# 3

# Motion in a Plane

## TOPIC 1 Vectors

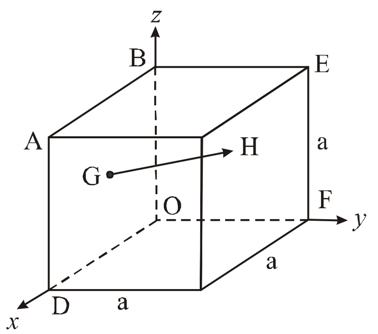
**1.** A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N-m. The value of $x$ is ______. **[NA Sep. 05, 2020 (I)]**

**2.** The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of 2 $\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is _______. **[NA 7 Jan. 2020 II]**

**3.** Let $|\vec{A_1}| = 3$, $|\vec{A_2}| = 5$ and $|\vec{A_1} + \vec{A_2}| = 5$. The value of $(2\vec{A_1} + 3\vec{A_2}) \bullet (3\vec{A_1} - 2\vec{A_2})$ is : **[8 April 2020 II]**

(a) – 106.5 (b) – 99.5
(c) –112.5 (d) –118.5

**4.** In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be: **[10 Jan. 2019 I]**




(a) $\frac{1}{2}a(\hat{k} - \hat{i})$ (b) $\frac{1}{2}a(\hat{i} - \hat{k})$
(c) $\frac{1}{2}a(\hat{j} - \hat{i})$ (d) $\frac{1}{2}a(\hat{j} - \hat{k})$

**5.** Two forces P and Q, of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is: **[10 Jan. 2019 II]**

(a) 120° (b) 60°
(c) 90° (d) 30°

**6.** Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the magnitude of $(\vec{A} - \vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is: **[10 Jan. 2019 II]**

(a) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (b) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$
(c) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (d) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

**7.** Let $\vec{A} = (\hat{i} + \hat{j})$ and $\vec{B} = (\hat{i} - \hat{j})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} = \vec{A}.\vec{B}$ is given by **[Online April 16, 2018]**

(a) $\sqrt{\frac{5}{9}}$ (b) $\sqrt{\frac{10}{9}}$
(c) $\sqrt{\frac{20}{9}}$ (d) $\sqrt{\frac{9}{12}}$

**8.** A vector $\vec{A}$ is rotated by a small angle Δθ radian (Δθ << 1) to get a new vector $\vec{B}$. In that case $|\vec{B} - \vec{A}|$ is : **[Online April 11, 2015]**

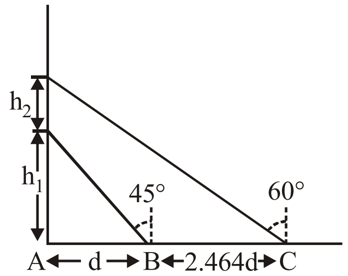
(a) $|\vec{A}|\Delta\theta$ (b) $|\vec{B}|\Delta\theta - |\vec{A}|$
(c) $|\vec{A}|\left(1 - \frac{\Delta\theta^2}{2}\right)$ (d) 0

**9.** If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is **[2004]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) π (d) $\frac{\pi}{4}$

## TOPIC 2 Motion in a Plane with Constant Acceleration

**10.** A balloon is moving up in air vertically above a point A on the ground. When it is at a height $h_1$, a girl standing at a distance $d$ (point B) from A (see figure) sees it at an angle 45º with respect to the vertical. When the balloon climbs up a further height $h_2$, it is seen at an angle 60º with respect to the vertical if the girl moves further by a distance 2.464 $d$ (point C). Then the height $h_2$ is (given tan 30º = 0.5774): **[Sep. 05, 2020 (I)]**




(a) 1.464 $d$ (b) 0.732 $d$
(c) 0.464 $d$ (d) $d$

**11.** Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ ms$^{-1}$, a particle moves in the $x$–$y$ plane with a constant acceleration of $(10\hat{i} + 4\hat{j})$ ms$^{-2}$. At time $t$, its coordiantes are (20 m, $y_0$ m). The values of $t$ and $y_0$ are, respectively : **[Sep. 04, 2020 (I)]**

(a) 2 s and 18 m (b) 4 s and 52 m
(c) 2 s and 24 m (d) 5 s and 25 m

**12.** The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$? **[9 April 2019 II]**

(a) 40 (b) 25 (c) 100 (d) 50

**13.** A particle moves from the point $(2.0\hat{i} + 4.0\hat{j})$ m, at t = 0, with an initial velocity $(5.0\hat{i} + 4.0\hat{j})$ ms$^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i} + 4.0\hat{j})$ ms$^{-2}$. What is the distance of the particle from the origin at time 2s? **[11 Jan. 2019 II]**

