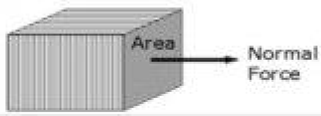
Stress and strain

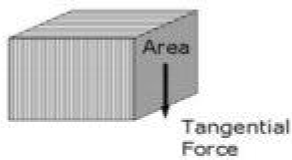
Stress = Force / Area

Normal stress



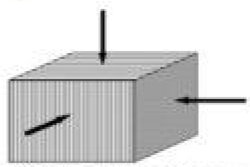
Normal stress = $\dfrac{\text{Normal force}}{\text{area}}$ $\sigma_n = \dfrac{F_n}{A}$

Shear stress



Shear stress = $\dfrac{\text{tangential force}}{\text{area}}$ $\sigma_t = \dfrac{F_t}{A}$

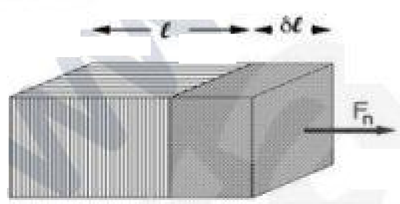
Bulk stress



Bulk Stress = $\dfrac{\text{normal inward force}}{\text{area}}$ $\sigma_B = P$

$$\textit{Tension strain}(e) = \frac{\Delta L}{L} = \frac{\textit{Change in length}}{\textit{Initial length}}$$

Normal strain

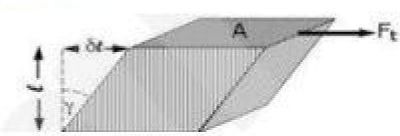


Normal Strain = $\dfrac{\text{change in normal length}}{\text{original normal length}}$

$\varepsilon_n = \dfrac{\delta \ell}{\ell}$

Since strain is m/m it is dimensionless.

Shear strain



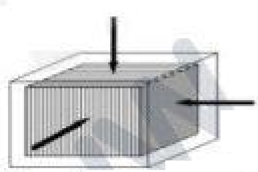
Shear Strain = $\dfrac{\text{tangential dislacement}}{\text{original normal length}}$

$\varepsilon_t = \dfrac{\delta t}{\ell} = \gamma(rad)$

Note 1: the volume of the solid is not changed by shear strain.

Note 2: the angle is is radians, not degrees.

Bulk strain



Bulk Strain = $\dfrac{-(\text{change in volume})}{\text{original volume}}$

$\varepsilon_B = \dfrac{-\delta V}{V}$

Brinell Hardness Number (BHN)

$$HB = \frac{\text{Load (kgf)}}{\text{Surface Area of Indentation (mm}^2\text{)}} = \frac{P}{\frac{\pi D}{2}\left(D - \sqrt{D^2 - d^2}\right)}$$

