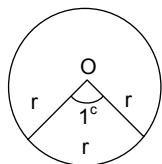


Trigonometry

SYNOPSIS

Systems of Measurement of Angle

- (i) **Sexagesimal system:** In this system, the angle is measured in degrees($^{\circ}$).
1 revolution = $360^{\circ} \Rightarrow$ 1 right angle = 90°
 $1^{\circ} = 60'$ (minutes) and $1' = 60''$ (seconds)
- (ii) **Centesimal system:** In this system, the angle is measured in grades.
1 revolution = 400 grades $\Rightarrow$ 1 right angle = 100^g
 $1^g = 100'$ (minutes) and $1' = 100''$ (seconds)
- (iii) **Circular system:** In this system, the angle is measured in radians. The angle subtended by an arc of length equal to the radius of a circle at its centre is said to have a measure of one radian. It is written as 1^c .
 $\therefore$ 1 revolution = $2\pi^c$



Relation between the Units of the Three Systems

When a rotating ray completes one revolution, the measure of angle formed about the vertex is 360° or 400^g or $2\pi^c$ so,

$$360^{\circ} = 400^g = 2\pi^c \text{ (or) } 90^{\circ} \\ = 100^g = \frac{\pi^c}{2}$$

Note:

- (i) $1^{\circ} = \frac{\pi}{180}$ radians = 0.0175 radians (approx)
- (ii) $1^c = \frac{180}{\pi}$ degrees = $57^{\circ}17'44''$ (approx)

Relation between the Ratios

- (i) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$ and $\cot\theta = \frac{1}{\tan\theta}$
- (ii) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

Values of Trigonometric Ratios for Specific Angles

θ	0°	30°	45°	60°	90°
Ratio					
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0

From the above table, we observe that

- (i) $\sin\theta = \cos\theta$, $\tan\theta = \cot\theta$ and $\sec\theta = \operatorname{cosec}\theta$ if $\theta = 45^\circ$
- (ii) $\sin\theta$ and $\tan\theta$ are increasing functions when $0^\circ \leq \theta \leq 90^\circ$
- (iii) $\cos\theta$ is a decreasing function when $0^\circ \leq \theta \leq 90^\circ$.

Trigonometric Ratios of Compound Angles

- (i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and $\sin(A - B) = \sin A \cos B - \cos A \sin B$.
- (ii) $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and $\cos(A - B) = \cos A \cos B + \sin A \sin B$.
- (iii) $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ and $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Also, by taking $A = B$ in the above relations, we get,

- (i) $\sin 2A = 2 \sin A \cos A$
- (ii) $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$.
- (iii) $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Signs of Trigonometric Ratios

- (i) If θ lies in the first quadrant, i.e., $0 < \theta < \pi/2$, then all the trigonometric ratios are taken positive.
- (ii) If θ lies in the second quadrant, i.e., $\pi/2 < \theta < \pi$, then only $\sin\theta$ and $\operatorname{cosec}\theta$ are taken positive and all the other trigonometric ratios are taken negative.
- (iii) If θ lies in the third quadrant, i.e., $\pi < \theta < \frac{3\pi}{2}$, then only $\tan\theta$ and $\cot\theta$ are taken positive and all the other trigonometric ratios are taken negative.

- (iv) If θ lies in the fourth quadrant, i.e., $\frac{3\pi}{2} < \theta < 2\pi$, then only $\cos\theta$ and $\sec\theta$ are taken positive and all the other trigonometric ratios are taken negative.

Heights and Distances

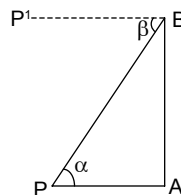
Let AB be a vertical line and PA and P'B be two horizontal lines as shown in the figure.

Let $\angle APB = \alpha$ and $\angle PBP' = \beta$.

Then,

α is called the angle of elevation of the point B as seen from the point P and

β is called the angle of depression of the point P as seen from the point B.



Note: Angle of elevation is always equal to the angle of depression.

Important notes:

- (i) $\sin^2\theta + \cos^2\theta = 1$
- (ii) $\sec^2\theta - \tan^2\theta = 1$
- (iii) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
- (iv) $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$ and $\tan(-\theta) = -\tan\theta$
- (v) Maximum value of $a \sin\theta + b \cos\theta = \sqrt{a^2 + b^2}$
Minimum value of $a \sin\theta + b \cos\theta = -\sqrt{a^2 + b^2}$

Solved Examples

- What is the sexagesimal measure of angle measuring $\frac{\pi^c}{3}$ and 300° ?