(a) 15 m (b) $20\sqrt{2}$ m
(c) 5 m (d) $10\sqrt{2}$ m

**14.** A particle is moving with a velocity $\vec{v}$ = K ($y\hat{i} + x\hat{j}$), where K is a constant. The general equation for its path is: **[9 Jan. 2019 I]**

(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) $y^2 = x^2$ + constant (d) $xy$ = constant

**15.** A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{i}$ m/s and moves in the $x$-$y$ plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})$ m/s². The $x$-coordinate of the particle at the instant when its $y$-coordinate is 32 m is D meters. The value of D is: **[9 Jan. 2020 II]**

(a) 32 (b) 50 (c) 60 (d) 40

**16.** A particle is moving along the $x$-*axis* with its coordinate with time '$t$' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the $y$-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$ s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then $v$ (in m/s) is____ **[NA 8 Jan. 2020 I]**

**17.** A particle moves such that its position vector $\vec{r}$ ($t$) = cos ω$t\,\hat{i}$ + sin ω$t\,\hat{j}$ where ω is a constant and $t$ is time. Then which of the following statements is true for the velocity $\vec{v}$ ($t$) and acceleration $\vec{a}$ ($t$) of the particle: **[8 Jan. 2020 II]**

(a) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin
(b) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$
(c) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$
(d) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

**18.** A particle is moving with velocity $\vec{v} = k(y\hat{i} + x\hat{j})$, where k is a constant. The general equation for its path is **[2010]**

(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) $xy$ = constant (d) $y^2 = x^2$ + constant

**19.** A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is : **[2009]**

(a) $7\sqrt{2}$ units (b) 7 units
(c) 8.5 units (d) 10 units

**20.** The co-ordinates of a moving particle at any time '$t$' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time '$t$' is given by **[2003]**

(a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$
(c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$

## TOPIC 3 Projectile Motion

**21.** A particle of mass $m$ is projected with a speed $u$ from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is: **[9 Jan. 2020 II]**

(a) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (b) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

(c) $\frac{5}{8}\frac{u^2}{g}$ (d) $2\sqrt{2}\frac{u^2}{g}$

**22.** The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle $\theta_0$ with speed $v_0$ then ($g = 10\ ms^{-2}$): **[12 April 2019 I]**

(a) $\theta_0 = \sin^{-1}\frac{1}{\sqrt{5}}$ and $v_0 = \frac{5}{3}\ ms^{-1}$

(b) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

(c) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{9}{3}\ ms^{-1}$

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

**23.** A shell is fired from a fixed artillery gun with an initial speed $u$ such that it hits the target on the ground at a distance R from it. If $t_1$ and $t_2$ are the values of the time taken by it to hit the target in two possible ways, the product $t_1t_2$ is : **[12 April 2019 I]**

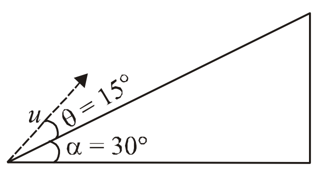
(a) R/4g (b) R/g (c) R/2g (d) 2R/g

**24.** Two particles are projected from the same point with the same speed $u$ such that they have the same range R, but different maximum heights, $h_1$ and $h_2$. Which of the following is correct ? **[12 April 2019 II]**

(a) $R^2 = 4\,h_1h_2$ (b) $R^2 = 16\,h_1h_2$

(c) $R^2 = 2\,h_1h_2$ (d) $R^2 = h_1h_2$

**25.** A plane is inclined at an angle $\alpha = 30^\circ$ with respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$, from the base of the plane, as shown in figure. The distance from the base, at which the particle hits the plane is close to : (Take $g = 10\ ms^{-2}$) **[10 April 2019 II]**




(a) 20 cm (b) 18 cm (c) 26 cm (d) 14 cm

**26.** A body is projected at t = 0 with a velocity $10\ ms^{-1}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\ ms^{-2}$, the value of R is: **[11 Jan. 2019 I]**

(a) 10.3 m (b) 2.8 m

(c) 2.5 m (d) 5.1 m

**27.** Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is: **[10 Jan. 2019 I]**

(a) 1 : 16 (b) 1 : 2 (c) 1 : 4 (d) 1 : 8

**28.** The initial speed of a bullet fired from a rifle is 630 m/s. The rifle is fired at the centre of a target 700 m away at the same level as the target. How far above the centre of the target ? **[Online April 11, 2014]**

(a) 1.0 m (b) 4.2 m (c) 6.1 m (d) 9.8 m

**29.** The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = (40\hat{i} + 50\hat{j})$ m at $t = 2$s. If the projectile was launched at an angle θ from the horizontal, then θ is (take $g = 10\ ms^{-2}$) **[Online April 9, 2014]**

