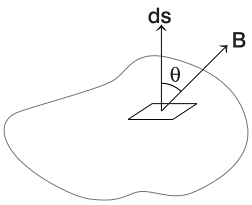
DAY TWENTY THREE

Electromagnetic Induction

Learning & Revision for the Day

- Magnetic Flux (ϕ_B)
- Faraday's Law of Electromagnetic Induction
- Motional Emf
- Rotational Emf
- Self-Induction
- Mutual Induction
- Combination of Inductors
- Eddy Currents

Magnetic Flux (ϕ_B)



The flux associated with a magnetic field is defined in a similar manner to that used to define electric flux. Consider an element of area $d\mathbf{s}$ on an arbitrary shaped surface as shown in figure. If the magnetic field at this element is $\mathbf{B}$, the magnetic flux through the element is,

$$d\phi_B = \mathbf{B}\cdot d\mathbf{s} = Bds\cos\theta$$

Here, $d\mathbf{s}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area ds and θ is the angle between $\mathbf{B}$ and $d\mathbf{s}$ at that element.

Magnetic flux is a scalar quantity. Outward magnetic flux is taken as positive (i.e. $\theta < 90°$) and inward flux is taken as negative (i.e. $\theta > 90°$).

SI unit of magnetic flux is 1 weber (1 Wb).

where, $1\text{ Wb} = 1\text{ T}\times 1\text{ m}^2 = 1\text{ T-m}^2$

Dimensional formula of magnetic flux is $[ML^2T^{-2}A^{-1}]$.

Faraday's Law of Electromagnetic Induction

This law states that, the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

Induced emf, $|e| = \dfrac{d\phi_B}{dt}$

- For N turns, $|e| = N\dfrac{d\phi_B}{dt}$

  However, if we consider the direction of induced emf, then

$$e = -N\frac{d\phi_B}{dt} = -\frac{Nd(BA\cos\theta)}{dt} = \frac{-NBA(\cos\theta_2 - \cos\theta_1)}{\Delta t}$$

- If the given electric circuit is a closed circuit having a total resistance R, then the induced current,

$$I = \frac{e}{R} = -\frac{N}{R}\frac{d\phi_B}{dt}$$

Induced charge, $dq = Idt = -\frac{N}{R} d\phi_B$

and induced power, $P = \frac{e^2}{R} = \frac{N^2}{R}\left(\frac{d\phi_B}{dt}\right)^2$

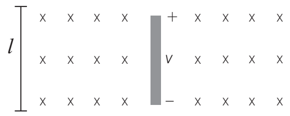
Lenz's Law

The negative sign in Faraday's equations of electromagnetic induction describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with the help of Lenz's law. This law states that **the direction of any magnetic induction effect is such as to oppose the cause of the effect**.

Later, we will see that Lenz's law is directly related to **energy conservation.**

Motional Emf

Let a conducting rod of length l be moving with a uniform velocity **v** perpendicular to a uniform magnetic field **B**, an induced emf is set up.



The magnitude of the induced emf will be

$$|e| = B\,l\,v$$

- If the rod is moving such that it makes an angle θ with the direction of the magnetic field, then

$$|e| = B\,l\,v \sin\theta$$

Hence, for the motion parallel to **B**, the induced emf is zero.
- When a conducting rod moves horizontally, then an induced emf is set up between its ends due to the vertical component of the earth's magnetic field. However, at the magnetic equator, induced emf will be zero, because $B_V = 0$.
- If during landing or taking off, the wings of an aeroplane are along the East-West direction, an induced emf is set up across the wings (due to the effect of B_H).

Motional Emf in a Loop

If a conducting rod moves on two parallel conducting rails, then an emf is induced whose magnitude is $|e| = B\,l\,v$

and the direction is given by the Fleming's right hand rule.

- Induced current, $|I| = \frac{|e|}{R} = \frac{B\,l\,v}{R}$
- Magnetic force, $F_m = BIl = \frac{B^2 l^2 v}{R}$

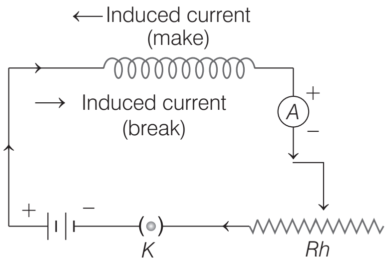
Rotational Emf

Let a conducting rod of length l rotate about an axis passing through one of its ends (that end may be fixed), with an angular velocity ω in a plane perpendicular to the magnetic field B, then an induced emf is set up between the ends of the rod, whose magnitude is given by

$$|e| = \frac{1}{2} B l^2 \omega$$

Self-Induction

Self-induction is the phenomenon due to which an induced emf is set up in a coil or a circuit whenever the current passing through it changes. The induced emf opposes the change that causes it and is thus known as **back emf**.



- Inductance is the inherent property of electrical circuits and is known as the **electrical inertia**.
- An inductor is said to be an **ideal inductor** if its resistance is zero.
- An inductor does not oppose current but opposes changes (growth or decay of current) in the circuit.

Self-Inductance

Flux linked with the coil is

$$N\phi_B \propto I \text{ or } N\phi_B = LI,$$

where the constant L is known as the **coefficient of self-induction** or **self-inductance** of the given coil.

It may be defined as the magnetic flux linked with the coil, when a constant current of 1 A is passed through it.

Induced emf due to self-induction,

$$e = -N\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

SI unit of inductance is **henry**.

Magnetic Potential Energy of an Inductor

- In building, a steady current in an electric circuit, some work is done by the emf of the source, against the self-inductance of the coil.

The work done, $W = \frac{1}{2} LI^2$
- The work done is stored as the magnetic potential energy of that inductor.

Thus, $U = \frac{1}{2} L I^2$

Formulae for Self-Inductance

- For a circular coil of radius R and N turns, the self-inductance,
$$L = \frac{1}{2}\mu_0 \pi N^2 R$$
- For a solenoid coil having length l, total number of turns N and cross-sectional area A,
$$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al \qquad \left[\text{where, } n = \frac{N}{l}\right]$$
- For a toroid of radius R and number of turns N,
$$L = \frac{1}{2}\mu_0 N^2 R$$
- For a square coil of side a and number of turns N,
$$L = \frac{2\sqrt{2}}{\pi}\mu_0 N^2 a$$

Mutual Induction

Mutual induction is the phenomenon due to which an emf is induced in a coil when the current flowing through a neighbouring coil changes.