Solution: $\therefore \frac{\pi^c}{3} = \frac{180^\circ}{3} = 60^\circ$

$$300^\circ = \frac{9}{10} \times 360 = 270^\circ$$

Hence, the sexagesimal measure of $\frac{\pi^c}{3}$ is 60° and 300° is 270° .

- Find the value of $\sin 75^\circ$.

Solution: We have, $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$\therefore \sin 75^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

3. Find the relation obtained by eliminating θ from the equations $x = r \cos \theta + s \sin \theta$ and $y = r \sin \theta - s \cos \theta$.

☞ **Solution:** Given, $x = r \cos \theta + s \sin \theta$
 $\Rightarrow x^2 = (r \cos \theta + s \sin \theta)^2 = r^2 \cos^2 \theta + 2rs \cos \theta \sin \theta + s^2 \sin^2 \theta$

Also $y = r \sin \theta - s \cos \theta \Rightarrow y^2 = r^2 \sin^2 \theta + s^2 \cos^2 \theta - 2rs \sin \theta \cos \theta$

$\Rightarrow x^2 + y^2 = r^2 (\cos^2 \theta + \sin^2 \theta) + s^2 (\sin^2 \theta + \cos^2 \theta)$
 $= r^2 (1) + s^2 (1) [\because \sin^2 \theta + \cos^2 \theta = 1] = r^2 + s^2$

Hence, the required relation is $x^2 + y^2 = r^2 + s^2$

4. If $\cos(A + B) = 1/2$ and $\sec B = \sqrt{2}$, then find A and B.

☞ **Solution:** Given, $\cos(A + B) = 1/2$
 $\cos(A + B) = \cos 60^\circ \Rightarrow A + B = 60^\circ \rightarrow (1)$
 $\sec B = \sqrt{2} = \sec 45^\circ \Rightarrow B = 45^\circ \rightarrow (2)$
 From (1) & (2), we have $A = 15^\circ$ and $B = 45^\circ$

5. If ABCD is a cyclic quadrilateral, then find the value of $\cos A \cos B - \cos C \cos D$.

☞ **Solution:** Given, ABCD is a cyclic quadrilateral
 $A + C = 180^\circ$ and $B + D = 180^\circ$ (1)
 $-\cos C \cos D = \cos A \cos B - \cos(180^\circ - A)$
 $\qquad\qquad\qquad \cos(180^\circ - B)$
 $= \cos A - \cos B - \cos A (-\cos A) (-\cos B)$
 $= \cos A \cos B - \cos A \cos B = 0$

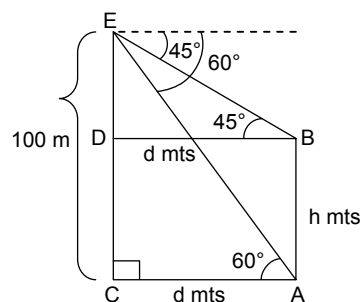
6. Express $\sqrt{3} \cos \theta - \sin \theta$ as a single trigonometric ratio.

☞ **Solution:** $\sqrt{3} \cos \theta - \sin \theta = 2 \left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right)$
 $= 2 (\cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ) = 2 (\cos(\theta + 30^\circ))$
 $\Rightarrow \sqrt{3} \cos \theta - \sin \theta = 2 \cos(\theta + 30^\circ)$

7. Simplify the following expression. $\sin 1^\circ \sin 2^\circ \dots \sin 181^\circ$

☞ **Solution:** We know that, $\sin 180^\circ = 0$
 $\sin 1^\circ \sin 2^\circ \sin 3^\circ \dots \sin 181^\circ = \sin 1^\circ \sin 2^\circ \dots \sin 180^\circ \sin 181^\circ = 0$

8. From the top of a building 100 m high, the angles of depression of the bottom and the top of another building just opposite to it are observed to be 60° and 45° respectively. Find the height of the building.