(a) $\tan^{-1}\frac{2}{3}$ (b) $\tan^{-1}\frac{3}{2}$

(c) $\tan^{-1}\frac{7}{4}$ (d) $\tan^{-1}\frac{4}{5}$

**30.** A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ m/s^2$, the equation of its trajectory is : **[2013]**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

**31.** The maximum range of a bullet fired from a toy pistol mounted on a car at rest is $R_0 = 40$ m. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity v = 20 m/s, on a horizontal surface ? ($g = 10\ m/s^2$) **[Online April 25, 2013]**

(a) 30° (b) 60° (c) 75° (d) 45°

**32.** A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is : **[Online April 22, 2013]**

(a) 4.4 m (b) 2.4 m (c) 3.6 m (d) 1.6 m

**33.** A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be **[2012]**

(a) $20\sqrt{2}$ m (b) 10 m

(c) $10\sqrt{2}$ m (d) 20 m

**34.** A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is $v$, the total area around the fountain that gets wet is: **[2011]**

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$ (c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

**35.** A projectile can have the same range '$R$' for two angles of projection. If '$T_1$' and '$T_2$' to be time of flights in the two cases, then the product of the two time of flights is directly proportional to. **[2004]**

(a) $R$ (b) $\frac{1}{R}$ (c) $\frac{1}{R^2}$ (d) $R^2$

**36.** A ball is thrown from a point with a speed '$v_0$' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ? **[2004]**

(a) No (b) Yes, 30°
(c) Yes, 60° (d) Yes, 45°

**37.** A boy playing on the roof of a 10 m high building throws a ball with a speed of 10m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? **[2003]**

$[g = 10\text{m/s}^2, \sin 30^o = \frac{1}{2}, \cos 30^o = \frac{\sqrt{3}}{2}]$

(a) 5.20m (b) 4.33m (c) 2.60m (d) 8.66m

## TOPIC 4 Relative Velocity in Two Dimensions & Uniform Circular Motion

**38.** A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of $ms^{-2}$) is of the order of: **[Sep. 06, 2020 (I)]**

(a) $10^{-3}$ (b) $10^{-4}$
(c) $10^{-2}$ (d) $10^{-1}$

**39.** When a carsit at rest, its driver sees raindrops falling on it vertically. When driving the car with speed $v$, he sees that raindrops are coming at an angle 60º from the horizontal. On furter increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45º. The value of β is close to: **[Sep. 06, 2020 (II)]**

(a) 0.50 (b) 0.41
(c) 0.37 (d) 0.73

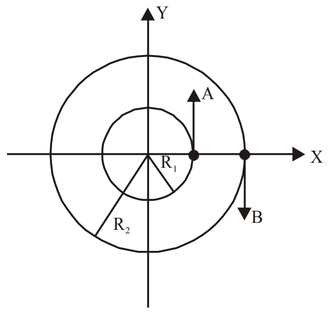
**40.** The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **[9 April 2019 I]**

(a) 90° (b) 150°
(c) 120° (d) 60°

**41.** Ship A is sailing towards north-east with velocity km/hr where points east and , north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in: **[8 April 2019 I]**

(a) 4.2 hrs. (b) 2.6 hrs.
(c) 3.2 hrs. (d) 2.2 hrs.

**42.** Two particles A, B are moving on two concentric circles of radii $R_1$ and $R_2$ with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure : **[12 Jan. 2019 II]**



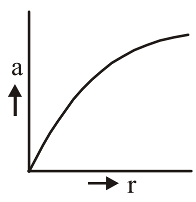

The relative velocity $\vec{v_A} - \vec{v_B}$ and $t = \frac{\pi}{2\omega}$ is given by:

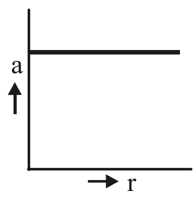
(a) $\omega(R_1 + R_2)\hat{i}$ (b) $-\omega(R_1 + R_2)\hat{i}$
(c) $\omega(R_2 - R_1)\hat{i}$ (d) $\omega(R_1 - R_2)\hat{i}$

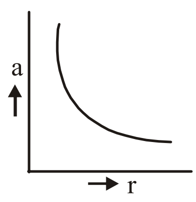
**43.** A particle is moving along a circular path with a constant speed of 10 $ms^{-1}$. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[Online April 10, 2015]**

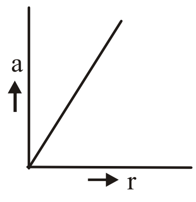
(a) $10\sqrt{3}$m/s (b) zero
(c) $10\sqrt{2}$m/s (d) 10 m/s

**44.** If a body moving in circular path maintains constant speed of 10 $ms^{-1}$, then which of the following correctly describes relation between acceleration and radius? **[Online April 10, 2015]**

(a) 


(b) 


(c) 


(d) 

# Hints & Solutions

**1.** **(195)**

Given : $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N

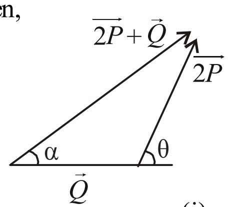
And, $\vec{r} = [(4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})] = 3\hat{i} + \hat{j} - 2\hat{k}$

Torque, $\tau = \vec{r} \times \vec{F} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

Magnitude of torque, $|\vec{\tau}| = \sqrt{195}$.