Mutual Inductance

Mutual inductance of a pair of coils is defined as the magnetic flux linked with one coil, when a constant current of unit magnitude, flows through the other coil.

Mathematically, $N\phi_{B_2} = MI_1$

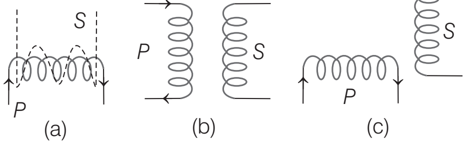
where, M is known as the **mutual inductance** for the given pair of coils.

Induced emf due to mutual inductance,
$$e_2 = -N\frac{d\phi_{B_2}}{dt} = -M\frac{dI_1}{dt}$$

Hence, mutual inductance for a pair of coils is numerically equal to the magnitude of induced emf in one coil when current in the other coil changes at a rate of 1 As^{-1}.

SI unit of mutual inductance M, is **henry.**

Mutual inductance of a pair of coils is maximum, when the two coils are wound on the same frame. However, mutual inductance is negligible when the two coils are oriented mutually perpendicular to each other (see figure). In this context, we define a term **coupling coefficient** k.



Coupling coefficient is given by
$$k = \frac{\text{Magnetic flux linked with secondary coil}}{\text{Magnetic flux developed in primary coil}}$$

It is observed that $0 \le k \le 1$.

For a pair of two magnetically coupled coils of self-inductances L_1 and L_2 respectively, the mutual inductance,
$$M_{12} = M_{21} = M = k\sqrt{L_1 L_2}$$
where, k is the coupling coefficient.

Formulae for Mutual Inductance

- Assuming the coupling coefficient $k = 1$ and medium to be a free space or air. Mutual inductance of a pair of concentric circular coils is
$$M = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}$$
where, r = radius of the coil (of small radius)
and R = radius of the coil (of larger radius).
- For a pair of two solenoid coils, wound one over the other, $M = \dfrac{\mu_0 N_1 N_2 A}{l}$

  For a pair of concentric coplanar square coils,
$$M = \frac{2\sqrt{2}\,\mu_0 N_1 N_2 a^2}{\pi b}$$
where, a = side of the smaller coil and b = side of the larger coil.
- For a given pair of coils, mutually coupled, then according to theorem of reciprocity,
$$M_{12} = M_{21} = M$$

Combination of Inductors

- If two coils of self-inductances L_1 and L_2 are placed quite far apart and are arranged in series, then their equivalent inductance,
$$L_s = L_1 + L_2$$
- If the coils are placed quite close to each other, so as to mutually affect each other, then their equivalent inductance,
$$L_s = L_1 + L_2 \pm 2M$$
Here, M has been written with ± sign depending on the fact whether currents in the two coils are flowing in same sense or opposite sense.
- If two coils of self-inductances L_1 and L_2 are connected in parallel, then equivalent inductance L_p is given by
$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_p = \frac{L_1 L_2}{L_1 + L_2}$$

Eddy Currents

Currents induced in the body of bulk of the conductors due to change in magnetic flux linked to them, are called the eddy currents. The production of eddy currents in a metallic conductor leads to a loss of electric energy in the form of heat energy.

Eddy currents can be minimised by taking the metal (generally soft iron) core in the form of a combination of thin laminated sheets or by slotting process.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The magnetic field in a certain region is given by $B = (40\,\hat{\mathbf{i}} - 18\,\hat{\mathbf{k}})$ gauss. How much flux passes through a loop of area $5\,\text{cm}^2$, in this region, if the loop lies flat on the *xy*-plane?

(a) -900×10^{-9} Wb (b) 900×10^{-9} Wb
(c) Zero (d) 9 Wb

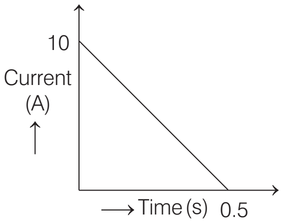
2 The flux linked with a coil at any instant *t* is given by $\phi = 10t^2 - 50t + 250$. The induced emf at $t = 3$ s is

(a) −190 V (b) −10 V (c) 10 V (d) 190 V

3 A coil having *n* turns and resistance $R\ \Omega$ is connected with a galvanometer of resistance $4R\ \Omega$. This combination is moved for time *t* seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is

(a) $\dfrac{W_2 - W_1}{5Rnt}$ (b) $-\dfrac{n(W_2 - W_1)}{5Rt}$
(c) $-\dfrac{(W_2 - W_1)}{Rnt}$ (d) $-\dfrac{n(W_2 - W_1)}{Rt}$

4 In a coil of resistance $100\ \Omega$, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is
→ **JEE Main 2017 (Offline)**



(a) 225 Wb (b) 250 Wb (c) 275 Wb (d) 200 Wb

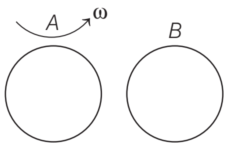
5 A coil has an area of $0.05\,\text{m}^2$ and has 800 turns. After placing the coil in a magnetic field of strength $4 \times 10^{-5}\ \text{Wb m}^{-2}$, perpendicular to the field, the coil is rotated by 90° in 0.1 s. The average emf induced is

(a) zero (b) 0.016 V (c) 0.01 V (d) 0.032 V

6 A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical suface through a contact. Then,

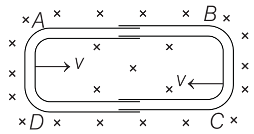
(a) a direct current flows in the ammeter *A*
(b) no current flows through the ammeter *A*
(c) an alternating sinusoidal current flows through the ammeter *A* with a time period $T = \dfrac{2\pi}{\omega}$
(d) a time varying non- sinusoidal current flows through the ammeter *A*

7 Coil *A* is made to rotate about a vertical axis (figure). No current flows in *B*, if *A* is at rest. The current in coil *A*, when the current in *B* (at $t = 0$) is counter clockwise and the coil *A* is as shown at this instant, ($t = 0$), is



