18 Polynomials

Polynomial functions are powerful tools that can help people make decisions every day. These functions can also help you design amazing monuments or build bridges for people to drive and walk across. You could even analyse for the FBI or create estimations of the future weather patterns.

As an environmentalist is it important not to waste material as well as getting as much out of the product as possible. You are being commissioned to develop a cardboard box, with no lid, that has the most volume. The problem is that the machine that produces the cardboard box has not been programmed yet with a polynomial function for the sample size of cardboard you have in stock. It is up to you to conserve the material and create a box with no lid with the most possible amount of volume.

An algebraic expression (f(x) of the form $f(x)=a_0+a_1x+a_2x^2+...+a^nx^n$, where $a_0, a_1, a_2 \ldots, a_n$ are real numbers and all the index of 'x' are non-negative integers is called a polynomials in *x*.

Degree of the Polynomial: Highest index of *x* in algebraic expression is called the degree of the polynomial, here $a_0, a_1x, a_2x^2 \ldots a_nx^n$, are called the terms of the polynomial and $z_0.a_1, a_2 \ldots, a_n$ are called various coefficients of the polynomial $f(x)$.

Note

A polynomial in *x* is said to be in standard form when the terms are written either in increasing order or decreasing order of the indices of *x* in various terms.

Different Types of Polynomials: Generally, we divide the polynomials in the following categories.

Based on Degrees: There are four types of polynomials based on degrees. These are listed below:

- **Linear Polynomials:** A polynomials of degree one is called a linear polynomial. The general formula of linear polynomial is $ax+b$, where *a* and *b* are any real constant and $a \neq 0$.
- **Quadratic Polynomials:** A polynomial of degree two is called a quadratic polynomial. The general form of a quadratic polynomial is ax^2+b+c, where $a \neq 0$.
- **Cubic Polynomials:** A polynomial of degree three is called a cubic polynomial. The general form of a cubic polynomial is ax^3+bx^2+cx+d, where $a \neq 0$.
- **Biquadratic (or quadric) Polynomials:** A polynomial of degree four is called a biquadratic (quadratic) polynomial. The general form of a biquadratic polynomial is $ax^4+bxj^3+cx^2+dx+e$, where $a \neq 0$.

Note: A polynomial of degree five or more than five does not have any particular name. Such a polynomial usually called a polynomial of degree five or six or ....etc.

Based on Number of Terms: There are three types of polynomials based on number of terms. These are as follows:

- **Monomial:** A polynomial is said to be monomial if it has only one term, e.g., $x, 9x^2, 5x^3$ all are monomials.
- **Binomial:** A polynomial is said to be binomial if it contains two terms, e.g., $2x^2+3x, \sqrt{3}x+5x^3, -8x^3+3$, all are binomials.
- **Trinomials:** A polynomial is said to be a trinomial it if contains three terms, e.g., $3x^3-8+\frac{5}{2}$.

  $\sqrt{7}x^{10}, 8x^4-3x^2, 5-7x+8x^9$, are all trinomials.

Note

A polynomial having four or more than four terms does not have particular Name. These are simply called polynomials.

Zero Degree Polynomial: Any non-zero number (constant) is regarded as polynomial of degree zero or zero degree polynomial. i.e. $f(x)=a$, where $a \neq 0$ is a zero degree polynomial, since we can write $f(x)=a$ as $f(x)=ax^0$.

Zero polynomial: A polynomial whose all coefficients are zeros is called as zero polynomial i.e. $f(x)=0$, we cannot determine the degree of zero polynomial.