**2.** **(90)** Given,




$|\vec{R}| = |\vec{P}| \Rightarrow |\vec{P} + \vec{Q}| = |\vec{P}|$

$P^2 + Q^2 + 2PQ.\cos\theta = P^2$

$\Rightarrow Q + 2P\cos\theta = 0$

$\Rightarrow \cos\theta = -\dfrac{Q}{2P}$ ..(i)

$\tan\alpha = \dfrac{2P\sin\theta}{Q + 2P\cos\theta} = \infty$ ($\because$ $2P\cos\theta + Q = 0$)

$\Rightarrow \alpha = 90°$

**3.** **(d)** Using,

$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\theta$

$5^2 = 3^2 + 5^2 + 2 \times 3 \times 5\cos\theta$

or $\cos\theta = -0.3$

$\left(2\vec{A_1} + 3\vec{A_2}\right).\left(3\vec{A_1} - 2\vec{A_2}\right) = 2A_1 \times 3A_1$

$+ (3A_2)(3A_1)\cos\theta - (2A_1)(2A_2)\cos\theta - 3A_2 \times 2A_2$

$= 6A_1^2 + 9A_1A_2\cos\theta - 4A_1A_2\cos\theta - 6A_2^2$

$= 6A_1^2 \; 6A_2^2 + 5A'_1A_2\cos\theta$

$= 6 \times 3^2 - 6 \times 5^2 + 5 \times 3 \times 5(-0.3)$

$= -118.5$

**4.** **(c)** From figure,

$\vec{r}_G = \dfrac{a}{2}\hat{i} + \dfrac{a}{2}\hat{k}$

$\vec{r}_H = \dfrac{a}{2}\hat{j} + \dfrac{a}{2}\hat{k}$

$\therefore \vec{r}_H - \vec{r}_G = \left(\dfrac{a}{2}\hat{j} + \dfrac{a}{2}\hat{k}\right) - \left(\dfrac{a}{2}\hat{i} + \dfrac{a}{2}\hat{k}\right) = \dfrac{a}{2}\left(\hat{j} - \hat{i}\right)$

**5.** **(a)** Using, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$4F^2 + 9F^2 + 12F^2\cos\theta = R^2$

When forces Q is doubled,

$4F^2 + 36F^2 + 24F^2\cos\theta = 4R^2$

$4F^2 + 36F^2 + 24F^2\cos\theta$

$= 4(13F^2 + 12F^2\cos\theta) = 52F^2 + 48F^2\cos\theta$

$\therefore \cos\theta = -\dfrac{12F^2}{24F^2} = -\dfrac{1}{2} \quad \Rightarrow \theta = 120^o$

**6.** **(a)** Let magnitude of two vectors $\vec{A}$ and $\vec{B}$ = a

$|\vec{A} + \vec{B}| = \sqrt{a^2 + a^2 + 2a^2\cos\theta}$ and

$|\vec{A} - \vec{B}| = \sqrt{a^2 + a^2 - 2a^2[\cos(180° - \theta)]}$

$= \sqrt{a^2 + a^2 - 2a^2\cos\theta}$

and accroding to question,

$|\vec{A} + \vec{B}| = n|\vec{A} - \vec{B}|$

or, $\dfrac{a^2 + a^2 + 2a^2\cos\theta}{a^2 + a^2 - 2a^2\cos\theta} = n^2$

$\Rightarrow \dfrac{\cancel{a^2}(1 + 1 + 2\cos\theta)}{\cancel{a^2}(1 + 1 - 2\cos\theta)} n^2 \Rightarrow \dfrac{(1 + \cos\theta)}{(1 - \cos\theta)} = n^2$

using componendo and dividendo theorem, we get

$\theta = \cos^{-1}\left(\dfrac{n^2 - 1}{n^2 + 1}\right)$

**7.** **(a)** If $\vec{C} = a\hat{i} + b\hat{j}$ then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ ..... (i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a - b = 1$ ..... (ii)

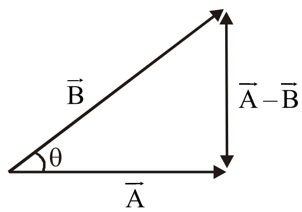
Solving equation (i) and (ii) we get

$a = \dfrac{1}{3}, b = \dfrac{2}{3}$

$\therefore$ Magnitude of coplanar vector, $|\vec{C}| = \sqrt{\dfrac{1}{9} + \dfrac{4}{9}} = \sqrt{\dfrac{5}{9}}$