(a) constant current clockwise
(b) varying current clockwise
(c) varying current counter clockwise
(d) constant current counter clockwise

8 One conducting U-tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field *B* is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed *v*, then the emf induced in the circuit in terms of *B*, *l* and *v*, where *l* is the width of each tube, will be



(a) *Blv* (b) −*Blv*
(c) zero (d) 2*Blv*

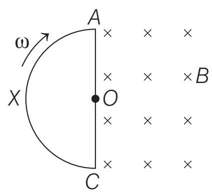
9 A boat is moving due to East in a region where the earth's magnetic field is $5.0 \times 10^{-5}\ \text{NA}^{-1}\text{m}^{-1}$ due to North and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is $1.50\ \text{ms}^{-1}$, the magnitude of the induced emf in the wire of aerial is → **AIEEE 2011**

(a) 1 mV (b) 0.75 mV
(c) 0.50 mV (d) 0.15 mV

10 A helicopter rises vertically with a speed of $10\ \text{ms}^{-1}$. If helicopter has a length of 10 m and the horizontal component of the earth's magnetic field is $1.5 \times 10^{-3}\ \text{Wbm}^{-2}$, the emf induced between the tip of the nose and the tail of the helicopter, is

(a) 0.15 V (b) 125 V
(c) 130 V (d) 5 V

11 The magnetic field as shown in the figure is directed into the plane of paper. *A X C A* is a semi-circular conducting loop of radius *a* with centre *O*. The loop rotates clockwise with velocity ω about an axis fixed at *O* and perpendicular to the plane of the paper. The resistance of the loop is *R*. The induced current is



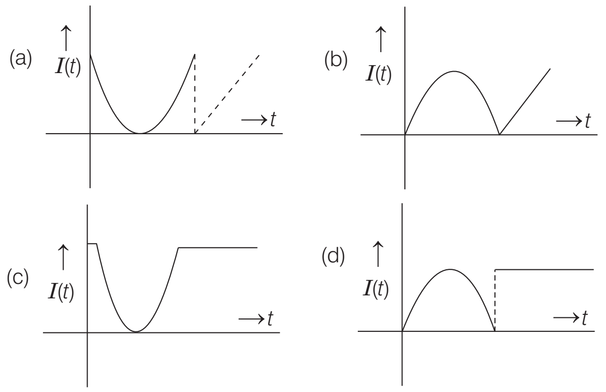
(a) $\dfrac{\omega r^2}{2R}$ (b) $-\dfrac{B\omega r^2}{2R}$ (c) $\dfrac{-2R}{B\omega r}$ (d) $\dfrac{2R}{\omega r^2}$

12 A metal rod of resistance 20 Ω is fixed along the diameter of a conducting ring of radius 0.1 m and lies on the xy-plane. There is a magnetic field $B = |50\,\mathbf{k}|$. The ring rotates with an angular velocity $\omega = 20\ \text{rads}^{-1}$ about its axis. An external resistance of 10 Ω is connected across the centre of the ring and the rim. The current through the external resistance is

(a) $\frac{1}{2}$ A (b) $\frac{1}{3}$ A (c) $\frac{1}{4}$ A (d) zero

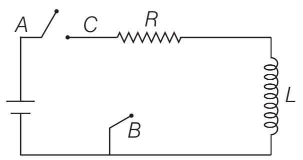
13 Two coils, x and y are kept in close vicinity of each other. When a varying current, $I(t)$ flows through coil x, the induced emf [$V(t)$] in coil Y, varies in the manner shown here. The variation if $I(t)$, with time can then be represented by the graph labelled as graph.

➞ **JEE Main (Online) 2013**



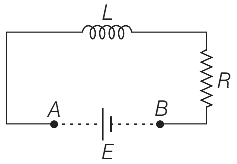
14 In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to

➞ **JEE Main 2014**



(a) $\frac{e}{1-e}$ (b) 1 (c) -1 (d) $\frac{1-e}{e}$

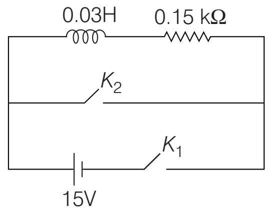
15 An inductor ($L = 100\,\text{mH}$), a resistor ($R = 100\ \Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time, the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 millisecond after the short circuit is



(a) $1/e$ A (b) e A (c) 0.1 A (d) 1 A

16 An inductor ($L = 0.03$ H) and a resistor ($R = 0.15\ \text{k}\Omega$) are connected in series to a battery of 15V emf in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1\text{ms}$, the current in the circuit will be ($e^5 \cong 150$)

➞ **JEE Main 2015**

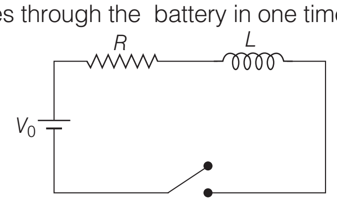


(a) 100 mA (b) 67 mA (c) 6.7 mA (d) 0.67mA

17 An inductor coil stores 32J of magnetic energy and dissipates it as heat at the rate of 320 W when a current of 4 A is passed through it. The time constant of the circuit is

(a) 0.2 s (b) 0.1 s (c) 0.3 s (d) 0.4 s

18 In series R-L circuit, switch is closed at $t = 0$. The charge which passes through the battery in one time constant is



(a) $\frac{V_0 e}{R\tau}$ (b) $\frac{V_0\, e}{Rt}$ (c) $\frac{R\tau}{V_0 e}$ (d) $\frac{V_0\,\tau}{Re}$

19 An uniformly wound solenoid of inductance 1.8×10^{-4}H and resistance 6 Ω is broken into two identical parts. These identical coils are then connected in parallel across a 15 V battery of negligible resistance. The time constant of the circuit is

(a) 3×10^{-5} s (b) 6×10^{-5} s (c) 1.5×10^{-5} s (d) 1.8×10^{-5} s

20 A uniformly wound solenoidal coil of self-inductance 1.8×10^{-4} H and a resistance of 6 Ω is broken up into two identical coils. These identical coils are then connected in parallel across a 120 V battery of negligible resistance. The time constant of the current in the circuit and the steady state current through the battery is

(a) 3×10^{-5} s, 8 A (b) 1.5×10^{-5} s, 8 A
(c) 0.75×10^{-4} s, 4 A (d) 6×10^{-5} s, 2 A

21 A coil is suspended in a uniform magnetic field with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil, it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to

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(a) development of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

Direction (Q. Nos. 22-26) *Each of these questions contains two statements : Statement I and Statement II. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

22 **Statement I** The mutual inductance of two coils is doubled if the self-inductance of the primary and the secondary coil is doubled.