Algebraic Identity

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a+b)(a-b)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a+b)$
- $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$
- $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Special case: if $a+b+c=0$ then $a^3 + b^3 + c^3 = 3abc.$

- $a^2 + b^2 = (a+b)^2 - 2ab,$ If $a+b$ and ab are given
- $a^2 + b^2 = (a-b)^2 + 2ab$ If $a-b$ and ab are given
- $a+b = \sqrt{(a-b)^2 + 4ab}$ If $a-b$ and ab are given
- $a-b = \sqrt{(a+b)^2 - 4ab}$ If $a+b$ and ab are given
- $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$ If $a + \frac{1}{a}$ is given
- $a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$ If $a - \frac{1}{a}$ is given
- $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ If $(a+b)$ and ab are given
- $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$ If $(a-b)$ and ab are given
- $a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$ If $a + \frac{1}{a}$ is given
- $a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right),$ If $\left(a - \frac{1}{a}\right)$ is given
- $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2 = \left[(a+b)^2 - 2ab\right]^2 - 2a^2b^2,$

  if $(a+b)$ and ab are given
- $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = \left[(a+b)^2 - 2ab\right](a+b)(a-b)$
- $a^5 + b^5 = (a^3 + b^3)(a^2 + b^2) - a^2b^2(a+b)$

Example 1. Find the value of: $36x^2 + 49y^2 + 84xy,$ when $x = 3, y = 6$

Solution: $36x^2 + 49y^2 + 84xy$

$= (6x)^2 + (7y)^2 + 2\times(6x)\times(7y)$

$= (6x + 7y)^2$

$= (6\times 3 + 7\times 6)^2$ [When $x = 3, y = 6$]

$= (18 + 42)^{21} = (60)^2 = 3600.$

Example 2. If $x^2 + \frac{1}{x^2} = 23,$ find the value of $\left(x + \frac{1}{x}\right).$

Solution: $x^2 + \frac{1}{x^2} = 23$ . . . (i)

$\Rightarrow$ $x^2 + \frac{1}{x^2} + 2 = 25$ [Adding 2 on both sides of (i)]

$\Rightarrow$ $(x^2) + \left(\frac{1}{x}\right)^2 + 2.x.\frac{1}{x} = 25$

$\Rightarrow$ $\left(x + \frac{1}{x}\right)^2 = (5)^2$

$\Rightarrow$ $x + \frac{1}{x} = 5$

Example 3. Prove that $a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right].$

Solution: Here, L.H.S.

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca\right]$

$= \frac{1}{2}\left[(a^2 - 2ab + b^2) + (b^2 - 2bx + c^2) + (c^2 - 2ca + a^2)\right]$

$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$ = RHS

Hence Proved.

Example 4. Evaluate:

(i) $(107)^2$ **(ii)** $(94)^2$ **(iii)** $(0.99)^2$

Solution: (i) $(107)^2 = (100 + 7)^2$

$= (100)^2 + (7)^2 + 2\times 100\times 7$

$= 10000 + 49 + 1400 = 11449$

(ii) $(94)^2 = (100 - 6)^2 = (100)^2 + (6)^2 - 2\times 100\times 6$

$= 10000 + 36 - 1200 = 8836$

(iii) $(0.99)^2 = (1 - 0.01)^2$

$= (1)^2 + (0.01)^2 - 2\times 1\times 0.01$

$= +1.0001 - 0.02 = 0.9801$

Note

We may extend the formula for squaring a binomial to the squaring of a trinomial as given below.

$$(a+b+c)^2 = [a+(b+c)]^2$$

$$= a^2 + (b+c)^2 + 2 \times a \times (b+c)$$

[Using the identity for the square of binomial]

$$= a^2 + b^2 + c^2 + 2bc + 2a(b+c)$$

[Using $(b+c)^2 = b^2 + c^2 + 2bc$]

$$= a^2 + b^2 + c^2 + 2bc + 2ab + 2ac$$

[Using the distributive law]

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$\therefore \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