where D: Diameter of the ball indenter,

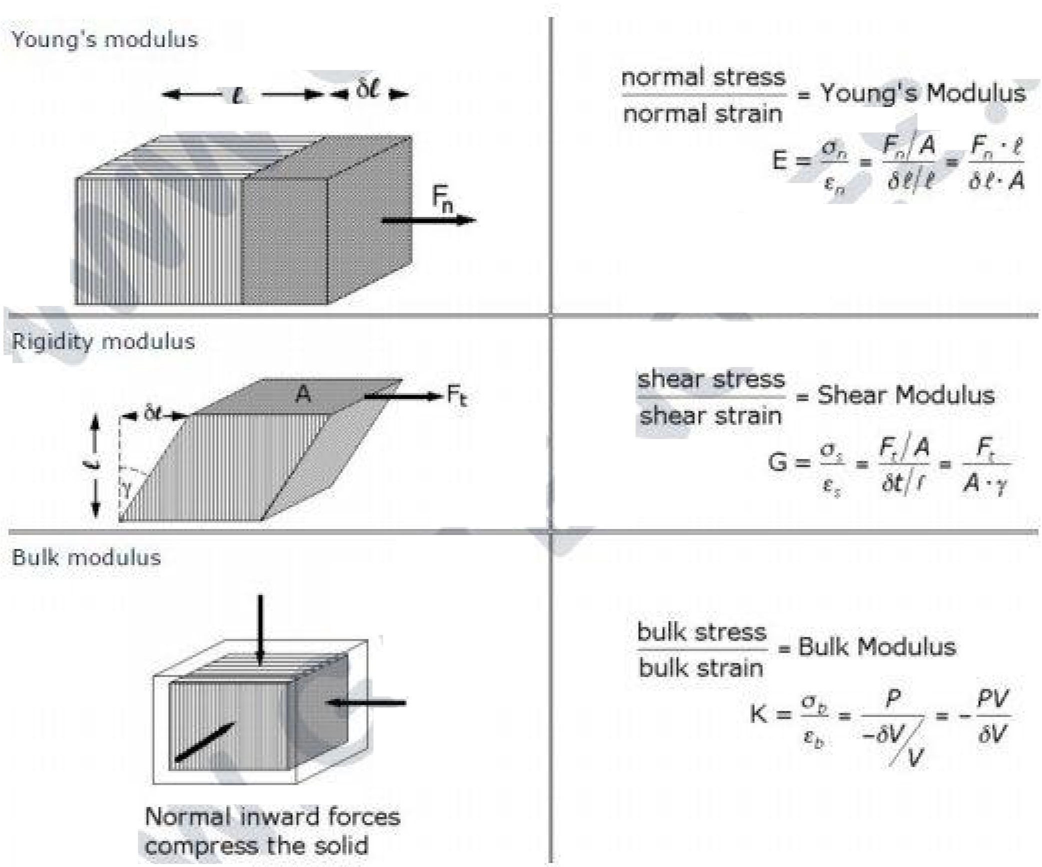
d: Diameter at the rim of the permanent impression,

P: Load.

Elastic constants:

$$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic Modulus} = \frac{\sigma}{\varepsilon}$$

where, P = Standard load, D = Diameter of steel ball, and d = Diameter of the indent.



Axial Elongation of Bar Prismatic Bar Due to External Load

$$\Delta = \frac{PL}{AE}$$

Elongation of Prismatic Bar Due to Self Weight

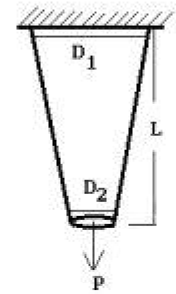
$$\Delta = \frac{PL}{2AE} = \frac{\gamma L^2}{2E}$$

Where γ is specific weight

Elongation of Tapered Bar

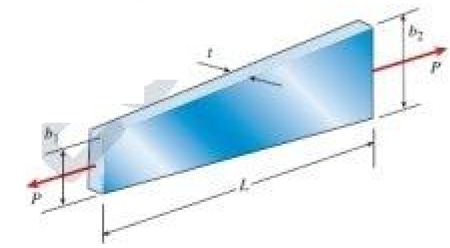
- **Circular Tapered**

$$\Delta = \frac{4PL}{\pi D_1 D_2 E}$$



- **Rectangular Tapered**

$$\Delta = \frac{PL\log_e\left(\frac{B_2}{B_1}\right)}{E.t(B_2 - B_1)}$$



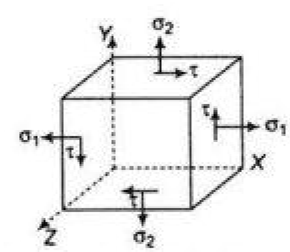
Stress Induced by Axial Stress and Simple Shear

- **Normal stress**

$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta + \tau \sin 2\theta$$

- **Tangential stress**

$$\sigma_1 = -\left(\frac{\sigma_1 + \sigma_2}{2}\right)\sin 2\theta + \tau \cos 2\theta$$



Induced stress body diagram

Principal Stresses and Principal Planes

- Major principal stress

$$\sigma_1' = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{2} + \tau^2}$$

- Major principal stress

$$\sigma_2' = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{2} + \tau^2}$$

$$\tan 2\theta_p = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$\sigma_1' + \sigma_2' = \sigma_1 + \sigma_2$$

$$when\ 2\theta_p = 0$$

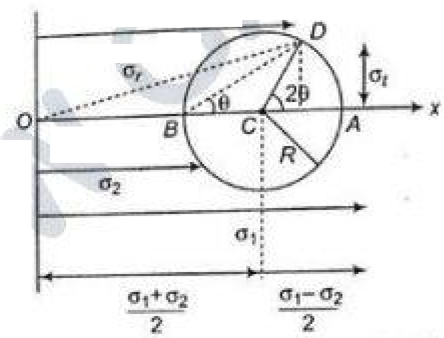
$$\Rightarrow \sigma_1' = \sigma_1\ and\ \sigma_2' = \sigma_2$$