Statement II Mutual inductance is proportional to the square root of self-inductance of primary and secondary coils.

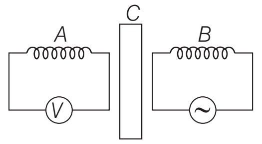
23 **Statement I** The energy stored in the inductor of 2 H, when a current of 10 A flows through it, is 100 J.

Statement II Energy stored in an inductor is directly proportional to its inductance.

24 **Statement I** An artificial satellite with a metal surface, is moving about the earth in a circular orbit. A current is induced when the plane of the orbit is inclined to the plane of the equator.

Statement II The satellite cuts the magnetic field of earth.

25 **Statement I** A coil A is connected to a voltmeter V and the other coil B is connected to an alternating current source. If a large copper sheet C is placed between the two coils, the induced emf in the coil A is reduced.



Statement II Copper sheet between the coils, has no effect on the induced emf in coil A.

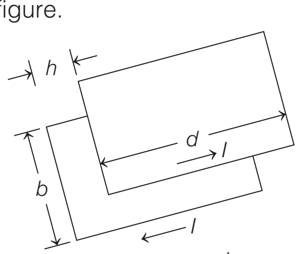
26 **Statement I** When a DC current is made to flow in a soft wire loop of arbitrary shape, it tend to acquire a circular shape.

Statement II Flux linked with a wire loop is maximum when loop is a circle.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The inductance per unit length of a double tape line as shown in the figure.



(a) $\frac{\mu_0 h}{b}$ (b) $\frac{b}{\mu_0 h}$ (c) $\frac{\mu_0 b}{h}$ (d) $\frac{hb}{\mu_0}$

2 An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

(a) 6.5 V (b) 7.4 V (c) 8.2 V (d) 9.3 V

3 A short circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns are quadrupled and the wire radius is halved, the electrical power dissipated in the coil, would be

(a) halved (b) the same (c) doubled (d) quadrupled

4 A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad/s in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed-loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating.

(a) 0.012 W (b) 0.1 W
(c) 0.018 W (d) 0.42 W

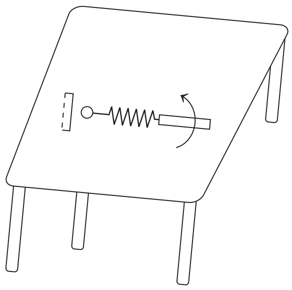
5 A long straight solenoid with cross-sectional radius a and number of turns per unit length n has a current varying with time as I As^{-1}. The magnitude of the eddy current as a function of distance r from the solenoid axis is

(a) $\frac{-n\mu_0 a^2 I}{2r}$ (b) $\frac{\mu_0 In}{2a}$ (c) $\frac{-na^2 I}{2\mu_0 r}$ (d) $\frac{\mu_0 I a}{2n}$

6 A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

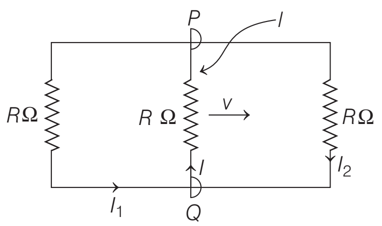
(a) 9.2×10^{-11} Wb (b) 6×10^{-11} Wb
(c) 3.3×10^{-11} Wb (d) 6.6×10^{-9} Wb

7 A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is



(a) $\frac{2B\omega l^3}{2}$ (b) $\frac{3B\omega l^3}{2}$ (c) $\frac{4B\omega l^2}{2}$ (d) $\frac{5B\omega l^2}{2}$

8 A rectangular loop has a sliding connector PQ of length l and resistance $R\ \Omega$ and it is moving with a speed v as shown. The setup is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(a) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

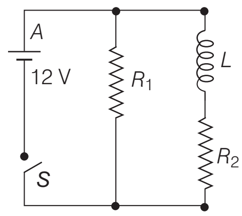
(c) $I_1 = I_2 = I = \frac{Blv}{R}$

(d) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

9 An ideal coil of 10 H is connected in series with a resistance of 5Ω and a battery of 5 V. 2s after the connection is made, the current flowing (in ampere) in the circuit is

(a) $(1-e)$ (b) e (c) e^{-1} (d) $(1-e^{-1})$

10 An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 4\ \Omega$ and $R_2 = 2\ \Omega$ are connected to battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is → **AIEEE 2009**



(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V

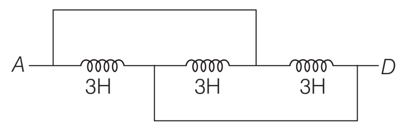
(c) $6(1-e^{-t/0.2})$V (d) $12e^{-5t}$ V

11 In a uniform magnetic field of induction B, a wire in the form of semi-circle of radius r rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is R, the mean power generated per period of rotation is

(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$

(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

12 The inductance between A and D is



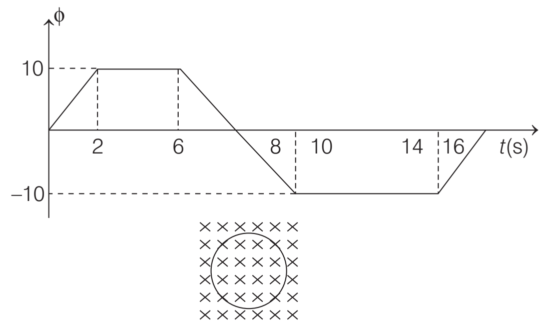
(a) 3.66 H (b) 9 H
(c) 0.66 H (d) 1 H

13 A loop, made of straight edges has six corners at $A(0,0,0), B(L,0,0), C(L,L,0), D(0,L,0), E(0,L,L)$ and $F(0,0,L)$. A magnetic field $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ T is present in the region. The flux passing through the loop $ABCDEFA$ (in that order) is

(a) B_0L^2 Wb (b) $2B_0L^2$ Wb

(c) $\sqrt{2}B_0L^2$ Wb (d) $4B_0L^2$ Wb

14 Magnetic flux in a circular coil of resistance $10\ \Omega$ changes with time as shown in figure. Cross indicates a direction perpendicular to paper inwards. Match the following.

