General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38 questions.** All questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE Sections – Section A, B, C, D and E.***

(iii) *In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – C, 2 questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

(This section comprises of 20 multiple choice questions (MCQs) of 1 mark each.)

(20 × 1 = 20)

1. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is

(A) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

(B) $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

(C) $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

(D) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

2. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

(A) $(0, 2)$

(B) $(-\infty, 2]$

(C) $[1, 2]$

(D) $[2, \infty)$

3. If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)\,dx$ is

(A) $\int_0^{2a} f\left(\frac{x}{2}\right) dx$

(B) $\int_0^{a} f(x)\,dx$

(C) $2\int_a^{0} f(x)\,dx$

(D) $2\int_0^{a} f(x)\,dx$

4. If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is

(A) -8

(B) 0

(C) 6

(D) 8

5. If $y = \sin^{-1}x, -1 \le x \le 0$, then the range of y is

(A) $\left(\frac{-\pi}{2}, 0\right)$

(B) $\left[\frac{-\pi}{2}, 0\right]$

(C) $\left[\frac{-\pi}{2}, 0\right)$

(D) $\left(\frac{-\pi}{2}, 0\right]$

6. If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is

(A) $\frac{-\pi}{3}$ only

(B) $\frac{\pi}{3}$ only

(C) $\frac{\pi}{6}$

(D) $\pm\frac{\pi}{3}$

7. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\overline{E}/\overline{F})$ is

(A) $\frac{P(\overline{E})}{P(\overline{F})}$

(B) $1 - P(\overline{E}/F)$

(C) $1 - P(E/F)$

(D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$

8. Which of the following can be both a symmetric and skew-symmetric matrix ?

(A) Unit Matrix

(B) Diagonal Matrix

(C) Null Matrix

(D) Row Matrix

9. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point (4, –3, 7) is :

(A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$

(B) $x = 3t + 4, y = t + 3, z = 2t + 7$

(C) $x = 3t + 4, y = t - 3, z = 2t + 7$

(D) $x = 3t + 4, y = -t + 3, z = 2t + 7$

10. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4\,AB + 3(AB + BA) - 4\,BA$, where A and B are both matrices of order 2×2. It is known that $A \neq B \neq I$ and $A^{-1} \neq B$.

Their answers are given as :

Abhay : $6\,AB$

Bina : $7\,AB - BA$

Chhaya : $8\,AB$

Devesh : $7\,BA - AB$

Who answered it correctly ?

(A) Abhay
(B) Bina
(C) Chhaya
(D) Devesh

11. A cylindrical tank of radius 10 cm is being filled with sugar at the rate of $100\,\pi$ cm^3/s. The rate, at which the height of the sugar inside the tank is increasing, is :

(A) 0.1 cm/s
(B) 0.5 cm/s
(C) 1 cm/s
(D) 1.1 cm/s

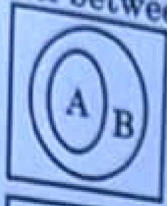
12. Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p} + \vec{q})$ will be a unit vector for what value of α ?

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{2}$
(D) $\frac{2\pi}{3}$

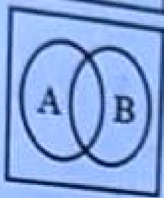
13. The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point ?

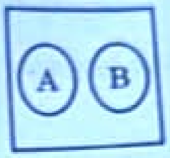
(A) $(1, -5, 6)$
(B) $(1, 5, 6)$
(C) $(1, -5, -6)$
(D) $(-1, -5, 6)$

14. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ?

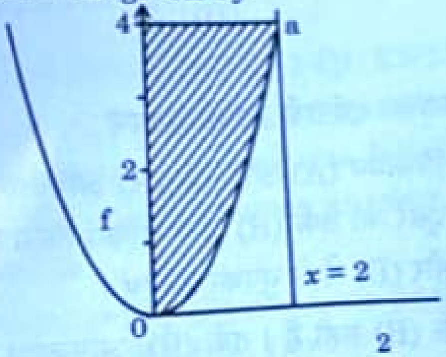
(A) 

(B) 

(C) 

(D) 

15. The area of the shaded region (figure) represented by the curves $y = x^2, 0 \leq x \leq 2$ and y-axis is given by



(A) $\int_0^2 x^2\, dx$

(B) $\int_0^2 \sqrt{y}\, dy$

(C) $\int_0^4 x^2\, dx$

(D) $\int_0^4 \sqrt{y}\, dy$

16. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ?