☞ **Solution:** Let the height of the building be h metres. Let $AC = BD = d$ metres

From $\triangle BDE$, $\tan 45^\circ = \frac{ED}{BD}$

$\Rightarrow 1 = \frac{100 - h}{d}$

$\Rightarrow d = 100 - h$ (1)

From $\triangle ACE$, $\tan 60^\circ = \frac{CE}{AC}$

$\Rightarrow \sqrt{3} = \frac{100}{d} \Rightarrow \sqrt{3} d = 100$

$\Rightarrow \sqrt{3} (100 - h) = 100$ (using (1))

Hence, the height of the tower is, $h = \frac{100(3 - \sqrt{3})}{3}$ m.

PRACTICE EXERCISE 5 (A)

Directions for questions 1 to 45: Select the correct alternative from the given choices.

1. The value of 144° in circular measure, is ____.

(1) $\frac{3\pi^c}{4}$ (2) $\frac{2\pi^c}{3}$
 (3) $\frac{4\pi^c}{5}$ (4) $\frac{5\pi^c}{6}$

2. The value of 60^g in circular measure, is ____.

(1) $\frac{\pi^c}{10}$ (2) $\frac{3\pi^c}{10}$
 (3) $\frac{2\pi^c}{5}$ (4) $\frac{\pi^c}{2}$

3. The value of 60° in centesimal system, is ____.

(1) $\frac{100^g}{3}$ (2) $\frac{200^g}{3}$
 (3) $\frac{140^g}{3}$ (4) $\frac{160^g}{3}$

4. The angle measuring $\frac{\pi^c}{4}$ when expressed in centesimal system, is ____.

(1) 50^g (2) 60^g
 (3) 75^g (4) 100^g

5. The value of $\frac{7\pi^c}{9}$ in sexagesimal measure is ____.

(1) 120° (2) 130°
 (3) 140° (4) 150°

6. A wheel makes 240 revolutions in one minute. The measure of the angle it describes at the centre in 15 seconds is ____.

(1) $60\pi^c$ (2) $120\pi^c$
 (3) $8\pi^c$ (4) π^c

7. If $\tan(89^\circ 16') = x$, then $\tan(269^\circ 16') =$ ____.

(1) $x/2$ (2) $x/3$
 (3) x (4) $2x$

8. The value of 45° in centesimal system, is ____.

(1) 25^g (2) 50^g
 (3) 75^g (4) 100^g

9. $\frac{\sin 36^\circ}{\cos 54^\circ} =$ ____.

(1) 0 (2) 1
 (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

10. $\sec^4\theta - \sec^2\theta$ in terms of $\tan\theta$ is ____.

(1) $\tan^4\theta - \tan^2\theta$
 (2) $\tan^4\theta + \tan^2\theta$
 (3) $\tan^2\theta - \tan^4\theta$
 (4) None of these

11. If $A + B = 60^\circ$, then the value of $\sin A \cos B + \cos A \sin B$ is ____.

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$
 (3) $\frac{\sqrt{3}}{2}$ (4) 1

12. The value of $\sin 30^\circ \cdot \sin 45^\circ \cdot \operatorname{cosec} 45^\circ \cdot \cos 30^\circ$ is ____.

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$
 (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{3}}{4}$

13. If $\tan\theta = \frac{5}{6}$ and $\tan\phi = \frac{1}{11}$, then $\theta + \phi =$ ____.

(1) 30° (2) 45°
 (3) 60° (4) 90°

14. If $\operatorname{cosec}\theta - \cot\theta = x$, then $\operatorname{cosec}\theta + \cot\theta =$ ____.

(1) x (2) $2x$
 (3) $1/x$ (4) $1/2x$

15. The value of $(\sin A - \cos A)^2 + (\sin A + \cos A)^2$ is ____.

(1) 1 (2) 3
 (3) 2 (4) 4

16. If $\sin A = \frac{\sqrt{3}}{2}$ and A is an acute angle, then find the value of $\frac{\tan A - \cot A}{\sqrt{3} + \operatorname{cosec} A}$.

(1) $-\frac{2}{5}$ (2) $\frac{2}{5}$
 (3) $\frac{2}{3+2\sqrt{3}}$ (4) -2

17. The simplified value of

$\operatorname{cosec}^2\alpha \left(1 + \frac{1}{\sec\alpha}\right) \left(1 - \frac{1}{\sec\alpha}\right)$ is ____.