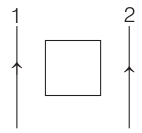


Column I	Column II
A. at $t = 1$ s	1. emf induced is zero
B. at $t = 5$s	2. emf induced in anti-clockwise direction
C. at $t = 9$s	3. emf induced in clockwise direction
D. at $t = 15$s	4. 2 A

Codes

	A	B	C	D		A	B	C	D
(a)	2	1	1	2	(b)	1	2	3	4
(c)	4	3	2	1	(d)	2	4	1	3

15 A square loop is symmetrically placed between two infinitely long current carrying wires in the same direction. Magnitude of currents in both the wires are same.Now match the following two columns.



Column I	Column II
A. Loop is moved towards right	1. Induced current in the loop is clockwise
B. Loop is moved towards left	2. Induced current in the loop is anti-clockwise
C. Wire-1 is moved towards left	3. Induced current in the loop is zero
D. Wire-2 is moved towards right	4. Induced current in the loop is non-zero

Codes

	A	B	C	D		A	B	C	D
(a)	1	2	1	2	(b)	1	3	2	4
(c)	1	2	3	4	(d)	4	2	2	4

ANSWERS

SESSION 1

1 (a)	**2** (b)	**3** (b)	**4** (b)	**5** (b)	**6** (b)	**7** (b)	**8** (d)	**9** (d)	**10** (a)
11 (b)	**12** (b)	**13** (c)	**14** (c)	**15** (a)	**16** (d)	**17** (a)	**18** (d)	**19** (a)	**20** (a)
21 (d)	**22** (a)	**23** (a)	**24** (a)	**25** (c)	**26** (b)				

SESSION 2

1 (a)	**2** (a)	**3** (b)	**4** (c)	**5** (a)	**6** (a)	**7** (d)	**8** (b)	**9** (d)	**10** (d)
11 (b)	**12** (d)	**13** (b)	**14** (a)	**15** (a)					

Hints and Explanations

SESSION 1

1 As loop is in the xy-plane, only the z-component of the magnetic field, is effective.

$B = -18 \text{ gauss} = -18 \times 10^{-4}$ T

$A = 5 \times 10^{-4}\,\text{m}^2$

$\phi = BA\cos 0° = -18 \times 10^{-4} \times 5 \times 10^{-4}$

$= -90 \times 10^{-8}$ Wb

$= -900 \times 10^{-9}$ Wb

2 $\phi = 10t^2 - 50t + 250$

From Faraday's law of electromagnetic induction,

$e = -d\phi/dt$

$\therefore \quad e = -[10 \times 2t - 50]$

$\therefore e|_{t=3\,\text{s}} = -[10 \times 6 - 50] = -10$ V

3 The rate of change of flux or emf induced in the coil is $e = -n\dfrac{d\phi}{dt}$.

$\therefore$ Induced current,

$$I = \frac{e}{R'} = -\frac{n}{R'}\frac{d\phi}{dt} \qquad \text{...(i)}$$

Given, $R' = R + 4R = 5R$,

$d\phi = W_2 - W_1, dt = t$

[here, W_1 and W_2 are flux associated with one turn]

Putting the given values in Eq. (i), we get

$$I = -\frac{n}{5R}\frac{(W_2 - W_1)}{t}$$

4 Induced constant, $I = \dfrac{e}{R}$

Here, e = induced emf $= \dfrac{d\phi}{dt}$

$$I = \frac{e}{R} = \left(\frac{d\phi}{dt}\right)\cdot\frac{1}{R}$$

$d\phi = IRdt$

$\Rightarrow \quad \phi = \int IRdt$

$\therefore$ Here, R is constant

$\therefore \quad \phi = R\int Idt$

$\int I \cdot dt$ = Area under I - t graph

$= \dfrac{1}{2} \times 10 \times 0.5 = 2.5$

$\therefore \quad \phi = R \times 2.5 = 100 \times 2.5$

$= 250$ Wb

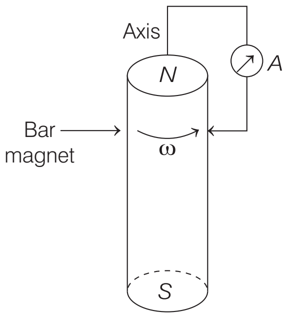
5 As from Faraday's rule,

$$e = N\frac{d\phi}{dt} = \frac{NAdB}{dt}$$

$= 800 \times 0.05 \times \dfrac{4 \times 10^{-5}}{0.1}$

$= 0.016$ V

6 When cylindrical bar magnet is rotated about its axis,no change in flux linked with the circuit takes place, consequently no emf induces and hence,no current flows through the ammeter A.



7 When the current in B(at $t = 0$) is counter clockwise and the coil A is considered above to it. The counter clockwise flow of the current in B is equivalent to North pole of magnet and magnetic field lines are emerging upward to coil A. When coil A start rotating at $t = 0$, the current in A is time varying along clockwise direction by Lenz's rule.

8 Relative velocity $= v - (-v) = 2v = \frac{dl}{dt}$

Now, $e = \frac{d\phi}{dt} \Rightarrow e = \frac{Bl\,dl}{dt}$ $\quad [\because \phi = BA]$

Induced emf, $e = 2Blv$ $\quad \left[\because \frac{dl}{dt} = 2v\right]$

9 $E_{\text{ind}} = B \times v \times l = 5.0 \times 10^{-5} \times 1.50 \times 2$

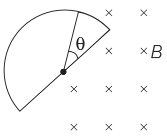
$= 10.0 \times 10^{-5} \times 1.5$

$= 15 \times 10^{-5} = 0.15\text{mV}$

10 $e = Bl\,v = B_H l v$

$= 1.5 \times 10^{-3} \times 10 \times 10 = 0.15 \text{ V}$

11 $A = \frac{\theta}{\pi} \frac{\pi r^2}{2} = \frac{\theta r^2}{2} = \frac{\omega t r^2}{2}$



Flux, $\phi = BA = B\frac{\omega t r^2}{2}$

Emf, $e = -\frac{d\phi}{dt} = -\frac{B\omega r^2}{2}$

$I = \frac{-B\omega r^2}{2R}$

After half rotation $A(t) = \pi r^2 - \frac{\omega t r^2}{2}$ will give same current but in opposite direction.