(A) The objective function maximizes the difference of the profit earned from products X and Y.

(B) The objective function measures the total production of products X and Y.

(C) The objective function maximizes the combined profit earned from selling X and Y.

(D) The objective function ensures the company produces more of product X than product Y.

17. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ?

(A) $A = B$

(B) $AB = BA$

(C) $A = 0$ or $B = 0$

(D) $A = I$ or $B = I$

18. If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then $(p - q)$ is

(A) 0

(B) 1

(C) 2

(D) 3

ASSERTION – REASON BASED QUESTIONS

Direction : Question number **19** and **20** are Assertion (A) and Reason (R) based questions. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A)** : $A = \text{diag}\,[3\ \ 5\ \ 2]$ is a scalar matrix of order 3×3.

Reason (R) : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

20. **Assertion (A)** : Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R) : The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.

SECTION – B

(This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.)

$(5 \times 2 = 10)$

21\. (a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

OR

(b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.

22\. Evaluate : $\int_0^{\frac{\pi}{4}} \sqrt{1+\sin 2x}\, dx$

23\. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on R.

24\. If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha} = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear.

25\. (a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$.

OR

(b) If $f(x) = \begin{cases} 2x-3, & -3 \le x \le -2 \\ x+1, & -2 < x \le 0 \end{cases}$

Check the differentiability of $f(x)$ at $x = -2$.

P.T.O.

SECTION – C

(This section comprises of 6 Short Answer (SA) type questions of 3 marks each.)

(6 × 3 = 18)

26. (a) Solve the differential equation $2(y + 3) - xy \dfrac{dy}{dx} = 0$; given $y(1) = -2$.

OR

(b) Solve the following differential equation :

$$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2.$$

27. Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n ∈ N." Find whether R is reflexive, symmetric and transitive or not.

28. Solve the following linear programming problem graphically :

Minimise $Z = x - 5y$

subject to the constraints :

$x - y \geq 0$

$-x + 2y \geq 2$

$x \geq 3,\ y \leq 4,\ y \geq 0$

29. (a) If $y = \log\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^2$, then show that $x(x + 1)^2 y_2 + (x + 1)^2 y_1 = 2$.

OR

(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that $\dfrac{dy}{dx} = \dfrac{-1}{(1+x)^2}$.

30. (a) A die with number 1 to 6 is biased such that $P(2) = \dfrac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B : $A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

P.T.O.

31. Find : $\int \frac{1}{x} \sqrt{\frac{x+a}{x-a}}\, dx$.

SECTION – D

(This section comprises of 4 Long Answer (LA) type questions of 5 marks each.)

(4 × 5 = 20)

32. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x – axis and the ordinates $x = -2$ and $x = 2$.

33. Find : $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)}\, dx$.

34. (a) Find the shortest distance between the lines :

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and}$$
$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

OR

(b) Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\,(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA′. Find the foot of perpendicular from point A on the line l.

35. (a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations :

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

OR

(b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}.

Hence, solve the system of linear equations :

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

SECTION - E

(This section comprises of 3 case study based questions of 4 marks each.)

(3 × 4 = 12)

36. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge y}, x \in S, y \in J\}$.

Based on the above, answer the following :

(i) How many relations can be there from S to J ? **1**

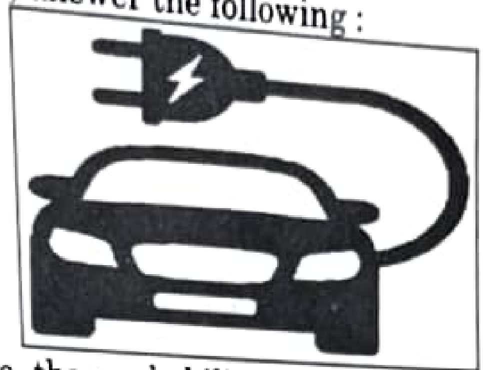
(ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective. **1**

(iii) (a) How many one-one functions can be there from set S to set J ? **2**

OR

(iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. **2**

Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated). Based on the above, answer the following :



(i) (a) What is the probability that a randomly selected car is an electric car ? 2

OR

(i) (b) What is the probability that a randomly selected car is a petrol car ? 2

(ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet ? 1

(iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ? 1

A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following :

(i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. 2

(ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. 2