Example 5. Evaluate : **(i)** $(1005)^3$ **(ii)** $(997)^3$

Solution: (i) $(1005)^3 = (1000+5)^3$

$$= (1000)^3 + (5)^3 + 3 \times 1000 \times 5 \times (1000+5)$$

$$= 1000000000 + 125 + 15000 \times (1000+5)$$

$$= 1000000000 + 125 + 15000000 + 75000$$

$$= 1015075125$$

(ii) $(997)^3 = (1000-3)^3$

$$= (1000)^3 - (3)^3 - 3 \times 1000 \times 3 \times (1000-3)$$

$$= 1000000000 - 27 - 9000 \times (1000-3)$$

$$= 1000000000 - 27 - 900000 + 27000$$

$$= 991026973$$

Example 6. If $x - \frac{1}{x} = 5$, find the value of $x^3 - \frac{1}{x^3}$

Solution: We have, $x - \frac{1}{x} = 5$ . . . (i)

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = (5)^3$ [Cubing both sides of (i)]

$\Rightarrow x^3 - \frac{1}{x^3} - 3x.\frac{1}{x}.\left(x - \frac{1}{x}\right) = 125$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 125$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 5 = 125$ [Substituting $\left(x - \frac{1}{x}\right) = 5$]

$\Rightarrow x^3 - \frac{1}{x^3} - 15 = 125 \quad \Rightarrow x^3 - \frac{1}{x^3} = (125+15) = 140$

Example 7. Simplify: $\dfrac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$.

Solution: Here $(a^2-b^2) + (b^2-c^2)^3 + (c^2-a^2) = 0$

$\therefore \quad (a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3 = 3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

Also, $(a-b) + (b-c) + (c-a) = 0$

$\therefore \quad (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$

$\therefore$ Given expression

$$= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$$

$$= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$$

$$= (a+b)(b+c)(c+a)$$

Example 8. Prove that:

$$(x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x).$$

Solution: Let $(x-y) = a, (y-z) = b$

and $(z-x) = c.$

Then, $a+b+c = (x-y) + (y-z) + (z-x) = 0$

$\therefore \quad a^3 + b^3 + c^3 = 3abc$

or $\quad (x-y)^3 + (y-z)^3 + (z-x)^3$

$= 3(x-y)(y-z)(z-x)$

Example 9. Find the value of $(28)^3 - (78)^3 + (50)^3$.

Solution: Let $a = 28, b = -78, c = 50$

Then, $a + b + c = 28 - 78 + 50 = 0$

$\therefore \quad a^3 + b^3 + c^3 = 3abc.$

So, $\quad (28)^3 + (-78)^3 + (50)^3 = 3 \times 28 \times (-78) \times 50$

$= -3,27,600$

Example 10. If $a+b+c = 9$ and $ab+bc+ac = 26$, find the value of $a^3 + b^3 + c^3 - 3abc$.

Solution: We have

$\Rightarrow \quad a+b+c = 9$ . . . (i)

$\Rightarrow \quad (a+b+c)^2 = 81$

[On squaring both sides of (i)]

$\Rightarrow \quad a^2 + b^2 + c^2 + 2(ab+bc+ac) = 81$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2 \times 26 = 81 \qquad [\therefore ab+bc+ac = 26]$

$\Rightarrow \quad a^2+b^2+c^2=(81-52)$

$\Rightarrow \quad a^2+b^2+c^2=29.$

Now, we have $a^3+b^3+c^3-3abc$

$$=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$$

$$=(a+b+c)\left[(a^2+b^2+c^2)-(ab+bc+ac)\right]$$

$$=9\times\left[(29-26)\right]=(9\times 3)=27$$

(b) A Special Product:

We have $(x+a)(x+b)=x(x+b)+a(x+b)$

$$=x^2+xb+ax+ab$$

$$=x^2+bx+ax+ab \quad [\therefore xb=bx]$$

$$=x^2+ax+bx+ab$$

$$=x^2+(a+b)x+ab$$

Thus, we have the following identity

$$(x+a)(x+b)=x^2+(a+b)x+ab.$$

Example 11. $\left(3a-\frac{1}{b}\right)^2-6\left(3a-\frac{1}{b}\right)+9+\left(c+\frac{1}{b}-2a\right)\left(3a-\frac{1}{b}-3\right)$