(1) 1 (2) 0
 (3) 2 (4) -1

18. If $\cos\alpha = \frac{12}{13}$ and $\sin\beta = \frac{4}{5}$, then find $\sin(\alpha + \beta)$.

- (1) $\frac{63}{65}$ (2) $\frac{58}{65}$
(3) $\frac{47}{65}$ (4) $\frac{39}{65}$

19. Eliminate θ from the equations, $x = 2 \cos\theta$ and $y = -2 \sin\theta$.

- (1) $x^2 - y^2 = 4$ (2) $x^2 - y^2 = 2$
(3) $x^2 + y^2 = 4$ (4) $x^2 + y^2 = 2$

20. If $3\cos^2 A = \cos 60^\circ + \sin^2 45^\circ$, then find the value of $\sec^2 A$.

- (1) 1 (2) 0
(3) 2 (4) 3

21. If $\frac{1 + \tan\theta}{1 - \tan\theta} = \sqrt{3}$, then find the value of θ .

- (1) 15° (2) 30°
(3) 45° (4) 60°

22. Find the value of $\sin 18^\circ \operatorname{cosec} 54^\circ \operatorname{cosec} 36^\circ \sec 72^\circ \cos 54^\circ \cos 36^\circ$.

- (1) 1 (2) 0
(3) 2 (4) -1

23. Find the relation obtained by eliminating θ from the equation $x = a \cos\theta + b \sin\theta$ and $y = a \sin\theta - b \cos\theta$.

- (1) $x^2 + y^2 = a^2 - b^2$
(2) $x^2 - y^2 = a^2 + b^2$
(3) $x^2 - y^2 = a^2 - b^2$
(4) $x^2 + y^2 = a^2 + b^2$

24. The simplified value of

$$\left(\frac{1 - \sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{1 + \sin\alpha} \right) \left(\sec\alpha + \frac{1}{\cot\alpha} \right) \text{ is } \underline{\hspace{2cm}}.$$

- (1) 0 (2) 1
(3) 2 (4) 3

25. If $\operatorname{cosec}\theta - \cot\theta = 2$, then find the value of $\operatorname{cosec}^2\theta + \cot^2\theta$.

- (1) $\frac{8}{15}$ (2) $\frac{15}{8}$
(3) $\frac{8}{17}$ (4) $\frac{17}{8}$

26. Solve for θ : $\frac{\cos^2\theta}{1 - \sin\theta} - \frac{3}{2} = 0$

- (1) 30° (2) 45°
(3) 60° (4) 90°

27. If $25\sin^2\theta + 10\cos^2\theta = 15$, then find $\cot^2\theta$.

- (1) 1 (2) 2
(3) 3 (4) 4

28. If $\cos\theta + \left(\frac{1}{\sqrt{3}}\right)\sin\theta = \frac{2}{\sqrt{3}}$, then find θ in circular measure.

- (1) $\frac{\pi^2}{2}$ (2) $\frac{\pi^2}{3}$
(3) $\frac{\pi^2}{4}$ (4) $\frac{\pi^2}{6}$

29. $\operatorname{cosec}(7\pi + \theta) \sin(8\pi + \theta) = \underline{\hspace{2cm}}$.

- (1) 0 (2) 1
(3) -1 (4) 2

30. If A and B are two supplementary angles, then $\sec^2 A - \tan^2 B = \underline{\hspace{2cm}}$.

- (1) 0 (2) 1
(3) -1 (4) 2

31. The possibility among the following is _____.

- (1) $\sin^2\theta = 4$ (2) $\cos^2\theta = 8$
(3) $\sin\theta = -8$ (4) $\operatorname{cosec}^2\theta = 4$

32. $\sin 25^\circ + \cos 115^\circ = \underline{\hspace{2cm}}$.

- (1) 1 (2) 0
(3) -1 (4) $\sqrt{3}/2$

33. Find the value of $\cos 420^\circ$.

- (1) 1 (2) 0
(3) -1 (4) $\frac{1}{2}$

34. Which of the following statements are false?

- (a) $\sin^2\theta = 1.44$ (2) $\cos^2\theta = 1.69$
(c) $\operatorname{cosec}^2\theta = 0.25$ (4) All of these

35. If $x = \tan 1^\circ + \tan 2^\circ + \dots + \tan 45^\circ$ and $y = -(\cot 46^\circ + \cot 47^\circ + \dots + \cot 89^\circ)$, then find the value of $x + y$.