12 Here, resistance of rod $= 20\,\Omega$,

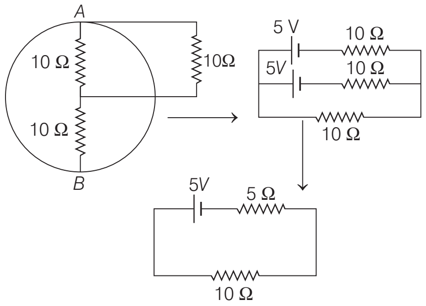
$r = 0.1$ m, $B = 50$ T, acting along the z-axis and $\omega = 20 \text{ rad s}^{-1}$.

Potential difference between the centre of the ring and the rim is

$V = \frac{1}{2} B\omega r^2$

$= \frac{1}{2} \times 50 \times 20 \times (0.1)^2 = 5 \text{ V}$

The equivalent circuit of the arrangement is shown in figures.



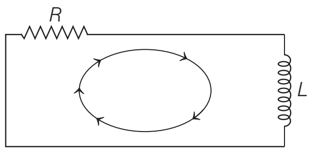
$\frac{1}{R_p} = \frac{1}{10} + \frac{1}{10} \Rightarrow \frac{2}{10} = \frac{1}{5} \Rightarrow R_p = 5\,\Omega$

Current through the external resistance,

$I = \frac{E}{R + r} = \frac{5}{10 + 5} = \frac{1}{3}$ A

13 Firstly, the current decreases due to electrical inertia goes to zero, but due to back emf induced in the easily, the induced current in the coil decreases off a point when back emf is equal to the applied emf induced in the another coil. The value of emf of two current is zero.Then, current is regularly increased, after that time became it is continuously by supplied by the source (variable).

14 After connecting C to B hanging the switch, the circuit will act like L-R discharging circuit.

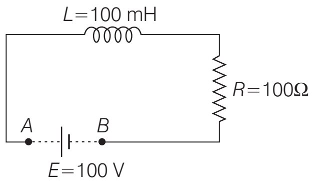


Applying Kirchhoff's loop equation,

$V_R + V_L = 0 \Rightarrow V_R = -V_L$

$\therefore \quad \frac{V_R}{V_L} = -1$

15 This is a combined example of growth and decay of current in an L-R circuit.



The current through circuit just before shorting the battery,

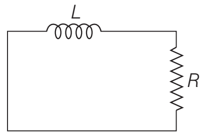
$I_0 = \frac{E}{R} = \frac{100}{100} = 1$ A

[as inductor would be shorted in steady state]

After this decay of current starts in the circuit, according to the equation,

$I = I_0 e^{-t/\tau}$

where, $\quad \tau = L/R$



$I = 1 \times e^{-(1 \times 10^{-3})/(100 \times 10^{-3}/100)} = \left(\frac{1}{e}\right)$ A

$[\because t = 1$ millisecond $= 1 \times 10^{-3}$ s and $L = 100 \times 10^{-3}$ H$]$

16 After long time inductor behaves as short-circuit.

At $t = 0$, the inductor behaves as short-circuited. The current

$I_0 = \frac{E_0}{R} = \frac{15\text{V}}{0.15 \text{ k}\Omega} = 100$ mA

As K_2 is closed, current through the inductor starts decay, which is given at any time t as

$I = I_0 e^{\frac{-tR}{L}} = (100 \text{ mA}) e^{\frac{-t \times 15000}{3}}$

At $\; t = 1$ ms

$I = (100 \text{ mA}) e^{-\frac{1 \times 10^{-3} \times 15 \times 10^3}{3}}$

$I = (100 \text{ mA}) e^{-5} = 0.6737$ mA

or $\; I = 0.67$ mA

17 $U = \frac{LI^2}{2}$ or $32 = \frac{L(4)^2}{2} = L = 4$H

$P = I^2R, R = \frac{320}{(4)^2} = 20\,\Omega$

$\therefore \quad \tau = \frac{L}{R} = \frac{4}{20} = 0.2$ s

18 $I = \frac{V_0}{R}(1 - e^{-t/\tau})$

$Q = \int I\,dt = \int_0^{\tau} \frac{V_0}{R}(1 - e^{-t/\tau})\,dt$

$\Rightarrow Q = \frac{V_0}{R}\tau + \tau(e^{-1} - 1)\frac{V_0}{R} = \frac{V_0 \tau}{Re}$

19 Inductance of each part,

$L_1 = L_2 = \frac{L}{2} = 0.9 \times 10^{-4}$ H

Resistance of each part,

$R_1 = R_2 = \frac{R}{2} = 3\,\Omega$

Time constant,

$\tau = \frac{L_1 || L_2}{R_1 || R_2} = \frac{L_1 L_2}{L_1 + L_2} \times \frac{R_1 + R_2}{R_1 R_2}$

$\tau = \frac{1.8 \times 10^{-4} \times 0.9 \times 10^{-4}}{1.8 \times 10^{-4} + 0.9 \times 10^{-4}} \times \frac{6 + 3}{6 \times 3}$

$= \frac{1.62 \times 10^{-4}}{5.4} = 0.3 \times 10^{-4} = 3 \times 10^{-5}$s

20 Since, the self-inductance in parallel is given by

$$\frac{1}{L_p} = \frac{1}{L} + \frac{1}{L} = \frac{2}{L} \Rightarrow L_p = \frac{L}{2}$$

and $L = \frac{1.8 \times 10^{-14}}{2} = 0.9 \times 10^{-4}$ H

$\therefore \quad L_p = 0.45 \times 10^{-4}$ H

Resistance of each part, $r = 6/2 = 3\,\Omega$

Now, $\frac{1}{r_p} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$\therefore \quad r_p = \frac{3}{2}\ \Omega$

So, the time constant of the circuit is given by

$$\tau = \frac{L_p}{r_p} = \frac{0.45 \times 10^{-4}}{3/2} = 3 \times 10^{-5}\ \text{s}$$

and the steady current is,

$$I = \frac{V}{r_p} = \frac{12}{3/2} = 8\ \text{A}$$

21 According to Lenz's law, electromagnetic induction takes place in the aluminium plate for which eddy current is developed. This causes loss in energy which results in damping of oscillatory motion of the coil.