**8.** (a) Arc length = radius × angle

So, $|\vec{B} - \vec{A}| = |\vec{A}|\Delta\theta$




**9.** **(c)** $\vec{A} \times \vec{B} - \vec{B} \times \vec{A} = 0 \Rightarrow \vec{A} \times \vec{B} + \vec{A} \times \vec{B} = 0$

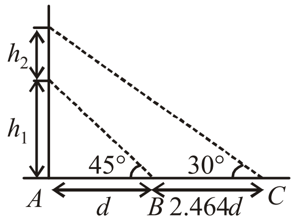
$\therefore \vec{A} \times \vec{B} = 0$

Angle between them is 0, π, or 2 π

from the given options, $\theta = \pi$

**10.** **(d)** From figure/ trigonometry,

$\frac{h_1}{d} = \tan 45°$ $\quad \therefore h_1 = d$




And, $\frac{h_1 + h_2}{d + 2.464d} = \tan 30°$

$\Rightarrow (h_1 + h_2) \times \sqrt{3} = 3.46d$

$\Rightarrow (h_1 + h_2) = \frac{3.46d}{\sqrt{3}} \Rightarrow d + h_2 = \frac{3.46d}{\sqrt{3}}$

$\therefore h_2 = d$

**11.** **(a)** Given : $\vec{u} = 5\hat{j}$ m/s

Acceleration, $\vec{a} = 10\hat{i} + 4\hat{j}$ and final coordinate $(20, y_0)$ in time $t$.

$S_x = u_x t + \frac{1}{2} a_x t^2$ $\quad [\because u_x = 0]$

$\Rightarrow 20 = 0 + \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 2$ s

$S_y = u_y \times t + \frac{1}{2} a_y t^2$

$y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 18$ m

**12.** **(d)** $\vec{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} - 40t\hat{j}$

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$

$\therefore a = \sqrt{30^2 + 40^2} = 50 \text{ m/s}^2$

**13.** **(b)** As $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i} + 4\hat{j})4$

$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$

[as $\vec{s}$ = change in position = $\vec{r}_f - \vec{r}_i$]

$\vec{r}_r = 20\hat{i} + 20\hat{j}$

$|\vec{r}_r| = 20\sqrt{2}$

**14.** **(c)** From given equation,

$\vec{V} = K\left(y\hat{i} + x\hat{j}\right)$

$\frac{dx}{dt} = ky$ and $\frac{dy}{dt} = kx$

Now $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y} = \frac{dy}{dx}, \Rightarrow ydy = xdx$

Integrating both side

$y^2 = x^2 + c$

**15.** **(c)** Using $S = ut + \frac{1}{2}at^2$

$y = u_y t + \frac{1}{2} a_y t^2$ (along $y$ Axis)

$\Rightarrow \quad 32 = 0 \times t + \frac{1}{2}(4)t^2$

$\Rightarrow \quad \frac{1}{2} \times 4 \times t^2 = 32$

$\Rightarrow t = 4\ s$

$S_x = u_x t + \frac{1}{2} a_x t^2$ (Along $x$ Axis)

$\Rightarrow \quad x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2 = 60$

**16.** **(580)**

| For pariticle '$A$' | For particle '$B$' |
|---|---|
| $X_A = -3t^2 + 8t + 10$ | $Y_B = 5 - 8t^3$ |
| $\vec{V}_A = (8 - 6t)\hat{i}$ | $\vec{V}_B = -24t^2\hat{j}$ |
| $\vec{a}_A = -6\hat{i}$ | $\vec{a}_B = -48t\hat{j}$ |

At $t = 1$ sec

$\vec{V}_A = (8 - 6t)\hat{i} = 2\hat{i}$ and $\vec{v}_B = -24\hat{j}$

$\therefore \vec{V}_{B/A} = -\vec{v}_A + \vec{v}_B = -2\hat{i} - 24\hat{j}$

$\therefore$ Speed of $B$ w.r.t. $A$, $\sqrt{v} = \sqrt{2^2 + 24^2}$

$= \sqrt{4 + 576} = \sqrt{580}$

$\therefore v = 580$ (m/s)

**17.** **(d)** Given, Position vector,

$\vec{r} = \cos\omega t\hat{i} + \sin\omega t\,\hat{j}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \omega(-\sin\omega t\hat{i} + \cos\omega t\,\hat{j})$

Acceleration,

$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\hat{i} + \sin\omega t\,\hat{j})$

$\vec{a} = -\omega^2\vec{r}$

$\therefore \vec{a}$ is antiparallel to $\vec{r}$

Also $\vec{v}.\vec{r} = 0 \quad \therefore \vec{v} \perp \vec{r}$

Thus, the particle is performing uniform circular motion.