- (1) 1 (2) 0
(3) -1 (4) $\sqrt{3}/2$

36. If $\sin 53^\circ 21' = 0.8$, then $\cos 36^\circ 39' = \underline{\hspace{2cm}}$.

- (1) 0.2 (2) 0.4
(3) 0.6 (4) 0.8

37. A tower subtends an angle θ at a point P on the same level as the foot of the tower, and from a point h m above P, the depression of the foot of the tower is α . The height of the tower is (in m)

- (1) $h \tan \theta \tan \alpha$ (2) $h \cot \theta \cot \alpha$
 (3) $h \tan \theta \cot \alpha$ (4) $h \cot \theta \tan \alpha$

38. A vertically straight tree 12 m tall, is broken by wind in such a fashion that its top just touches the ground making an angle with the ground such that the cosine of the angle is 0.6 . At what height from the top did the tree break? (in m)

- (1) $20/3$ (2) $10/3$
 (3) $16/3$ (4) $8/3$

39. The angles of depression of the top and the bottom of a 10 m tall building, observed from the top of a tower

are 30° and 45° respectively. Find the height of the tower. (in m)

- (1) $20 + 5\sqrt{2}$ (2) $12 + 10\sqrt{2}$
 (3) $15 + 5\sqrt{3}$ (4) $18 + 3\sqrt{3}$

40. The angles of depression of two points from the top of the tower are 30° and 60° . If the height of the tower is 30 m, then find the maximum possible distance between the two points. (in m)

- (1) $20\sqrt{3}$ (2) $30\sqrt{3}$
 (3) $40\sqrt{3}$ (4) $50\sqrt{3}$

PRACTICE EXERCISE 5 (B)

Directions for questions 1 to 40: Select the correct alternative from the given choices.

1. If the angles of a quadrilateral are in the ratio $1 : 2 : 3 : 4$, then the smallest angle in the centesimal system is _____.

- (1) 30^g (2) 40^g
 (3) 50^g (4) 60^g

2. $\frac{\pi^c}{5}$ in sexagesimal measure, is _____.

- (1) 18° (2) 36°
 (3) 54° (4) 72°

3. 160^g in circular measure, is _____.

- (1) $\frac{2\pi^c}{5}$ (2) $\frac{3\pi^c}{5}$
 (3) $\frac{4\pi^c}{5}$ (4) π^c

4. 30° in centesimal measure, is _____.

- (1) $\frac{50^g}{3}$ (2) $\frac{100^g}{3}$
 (3) $\frac{160^g}{3}$ (4) $\frac{200^g}{3}$

5. The value of $\frac{6\pi^c}{5}$ in sexagesimal measure is

- (1) 144° (2) 216°
 (3) 240° (4) 120°

6. A wheel makes 12 revolutions per hour. The radians it turns through in 20 minutes is _____.

- (1) $8\pi^c$ (2) $16\pi^c$
 (3) $24\pi^c$ (4) $32\pi^c$

7. The value of 108° in circular measure is _____.

- (1) $5\pi^c/3$ (2) $3\pi^c/5$
 (3) $\pi^c/5$ (4) $6\pi^c/7$

8. $1^\circ =$ _____ (approximately).

- (1) 0.0175^c (2) 57^c
 (3) 45^c (4) 1.75^c

9. The value of $\frac{\tan 29^\circ + \tan 31^\circ}{1 - \tan 29^\circ \tan 31^\circ}$ is _____.

- (1) 1 (2) $\frac{1}{\sqrt{3}}$
 (3) $\sqrt{3}$ (4) 0

10. $\sin^2 85^\circ + \sin^2 5^\circ =$ _____.

- (1) 0 (2) 1
 (3) $1/2$ (4) $2/3$

11. If $\operatorname{cosec}(20^\circ + x) = \sec(50^\circ + x)$, the value of x is

- (1) 10° (2) 20°
 (3) 30° (4) 40°

12. If $\sin \theta = 1/2$ and $0^\circ < \theta < 90^\circ$, then $\cos 2\theta =$ _____.

- (1) 0 (2) 1
 (3) $\frac{1}{2}$ (4) $\frac{\sqrt{3}}{2}$

13. ABC is a right isosceles triangle, right angled at B. Then $\sin^2 A + \cos^2 C =$ _____.

- (1) 0
(2) $\frac{1}{2}$
(3) $\frac{1}{\sqrt{2}}$
(4) 1

14. The value of $\frac{\sin 20^\circ \cos 70^\circ + \cos 20^\circ \sin 70^\circ}{\sin 23^\circ \operatorname{cosec} 23^\circ + \cos 23^\circ \sec 23^\circ}$ is