22 If two coils of inductances L_1 and L_2 are joined together, then their mutual inductance is given by

$$M = k\sqrt{L_1 L_2}$$

It is clear from the relation, if the self-inductance of the primary and the secondary coil is doubled, the mutual inductance of the coils, will also be doubled.

23 The energy stored in the inductor is given by

$$U = \frac{1}{2} L I_0^2 = \frac{1}{2} \times 2 \times (10)^2 = 100\ \text{J}$$

It is obvious that energy stored in the inductor, is directly proportional to its inductance.

24 It is concept of eddy current losses.

25 In the absence of the copper sheet, induced emf will be produced in the coil A due to the mutual induction between the coils A and B. As a result, voltmeter will show deflection depending on the magnitude of the induced emf.

When the copper sheet is placed between the two coils, eddy currents will be setup in the coil. Since, the eddy currents have an opposing effect, the magnetic flux linked with A due to eddy current will always be opposite to that due to the alternating current through B. Thus, induced current will be reduced.

26 Each section of wire repels diametrically opposite section as current flows in opposite direction.

SESSION 2

1 Neglecting end effects of magnetic field, we have

$$B = \frac{\mu_0 I}{b}$$

Flux ϕ per unit length of the plates is

$$\frac{\mu_0 I}{b} \times h \times 1 = \frac{\mu_0 h I}{b}$$

Also, $\phi = LI \Rightarrow L = \frac{\mu_0 h}{b}$

2 Given, length of solenoid $l = 30$ cm $= 30 \times 10^{-2}$ m

Area of cross-section $A = 25\ \text{cm}^2 = 25 \times 10^{-4}\ \text{m}^2$

Number of turns $N = 500$

Current $I_1 = 2.5$ A, $I_2 = 0$

Brief time $dt = 10^{-3}$ s

Induced emf in the solenoid

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) \qquad (\because \phi = BA)$$

Magnetic field induction B at a point well inside the long solenoid carrying current I is

$$B = \mu_0 n I$$

$$\left(\text{where, } n = \text{Number of turns per unit length} = \frac{N}{l}\right)$$

$$e = NA\frac{dB}{dt} = A\frac{d}{dt}\left(\mu_0 \frac{N}{l} I\right)$$

$$= A\frac{\mu_0 N}{l} \cdot \frac{dI}{dt}$$

$$e = 500 \times 25 \times 10^{-4} \times 4 \times 3.14 \times 10^{-7} \times \frac{500}{30 \times 10^{-2}} \times \frac{2.5}{10^{-3}}$$

$$= 6.5\ \text{V}$$

3 The magnitude of the induced voltage is proportional to the rate of change of magnetic flux which, in turns depends on the number of turns in the coil, i.e. $V \propto n$.

So, resistance of a wire is given by

$$R = \frac{\rho l}{\pi r^2} \qquad [A = \pi r^2]$$

i.e. $R \propto \frac{l}{r^2}$

[ρ is a resistivity of a wire]

$$\therefore \quad P = \frac{V^2}{R} \propto \frac{n^2}{l/r^2} \Rightarrow P = \frac{(nr)^2}{l}$$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{l_1}{l_2}\right)$$

or $\frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2 \times \left(\frac{r_2}{r_1}\right)^4$

$$\left[\text{Given, } \frac{n_2}{n_1} = 4 \text{ and } \frac{r_2}{r_1} = \frac{1}{2}\right]$$

So, $\frac{P_2}{P_1} = (4)^2 \times \left(\frac{1}{2}\right)^4 = 16 \times \frac{1}{16} = 1$

4 Average induced emf

$$e_{av} = \frac{1}{T}\int_0^{2\pi} e\,dt = \frac{1}{T}\int_0^{\pi} NBA\,\omega \sin\omega t\,dt$$

$$e_{av} = \frac{1}{T} \cdot NAB\,\omega\left[\frac{\cos\omega t}{\omega}\right]_0^{2\pi}$$

$$= \frac{NBA}{T}[\cos 2\pi - \cos 0°]$$

$$e_{av} = \frac{NBA}{T}[1 - 1] = 0$$

For full cycle average emf, $e_{av} = 0$

Average power loss due to heating

$$= \frac{E_0 I_0}{2} = \frac{0.603 \times 0.0603}{2} = 0.018\ \text{W}$$

5 $B = n\mu_0 I$ and $\oint E \cdot dl = -\frac{d\phi}{dt}$

For $r < a$, $E\,(2\pi r) = -\pi r^2\ n\mu_0 I$

or $E = -\frac{n\mu_0 I r}{2} \qquad (\text{for } r < a)$

where, $I = \frac{dI}{dt}. \qquad (\text{for } r > a)$

$$E(2\pi r) = -\pi a^2 n\mu_0 I \Rightarrow E = \frac{-n\mu_0 a^2 I}{2r}$$

6 Mutual inductance of two coils,

$$M = \frac{\mu_0 R_1^2 \pi R_2^2}{2(R_1^2 \times x^2)^{3/2}}$$

Flux through the bigger coil,

$$M = \frac{\mu_0}{4\pi} \cdot \frac{\pi^2 R_1^2 R_2^2}{(R_1^2 + x^2)}$$

Substituting the values

$$M = \frac{\mu_0 (2)(20 \times 10^{-2})^2}{2[(0.2)^2 + (0.15)^2]} \times \pi(0.3 \times 10^{-2})^2$$