**18.** **(d)** $v = k(yi + xj)$

$v = kyi + kxj$

$$\frac{dx}{dt} = ky, \frac{dy}{dt} = kx$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{kx}{ky}$$

$ydy = xdx$ ...(*i*)

Integrating equation (i)

$$\int ydy = \int x \cdot dx$$

$$y^2 = x^2 + c$$

**19.** **(a)** Given $\vec{u} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, $t = 10$s

From 1st equatoin of motion.

$$a = \frac{v-u}{t}$$

$\therefore v = at\ tu$

$$\Rightarrow v = (0.4\hat{i} + 0.3\hat{j}) \times 10 + (3\hat{i} + 4\hat{j})$$

$$\Rightarrow 4\hat{i} + 3\hat{j} + 3\hat{j} + 4\hat{j}$$

$$\Rightarrow v = 7\hat{i} + 7\hat{j}$$

$$\Rightarrow |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ unit.}$$

**20.** **(b)** Coordinates of moving particle at time '*t*' are $x = \alpha t^3$ and $y = \beta t^3$

$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

**21.** **(a)** Using principal of conservation of linear momentum for horizontal motion, we have

$2mv_x = mu + mu \cos 60°$

$$v_x = \frac{3u}{4}$$

For vertical motion

$$h = 0 + \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$$

Let $R$ is the horizontal distance travelled by the body.

$$R = v_x T + \frac{1}{2}(0)(T)^2 \text{ (For horizontal motion)}$$

$$R = v_x T = \frac{3u}{4} \times \sqrt{\frac{2h}{g}}$$

$$\Rightarrow R = \frac{3\sqrt{3}u^2}{8g}$$

**22.** **(c)** Given, $y = 2x - 9x^2$

On comparing with,

$$y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta},$$

We have,

$$\tan\theta = 2 \text{ or } \cos\theta = \frac{1}{\sqrt{5}}$$

$$\text{and } \frac{g}{2u^2\cos^2\theta} = 9 \text{ or } \frac{10}{2u^2(1/\sqrt{5})^2} = 9$$

$\therefore$ $u = 5/3$ m/s

**23.** **(d)** $R$ will be same for $\theta$ and $90° - \theta$.

Time of flights:

$$t_1 = \frac{2u\sin\theta}{g} \text{ and}$$

$$t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\text{Now, } t_1 t_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right)$$

$$= \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$$

**24.** **(b)** For same range, the angle of projections are : $\theta$ and $90° - \theta$. So,

$$h_1 = \frac{u^2\sin^2\theta}{2g} \text{ and}$$

$$h_2 = \frac{u^2\sin^2(90° - \theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$\text{Also, } R = \frac{u^2\sin 2\theta}{g}$$

$$h_1 h_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g}$$

$$= \frac{u^2}{16}\frac{u^2(2\sin\theta\cos\theta)^2}{g^2}$$

$$= \frac{R^2}{16}$$

or $R^2 = 16\,h_1 h_2$

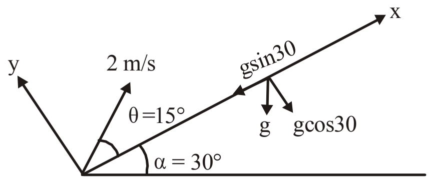
**25.** **(a)** On an inclined plane, time of flight (T) is given by

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Substituting the values, we get

$$T = \frac{(2)(2\sin 15°)}{g\cos 30°} = \frac{4\sin 15°}{10\cos 30°}$$

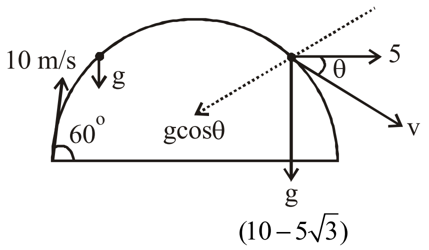
$$\text{Distance, S} = (2\cos 15°)T - \frac{1}{2}g\sin 30°(T)^2$$



$$= (2\cos 15°)\frac{4}{10}\frac{\sin 15°}{10\cos 30°} - \left(\frac{1}{2}\times 10\sin 30°\right)\frac{16\sin^2 15°}{100\cos^2 30°}$$

$$= \frac{16\sqrt{3}-16}{60} \simeq 0.1952\text{m} \simeq 20\text{cm}$$

**26. (b)**




Horizontal component of velocity

$v_x = 10\cos 60° = 5$ m/s

vertical component of velocity

$v_y = 10\cos 30° = 5\sqrt{3}$ m/s

After t = 1 sec.