Principal Strain

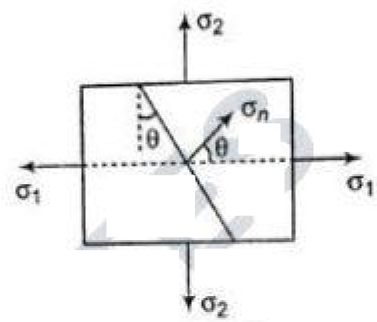
$$\varepsilon_I = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$\varepsilon_{II} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

Mohr's Circle-



Mohr's circle for plane stress and strain



Different stress diagram

STRAIN ENERGY
Energy Methods:

(i) Formula to calculate the strain energy due to axial loads (tension):

U = ∫ P^2 / (2AE)dx limit 0 toL

Where, P = Applied tensile load, L = Length of the member , A = Area of the member, and E = Young'smodulus.

(ii) Formula to calculate the strain energy due tobending:

U = ∫ M^2 / (2EI) dx limit 0 toL

Where, M = Bending moment due to applied loads, E = Young's modulus, and I = Moment of inertia.

(iii) Formula to calculate the strain energy due totorsion:

U = ∫ T^2 / (2GJ) dx limit 0 toL

Where, T = Applied Torsion , G = Shear modulus or Modulus of rigidity, and J = Polar moment ofinertia

(iv) Formula to calculate the strain energy due to pureshear:

$$U = K \int V^2 / (2GA)\ dx \quad \text{limit 0 to L}$$

Where, V= Shearload

G = Shear modulus or Modulus of rigidity

A = Area of cross section.

K = Constant depends upon shape of cross section.

(v) Formula to calculate the strain energy due to pure shear, if shear stress isgiven:

$$U = \tau^2 V / (2G)$$

Where, τ = ShearStress

G = Shear modulus or Modulus of rigidity

V = Volume of the material.

(vi) Formula to calculate the strain energy , if the moment value isgiven:

$$U = M^2 L / (2EI)$$

Where, M = Bending moment

L = Length of the beam

E = Young'smodulus

I = Moment ofinertia

(vii) Formula to calculate the strain energy , if the torsion moment value isgiven:

$$U = T^2 L / (2GJ)$$

Where, T = AppliedTorsion

L = Length of the beam

G = Shear modulus or Modulus of rigidity

J = Polar moment of inertia

(viii) Formula to calculate the strain energy, if the applied tension load isgiven:

$$U = P^2L / (2AE)$$

Where,

P = Applied tensile load.

L = Length of the member

A = Area of the member

E = Young's modulus.

(ix) Castigliano's first theorem:

$$\delta = \partial U / \partial P$$

Where, δ = Deflection, U= Strain Energy stored, and P = Load

(x) Formula for deflection of a fixed beam with point load at centre:

$$\delta = - wl^3 / 192EI$$

This defection is ¼ times the deflection of a simply supported beam.

(xi) Formula for deflection of a fixed beam with uniformly distributed load:

$$\delta = - wl^4 / 384EI$$

This defection is 5 times the deflection of a simply supported beam.

(xii) Formula for deflection of a fixed beam with eccentric point load:

$$\delta = - wa^3b^3 / 3 EIl^3$$

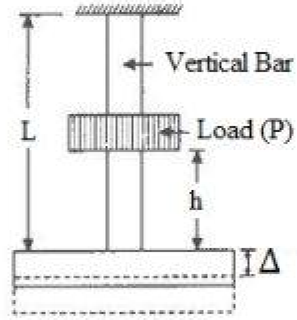
Stresses due to

- **Gradual Loading:-**

$$\sigma = \frac{F}{A}$$

- **Sudden Loading:-**

$$\sigma = 2F/A$$



- **Impact Loading:-**

$$\sigma = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2AEh}{PL}}\right)$$