On solving,

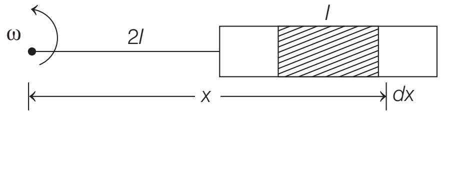
$$M = 9.216 \times 10^{-11}$$

$$= 9.216 \times 10^{-11} \approx 9.2 \times 10^{-11}\ \text{Wb}$$

7 Consider a small element of length dx as shown below emf induced due to whole rod

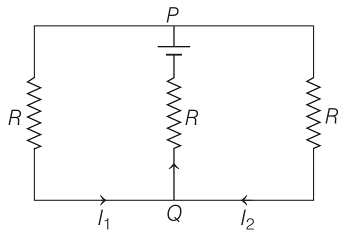
$$e = \int_{2l}^{3l} (\omega x) B dx = B\omega \frac{[(3l)^2 - (2l)^2]}{2}$$

$$= \frac{5Bl^2\omega}{2}$$



8 A moving conductor is equivalent to a battery of emf = vBl (motion emf)

Equivalent circuit, $I = I_1 + I_2$



Applying Kirchhoff's law,

$$I_1 R + IR - vBl = 0 \quad ...(i)$$

$$I_2 R + IR - vBl = 0 \quad ...(ii)$$

Adding Eqs. (i) and (ii), we get

$$2IR + IR = 2vBl$$

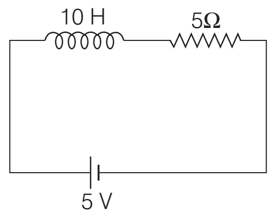
$$I = \frac{2vBl}{3R}$$

$$I_1 = I_2 = \left|\frac{Blv}{3R}\right|$$

9 Rise of current in L-R circuit is given by

$$I = I_0(1 - e^{-t/\tau})$$

where, $I_0 = \frac{E}{R} = \frac{5}{5} = 1\,\text{A}$

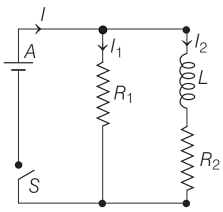


Now, $\tau = \frac{L}{R} = \frac{10}{5} = 2\,\text{s}$

After 2s, i.e. at $t = 2\text{s}$

Rise of current $I = (1 - e^{-1})\,\text{A}$

10 $I_1 = \frac{E}{R_1} = \frac{12}{4} = 3\,\text{A}$



$\because$ Potential drop $= E - I_2 R_2$

$$I_2 = I_0(1 - e^{-T/t_c})$$

[current as a function of time]

$$\Rightarrow \quad I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\,\text{A}$$

and $$t_c = \frac{L}{R_2} = \frac{400 \times 10^{-3}}{2} = 0.2$$

$$I_2 = 6(1 - e^{-t/0.2})$$

$$\Rightarrow \quad I_2 = 6(1 - e^{-5t})$$

Potential drop across

$$L = E - R_2 I_2$$

$$= 12 - 2 \times 6(1 - e^{-5t})$$

$$= 12e^{-5t}\,\text{V}$$

11 The flux associated with coil of area A and magnetic induction B is

$$\phi = BA\cos\theta = \frac{1}{2}B\pi r^2 \cos\omega t \quad \left[\because A = \frac{1}{2}\pi r^2\right]$$

$$\therefore \quad e_{\text{induced}} = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}\left(\frac{1}{2}B\pi r^2 \cos\omega t\right)$$

$$= \frac{1}{2}B\pi r^2 \omega \sin\omega t$$

$$\therefore \text{ Power, } P = \frac{e^2_{\text{induced}}}{R} \quad [\because P = V^2/R]$$

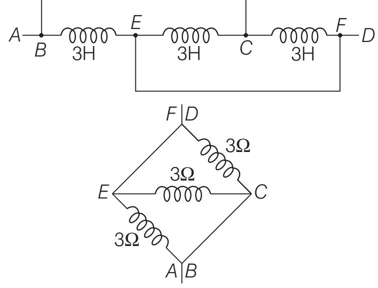
$$= \frac{B^2\pi^2 r^4 \omega^2 \sin^2\omega t}{4R}$$

Hence, $P_{\text{mean}} = \langle P \rangle$

$$= \frac{B^2\pi^2 r^4\omega^2}{4R} \cdot \frac{1}{2} \quad \left[\because \langle \sin^2\omega t \rangle = \frac{1}{2}\right]$$

$$= \frac{(B\pi r^2\omega)^2}{8R}$$

12

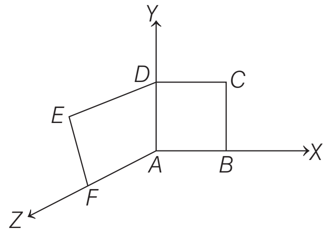


Here, inductors are in parallel.

$$\therefore \quad \frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

or $\quad L = 1\,\text{H}$

13 Also, the magnetic flux linked with uniform surface of area A in uniform magnetic field is given by



$$\phi = \mathbf{B} \cdot \mathbf{A}$$

$$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2$$

$$= (L^2\,\hat{\mathbf{k}} + L^2\,\hat{\mathbf{i}})$$

and $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})\text{T}$

Now,

$$\phi = \mathbf{B} \cdot \mathbf{A}$$

$$= B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}}) \cdot (L^2\,\hat{\mathbf{k}} + L^2\,\hat{\mathbf{i}})$$

$$= 2B_0 L^2\,\text{Wb}$$

14 A → 2, B → 1, C → 1, D → 2

A. At $t = 1\,\text{s}$, flux is increasing in the inward direction, hence induced emf will be in anti-clockwise direction.

B. At $t = 5\,\text{s}$, there is no change in flux,so induced emf is zero.

C. At $t = 9\,\text{s}$, flux is constant, hence induced emf will be zero.

D. At $t = 15\,\text{s}$, flux is decreasing in upward direction, so induced emf will be in anti-clockwise direction.

15 A - 1, B - 2, C - 1, D - 2

When loop is moved towards right, upward flux increases, so current in loop is clockwise. When loop is moved towards left, downward flux increases, so current in loop is anti-clockwise. When wire 1 is moved left, upward flux through loop increases, so current is clockwise. When wire 2 is moved right, downward flux increases, so current is anti-clockwise.