- (1) 0
(2) 1
(3) 2
(4) $\frac{1}{2}$

15. The value of $4(\sin^4 30^\circ + \cos^4 30^\circ) - 3(\cos^2 45^\circ + \sin^2 90^\circ)$ is ____.

- (1) $-\frac{1}{2}$
(2) -2
(3) 2
(4) $\frac{1}{2}$

16. If $\sin^4 \theta + \cos^4 \theta = \frac{1}{2}$, then the value of $\sin \theta \cos \theta$ is

- (1) $\pm \frac{1}{8}$
(2) $\pm \frac{1}{4}$
(3) ± 1
(4) $\pm \frac{1}{2}$

17. If $\sin \theta = \frac{3}{5}$ and θ is acute, then find the value of

$$\frac{\tan \theta - 2 \cos \theta}{3 \sin \theta + \sec \theta}.$$

- (1) $\frac{15}{61}$
(2) $\frac{17}{61}$
(3) $-\frac{15}{61}$
(4) $-\frac{17}{61}$

18. If $\sin(A + B) = \frac{\sqrt{3}}{2}$ and $\cot(A - B) = 1$, then find A.

- (1) $27\frac{1^\circ}{2}$
(2) $35\frac{1^\circ}{2}$
(3) $52\frac{1^\circ}{2}$
(4) $55\frac{1^\circ}{2}$

19. Find the value of $\cos 15^\circ$.

- (1) $\frac{\sqrt{3}+1}{2\sqrt{2}}$
(2) $\frac{\sqrt{3}-1}{2\sqrt{2}}$
(3) $\frac{\sqrt{2}+1}{2\sqrt{3}}$
(4) $\frac{\sqrt{2}-1}{2\sqrt{3}}$

20. If $\tan \theta - \cot \theta = 7$, find the value of $\tan^3 \theta - \cot^3 \theta$.

- (1) 284
(2) 296
(3) 345
(4) 364

21. If $\tan \theta + \cot \theta = 2$, find the value of $\tan^{1025} \theta + \cot^{1025} \theta$.

- (1) 0
(2) 1
(3) 2
(4) $\frac{1}{2}$

22. If $\sin \theta + \operatorname{cosec} \theta = 2$, find the value of $\cot \theta + \cos \theta$.

- (1) 0
(2) $\frac{1}{2}$
(3) 1
(4) 2

23. If $\sin(A + B) = \frac{\sqrt{3}}{2}$ and $\cot(A - B) = \sqrt{3}$, then find the value of

- (1) 15°
(2) 30°
(3) 10°
(4) 20°

24. $\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$ ____.

- (1) $\frac{1 - \cos A}{\sin A}$
(2) $\frac{1 + \sin A}{\cos A}$
(3) $\frac{1 - \sin A}{\cos A}$
(4) $\frac{1 + \cos A}{\sin A}$

25. If $\sec \theta + \tan \theta = 2$, then find the value of $\sin \theta$, where $0^\circ < \theta < 90^\circ$.

- (1) $\frac{2}{5}$
(2) $\frac{3}{5}$
(3) $\frac{4}{5}$
(4) 1

26. $\sin \theta - \sqrt{3} \cos \theta =$

- (1) $\sin(\theta + 60^\circ)$
(2) $\sin(\theta - 60^\circ)$
(3) $2 \sin(\theta + 60^\circ)$
(4) $2 \sin(\theta - 60^\circ)$

27. Find the value of $\tan(22\frac{1}{2}^\circ)$.

- (1) $\sqrt{2} - 1$
(2) $\sqrt{2} + 1$
(3) $\sqrt{3} - 1$
(4) $\sqrt{3} + 1$

28. $[\sin(x - \pi) + \cos(x - \frac{\pi}{2})] \cdot \cos(x - 2\pi) =$ ____.

- (1) 0
(2) 1
(3) $\frac{1}{\sqrt{2}}$
(4) $\frac{\sqrt{3}}{2}$

29. If ABCD is a cyclic quadrilateral, then $\tan A + \tan C$ is

- (1) 0
(2) 1
(3) -1
(4) 2

30. $[\sin \alpha + \sin(180^\circ - \alpha) + \sin(180^\circ + \alpha)] \operatorname{cosec} \alpha =$ ____.