Deflection, $$If\ \Delta_{st} = \frac{PL}{AE}$$

$$\Delta = \Delta_{st} + \sqrt{(\Delta_{st})^2 + 2h\Delta_{st}}$$

$$if\ h\ is\ very\ small\ then\ \Delta \approx \sqrt{2h\Delta_{st}}$$

Thermal Stresses:-

$$\Delta L = \alpha L \Delta T$$
$$\sigma = \alpha E \Delta T$$

When bar is not totally free to expand and can be expand free by "a"

$$\sigma = E\alpha\Delta T - \frac{aE}{L}$$

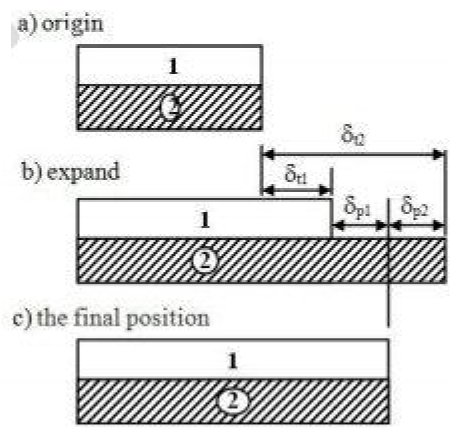
Temperature Stresses in Taper Bars:-

$$\text{Stress} = \alpha L \Delta T = \frac{4PL}{\pi d_1 d_2 E}$$

Tempertaure Stresses in Composite Bars

$$\delta t_2 = \delta_{t1} + \delta_{p1} + \delta_{p2}$$
$$\delta t_2 - \delta_{t1} = \delta_{p1} + \delta_{p2}$$
$$\Delta t\,(\alpha_1 L_1 - \alpha_2 L_2) = P\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right)$$

$$P = \frac{\Delta t\,(\alpha_2 - \alpha_1)}{\left(\frac{1}{A_1E_1} + \frac{1}{A_2E_2}\right)}$$



Hooke's Law (Linear elasticity):

Hooke's Law stated that within elastic limit, the linear relationship between simple stress and strain for a bar is expressed by equations.

$$\sigma \propto \varepsilon,$$

$$\sigma = E\varepsilon$$

$$\frac{P}{A} = E\frac{\Delta L}{L}$$

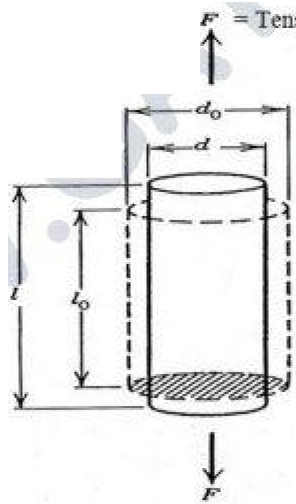
Where, E = Young's modulus of elasticity

P = Applied load across a cross-sectional area

Δl = Change in length

l = Original length

Poisson's Ratio:



$$\text{Poisson Ratio} = \frac{\text{Lateral Strain}}{\text{Linear Strain}}$$

$$\text{Linear strain} = \frac{\Delta l}{l} = \frac{l - l_o}{l_o}$$

$$\text{Lateral Strain} = \frac{\Delta d}{d_o} = \frac{d - d_o}{d_o}$$

Volumetric Strain:

$$\text{Volumetric Strain} = \frac{\text{Change in Volume}(\delta V)}{\text{Original Volume}(V)}$$

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

Futher Volumetric strain $= \epsilon_1 + \epsilon_2 + \epsilon_3$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{E} - \frac{2\mu(\sigma_1 + \sigma_2 + \sigma_3)}{E}$$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}$$

hence the

$$\boxed{\text{Volumetric strain} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}}$$

Relationship between E, G, K and μ:

- **Modulus of rigidity:-**

$$Modulus\ of\ rigidity, G = \frac{shear\ stress}{shear\ stress} = \frac{\tau}{\gamma}$$

- **Bulk modulus:-**

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

$$K = -\frac{dP}{\frac{dV}{V}} = -V\frac{dP}{dV}$$

Negative sign shows decrease in volume.

