Mechanical Properties of Solids

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Young's modulus is a measure of a solid's stiffness or resistance to elastic deformation under load. It relates stress to strain along an axis or line. The basic principle is that a material undergoes elastic deformation when it is compressed or extended, returning to its original shape when the load is removed. More deformation occurs in a flexible material compared to that of a stiff material. The Young's modulus of a wire is a measure of its stiffness and is defined as the ratio of stress to strain.

(A) If there are two wires of the same material and the same length while the diameter of the second wire is two times the diameter of the first wire, then what will be the ratio of extension produced in the wires by applying the same load?

(B) The Young's modulus of a wire of length L and radius r is Y. If the length is reduced to half, what will be its Young's modulus?

(C) If we can easily stretch rubber and most elastic bands are made of rubber then why can't we use rubber for the manufacturing of springs?

Ans. (A) Both wires are of the same materials, so both will have the same Young's modulus, and let it be Y.

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A\frac{\Delta L}{L}}$$

F = applied force

A = area of cross-section of a wire

Now, $Y_1 = Y_2$

$$\frac{FL}{(A_1)(\Delta L_1)} = \frac{FL}{(A_2)(\Delta L_2)}$$

Since, load and length are the same for both.

$$r_1^2 \Delta L_1 = r_2^2 \Delta L_2$$

$$\left(\frac{\Delta L_1}{\Delta L_2}\right) = \left(\frac{r_2}{r_1}\right)^2 = 4$$

$$\Delta L_1 : \Delta L_2 = 4 : 1$$

(B) Young's modulus will remain the same, as it is constant for a particular material and doesn't depend on the physical dimension of the wire.
(C) A spring needs a large restoring force when it is deformed, which in turn depends upon the elasticity of the material of the spring. Since, Young's modulus of elasticity of steel is more than that of rubber, steel is preferred in making the springs.

2. Most of us would have seen a crane used for lifting and moving heavy loads. The crane has a thick metallic rope. The maximum load that can be lifted by the rope must be specified. This maximum load under any circumstances should not exceed the elastic limit of the material of the rope. By knowing this elastic limit and the extension per unit length of the material, the area of cross-section of the wire can be evaluated. From this, the radius of the wire can be calculated.

(A) The work done when a wire of length I and area of cross-section A is made of material of young's modulus Y is stretched by an amount x is:

(a) $\frac{YAx^2}{L}$ (b) $\frac{YAx^2}{2L}$

(c) $\frac{YAx}{2L}$ (d) $\frac{YA}{2L}$

(B) The Young's modulus of steel is 2.0 x 1011 w/m². If the interatomic spacing for the metal is 2.8 x 10-10 m, find the increase in the interatomic spacing for a force of 10° N/m².
(a) 0.014 Å
(b) 0.020 Å
(c) 0.025 Å
(d) 0.030 Å
(C) Assertion (A): The shape and size of rigid body remain unaffected under the effect of external forces
Reason (R): The distance between two particles remains constant in rigid body.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.
(D) A wire can support a load Mg without breaking. If it is cut into two equal parts then each wire can support the load of:

(a) $\frac{Mg}{4}$ (b) $\frac{Mg}{2}$

(c) Mg (d) 2Mg

(E) Two wires are made of the same metal. The length of the first wire is half that of the second wire and its diameter is double that of the second wire. If equal loads are applied on both wires, the ratio of increase in their lengths is:

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{10}$ (d) $\frac{1}{8}$

Ans.

(A) *(b)* $\frac{YAx^2}{2L}$

Explanation: Young's Modulus

$$Y = \frac{\text{Normal Stress}}{\text{Longitudnal Strain}} = \frac{\frac{F}{A}}{\frac{l}{L}}$$

F = Force
A = Area

l = change in length
L = original Length
x = change in Length (Given)

$$\text{Average extension} = \frac{0+x}{2} = \frac{x}{2}$$

Now, Work Done = Force.

$$\text{Average extension}Y = \frac{FL}{Al}$$

$$F = \frac{YAl}{L}$$

$$\text{Work done} = \frac{YAl}{L} \cdot \frac{x}{2}$$

$$= \frac{YAx}{L} \cdot \frac{x}{2}$$

[$l = x$ (given)]

$$\text{Work done} = \frac{YAx^2}{2L}$$

(B) (a) 0.014 Å

Explanation: Given, Y = 2.0 x 10^{11}N/m^2
L = 2.8 × 10^{-10} m
F = force
A = Area
Al charge in length

$$\frac{F}{A} = \frac{10^9 N}{m^2}$$

Force constant, $K = \frac{F}{\Delta l}$

So, Y = Modulus of elasticity

$$= \frac{F \times l}{A \times \Delta l}$$

Or $\Delta l = \frac{F \times l}{A \times \Delta l}$

$$\Delta l = \frac{10^9 \times 2.8 \times 10^{-10}}{2 \times 10^{11}}$$

$$= 1.4 \times 10^{-12} m$$

Or $\Delta l = 0.014$ Å

(C) (a) Both A and R are true R is the correct explanation of A.

Explanation: Any entity in which the separation between any two particles is constant is referred to as a rigid body. Hence, the size and shape of the body stay the same even in the presence of external forces due to the constant spacing between all particles. A soft substance, however, may alter in size and shape when subjected to external pressures.

(D) (c) Mg

Explanation: Maximum load supported by the cable is directly proportional to the breaking stress.

Since,

Breaking stress = $\frac{F}{A}$

Where, F is the force and A is the cross- sectional area.

As we see that the breaking stress is independent of the length of the cable. So, if the cable is cut in two equal parts, the maximum load that can be supported by either part of the cable remains the same as before.

(D)

(d) $\frac{1}{8}$

Explanation:

$$Y = \frac{F}{\pi(2r)^2} \times \frac{\frac{L}{2}}{\Delta L_1}$$

$$= \frac{F}{\pi r^2} \times \frac{L}{\Delta L_2}$$

$$= \frac{\Delta L_1}{\Delta L_2} = \frac{1}{8}$$