Solution: $\left(3a-\frac{1}{b}\right)^2-6\left(3a-\frac{1}{b}\right)+9+\left(c+\frac{1}{b}-2a\right)\left(3a-\frac{1}{b}-3\right)$

$$=\left(3a-\frac{1}{b}\right)^2-2.3\left(3a-\frac{1}{b}\right)+(3)^2+\left(c+\frac{1}{b}-2a\right)\left(3a-\frac{1}{b}-3\right)$$

$$=\left(3a-\frac{1}{b}-3\right)^2+\left(c+\frac{1}{b}-2a\right)\left(3a-\frac{1}{b}-3\right)$$

$$=\left(3a-\frac{1}{b}-3\right)\left[3a-\frac{1}{b}+3+\frac{1}{b}-2a\right]$$

$$=\left(3a-\frac{1}{b}-3\right)\left[a+c-3\right]$$

Example 12. Factorise : $4x^2+\frac{1}{4x^2}+2-9y^2.$

Solution: $4x^2+\frac{1}{4x^2}+2-9y^2.$

$$=(2x)^2+2.(2x).\left(\frac{1}{2x}\right)+\left(\frac{1}{2x}\right)^2-(3y)^2$$

$$=\left(2x+\frac{1}{2x}\right)^2-(3y)^2$$

$$=\left(2x+\frac{1}{2x}+3y\right)\left(2x+\frac{1}{2x}=3y\right)$$

Example 13. Factorise:

$$a^4+\frac{1}{a^4}-3.$$

Solution: $(a^2)^2+\left(\frac{1}{a^2}\right)^2-2.(a^2)\left(\frac{1}{a^2}\right)-1$

$$=\left(a^2-\frac{1}{a^2}\right)^2-(1)^2$$

$$=\left(a^2-\frac{1}{a^2}+1\right)\left(a^2-\frac{1}{a^2}-1\right)$$

Example 14. Factorise:

$$64a^{13}b+343ab^{13}.$$

Solution: $64a^{13}b+343ab^{13}$

$$=ab[64a^{112}+343b^{12}]$$

$$=ab\left[(4a^4)^3+(7b^4)^3\right]$$

$$=ab\left[4a^4+7b^4\right]\left[(4a^4)^2-(4a^4)(7b^4)+(7b^4)^2\right]$$

$$=ab\left[4a^4+7b^4\right]\left[16a^8-28a^4b^4+49b^8\right]$$

Example 15. Factorise:

$$x^3-6x^2+32$$

Solution: $x^3+32-6x^2$

$$=x^3+8+24-6x^2$$

$$=\left[(x)^3+(2)^3\right]+6\left[4-x^2\right]$$

$$=(x+2)\left[x^2-2x+4\right]+6[2+x][2-x]$$

$$=(x+2)\left[x^2-2x+4+6(2-x)\right]$$

$$=(x+2)\left[x^2-2x+4+12-6x\right]$$

$$=(x+2)\left[x^2-8x+16\right]$$

$$=(x+2)(x-4)^2$$

Multiple Choice Questions

1. The product of $(x+a)(x+b)$ is:

a. $x^2+(a+b)x+ab$ **b.** $x^2-(a-b)x+ab$

c. $x^2+(a-b)x+ab$ **d.** $x^2+(a-b)x-ab$

2. The value of 150×98 is:

a. 10047 **b.** 14800 **c.** 14700 **d.** 10470

3. The expansion of $(x+y-z)^2$ is:

a. $x^2+y^2+z^2+2xy+2yz+2zx$

b. $x^2+y^2-z^2-2xy+yz+2zx$

c. $x^2+y^2+z^2+2xy-2yz-2zx$

d. $x^2+y^2-z^2+2xy-2yz-2zx$

4. The value of $(x+2y+2z)^2+(x-2y-2z)^2$ is:

a. $2x^2+8y^2+8z^2$ **b.** $2x^2+8y^2+8z^2+8xyz$

c. $2x^2+8y^2+8z^2-8xyz$ **d.** $2x^2+8y^2+8z^2+16xyz$

5. The value of $25x^2+16y^2+40xy$ at $x=1$ and $y=-1$ is:

a. 81 **b.** –49

c. 1 **d.** None of these

6. On simplifying $(a+b)^3+(a-b)^3+6a\,(a^2-b^2)$ we get:

a. $8a^2$ **b.** $8a^2b$ **c.** $8a^3b$ **d.** $8a^3$

7. Find the value of $\dfrac{a^3+b^3+c^3-3abc}{ab+bc+ca-a^2-b^2-c^2}$, when $a=-5, 5=-6, c=10$.

a. 1 **b.** –1 **c.** 2 **d.** -2

8. If $(x+y+z)=1, xy+yz+zx=-1xyz=-1$ then value of $x^3+y^3+z^3$ is:

a. –1 **b.** 1 **c.** 2 **d.** –2

9. In method of factorisation of an algebraic expression. Which of the following statement is false?

a. Taking out a common factor from two or more terms.

b. Taking out a common factor from a group of terms.

c. By using remainder theorem.

d. By using standard identities.

10. Factors of $(a+b)^3-(a-b)^3$ is:

a. $2ab(3a^2+b^2)$ **b.** $ab(3a^2+b^2)$

c. $2b(3a^2+b^2)$ **d.** $3a^2+b^2$

11. Degree of zero polynomial is:

a. 0 **b.** 1

c. Both 0 and 1 **d.** Not defined

12. Factors of $(42-x-x^2)$ are:

a. $(x-7)(x-6)$ **b.** $(x+7)(x-6)$

c. $(x-7)(x+6)$ **d.** $(x+7)(x+6)$

13. Factors of $\left(x^2+\dfrac{x}{6}-\dfrac{1}{6}\right)$ are:

a. $\dfrac{1}{6}(2x+1)(3x+1)$ **b.** $\dfrac{1}{6}(2x+1)(3x-1)$

c. $\dfrac{1}{6}(2x-1)(3x-1)$ **d.** $\dfrac{1}{6}(2x-1)(3x+1)$

14. Factors of polynomial $x^3-3x^2-10x+2x$ are:

a. $(x-2)(x+3)(x-4)$ **b.** $(x+2)(x+3)(x+4)$

c. $(x+2)(x-3)(x-4)$ **d.** $(x-2)(x-3)(x-4)$

15. If $(x+a)$ is a factor of x^2+px+q and x^2+mx+n then the value of a is:

a. $\dfrac{m-p}{n-p}$ **b.** $\dfrac{n-q}{m-p}$ **c.** $\dfrac{n+q}{m+p}$ **d.** $\dfrac{m+p}{n+q}$

16. The HCF of $(6x^4-13x^3+6x^2)$ and $(8x^4-36x^3+54x^2-27x)$ is:

a. $x(2x+3)$ **b.** $x^2(2x-3)$ **c.** $x(3-2x)$ **d.** $x(2x-3)$

17. There are four polynomials $P(x), Q(x), R(x)$. If the HCF of each pair is $(x+3)^4$ and the LCM of all the four polynomials is $(x-1)(x-2)(x-3)$ then the product of four polynomials is ____.

a. $(x+3)^4(x-1)(x-2)(x-3)$

b. $(x+3)(x-1)(x-2)(x-3)$

c. $(x+3)(x-1)^4(x-2)^4(x-3)^4$

d. $(x+3)^4(x-1)^4(x-2)^4(x-3)^4$

18. Which of the following expressions is a rational expression?

a. $x^3-\sqrt{3}x^2+\sqrt{5}x-11$ **b.** $\dfrac{x^2+3}{2\sqrt{x}-1}$

c. $\dfrac{5x^2-\sqrt{6}x+7}{x+3}$ **d.** $\dfrac{\sqrt{2}x^2-4\sqrt{x}+5}{x-\sqrt{2}}$