$$E = 2G(1+\mu) = 3K(1-2\mu)$$

$$E = \frac{9KG}{G+3K}$$

$$\mu = \frac{3K-2G}{G+3K}$$

- **Shear Stress in Rectangular Beam**

Compound Stresses

- **Equation of Pure Bending**

$$\frac{\sigma_a}{y} = \frac{M}{I} = \frac{E}{R}$$

- **Section Modulus**

$$z = \frac{I}{y_{max}} \Rightarrow \frac{M}{I} = \frac{\sigma}{y}$$

$$M = \sigma_{max}\frac{I}{y_{max}} \Rightarrow M = \sigma_{max} x z$$

- **Shearing Stress**

$$\tau = \frac{VA\bar{y}}{Ib}$$

Where,

V = Shearing force

$A\bar{y}$=First moment of area

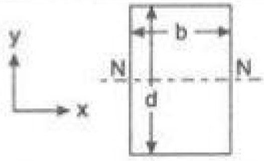
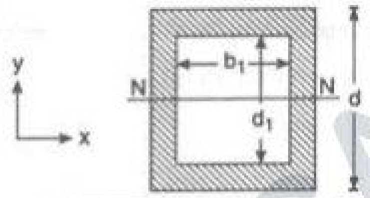
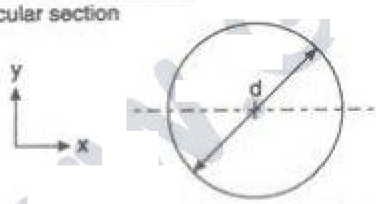
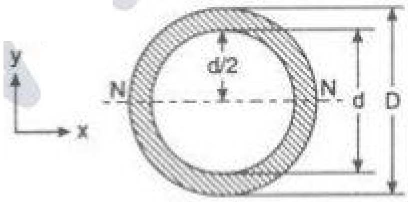
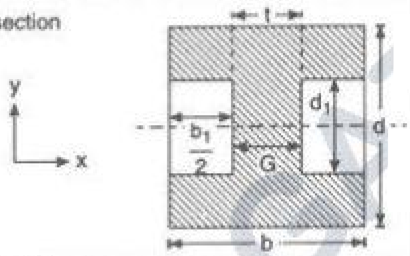
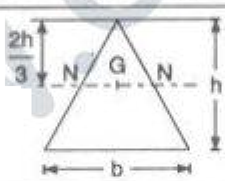
$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$

$$\tau_{max} = 1.5\tau_{avg}$$

- **Shear Stress Circular Beam**

$$\tau_{max} = \frac{4V}{3A} = \frac{4}{3}\tau_{av}$$

Moment of Inertia and Section Modulus

Table 11.2.1			
Type of section	**Moment of Inertia**	y_{max}	**Section modulus (Z)**
Rectangle or paralleogram 	$I_{xx} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{bd^2}{6}$ $Z_{yy} = \frac{db^2}{6}$
Hollow rectangular section 	$I_{xx} = \frac{bd^3}{12} - \frac{b_1d_1^3}{12}$ $I_{yy} = \frac{db^3}{12} - \frac{d_1b_1^3}{12}$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{1}{6d}(bd^3 - b_1d_1^3)$ $Z_{yy} = \frac{1}{6b}(db^3 - d_1b_1^3)$
Circular section 	$I_{xx} = \frac{p}{64}d^4$ $I_{yy} = \frac{p}{64}d^4$	$\frac{d}{2}$ $\frac{d}{2}$	$Z_{xx} = \frac{p}{32}d^3$ $Z_{yy} = \frac{p}{32}d^3$
Hollow curcular section 	$I_{xx} = I_{yy} = I$ $I_{yy} = \frac{p}{64}(D^4 - d^4)$	$\frac{D}{2}$	$Z_{xx} = Z_{yy} = Z$ $Z = \frac{p}{32D}(D^4 - d^4)$
I-section 	$I_{xx} = \frac{bd^3}{12} - \frac{b_1d_1^3}{12}$ $I_{yy} = \frac{db^3}{12} - \frac{d_1b_1^3}{12}$ or $I_{xx} = \frac{1}{12}(bd^3 - (b - t)d_1^3)$	$\frac{d}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{1}{6d}(bd^3 - b_1d_1^3)$ $Z_{yy} = \frac{1}{6b}(db^3 - d_1b_1^3)$
Triangle 	$I_G = \frac{bh^3}{36}$	$\frac{2}{3}h$	$Z_G = \frac{bh^2}{24}$

- **Direct Stress**

$$\sigma = \frac{P}{A}$$

where P = axial thrust, A = area of cross-section

- **Bending Stress** $\sigma_b = \frac{My}{I}$

where M = bending moment, y- distance of fibre from neutral axis, I = moment of inertia.