Horizontal component of velocity $v_x = 5$ m/s

Vertical component of velocity

$v_y = |(5\sqrt{3}-10)|\ \text{m/s} = 10-5\sqrt{3}$

Centripetal, acceleration $a_n = \frac{v^2}{R}$

$$\Rightarrow R = \frac{v_x^2+v_y^2}{a_n} = \frac{25+100+75-100\sqrt{3}}{10\cos\theta} \quad ...(i)$$

From figure (using (i))

$$\tan\theta = \frac{10-5\sqrt{3}}{5} = 2-\sqrt{3} \Rightarrow \theta = 15°$$

$$R = \frac{100(2-\sqrt{3})}{10\cos 15} = 2.8\text{m}$$

**27. (a)** As we know, range $R = \frac{u^2\sin 2\theta}{g}$

and, area $A = \pi R^2$

$\therefore A \propto R^2$ or, $A \propto u^4$

$$\therefore \frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$$

**28. (c)** Let '$t$' be the time taken by the bullet to hit the target.

$\therefore$ 700 m = 630 ms$^{-1}$ $t$

$$\Rightarrow t = \frac{700\text{m}}{630\text{ms}^{-1}} = \frac{10}{9}\text{sec}$$

For vertical motion,

Here, $u = 0$

$$\therefore h = \frac{1}{2}gt^2$$

$$= \frac{1}{2}\times 10\times\left(\frac{10}{9}\right)^2$$

$$= \frac{500}{81}\text{m} = 6.1\text{ m}$$

Therefore, the rifle must be aimed 6.1 m above the centre of the target to hit the target.

**29. (c)** From question,

Horizontal velocity (initial),

$$u_x = \frac{40}{2} = 20\text{m/s}$$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2}gt^2$

$$\Rightarrow u_y \times 2 + \frac{1}{2}(-10)\times 4$$

or, $50 = 2u_y - 20$

or, $u_y = \frac{70}{2} = 35\text{m/s}$

$$\therefore \tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$$

$$\Rightarrow \text{Angle } \theta = \tan^{-1}\frac{7}{4}$$

**30. (b)** From equation, $\vec{v} = \hat{i} + 2\hat{j}$

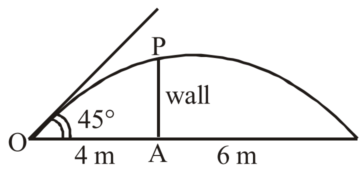
$\Rightarrow x = t$ ...(i)

$y = 2t - \frac{1}{2}(10t^2)$ ...(ii)

From (i) and (ii), $y = 2x - 5x^2$

**31. (b)**

**32. (b)**




As ball is projected at an angle 45° to the horizontal therefore Range = 4H

or $10 = 4H \Rightarrow H = \frac{10}{4} = 2.5\text{m}$

($\because$ Range = 4 m + 6 m = 10m)

Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$

$$\therefore u^2 = \frac{H\times 2g}{\sin^2\theta} = \frac{2.5\times 2\times 10}{\left(\frac{1}{\sqrt{2}}\right)^2} = 100$$

or, $u = \sqrt{100} = 10\text{ ms}^{-1}$

Height of wall PA

$$= OA\tan\theta - \frac{1}{2}\frac{g(OA)^2}{u^2\cos^2\theta}$$

$$= 4 - \frac{1}{2}\times\frac{10\times 16}{10\times 10\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}} = 2.4\text{m}$$

**33.** **(d)** $R = \frac{u^2 \sin^2 \theta}{g}$, $H = \frac{u^2 \sin^2 \theta}{2g}$

$H_{max}$ at $2\theta = 90°$

$H_{max} = \frac{u^2}{2g}$

$\frac{u^2}{2g} = 10 \Rightarrow u^2 = 10g \times 2$

$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R_{max} = \frac{u^2}{g}$

$R_{max} = \frac{10 \times g \times 2}{g} = 20$ metre

**34.** **(a)** Let, total area around fountain

$A = \pi R_{max}^2$ ...(i)

Where $R_{max} = \frac{v^2 \sin 2\theta}{g} = \frac{v^2 \sin 90°}{g} = \frac{v^2}{g}$ ...(ii)

From equation (i) and (ii)

$A = \pi \frac{v^4}{g^2}$

**35.** **(a)** A projectile have same range for two angle
Let one angle be θ, then other is 90° – θ

$T_1 = \frac{2u \sin \theta}{g}, T_2 = \frac{2u \cos \theta}{g}$

then, $T_1 T_2 = \frac{4u^2 \sin \theta \cos \theta}{g} = 2R$

$(\because R = \frac{u^2 \sin^2 \theta}{g})$

Thus, it is proportional to $R$. (Range)