- (1) 0
(2) 1
(3) 2
(4) 3

31. Which of the following functions is never negative?

- (1) $\sin x^2 + \cos y^2$
(2) $\sin x^4 + \cos y^4$
(3) $\sin |x| - \cos |x|$
(4) $(\sin x + \cos x)^2$

32. If $\operatorname{cosec} \theta = -\sqrt{2}$ and $\tan \theta = -1$, $\cos \theta$ is
 (1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$
 (3) 1 (4) $\frac{1}{2}$
33. If $\sin A = 3/5$ and A is not in the first quadrant, then find $\frac{\cos A + \sin 2A}{\tan A + \sec A}$.
 (1) $16/25$ (2) $17/25$
 (3) $22/25$ (4) $19/25$
34. If $\cos \theta_1 + \cos \theta_2 + \cos \theta_3 = 3$, find $\sin \theta_1 + \sin \theta_2 + \sin \theta_3$.
 (1) 0 (2) 1
 (3) 2 (4) 3
35. $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 90^\circ =$
 (1) $17/2$ (2) $19/2$
 (3) $21/2$ (4) $23/2$
36. $\sin^2 (43^\circ 24') + \sin^2 (46^\circ 36') =$ _____.
 (1) 0.3 (2) 0.5
 (3) 1.2 (4) 1
37. From a point at a height h m above a lake, the angle of elevation of a cloud is α and the angle of depression of its reflection in the lake is β . The height of the cloud above the surface of the lake is
 (1) $\frac{h \cos(\alpha + \beta)}{\cos(\alpha - \beta)} m$ (2) $\frac{h \sin(\alpha + \beta)}{\cos(\alpha - \beta)} m$
 (3) $\frac{h \sin(\alpha - \beta)}{\cos(\alpha + \beta)} m$ (4) $\frac{h \sin(\alpha + \beta)}{\sin(\beta - \alpha)} m$
38. A man on the top of a rock observed a boat coming towards the rock with a uniform speed. It takes 15 minutes for the angle of depression to change from 30° to 60° , then what time will the boat take to reach the shore?
 (1) 5 minutes (2) 10 minutes
 (3) $7\frac{1}{2}$ minutes (4) $2\frac{1}{2}$ minutes
39. If the sun ray's inclination increases from 45° to 60° , the length of the shadow of a tower decreases by 50 m. Find the height of the tower.
 (1) $25(\sqrt{3} + 1)$
 (2) $25(3 + \sqrt{3})$
 (3) $50(\sqrt{3} + 1)$
 (4) $30(3 + \sqrt{3})$
40. AB is a vertical pole. The end A is on the level ground. C is the mid point of AB. P is a point on the level ground such that the portion BC subtends an angle θ at P. If $AP = nAB$, then the value of $\cot \theta$ is
 (1) $\frac{2n^2 + 1}{n}$
 (2) $\frac{n}{2n^2 + 1}$
 (3) $\frac{2n^2 + 1}{2n}$
 (4) $\frac{2n}{2n^2 + 1}$

ANSWER KEYS

PRACTICE EXERCISE 5 (A)

1. 3	2. 2	3. 2	4. 1	5. 3	6. 2	7. 3	8. 2	9. 2	10. 2
11. 3	12. 4	13. 2	14. 3	15. 3	16. 2	17. 1	18. 1	19. 3	20. 4
21. 1	22. 1	23. 4	24. 3	25. 4	26. 1	27. 2	28. 4	29. 3	30. 2
31. 4	32. 2	33. 4	34. 4	35. 1	36. 4	37. 3	38. 1	39. 3	40. 3

PRACTICE EXERCISE 5 (B)

1. 2	2. 2	3. 3	4. 2	5. 2	6. 1	7. 2	8. 1	9. 3	10. 2
11. 1	12. 3	13. 4	14. 4	15. 2	16. 4	17. 4	18. 3	19. 1	20. 4
21. 3	22. 1	23. 1	24. 4	25. 2	26. 4	27. 1	28. 1	29. 1	30. 2
31. 4	32. 1	33. 3	34. 1	35. 2	36. 4	37. 4	38. 3	39. 2	40. 1