- **Torsional Shear Stress** $\tau = \frac{Tr}{J}$

where T = torque, r = radius of shaft, J = polar moment of inertia.

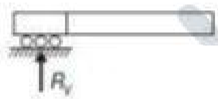
Equivalent Torsional Moment $\sqrt{M^2 + T^2}$

Equivalent Bending Moment $M + \sqrt{M^2 + T^2}$

Support: Supports are used to provide suitable reactions (Resisting force) to beams or any body. Following types of supports are used

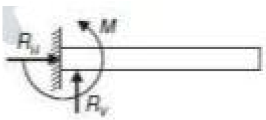
1. Simple support

2. Roller support



3. Hinged (Pin) support

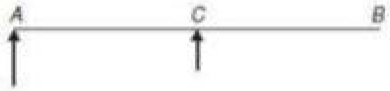
4. Fixed support



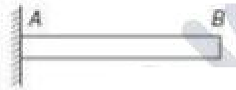
Types of Beams

1. Simply supported beams

2. Over hanging beam



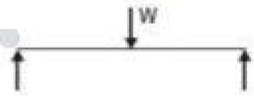
3. Cantilever beams



4. Continuous beams

Types of Loads

1. Point load



2. Uniformly distributed load (UDL)

Value of UDL = w × L KN point of application → mid point of AB

3. Uniformly varying load (UVL)

Value of UVL = ½ × W × L KN point of application = CG of triangle formed

⇒ $\frac{2}{3}$ L from A, $\frac{L}{3}$ from B

Shear force and Bending Moment Relation $\frac{dV}{dx} = -M$

Load	0	0	Constant
Shear	Constant	Constant	Linear
Moment	Linear	Linear	Parabolic
Load	0	Constant	Linear
Shear	Constant	Linear	Parabolic
Moment	Linear	Parabolic	Cubic

Euler's Buckling Load

$$P_{Critical} = \frac{\pi^2 EI}{l_{equi}^2}$$

For both end hinged $l_{equi} = l$
For one end fixed and other free $l_{equi} = 2l$
For both end fixed $l_{equi} = l/2$
For one end fixed and other hinged $l_{equi} = l/\sqrt{2}$

Slenderness Ratio (λ)

$$\lambda = \frac{L_\theta}{r_{min}}$$

$L_\theta = Effective\ length$

$r_{min} = \sqrt{(I_{min}/A)}$

r_{min} = Least radius of gyration

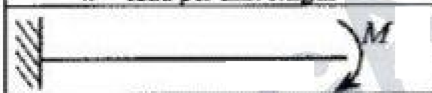
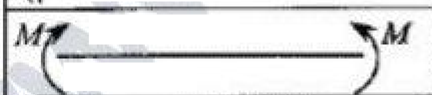
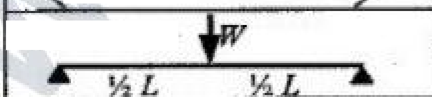
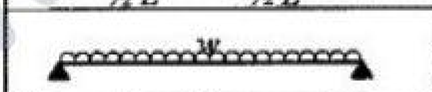
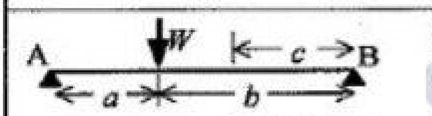
Rankine's Formula for Columns

$$\frac{1}{P_R} = \frac{1}{P_{cs}} + \frac{1}{P_E}$$

- P_R = Crippling load by Rankine's formula
- $P_{cs} = \sigma_{cs} A$ = Ultimate crushing load for column
- $P_E = \frac{\pi^2 El}{l_{eff}^2} =$ Crippling load obtained by Euler's formula