**36.** **(c)** Yes, Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man.

$\frac{v_o}{2} = v_o \cos \theta$

or $\theta = 60°$

**37.** **(d)** Horizontal range is required

$R = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2 \times 30°)}{10} = 5\sqrt{3} = 8.66\,\text{m}$

**38.** **(a)** Here, $R = 0.1$ m

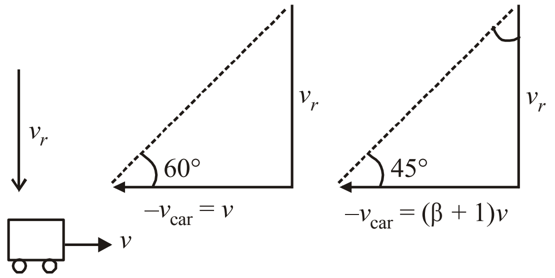
$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105$ rad /s

Acceleration of the tip of the clock second's hand,

$a = \omega^2 R = (0.105)^2 (0.1) = 0.0011 = 1.1 \times 10^{-3}$ m/s$^2$

Hence, average acceleration is of the order of $10^{-3}$.

**39.** **(d)** The given situation is shown in the diagram. Here $v_r$ be the velocity of rain drop.




When car is moving with speed $v$,

$\tan 60° = \frac{v_r}{v}$ ...(i)

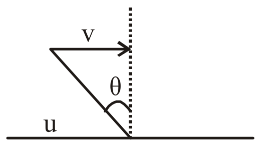
When car is moving with speed $(1+\beta)v$,

$\tan 45° = \frac{v_r}{(\beta+1)v}$ ...(ii)

Dividing (i) by (ii) we get,

$\sqrt{3}v = (\beta+1)v \Rightarrow \beta = \sqrt{3} - 1 = 0.732.$

**40.** **(c)** $\sin \theta = \frac{u}{v} = \frac{2}{4} = \frac{1}{2}$

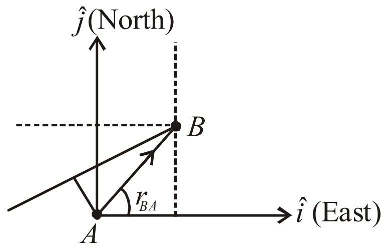



or $\theta = 30°$

with respect to flow,

$= 90° + 30° = 120°$

**41.** **(b)**




$\vec{v}_A = 30\hat{i} + 50\hat{j}$ km/hr

$\vec{v}_B = (-10\hat{i})$ km/hr

$r_{BA} = (80\hat{i} + 150\hat{j})$ km

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -10\hat{i} - 30\hat{i} - 50\hat{i} = 40\hat{i} - 50\hat{j}$

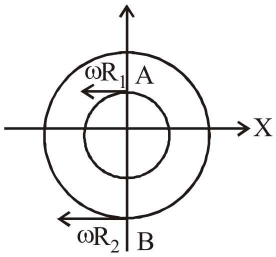
$t_{minimum} = \frac{|(\vec{r}_{BA}) \cdot (\vec{v}_{BA})|}{|(\vec{v}_{BA})|^2}$

$= \frac{|(80\hat{i} + 150\hat{j})(-40\hat{i} - 50\hat{j})|}{(10\sqrt{41})^2}$

$\therefore t = \frac{10700}{10\sqrt{41} \times 10\sqrt{41}} = \frac{107}{41} = 2.6$ hrs.

**42.** **(c)** From, $\theta = \omega t = \omega \dfrac{\pi}{2\omega} = \dfrac{\pi}{2}$

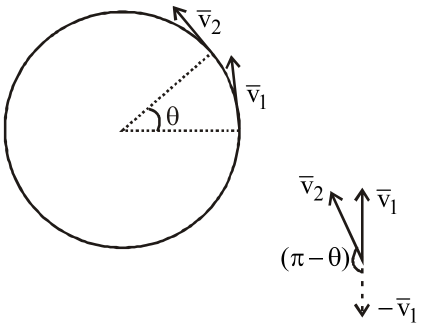
So, both have completed quater circle




Relative velocity,

$$v_A - v_B = \omega R_1\left(-\hat{i}\right) - \omega R_2\left(-i\right) = \omega\left(R_2 - R_1\right)i$$

**43.** **(d)**




Change in velocity,

$$|\Delta\bar{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos\left(\pi - \theta\right)}$$

$$= 2v\sin\frac{\theta}{2} \qquad \left(\because |\vec{v}_1| = |\vec{v}_2|\right) = v$$

$$= (2 \times 10) \times \sin(30°) = 2 \times 10 \times \frac{1}{2}$$

$$= 10 \text{ m/s}$$

**44.** **(c)** Speed, $V$ = constant (from question)

Centripetal acceleration,

$$a = \frac{V^2}{r}$$

$ra$ = constant

Hence graph (c) correctly describes relation between acceleration and radius.