19. If $x^4-2x^3+3x^2-mx+5$ is exactly divisible by $x-3$, then $m=?$

a. –4 **b.** –40

c. $\frac{-40}{3}$ **d.** None

20. If $f(x)=x^2+5x+p$ and $g(x)=x^2+3x+q$ have a common factor, then $(p-q)^2=?$

a. $2(5p-3q)$ **b.** $2(3p-5q)$

c. $3p-5q$ **d.** $5p-3q$

21. Simplify $\frac{3x+2}{x^2-16}+\frac{x-5}{(x+4)^2}$.

a. $\frac{4x^2+5x+28}{x^3+4x^2+16x-64}$ **b.** $\frac{4x^2+5x+28}{x^3+4x^2-16x-64}$

c. $\frac{4x^2+5x+28}{x^3-4x^2-16x-64}$ **d.** $\frac{4x^2+5x+28}{x^3+4x^2-16x+64}$

22. If $P=\frac{1+2x}{1-2x}$ and $Q=\frac{1-2x}{1+2x}$ then $\frac{P-Q}{P+Q}=$ ___

a. $\frac{4x}{1+4x^2}$ **b.** $\frac{1+4x^2}{4x}$

c. $-\frac{4x}{1+4x^2}$ **d.** $\frac{-(1+4x^2)}{4x}$

23. What should be subtracted from $\left(\frac{2x^2+2x-7}{x^2+x-6}\right)$ to get $\left(\frac{x-1}{x+2}\right)$?

a. $\frac{x-2}{x-3}$ **b.** $\frac{x-2}{x+3}$

c. $\frac{x+2}{x-3}$ **d.** $\frac{x+2}{x+3}$

24. The additive inverse of $3x-4+\frac{x}{2x-1}$ is:

a. $\frac{6x^2-10x+4}{2x-1}$ **b.** $-3x+4-\frac{x}{2x-1}$

c. $-3x+4+\frac{x}{2x-1}$ **d.** $-3x+4-\frac{x}{1-2x}$

25. If (x + 1) is a factor of $f(x)=a_0\cdot x^n+a_1\cdot x^{n-1}+a_2\cdot x^{n-2}+...+a_n=0$, then:

a. $a_0+a_1+a_2+\ldots+a_n=0$

b. $a_0+a_2+a_4+\ldots=0$

c. $a_1+a_2+a_3+\ldots=0$

d. $a_0+a_2+a_4+\ldots=a_1+a_3+a_5+\ldots$

26. The product of x^3+2x^2-3x+4 and $2x^2-5x+1$ is ___.

a. $2x^5-x^4-15x^3+25x^2-23x-4$

b. $2x^5-x^4-15x^3-25x^2-23x-4$

c. $2x^5-x^4-15x^3+25x^2-23x+4$

d. $2x^5-x^4-15x^3-23x-4$

27. If $g(x)=x^6+3x^4-24x^2+3$, find $g(1),g(2)$ and $g(3)$.

a. $g(1)=-17,g(2)=19,g(3)=759$

b. $g(1)=-17,g(2)=-19,g(3)=759$

c. $g(1)=-17,g(2)=-19,g(3)=-759$

d. $g(1)=17,g(2)=19,g(3)=759$

28. Find $f(4),f(-5),f(3.2)$ if $f(x)=6.2x^2-4x^3 4.28$.

a. $f(4)=-152.52,f(-5)=659.28,f(3.2)=-63.304$

b. $f(4)=-152.52,f(-5)=659.27,f(3.2)=-63.304$

c. $f(4)=-152.53,f(-5)=659.28,f(3.2)=63.304$

d. $f(4)=-152.53,f(-5)=-659.28,f(3.2)=63.304$

29. Find the value of "*k*" if the expressions $p(x)=kx^3+4x^2+3x-4$ and $q(x)=x^3-4x+k$ leave the same reminder when divided by $(x-3)$.