Deflection in different Beams

BEAM BENDING

L = overall length W = point load, M = moment w = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	WL
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	M
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
 $a \leq b, \quad c = \sqrt{\frac{1}{3}b(L+a)}$	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a}\theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)

Torsion

$$\frac{\tau_l}{r} = \frac{T}{J} = \frac{G\theta}{l}$$

Where, T = Torque,

- J = Polar moment of inertia
- G = Modulus of rigidity,
- θ = Angle of twist
- L = Length of shaft,

Total angle of twist

$$\theta = \frac{Tl}{GJ}$$

- GJ = Torsional rigidity
- $\frac{GJ}{l}$ = Torsional stiffness
- $\frac{l}{GJ}$ = Torsional flexibility
- $\frac{EA}{l}$ = Axial stiffness
- $\frac{l}{EA}$ = Axial flexibility

Moment of Inertia About polar Axis

- Moment of Inertia About polar Axis

$$J = \frac{\pi d^4}{32}, \tau_{max} = \frac{16T}{\pi d^3}$$

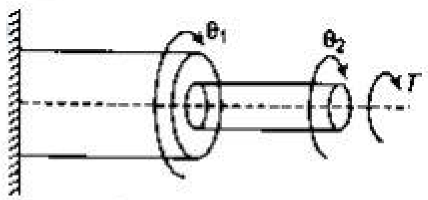
- For hollow circular shaft

$$J = \frac{\pi}{32}(d_0^4 - d_i^4)$$

Compound Shaft

- **Series connection**

$$\theta = \theta_1 + \theta_2$$
$$T = T_1 = T_2$$



Series connection

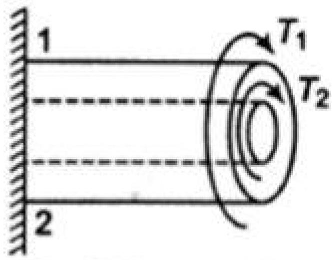
$$\theta = \frac{TL_1}{G_1J_1} + \frac{TL_2}{G_2J_2}$$

Where,

θ_1 = Angular deformation of 1st shaft

θ_2 = Angular deformation of 2nd shaft

- **Parallel Connection**



Parallel connection

$$\theta_1 = \theta_2$$
$$T = T_1 + T_2$$

$$\frac{T_1 L}{G_1 J_1} = \frac{T_2 L}{G_2 J_2}$$

Strain Energy in Torsion

$$U = \frac{1}{2} T\theta = \frac{1}{4} \frac{T^2 L}{GJ}$$

For solid shaft,

$$U = \frac{\tau^2}{4G} \times \text{Volume of shaft}$$

For hollow shaft,

$$U = \frac{\tau^2}{4G}\left(\frac{D^2 + d^2}{D^2}\right) \times \text{Volume of shaft}$$

Thin Cylinder

- **Circumferential Stress /Hoop Stress**

$$\sigma_h = \frac{pd}{2t} \Rightarrow \sigma_h = \frac{pd}{2tn}$$

η = Efficiency of joint

- **Longitudinal Stress**

$$\sigma_l = \frac{pd}{4t} \Rightarrow \sigma_l = \frac{pd}{4t\eta}$$

- **Hoop Strain**

$$\varepsilon_h = \frac{pd}{4tE}(2 - \mu)$$

- **Longitudinal Strain**

$$\varepsilon_L = \frac{pd}{4tE}(1 - 2\mu)$$

- **Ratio of Hoop Strain to Longitudinal Strain**

$$\varepsilon_v = \frac{pd}{4tE}(5 - 4\mu)$$

Stresses in Thin Spherical Shell

- **Hoop stress/longitudinal stress**

$$\sigma_L = \sigma_h = \frac{pd}{4t}$$

- **Hoop stress/longitudinal strain**

$$\varepsilon_L = \varepsilon_h = \frac{pd}{4tE}(1-\mu)$$

- **Volumetric strain of sphere**

$$\varepsilon_V = \frac{3pd}{4tE}(1-\mu)$$

Thickness ratio of Cylindrical Shell with Hemisphere Ends

$$\frac{t_2}{t_1} = \frac{1-v}{2-v}$$

Where v=Poisson Ratio