways.
(iii) Triangulation is used for filling in the minute details of an area.
Of the above statements, the correct one(s) is (are):
(a) (ii) only
(b) (i) and (iii)
(c) (ii) and (iii)
(d) (iii) only

Q.2 Choose the correct statement:
(a) Small angles are indispensable in triangulation
(b) Triangulation is most suitable for flat terrains.
(c) In triangulation, the length of only one side is measured.
(d) Limiting strength of figure for first and third order triangulation are 70 and 50 respectively.

Q.3 For a distance of 42 km in triangulation, which type of signal is used?
(a) Heliotrope
(b) Heliograph
(c) Beacon
(d) All of the above

Q.4 In triangulation, the towers used are known as:
(a) Bibly
(b) Heliotropes
(c) Hunter
(d) None of the above

Q.5 The purpose of providing station marks in triangulation is:
(a) a level surface mark
(b) detailed description in order to recover the station even after a long time
(c) For having a good target point
(d) All of the above

Q.6 Which of the following statement is false?
(a) For surveying long narrow area, triangulation is more suitable then traversing.
(b) Triangulation enables to cover a large area in a short time.
(c) Triangulation is best suited for hilly areas.
(d) Only one linear measurement that is made in triangulation is the measurement of base line.

Q.7 Which of the following statement is correct?
(a) Heliotrope is a luminous signal.
(b) Beacon lamp is a type of electric signal.
(c) Three legged signal is preferred over four legged signal.
(d) All of the above

Q.8 Consider the following statement:
(i) In triangulation, no angle should be less than 30°.
(ii) For highway triangulation, a triangle is the most suited figure.
(iii) For runway triangulation, braced quadrilateral is the most suited figure.
Of the above statements, the correct one(s) is(are):
(a) (i) only
(b) (i) and (iii)
(c) (i) and (ii)
(d) (ii) and (iii)

Q.9 Among the following statements, the correct statement is:
(a) Tellurometer is the most widely used third order triangulation.
(b) Flexible method of base line measurement is used in Wheeler's method.
(c) Satellite technology is used in Jaderin's method of base line measurement.
(d) None of the above.

Q.10 Which of the following instrument is used for measurement of bases in India by the Survey of India?
(a) Hunter's short base
(b) Colby apparatus
(c) Jaderin's apparatus
(d) Tellurometer

Q.11 The best shape of triangle that can be used in triangulation is:
(a) Equilateral triangle
(b) Isosceles triangle with base angle of 30°
(c) Isosceles triangle with base angle of 56°14'
(d) Right angled isosceles triangle

Q.12 The most commonly used figure in triangulation is
(a) Triangle
(b) Quadrilateral with centre point
(c) Pentagon
(d) Braced quadrilateral

Q.13 For India, geodetic survey was done using
(a) Plane table traverse
(b) Trilateration
(c) Triangulation
(d) Compass traversing

Q.14 The most precise instrument for measurement of base line is
(a) EDM
(b) Chain
(c) Tacheometer
(d) Tape

Answers

1. (a)	2. (c)	3. (d)	4. (a)	5. (d)
6. (a)	7. (d)	8. (c)	9. (d)	10. (b)
11. (c)	12. (d)	13. (c)	14. (a)	

Student's Assignments

Ex.1 A base line was measured in three bays and the following observations were obtained:

Bay	Measured length (m)	Temperature (°C)	Difference in level (m)	Tension (N)
1.	29.835	22	+0.065	195
2.	29.843	23	+0.354	195
3.	29.882	21	–0.215	195

Determine the correct length reduced to mean sea level after applying all the corrections. Mass of tape is 0.036 kg/m, cross-sectional area is 2.95 mm^2. Coefficient of linear expansion is 8×10^{-7} per °C, Young's modulus of elasticity is 1.5×10^5 N/mm^2, mean elevation of base line 153.25 m. The tape was standardized at 25°C at a pull of 100 N. Radius of earth is 6367 km.

Ex.2 A base line was measured with a steel tape which was exactly 30 m at 20°C under a pull of 100 N. The measured length of the base line came out to be 1495 m when the temperature was 29°C under a pull of 135 N. Determine the correct length of the base line. Cross-sectional area of the tape is 2.5 mm^2 coefficient of linear expansion is 3.7×10^{-6} per °C and Young's modulus is 1.5×10^5 N/mm^2.

Ans. 1495.189 m

■■■■