a. $k=-2$ **b.** $k=2$

c. $k=-1$ **d.** $k=3$

30. For the expression $f(x)=x^3+ax^2+bx+c$, if $f(1)=f(2)=0$ and $f(4)=f(0)$. Find the values of a,b and c.

a. $a=-9,b=20$ and $c=-12$

b. $a=-8,b=20$ and $c=-12$

c. $a=9,b=-20$ and $c=-12$

d. $a=-8,b=-20$ and $c=12$

ANSWERS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
a	c	c	d	c	d	a	b	c	c
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
d	c	b	a	b	d	a	a	d	b
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
b	a	d	b	d	c	a	a	c	a

SOLUTIONS

1. **(a)** $(x+a)(x+b)=x(x+b)+a(x+b)$

$=x^2+bx+ax+ab$

$=x^2+(a+b)x+ab.$

2. **(c)** $150\times(100-2)$

$=15000-300=14700.$

5. **(c)** $25(1)^2+16(-1)^2+40(1)(-1)$

$=25+16-40=41-40=1.$

10. **(c)** $x^3-y^3=(x-y)(x^2+xy+y^2).$

16. **(d)** $6x^4-13x+6x^2=x^2(6x^2-13x+6)$

$=x^2(2x-3)(3x-2)$

$=8x^4-36x^3+54x^2-27x$

$=x(8x^3-36x^2+54x-27)$

$=x(2x-3)^3$

So, HCF = $x(2x-3)$

17. **(a)** We have, product of 'n' polynomials

= (HCF of each pair)^n × LCM

So, product of polynomials

$=(x+3)^4(x-1)(x-2)(x-3).$

18. **(a)** By the definition of rational expression

$\Rightarrow\ x^3-3x^2+\sqrt{x}-11=\dfrac{x^3-\sqrt{3}x^2+\sqrt{5}x-11}{1}$

is a rational expression.

19. **(d)** Let $f(x)=x^4-2x^3+3x^2-mx+5$

Since it is exactly divisible by $(x-3)$,

we have $f(3)=0$

$\Rightarrow\ (3)^4-2(3)^3+3(3)^2-m(3)+5=0$

$\Rightarrow\ 81-54+27-3m+5=0$

$\Rightarrow\ m=\dfrac{59}{3}$

20. **(b)** Let the common factor be $x-k$,

we have, $f(x)=g(k)=0$

$\Rightarrow\ k^2+5k+p=k^2+3k+q$

$k=\dfrac{q-p}{2}$

Substituting "k" in $x^2+5x+p=0$

$\Rightarrow\ x^2+5x+p=0$

$\Rightarrow\ \left(\dfrac{q-p}{2}\right)^2+\left(\dfrac{q-p}{2}\right)+p=0$

$\therefore\ (p-q)^2=2(3p-5q).$

21. **(b)** $\dfrac{3x+2}{x^2-16}+\dfrac{x-5}{(x+4)^2}$

$=\dfrac{(3x+2)(x+4)+(x-5)(x-4)}{(x-4)(x+4)^2}$

$=\dfrac{4x^2+5x+28}{x^3+4x^2-16x-64}.$

22. **(a)** $\dfrac{P}{Q}=\dfrac{(1+2x)^2}{(1-2x)^2}$

By componendo and dividendo rule

$=\dfrac{P-Q}{P+Q}=\dfrac{(1+2x)^2-1(1-2x)^2}{(1+2x)^2+(1-2x)^2}$

$=\dfrac{8x}{2(1+4x^2)}=\dfrac{4x}{1+4x^2}$

23. **(d)** $\dfrac{2x^2+2x-7}{x^2+x-6}-\dfrac{x-1}{x-2}=\dfrac{2x^2+2x-7}{(x-2)(x+3)}=\dfrac{x+2}{x-2}$

$=\dfrac{2x^2+2x-7-x^2-2x+3}{(x-2)(x+3)}$

$=\dfrac{x^2-4}{(x-2)(x+3)}=\dfrac{x+2}{x+3}.$

24. **(b)** Additive inverse of

$3x-4+\dfrac{x}{2x-1}$ is $-3x+4-\dfrac{x}{2x-1}.$

25. **(d)** Since $(x+1)$ is a factor we have

$f(-1)=0\Rightarrow a_0\cdot(-1)^n+a_1\cdot(-1)^{n-1}+a_2\cdot(-1)^{n-2}+\ldots+a_n=0$

$a_0+a_2+a_4+\ldots=a_1+a_3+a_5+\ldots$

where 'n' is even (or) odd number whatever it may be.

26. **(c)** $\left(x^3+2x^2-3x+4\right)\left(2x^2-5x+1\right)$

$=x^3\left(2x^2-5x+1\right)+2x^2\left(2x^2-5x+1\right)-3x\left(2x^2-5x+1\right)+4\left(2x^2-5x+1\right)$

$=2x^5-5x^4+x^3+4x^4-10x^3+2x^2-6x^3+15x^2-3x+$
$8x^2-20x+4$

$=2x^5+\left(-5x^4+4x^4\right)+4x^4+\left(x^3-10x^3-6x^3\right)+$
$\left(2x^2+16x^2+8x^2\right)-3x-20x+4$

$=2x^5-x^4-15x^3+25x^2-23x+4.$

27. **(a)** Given that $g(x)=x^6+3x^4-24x^2+3$

$\Rightarrow\ g(1)=(1)^6+3(1)^4-24(1)^2+3=1+3-24+3=-17$

$\Rightarrow\ g(2)=(2)^6+3(2)^4-24(2)^2+3=64+48-96+3=19$

$\Rightarrow\ g(3)=(3)^6+3(3)^4-24(3)^2+3=729+243-216+3=759.$

28. **(a)** Given that $f(x)=6.2x^2=4x^3+4.28$

$\Rightarrow\ f(4)=6.2(4)^2-4(4)^3+4.28=99.2-256+4.28=-152.52$

$\Rightarrow\ f(-5)=6.2(-5)^2-4(-5)^3+4.28=659.28$

$\Rightarrow\ f(3.2)=6.2(3.2)^2-4(3.2)^3+4.28=-63.304$

29. **(c)** $p(x)=kx^3+4x^2+3x-4$ is divided by $(x-3)$, the remainder is $p(3)$.

$\therefore\ p(3)=k(3)^3+4(3)^2+3(3)-4=27k+36+9-4$

$=27k+41$ . . . (*i*)

Again $q(x)=x^3-4x+k$ is divided by $(x-3)$, the remainder is $q(3)$.

$\therefore\ p(3)=(3)^3-4(3)+k=27-12+k=15+k$ . . . (*ii*)

But given that $p(x)$ and $q(x)$ leave the same remainder when divided by $(x-3)$

$\Rightarrow\ p(3)=q(3)$

we have $27k+41=15+k$ [from (*i*) and (*ii*)]

$\Rightarrow\ k=-1.$

30. **(a)** Given that $f(x)=x^3+ax^2+bx+c,$

since it is a cubic expression it has three factors.

$f(1)=f(2)=0$

$\Rightarrow$ $(x-1)$ and $(x-2)$ are factors of $f(x)$ as per remainder theorem.

$\because$ $f(x)$ is a third degree it will have three linear factors.

Let the third factor be $(x-k)$.

$\therefore\ f(x)=x^3+x^2+bx+c=(x-1)(x-2)(x-k)$

Now, $f(4)-(4-1)(4-2)(4-k)=24-6k$

$\Rightarrow\ f(0)=(0-1)(0-2)(0-k)=-2k$

given that $f(4)=f(0)$

we have $\Rightarrow\ 24-6k=-2k\Rightarrow k=6$

As $f(x)=x^3+ax^2+bx+c=(x-1)(x-2)(x-k)$

But as $k=6,$ putting the value of '*k*' in $f(x)$ we have

$\Rightarrow\ f(x)=x^3+ax^2+bx+c=(x-1)(x-2)(x-6)$

$=x^3=9x^2+20x-12$

Comparing the coefficients we have

coefficient of $x^2=a=-9$; coefficient of $x=b=20$

constant term $=c=-12$

$\therefore\ a=-9, b=20, c=-12.$

□□□